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On E_c-Continuous and δ-$ß_c$-Continuous Mappings in Topological Spaces Via E_c-open and δ-$ß_c$-open sets

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Abstract
In the present paper, new concepts of generalized continuous mappings, namely E_c and δ-$ß_c$-continuous mappings have been introduced and studied by using a new generalized of open sets E_c and δ-$ß_c$-open sets ,respectively. Several characterizations and fundamental properties of these forms of generalized continuous mappings are obtained. Moreover, the graphs of E_c-continuous and δ-$ß_c$-continuous mappings have been investigated. In addition, the relationships among E_c-continuous and δ-$ß_c$-continuous mappings and other well-known forms of generalized continuous mappings in topological spaces are also discussed.
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الدوال $-E_c$ والمستمرة $-ß_c-\delta$ في الفضاءات التبولوجية بأستعمال المجموعات المفتوحة $-E_c$ والمفتوحة$-ß_c-\delta$المستمرة

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الخلاصة:
في البحث المُقدم ، مفاهيم جديدة من الدوال المستمرة المعممة والتي يطلق عليها $-E_c$ والمستمرة $-ß_c-\delta$ تم تقديمها ودراستها وذلك بأستعمال مجموعات مفتوحة معممة جديدة $-E_c$ والمفتوحة $-ß_c-\delta$الدوال المستمرة وهي المجموعات المفتوحة على التوالي.عدة خصائص وخواص أساسية تتعلق بهذه الأصناف من الدوال المستمرة المعممة تم الحصول عليها. أضافة الى ذلك تمت مناقشة البيانات لهذه الدوال المستمرة. كذلك فان العلاقات بين هذه الاصناف من الدوال المستمرة المعممة والدوال المعروفة الأخرى من الدوال المستمرة في الفضاءات التبولوجية تمت مناقشتها.

1. INTRODUCTION

Generalized open and closed sets play very a prominent role in general topology and its applications. Many topologists are focusing their research on these topics and this has mounted to many important and useful results. Indeed, a significant theme in General

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topology, Real analysis and many other branches of mathematics concerns the variously modified forms of continuity utilizing generalized open and closed sets. Several variants of continuity and generalized continuity occur in the lore of mathematical literature and applications of mathematics. New classes of generalized open sets in a topological space, called *δ-ß*-open sets or e^*-open sets and some of their properties have been obtained by E. Hatir and T. Noiri [1] and Erdal. E. [2]. In [3] Hariwan. Z. I. presented a new class of ƀ-open sets called $Ɓ_c$-open, this class of sets lies strictly between the classes of θ-semi open sets and ƀ-open sets. Moreover, Alias B. K., Zanyar A. A. [4] introduced a new class of sets, called S_c-open sets and investigated some properties of S_c-continuity and Zanyar A. A., [5] introduced a new class of sets called P_c-open sets, and investigated some properties of P_c-continuity. Ayman. Y. M., [6] also studied new classes of sets called β_c-Open sets which contained in the class of β-open sets and contains the class of $Ɓ_c$-open sets . He introduced β_c-continuous functions as a new class of continuous functions and gave characterizations of these functions.

Recently, many researchers have studied different forms of generalized continuous mappings. Humadi, N. K. and Ali, H. J. [7] introduced new classes of functions called perfectly supra continuous functions, supra continuous, supra open and supra closed functions.

As well a new strong form of continuity and several fundamental properties concerning this type of continuous mapping were obtained by Alaa M. F. Al. Jumaili,…etc in [8].

The motivation of the present paper is to introduce and study new classes of generalized continuous mappings called E_c and δ-$ß_c$-continuous mappings utilizing the new generalized of open sets called, E_c and δ-$ß_c$ -open sets, respectively. Some essential characterizations and interesting properties are obtained. Furthermore, the relationships among E_c and δ-$ß_c$-continuous mappings and other well-known types of generalized continuous mappings are discussed.

