DOI: 10.24996/ijs.2021.62.7.26

ISSN: 0067-2904

Replacing the Content in e-Abacus Diagram II

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Received: 12/8/2020 Accepted: 23/1/2021

Abstract

In our normal life, we sometimes need a process of replacing something with another to get out of the stereotype. From this point of view, Mahmood's attempted in the year 2020 to replace the content in the first main e-abacus diagram. He found the general rule for finding the value of the new partition after the replacement from the original partition. Here we raise the question: Can we find the appropriate mechanisms for the remainder of the main e-abacus diagram?

Keywords: Partition theory, Composition, Partition, e-abacus diagram

استبدال المحتوى في المخطط المعداد من النمط e – II

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الخلاصة:

في حياتنا العامة نحتاج أحيانا الى عملية استبدال شيء باخر ومن هذا المنطلق كانت فكرة محمود في العام 2000 الى استبدال المحتوى للمخطط المعداد الأول من النمط e حيث وجد فيها الالية المناسبة لهذه الحالة فقط فحاولنا ان نجد العلاقات المناسبة لبقية المخططات وكان السؤال الأهم هل من قواعد نعرف من خلالها قيمة التجزئة الناتجة من عملية الاستبدال مباشرة من قيمة التجزئة الرئيسية؟

1. Introduction

Since the emergence of the topic of e-abacus diagram by James [1], many researchers have been studying many of the traits that exist in the first place, as well as studying some changes that can be used in many areas. Fayers [2-3], Mathas [4], and others presented many relationships that made this topic the focus of much interest. Here we study the possibility of the replacement of content within this diagram and knowing the general behavior and its effects on the mathematical concepts of this topic.

Let r be a nonnegative integer. "The **composition** $\delta = (\delta_1, \delta_2, \dots, \delta_n) = (\theta_1{}^{\gamma_1}, \theta_2{}^{\gamma_2}, \dots, \theta_m{}^{\gamma_m})$, where γ_z is the number of times that θ_z appeared, *z=1, 2, …,* m of r, which is the sequence of non-negative integers such that $|\delta| = \sum_{j=1}^{n} \delta_j = \sum_{z=1}^{m} \theta_z{}^{\gamma_z} = r$." The composition is called a **partition** of r if $\delta_j \geq \delta_{j+1}, \forall\, j \geq 1$. δ is fixed as a partition of r and we define that $\beta_i = \delta_i + b - i, 1 \leq i \leq b$. The set $\{\beta_1, \beta_2, \cdots, \beta_b\}$ is said to be the set of β - number for δ [4]. Let e be a positive integer number greater than or equal to 2, then we can represent numbers by a diagram called ***e*-abacus diagram**[1], as shown in table 1.

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Table 1- *e*-abacus diagram

Runner 1	**Runner 2**	⋯	**Runner *e***
0	1	⋯	*e*-1
e	*e*+1	⋯	2*e*-1
2*e*	2*e*+1	⋯	3*e*-1
⋮	⋮	⋯	⋮

where every β will be represented by a star ($*$) and the rest of the sites are denoted by (-). In fact, the definition of *e*-abacus diagram will lead us to the fact of the presence of an infinite number of diagrams that are all suitable for any partition according to the value of *e*. For example, if $\delta = (3,3,3,2,1,1) = (3^3, 2, 1^2)$ and if we chose *e*=3, then we have many of *e*-abacus diagrams of this partition, as follows:

* - *	* * *	* * *	- * *
* - *	- * *	* * -	- * -
- * *	- * -	* * -	* * *
* - -	* * *	* - *	
		* * -	

To develop a specific method to control the number of these diagrams, Mohammed [5] provided the following: "For any partition of *r* with *n* parts, let $b_g = n + (g-1)$ where $g = 1,2,\ldots,e$, is said to be **guides** for this partition". For example, let $\delta = (5^2, 4, 3^3,1)$, then the diagrams in tables 2 and 3 hold:

Table 2- 2-abacus diagrams for $(5^2, 4, 3^3,1)$

	$b_1 = 7$	$b_2 = 8$	$b_1 + 2$	$b_2 + 2$	$b_1 + 4$	$b_2 + 4$	
0 1	- *	* -	* *	* *	* *	* *	
2 3	- -	* -	- *	* -	* *	* *	
4 5	* *	- *	- -	* -	- *	* -	
6 7	* -	* *	* *	- *	- -	* -	
8 9	* -	- *	* -	* *	* *	- *	
10 11	* *	- *	* -	- *	* -	* *	
14 15		* --	* *	- *	* -	- *	⋱
16 17			- -	* -	* *	- *	
18 19				- -	- -	* -	⋱
20 21					- -	- -	
						- -	⋱