2. PRELIMINARIES

Throughout this paper, *(X, T) and (Y, T^*) (or simply X and Y)* mean topological spaces on which no separation axioms are assumed unless explicitly stated. For any subset *A* of *X*, The closure and interior of *A* are denoted by *Cl(A)* and *Int(A)*, respectively.

We recall the following required definitions and results of generalized open sets, which will be used throughout this paper.

Definition 2.1: Let *(X, T)* be a topological space (Top-Sp, briefly) a subset *A* of *X* is said to be:

(a) Regular-open (resp. regular closed) [9] if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$).

(b) *δ*-open [10] if for each $x \in A$ there exists a regular open set *V* (s. t) $x \in V \subseteq A$. The *δ*-interior of *A* is the union of all regular open sets contained in *A* and is denoted by $Int_\delta(A)$. The subset *A* is called ***δ***-open [10] if $A = Int_\delta(A)$. A point $x \in X$ is called a *δ*-cluster point of *A* [10] if $A \cap Int(Cl(V)) \neq \varphi$, for each open set *V* containing *x* . The set of all *δ*-cluster points of *A* is called the ***δ***-closure of *A* and is denoted by $Cl_\delta(A)$. If $A = Cl_\delta(A)$), then A is said to be *δ*-closed [10]. The complement of ***δ***- closed set is said to be *δ*-open set.

Definition 2.2: A subset *A* of a Top. SP *X* is called *δ*-open set [10] if for all $x \in A$ there exists an open set *G* (s. t), $x \in G \subseteq Int(Cl(G)) \subseteq A$. The family of all ***δ***-open sets in *X* is denoted by *δΣ(X, T)*.

Remark 2.3: The family of all regular open sub-sets of *X* that contains a point $x \in X$ is denoted by *RΣ(X, x)*, The family of all regular open sets in *X* is denoted *RΣ(X, T)*.

Definition 2.4: Let *(X, T)* be a Top. Sp. Then we have the following:

(a) A sub-set *A* of a space *X* is called *E*-open [11] if $A \subseteq Cl(\delta\text{-}Int(A)) \cup Int(\delta\text{-}Cl(A))$. The complement of an *E*-open set is called *E*-denoted. The intersection of all *E*-closed sets containing *A* is called the *E*-closure of *A* [11] and is denoted by *E-Cl(A).* The union of all *E*-open sets of *X* contained in *A* is called the *E*-interior [11] of *A* and is denoted by *E-Int*(*A*) .

(b) A subset *A* of a space *X* is called *δ-ß*-open [1] or e^*-open [2], if $A \subseteq Cl(Int(\delta\text{-}Cl(A)))$, the complement of a *δ-ß*-open set is called *δ-ß*-closed. The intersection of all *δ-ß*-closed sets containing *A* is called the *δ-ß*-closure of *A* [6] and is denoted by *δ-ß*-Cl(*A*). The union of all *δ-ß*-open sets of *X* contained in *A* is called the *δ-ß*-interior [1] of *A* and is denoted by *δ-ß-Int(A)* .

Remark 2.5: The family of all *E*-open (resp. *E*-closed, *δ-ß*-open, *δ-ß*-closed) subsets of *X* containing a point $x \in X$ is denoted by $E\Sigma(X, x)$ (resp. $EC(X, x)$, $\delta\text{-}\beta\Sigma(X, x)$, $\delta\text{-}\beta C(X, x)$). The family of all *E-open* (resp. *E*-closed, *δ-ß*-open, *δ-ß*-closed) sets in *X* is denoted via $E\Sigma(X, T)$ (*resp*. $EC(X, T)$, $\delta\text{-}\beta\Sigma(X, T)$, $\delta\text{-}\beta C(X, T)$).

Definition 2.6: Let *(X, T)* be a Top. SP, then we have the following:

(a) A subset *A* of *X* is said to be *θ*-open set [10] if for all $x \in A$ there exists an open set *G* such that $x \in G \subseteq Cl(G) \subseteq A$ that means a point $x \in X$ is called a θ-cluster point of *A* if $Cl(V) \cap A \neq \varphi$ for every open subset *V* of *X* containing *x*. The set of all θ-cluster points of *A* is called the *θ*-closure of *A* and is denoted by $Cl_\theta(A)$. If $A = Cl_\theta(A)$, then *A* is said to be *θ*-closed [10]. The complement of a *θ*-closed set is said to be *θ*-open. The family of all *θ*-open set s in *X* is denoted by $\theta\Sigma(X, T)$.

(b) A subset *A* of *X* is said to be *θ*-Semi-open [12] if for all $x \in A$ there exists a semi-open set *G* (s. t), $x \in G \subseteq Cl(G) \subseteq A$. The family of all *θ*-Semi-open sets in *X* is denoted by $\theta S\Sigma(X, T)$.

Remark 2.7: The collection of *θ*-open sets in a Top. SP. *X* forms a topology T_θ which is coarser than *T*. In addition, the family of *δ*-open set in a Top-sp *X* forms a topology T_δ (s. t) $T_\delta \subseteq T$.

Proposition 2.8: [13] A Top. SP. *(X, T) is* Regular *iff* $T_\theta = T$.

Definitions 2.9: A Top. Sp. *(X, T)* is said to be:

(a) An extremely disconnected (A locally indiscrete) [14] if the closure of every open set of *X* is open in *X* (if and only if every open set is closed).

(b) A regular space [15] if for each $x \in X$ and for each open set *G* containing *x*, there exists an open set *K* (s. t), $x \in K \subseteq Cl(K) \subseteq G$.

(c) Alexandroff space [16] if any arbitrary intersection of open sets is open.

Remark 2.10: A space *X* is Alexandroff space if and only if an arbitrary union of closed sets is closed.

Remark 2.11: We have the following diagram in which the converses of implications need not be true, see the examples in [17], [11] and [2] .

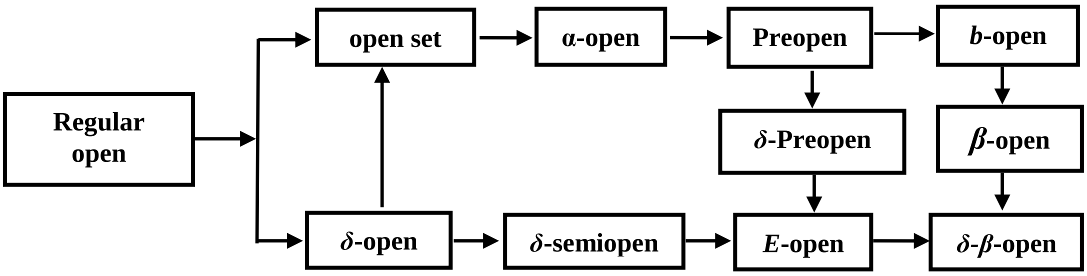

Figure 2.1-The relationships among some well-known generalized open sets in Top-Sp.

Lemma 2.12: [18] Let *X* be a space and *A*, $B \subseteq X$, if $A \in \delta\Sigma(X)$ and $B \in E\Sigma(X)$(resp. $B \in \delta\text{-}\beta\Sigma(X)$) then, $A \cap B \in E\Sigma(X)$(resp. $A \cap B \in \delta\text{-}\beta\Sigma(X)$).

Lemma 2.13: [19] If $A \subseteq X^* \subseteq X$ and $X^* \in E\Sigma(X, T)$ (*resp.* $X^* \in \delta\text{-}\beta\Sigma(X, T)$),then $A \in E\Sigma(X, T)$ (*resp*. $A \in \delta\text{-}\beta\Sigma(X, T)$) $\Leftrightarrow A \in E\Sigma(X^*, T)$ (*resp*. $A \in \delta\text{-}\beta\Sigma(X^*, T)$).

Lemma 2.14: [11, 17] If $U \in E\Sigma(X)$ (*resp*. $\delta\text{-}\beta\Sigma(X)$) and $V \in E\Sigma(Y)$ (*resp*. $\delta\text{-}\beta\Sigma(Y)$), then $U \times V \in E\Sigma(X \times Y)$ (*resp*. $\delta\text{-}\beta\Sigma(X \times Y)$).