Table 3-3-abacus diagrams for $(5^2, 4, 3^3,1)$

	$b_1 = 7$	$b_2 = 8$	$b_3 = 9$	$b_1 + 3$	$b_2 + 3$	$b_3 + 3$	
0 1 2	- * -	* - *	* * -	* * *	* * *	* * *	
3 4 5	- * *	- - *	* - -	- * -	* - *	* * -	
6 7 8	* - *	* * -	* * *	- * *	- - *	* - -	⋱
9 10 11	- * *	* - *	- * -	* - *	* * -	* * *	
12 13 14		* - -	* * -	- * *	* - *	- * -	⋱
15 16 17			- - -	- - -	* - -	* * -	
⋮ ⋮ ⋮				- - -	- - -	- - -	⋱
					- - -	- - -	
						- - -	

Any *e*-abacus diagram for each guide is said to be **main or guide *e*-abacus diagram**. Then there exist *e* of these diagrams; see a previous article [6] for more information about the technology that these diagrams take to become in this order.

Mahmood [7] submitted an idea to replace the content in *e*-abacus diagram, denoted by e^{rep}; Table 5 shows this idea for the above example.

Table 4- e^{rep}-abacus diagram

b_1, $e=2$ →	2^{rep}	$b_1, e=3$ →	3^{rep}	$b_1, e=5$ →	5^{rep}
- *	* -	- * -	* - *	- * - - *	* - * * -
- -	* *	- * *	* - -	* * - * -	- - * - *
* *	- -	* - *	- * -	* * - - -	- - * * *
* -	- *	- * *	* - -		
* -	- *				
* *	- -				
$(5^2,4,3^3,1)$	$(5,4,1^2)$	$(5^2,4,3^3,1)$	$(5,4,1^2)$	$(5^2,4,3^3,1)$	$(7^3,5,4,1^2)$

Through this, he was able to know the general rule for finding the value of partition after the proposed replace process, but only for the case of b_1 (or for the first main e-abacus diagram). In this paper, we will replace the content and find the value of partition for any value greater than or equal to b_g.

2. Explanation of the phenomenon

The location of (δ_1) in the row k and column l where $(1 \leq l \leq e)$ will play a fundamental role in the end position of the original diagram. We will then replace it considering what it is there.

$$\text{Location of } (\delta_1) = \delta_1 + ((\text{no. of parts of } \delta) - 1) \tag{1}$$

Which is exactly equal to

$$(k-1)e + (l-1) \tag{2}$$

(Note that, any part of power 0 **or** 0 of power ≥ 1 is not appearing here). (3)

<u>Case I:</u> In this case, we will use all the relationships listed by Mahmood in [6] and [7]:

<u>Rule (2.1)</u>: The partition after replacing the content in1st *e*-abacus diagram will be $((\gamma_1+\gamma_2+\cdots+\gamma_m)^{e-l},\ (\gamma_2+\gamma_3+\cdots+\gamma_m)^{\theta_1-\theta_2},\cdots,\ (\gamma_{m-1}+\gamma_m)^{\theta_{m-2}-\theta_{m-1}},\ (\gamma_m)^{\theta_{m-1}-\theta_m})$.

Proof: Through (1) and (2) in above, this location in the original diagram will be a $(*)$ and, after the replacement, it will turn into a (-). Then we have three possibilities that depend entirely on *e-l*: "The first possibility is that, if a $(*)$ is found after it, then it is calculated, unless this is in the condition (3) that makes *e-l* equal to zero, which is the second possibility. Otherwise, it indicates the existence of (-) only and this is the last possibility, which has no effect on mathematical relationships."

<u>Rule (2.2):</u> The partition after the replacement of the content in 2nd *e*-abacus diagram will be $((1+\gamma_1+\cdots+\gamma_m)^{(e-l)+(e+1)},\ (1+\gamma_2+\cdots+\gamma_m)^{\theta_1-\theta_2},\cdots,\ (1+\gamma_m)^{\theta_{m-1}-\theta_m},\ 1^{\theta_m})$.

Proof: By using all the remarks in [6], we notice that one $*$ is always added to the top of the first column, as well as a (-) to the bottom in the case of *e*=2, two (-) in the case of *e*=3, …, and so on downwards until adding *t-1*of the (-) when *e=t* in the second main diagram. After the replacement, all of this will change to the opposite, which explains the existence of the number *1* that is always added to the relation and likewise in the first term of power *(e-l) +(e-1)*

<u>Rule (2.3):</u> The partition after the replacement of the content in the 3rd *e*-abacus diagram will be $((2+\gamma_1+\cdots+\gamma_m)^{(e-l)+(2e-2)},\ (2+\gamma_2+\cdots+\gamma_m)^{\theta_1-\theta_2},\cdots,\ (2+\gamma_m)^{\theta_{m-1}-\theta_m},\ 2^{\theta_m})$.