Theorem 2.15: [20] Let (Y, T_Y) be a sub-Sp of (X, T). If A is a closed sub-set in Y and $Y \subseteq X$, then A is closed in X.

Theorem 2.16: [15] Let (X, T_X) and (Y, T_Y) be Top. Sp, and $X \times Y$ be the product topology, and let (a, b) be any point in the product Top. Sp, $X \times Y$. Then, we have the sub-space $X \times \{b\}$ is homeomorphic to X and the sub-Sp $\{a\} \times Y$ is homeomorphic to Y.

Proposition 2.17: [15] every regular, T_1-Sp. is Urysohn and every Urysohn space is Hausdorff space.

Proposition 2.18: [11, 17] the following properties hold for a space X:

(a) The arbitrary union of any family of E-(*resp.* δ-ß)-open sets in X, is an E-(*resp.* δ-ß)-open set.

(b) The arbitrary intersection of any family of E-(*resp.* δ-ß)- closed sets in X, is an E-(*resp.* δ-ß)-closed set.

3. CHARACTERIZATIONS OF (E_c) AND $(\delta$-ß$_c)$-OPEN SETS

In this section, new classes of E-open and δ-ß-open sets called E_c-open and δ-ß$_c$-open sets are studied and several characterizations concerning these forms of generalized open sets are obtained. Furthermore, the relations among E_c-(*resp.* δ-ß$_c$)-open sets and other forms of generalized open sets are discussed.

Definition 3.1: Let (X, T) be a Top-sp. A subset A of X is said to be:

(a) B_c - open [3] if for all $x \in A \in BO(X, T)$, there exists a closed set F (s. t), $x \in F \subseteq A$.

(b) S_c - open [4] if for all $x \in A \in SO(X, T)$, there exists a closed set F (s. t), $x \in F \subseteq A$.

(c) P_c-open [5] if for all $x \in A \in PO(X, T)$, there exists a closed set F (s. t), $x \in F \subseteq A$.

(d) ß$_c$-open [6] if for all $x \in A \in$ ß$O(X, T)$, there exists a closed set F (s. t), $x \in F \subseteq A$.

The family of all B_c-open , S_c-open, P_c-open and ß$_c$-open sets in X are denoted by $BC\Sigma(X)$, $SC\Sigma(X)$, $PC\Sigma(X)$ and ß$C\Sigma(X)$,respectively.

Definition 3.2: Let (X, T) be a Top. SP. A sub-set A of X is said to be:

(a) E_c-open set if for all $x \in A \in E\Sigma(X, T)$, there exists a closed set F (s. t), $x \in F \subseteq A$. The family of all E_c-open sub-sets of (X, T) is denoted by $EC\Sigma(X, T)$ or $EC\Sigma(X)$.

(b) δ-ß$_c$-open set if for all $x \in A \in \delta\text{-}\beta\Sigma(X,T)$, there exists a closed set F (s. t), $x \in F \subseteq A$. The family of all δ-ß$_c$ -open subsets of (X, T) is denoted by $\delta\text{-}\beta C\Sigma(X, T)$ or $\delta\text{-}\beta C\Sigma(X)$.

A sub-set F of a space (X, T) is said to be E_c (*resp.* δ-ß$_c$)-closed set when $X \setminus F \in EC\Sigma(X, T)$ (*resp.* $\delta\text{-}\beta C\Sigma(X,T)$).

Remark 3.3: The family of all E_c-(*resp.* δ-ß$_c$)-closed sub-sets of (X, T) is denoted by $ECC(X, T)$ OR $ECC(X)$ (*resp.* δ-ß$CC(X, T)$ OR δ-ß$CC(X)$).

Theorem 3.4: Let (X, T) be a Top. Sp. A subset A of X is E_c-(*resp.* δ-ß$_c$)-open set if and only if A is E- (*resp.* δ-ß)-open set it is a union of closed sets. That is $A = \bigcup F_\lambda$ where A is E_c-(*resp.* δ-ß$_c$)-open set and F_λ closed sets for all λ.

Proof: Let (X, T) be a Top-Sp. and A be a E_c -(*resp.* δ-ß$_c$)-open set. Then, A is E- (*resp.* δ-ß)-open and for all $\lambda \in A$ there exists a closed F_λ in X that means $\lambda \in F_\lambda \subseteq A$ implies $\bigcup_{(\lambda \in A)} F_\lambda \subseteq A \subseteq \bigcup_{(\lambda \in A)} F_\lambda$. Hence, $A = \bigcup_{(\lambda \in A)} F_\lambda$ where F_λ closed sets for all $\lambda \in A$. The converse is directly followed from the definition of E_c- (*resp.* δ-ß$_c$)-open sets.

Remark 3.5: From the respective definitions, the relationships among E_c-(*resp.* δ-ß$_c$)-open sets and other well-known forms of generalized open sets are shown in the following figure:

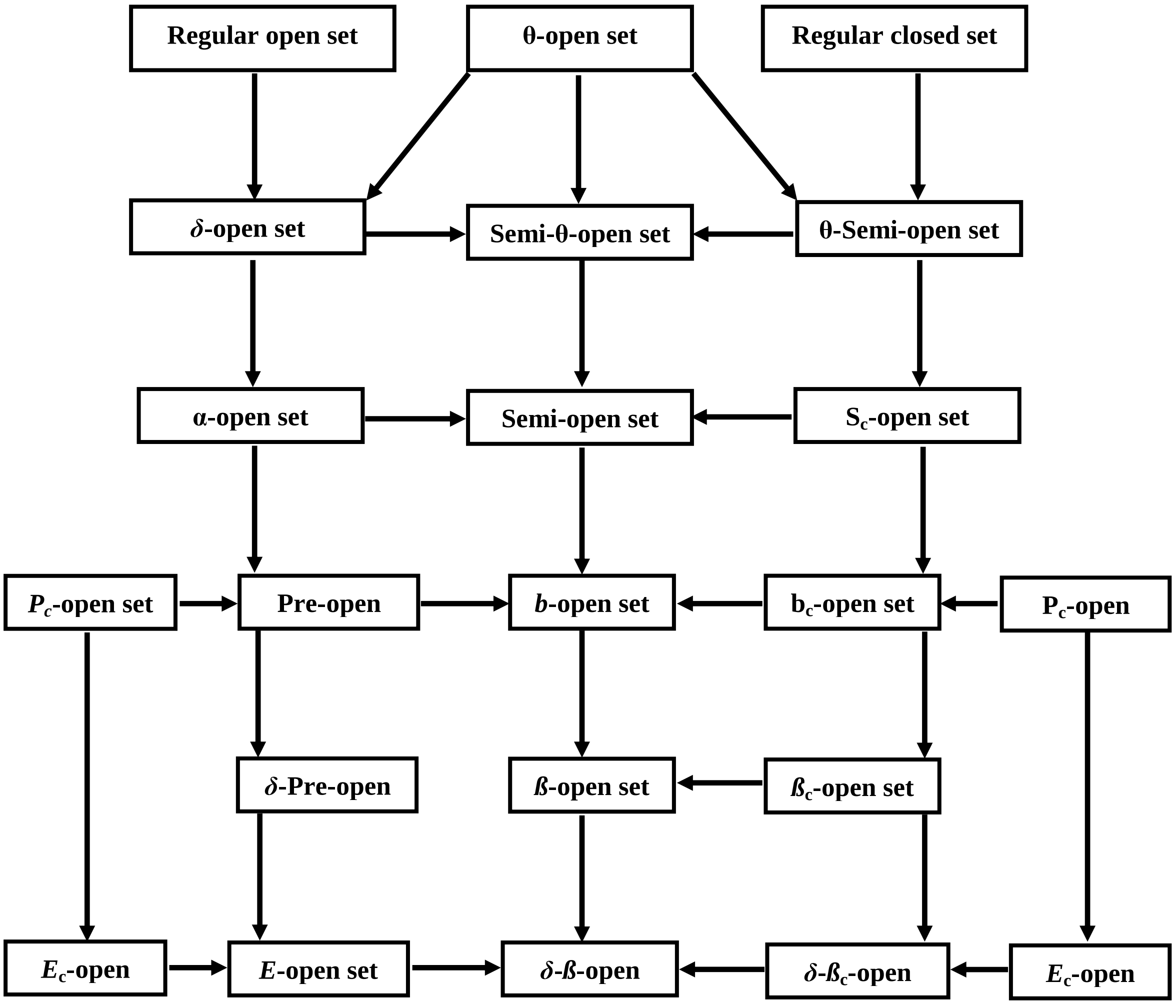

Figure 3.1-The relationships among E_c-*(resp.* δ-$ß_c$*)*-open sets and other well-known types of generalized open sets

However, none of these implications is reversible as shown via examples of [2,3,4,5,6 , 11] and the following examples:

Examples 3.6: Let $X = \{x, y, w, z\}$ and let $T = \{\varphi, \{x\}, \{w\}, \{x, y\}, \{x, w\}, \{x, y, w\}, \{x, w, z\}, X\}$. Then the family of all closed subsets is: $T^c = \{X, \{y, w, z\}, \{x, y, z\}, \{w, z\}, \{y, z\}, \{z\}, \{y\}, \varphi\}$.Thus:

i. The set $\{y, w\}$ is E-open. However, it is not E_c-open set.

ii. The set $\{y, w, z\}$ is E_c-open set . However, it is not S_c-open and not P_c-open, it is also neither b_c-open nor $ß_c$-open set.

iii. The set $\{y, z\}$ is δ-$ß_c$-open set but it is not E_c-open not S_c-open. Also, it is not P_c-open and neither b_c-open nor $ß_c$-open set.

iv. Let $X = \{x, y, w, z, s\}$ and let $T = \{\varphi, \{x, y\}, \{w, z\}, \{x, y, w, z\}, X\}$. Then the family of all closed subsets is: $T^c = \{X, \{w, z, s\}, \{x, y, s\}, \{s\}, \varphi\}$.Then, the sub set $\{x, s\}$ is δ-$ß$-open but it is not δ-$ß_c$-open set.

Theorem 3.7: If a Sp. *(X, T)* is T_1-Sp, then the families $E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)* are identical to the families $EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*)* that means $E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)* $= EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*).*