Proof: This is similar to the Rule (2.2), except that we always in front add two ($*$) from the top in the first column and we add from the bottom two (-) if *e=3*, four (-) if *e=4*, …, ((*2t-4*) of (-)) when *e=t*)

For example, if $\delta = (4^3, 2^2, 1^5)$, if we chose *e*=4, then we have four main *e*-abacus diagrams, as follows:

1st

-	*	*	*
*	*	-	*
*	-	-	*
*	*	-	-

4^{rep} of 1st

*	-	-	-
-	-	*	-
-	*	*	-
-	-	*	*

2nd

*	-	*	*
*	*	*	-
*	*	-	-
*	*	*	-
-	-	-	-

4^{rep} of 2nd

-	*	-	-
-	-	-	*
-	-	*	*
-	-	-	*
*	*	*	*

$(11^5, 8^2, 6, 1)$

3rd

*	*	-	*
*	*	*	*
-	*	*	-
-	*	*	*
-	-	-	-
-	-	-	-

4^{rep} of 3rd

-	-	*	-
-	-	-	-
*	-	-	*
*	-	-	-
*	*	*	*
*	*	*	*

$(12^8, 9^2, 7, 2)$

4th

*	*	*	-
*	*	*	*
*	-	*	*
-	-	*	*
*	-	-	-
-	-	-	-
-	-	-	-

4^{rep} of 4th

-	-	-	*
-	-	-	-
-	*	-	-
*	*	-	-
-	*	*	*
*	*	*	*
*	*	*	*

Then, by using the same prove of Rules (2.2) and (2.3), we have the following:

Rule (2.4): The partition after the replacement of the content for any X e-abacus diagram, it will be

$$\left(\left((X-1)+\gamma_1+\cdots+\gamma_m\right)^{(e-l)+((X-1)e-(X-1))}, \left((X-1)+\gamma_2+\cdots+\gamma_m\right)^{\theta_1-\theta_2}, \cdots, \left((X-1)+\gamma_{m-1}+\gamma_m\right)^{\theta_{m-2}-\theta_{m-1}}, \left((X-1)+\gamma_m\right)^{\theta_{m-1}-\theta_m}, (X-1)^{\theta_m}\right)$$

Case II: According to [6], there is a certain mechanism to find the main e-abacus diagram, where we notice that in the end there is always the presence of a (-) which does not affect the value of partition at all, but it will make a large change in the case of substitution of the content as these will be in the form of beads. So, we have two cases. The first case is that we keep (-) as it is. The second case is to delete it basically before replacing. It might be natural that there is a question about case I, which is "if there exist (-) after the last $*$ in a given diagram, why do we reposition it in the next diagram since it will not basically affect the main partition?". This question obliged us to study this case and, before going into its details, we will give an example that will clarify what we mean:

$\delta = (4^3, 2^2, 1^5)$

Case I

1st

-	*	*	*
*	*	-	*
*	-	-	*
*	*	-	-

2nd

*	-	*	*
*	*	*	-
*	*	-	-
*	*	*	-
-	-	-	-

3rd

*	*	-	*
*	*	*	*
-	*	*	-
-	*	*	*
-	-	-	-
-	-	-	-

4th

*	*	*	-
*	*	*	*
*	-	*	*
-	-	*	*
*	-	-	-
-	-	-	-
-	-	-	-

$\delta = (4^3, 2^2, 1^5)$

Case II

1st

-	*	*	*
*	*	-	*
*	-	-	*
*	*	-	-

2nd

*	-	*	*
*	*	*	-
*	*	-	-
*	*	*	-

3rd

*	*	-	*
*	*	*	*
-	*	*	-
-	*	*	*

4th

*	*	*	-
*	*	*	*
*	-	*	*
-	-	*	*
*	-	-	-

Through the above, we note that the difference between the two cases is that, in Case I, we have (-) at the end of the all main diagrams, which will have a significant impact in calculating the general rule after the replacement. In Case II, it will behave differently to the first case! Consequently, we must study the general behavior in every main diagram as follows:

i- Let l_g be the number of columns in main e-abacus diagram where $g = 1, 2, \dots, e$.

ii- Location of Location of (δ_1) $= \delta_1$ +((no. of parts of δ) -1) = $(k-1)e + (l_1 - 1)$.

iii- Where $h = 1, 2, \dots, e-1$, then $l_{h+1} = \begin{cases} 1 & if\ l_h = e \\ l_h + 1 & otherwise \end{cases}$.