Proof: Let (X, T) be a Top-sp and A be any subset of a space. X *(s. t)* $A \in E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)*, there are two cases, *if* $A =, \varphi$ so $A \in EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*), if* $A \neq \varphi$, Since a sp. X is T_1, then every singleton is closed set and hence $\forall$ $x \in$ A we get $x \in \{x\} \subseteq A$. thus, $A \in EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*),* so $E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)* $\subseteq$ $EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*),* but generally, $EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*)* $\subseteq E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)*, therefore: $E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)* $= EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*).*

Now we show that in any Top-Sp(X, T) the arbitrary unions of E_c (*resp*. δ-$ß_c$)-open sets is E_c (*resp*. δ-$ß_c$)-open.

Theorem 3.8: Let (X, T) be a Top. SP and $\{A_\lambda: \lambda \in \Delta\}$ be a family of E_c (*resp*. δ-$ß_c$)-open sets in a sp X. Then $\bigcup \{A_\lambda: \lambda \in \Delta\}$ is E_c (*resp*. δ-$ß_c$)-open.

Proof: suppose that $\{A_\lambda: \lambda \in \Delta\}$ be a collection of E_c (*resp*. δ-$ß_c$)-open set for all $\lambda \in \Delta$, then A_λ is E (*resp*. δ-$ß$)-open set $\forall$ λ and so via (Proposition 2.18), $\bigcup \{A_\lambda: \lambda \in \Delta\}$ is E-(*resp*. δ-$ß$)-open. If $x \in \bigcup \{A_\lambda: \lambda \in \Delta\}$, *there exits* $\lambda \in \Delta$ *such that* $x \in A_\lambda$. Since A_λ is E_c-(*resp*. δ-$ß_c$)- open for all $\lambda \in \Delta$, there exits a closed set F *such that* $x \in F \subseteq A_\lambda \subseteq \bigcup_{(\lambda \in \Delta)} A_\lambda \Longrightarrow x \in F \subseteq \bigcup_{(\lambda \in \Delta)} A_\lambda$. so $\bigcup\{A_\lambda: \lambda \in \Delta\}$ is E_c-(*resp*. δ-$ß_c$)-open set.