Now, we obtained the following rules by using (i-iii) in above and (2.1-2.3), unless the last rows are with (-):

Rule (2.5): The partition after the replacement of the content in the 1st e-abacus diagram will be
$((\gamma_1 + \gamma_2 + \gamma_3 + \cdots + \gamma_m)^{e-l_1},$
$(\gamma_2 + \gamma_3 + \cdots + \gamma_m)^{\theta_1-\theta_2}, \cdots, (\gamma_{m-1} + \gamma_m)^{\theta_{m-2}-\theta_{m-1}}, (\gamma_m)^{\theta_{m-1}-\theta_m})$.

Rule (2.6): The partition after the replacement of the content in the 2nd e-abacus diagram will be
$((1 + \gamma_1 + \gamma_2 + \gamma_3 + \cdots +$
$\gamma_m)^{e-l_2}, (1 + \gamma_2 + \gamma_3 + \cdots + \gamma_m)^{\theta_1-\theta_2}, \cdots, (1 + \gamma_{m-1} + \gamma_m)^{\theta_{m-2}-\theta_{m-1}}, (1 + \gamma_m)^{\theta_{m-1}-\theta_m}, 1^{\theta_m})$.

Rule (2.7): The partition after the replacement of the content in the 3rd e-abacus diagram will be
$((2 + \gamma_1 + \gamma_2 + + \cdots + \gamma_m)^{C_3}, (2 + \gamma_2 + \gamma_3 + \cdots + \gamma_m)^{\theta_1-\theta_2}, \cdots, (2 + \gamma_{m-1} + \gamma_m)^{\theta_{m-2}-\theta_{m-1}}, (2 +$
$\gamma_m)^{\theta_{m-1}-\theta_m}, 2^{\theta_m})$,

where $C_3 = \begin{cases} 0 & if\ e - l_3 = 0, \\ e - 1 & if\ e - l_2 = 0, \\ e - l_3 & otherwise. \end{cases}$

In general, we have the following rule:

Rule (2.8): The partition after the replacement the content in any $(X+1)$, the e-abacus diagram will be
$((X + \gamma_1 + \gamma_2 + + \cdots +$
$\gamma_m)^{C_{X+1}}, (X + \gamma_2 + \gamma_3 + \cdots + \gamma_m)^{\theta_1-\theta_2}, \cdots, (X + \gamma_{m-1} + \gamma_m)^{\theta_{m-2}-\theta_{m-1}}, (X + \gamma_m)^{\theta_{m-1}-\theta_m}, X^{\theta_m})$,

where $C_{X+1} = \begin{cases} 0 & if\ e - l_{X+1} = 0, \\ e - 1 & if\ e - l_X = 0, \\ e - l_{X+1} & otherwise. \end{cases}$

Then, without using any diagrams, we have the applications of rules of an example arbitrary, as presented in Tables- 5 and 6.

Table 5- The rules of Case I and Case II for $(5^6, 4^3, 2^7, 1^8)$ where e=3

	$\boldsymbol{e = 3}$					
	Case I			**Case II**		
	1st $\boldsymbol{l = 2}$	**2nd** $\boldsymbol{l = 2}$	**3rd** $\boldsymbol{l = 2}$	**1st** $\boldsymbol{l_1 = 2}$	**2nd** $\boldsymbol{l_2 = 3}$	**3rd** $\boldsymbol{l_3 = 1}$
$\boldsymbol{(5^6, 4^3, 2^7, 1^8)}$	$(6+3+7+8)^{3-2} = 24^1$	$(1+6+3+7+8)^{(3-2)+(3-1)} = 25^3$	$(2+6+3+7+8)^{(3-2)+(2(3)-2)} = 26^5$	$(6+3+7+8)^{3-2} = 24^1$	$(1+6+3+7+8)^{(3-3)} = 25^0$	$(2+6+3+7+8)^{(3-1)} = 26^2$
	$(3+7+8)^{5-4} = 18^1$	$(1+3+7+8)^{5-4} = 19^1$	$(2+3+7+8)^{5-4} = 20^1$	$(3+7+8)^{5-4} = 18^1$	$(1+3+7+8)^{5-4} = 19^1$	$(2+3+7+8)^{5-4} = 20^1$
	$(7+8)^{4-2} = 15^2$	$(1+7+8)^{4-2} = 16^2$	$(2+7+8)^{4-2} = 17^2$	$(7+8)^{4-2} = 15^2$	$(1+7+8)^{4-2} = 16^2$	$(2+7+8)^{4-2} = 17^2$
	$8^{2-1} = 8^1$	$(1+8)^{2-1} = 9^1$	$(2+8)^{2-1} = 10^1$	$8^{2-1} = 8^1$	$(1+8)^{2-1} = 9^1$	$(2+8)^{2-1} = 10^1$
		1^1	2^1		1^1	2^1
$\boldsymbol{3^{rep}}$	$\boldsymbol{(24, 18, 15^2, 8)}$	$\boldsymbol{(25^3, 19, 16^2, 9, 1)}$	$\boldsymbol{(26^5, 20, 17^2, 10, 2)}$	$\boldsymbol{(24, 18, 15^2, 8)}$	$\boldsymbol{(19, 16^2, 9, 1)}$	$\boldsymbol{(26^2, 20, 17^2, 10, 2)}$