Theorem 3.9: Let (X,T) be a Top. Sp and $\{A_\lambda: \lambda \in \Delta\}$ be a family of E_c-(*resp*. δ-$ß_c$)-closed sets in a space X. Then $\bigcap \{A_\lambda: \lambda \in \Delta\}$ is E_c- (*resp*. δ-$ß_c$)-closed.

Proof: The proof is obvious, it follows from Theorem 3.8 and using De Morgan's Law.

Theorem 3.10: A subset A of a space (X, T) is E_c (*resp*. δ-$ß_c$)-open set if and only if for all $x \in A$ $\exists$ a E_c -(*resp*. δ-$ß_c$)-open set B (s. t) $x \in B \subseteq A$.

Proof: The proof is obvious it follows from the Definition (3.2) and Theorem 3.9.

Remark 3.11: If a space (X, T) is T_1-Space, then $T \subseteq EC\Sigma(X)$ ***(****resp.* δ-$\beta C\Sigma(X)$***),*** since every open set is E-(*resp*. δ-$ß$)-open.

Theorem 3.12: Let A be a sub set of a space X, if A is a θ-open set. Then A is E_c- (*resp*. δ-$ß_c$)-open set.

Proof: suppose that A is θ-open set in X, then for all $x \in A$ there exists an open set G_x (s. t): $x \in G_x \subseteq Cl(G_x) \subseteq A$, so $\bigcup_{(x \in A)}\{x\} \in \bigcup_{(x \in A)} G_x \subseteq \bigcup_{(x \in A)} Cl(G_x) \subseteq A \Longrightarrow A = \bigcup_{(x \in A)} G_x$, which means A is a union of open sets and therefore A is open set, thus A is E- (*resp*. δ-$ß$)-open, as well $A = \bigcup_{(x \in A)} Cl(G_x)$ which is a union of closed sets, hence via Theorem (3.4) we get A is E_c- (*resp*. δ-$ß_c$)-open set.

The following example explains that the converse of Theorem-(3.12) need not be true in general .

Example 3.13: because any space X with the co-finite topology is T_1, so the families of $E\Sigma(X)$ *(resp.* δ-$\beta\Sigma(X)$*)* are identical to the families of $EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*),* therefore any open set G is E_c- (*resp*. δ-$ß_c$)-open . However, it is not θ-open set since $\overline{G} = X$, for all open sub-set G of X.

Theorem 3.14: Every regular closed subset in a space (X, T) is δ-$ß_c$-open set.

Proof: suppose that A is regular closed sub-set in X, thus $A = Cl(\text{Int}(A))$, but $\text{Cl}(\text{Int}(A)) \subseteq Cl(\text{Int}(\delta\text{-}Cl(A))) \Longrightarrow A$ is δ-$ß$-open. Now, since A is closed, then via definition-(3.2), A is δ-$ß_c$-open set.

Theorem 3.15: If X is a locally indiscrete space, then every semi-open set is δ-$ß_c$-open set.

Proof: Assume that A is semi-open sub-set in X, thus $A \subseteq Cl(Int(A)) \subseteq Cl(Int(Cl(A))) \subseteq Cl(Int(\delta\text{-}Cl(A))) \Longrightarrow A$ is δ-$ß$-open set. Since X is locally indiscrete $\Longrightarrow Int(A)$ is closed and $A \subseteq Cl(Int(A)) = Int(A) \Longrightarrow A$ is open set and for all $x \in A$ *implies* $x \in Int(A) \subseteq A$. Thus, via Definition 3.2-part-ii, we get A is δ-$ß_c$-open set.