Table 6- The rules of Case I and Case II for $(5^6, 4^3, 2^7, 1^8)$ where *e*=4

	$e=4$							
	Case I				**Case II**			
	1st $l=1$	2nd $l=1$	3rd $l=1$	4th $l=1$	1st $l_1=1$	2nd $l_2=2$	3rd $l_3=3$	4th $l_4=4$
$(5^6, 4^3, 2^7, 1^8)$	$(6+3+7+8)^{4-1}$ $=24^3$	$(1+6+3+7+8)^6$ $=25^6$	$(2+6+3+7+8)^9$ $=26^9$	$(3+6+3+7+8)^{12}$ $=27^{12}$	$(6+3+7+8)^{4-1}$ $=24^3$	$(1+6+3+7+8)^{(4-2)}$ $=25^2$	$(2+6+3+7+8)^{4-3}$ $=26^1$	$(3+6+3+7+8)^{4-4}$ $=27^0$
	$(3+7+8)^{5-4}=18^1$	$(1+3+7+8)^{5-4}=19^1$	$(2+3+7+8)^{5-4}=20^1$	$(3+3+7+8)^{5-4}=21^1$	$(3+7+8)^{5-4}=18^1$	$(1+3+7+8)^{5-4}=19^1$	$(2+3+7+8)^{5-4}=20^1$	$(3+3+7+8)^{5-4}=21^1$
	$(7+8)^{4-2}=15^2$	$(1+7+8)^{4-2}=16^2$	$(2+7+8)^{4-2}=17^2$	$(3+7+8)^{4-2}=18^2$	$(7+8)^{4-2}=15^2$	$(1+7+8)^{4-2}=16^2$	$(2+7+8)^{4-2}=17^2$	$(3+7+8)^{4-2}=18^2$
	$8^{2-1}=8^1$	$(1+8)^{2-1}=9^1$	$(2+8)^{2-1}=10^1$	$(3+8)^{2-1}=11^1$	$8^{2-1}=8^1$	$(1+8)^{2-1}=9^1$	$(2+8)^{2-1}=10^1$	$(3+8)^{2-1}=11^1$
		1^1	2^1	3^1		1^1	2^1	3^1
4^{rep}	$(\mathbf{24^3}, \mathbf{18}, \mathbf{15^2}, \mathbf{8})$	$(\mathbf{25^6}, \mathbf{19}, \mathbf{16^2}, \mathbf{9}, \mathbf{1})$	$(\mathbf{26^9}, \mathbf{20}, \mathbf{17^2}, \mathbf{10}, \mathbf{2})$	$(\mathbf{27^{12}}, \mathbf{21}, \mathbf{18^2}, \mathbf{11}, \mathbf{3})$	$(\mathbf{24^3}, \mathbf{18}, \mathbf{15^2}, \mathbf{8})$	$(\mathbf{25^2}, \mathbf{19}, \mathbf{16^2}, \mathbf{9}, \mathbf{1})$	$(\mathbf{26^1}, \mathbf{20}, \mathbf{17^2}, \mathbf{10}, \mathbf{2})$	$(\mathbf{21}, \mathbf{18^2}, \mathbf{11}, \mathbf{3})$

3. Conclusions

This paper has reached several conclusions. First, each diagram in e^{rep} gives a new different partition in the main *e*-abacus diagram.

Second, it is possible to have a similarity in the value of partition in e^{rep} only for the case 1st *e*-abacus diagram.

Third, for the first time, the content replacement method is used and the insights of this method are carefully studied,. This will provide later the possibility of adopting it as a type of encoding or encryption in many applications on the topic of partition, thus opening new horizons for scientific research in this direction. See [8-15].

Acknowledgment

We thank the University of Mosul/College of Education for Pure Science for their moral support during the preparation of this research.

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