Theorem 3.16: Let X be a Top. Sp, if X is Regular space, then every open set is a E_c -(*resp*. δ-$ß_c$)-open set.

Proof: suppose that A is any open subset of X, so A is E- (*resp*. δ-$ß$)-open. There are two cases, namely *if* $A = \varphi$, thus $A \in EC\Sigma(X)$ *(resp.* δ-$\beta C\Sigma(X)$*), if* $A = \varphi$, since X is regular, then via Definition (2.9), for all $x \in A \subseteq X$ there exists An open set G (s. t), $x \in G \subseteq Cl(G) \subseteq A$. So, $x \in Cl(G) \subseteq A$. Therefore, via Definition (3.2), we have $T \in EC\Sigma(X)$ *(resp.* $\delta\text{-}\beta C\Sigma(X)$*).*

Theorem 3.17: Let (X, T) be a Top. Sp and $A, B \subseteq X$. If $A \in EC\Sigma(X)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*),* and B is δ-clopen, then $A \cap B \in EC\Sigma(X)$(resp. $A \cap B \in \delta\text{-}\beta C\Sigma(X)$)

Proof: Suppose $A \in EC\Sigma(X)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*),* and B is δ clopen , thus A is E-(*resp.* δ-ß)-open and B is δ-open, so via Lemma-(2.12), we get $A \cap B \in E\Sigma(X)$(resp. $A \cap B \in \delta\text{-}\beta\Sigma(X)$) . Now let $x \in A \cap B$, implies $x \in A$ and $x \in B$ so, since A is E_c (*resp.* δ-ß$_c$) open implies that there exists a closed set F (s. t) $x \in F \subseteq A$,thus $x \in F \cap B \subseteq A \cap B$. Since B is δ-clopen set, hence B is δ-closed and consequently B is closed set , this implies $F \cap B$ is closed set. Therefore, $A \cap B \in EC\Sigma(X)$(resp. $A \cap B \in \delta\text{-}\beta C\Sigma(X)$).

Theorem 3.18: Let (X, T) be a Top. Sp and $A \subseteq X$. If A is a clopen set, then $A \in EC\Sigma(X)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*)*

Proof: It is straightforward, thus it is omitted .

Theorem 3.19: The following statements are equivalent for a subset A of a space (X, T) :

i) A is clopen set.

ii) *A is* E_c-(*resp.* δ-ß$_c$)-open set and closed.

iii) *A is E-(resp.* δ-ß)-open set and closed.

Proof: The proofs are obvious, thus they are omitted.

Theorem 3.20: let (Y, T_Y) be a subspace of a space X, if $A \in EC\Sigma(X)$ *(resp.* A $\delta\text{-}\beta C\Sigma(X)$*)* and $A \subseteq Y$ (s. t) Y is E-(*resp.* δ-ß)-open, then $A \in EC\Sigma(Y)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(Y)$*).*

Proof: Let $A \in EC\Sigma(X)$ *(resp.* $\delta\text{-}\beta C\Sigma(X)$*)* $\Longrightarrow A \in E\Sigma(X)$ *(resp.* $\delta\text{-}\beta\Sigma(X)$*).* Since $A \subseteq Y$ and Y is E-(*resp*. δ-ß)-open, thus via Lemma-(2.13) A is E-(*resp*. δ-ß)-open in sub-Sp Y, as well.

For all $x \in A$ there exists a closed set $F \in X$ *(s. t)* $x \in F \subseteq A$. Since $A \subseteq Y \Longrightarrow F$ is closed set-in sub-Sp Y. Hence $A \in EC\Sigma(Y)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(Y)$*)*

Theorem 3.21: let (Y, T_Y) be a subspace of a space X, if $A \in EC\Sigma(Y)$ ***(****resp.* $A \in \delta\text{-}\beta C\Sigma(Y)$***)*** and $A \subseteq Y$ and Y is clopen, then $A \in EC\Sigma(X)$ ***(****resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*).*

Proof: Let $A \in EC\Sigma(Y)$ ***(****resp.* $\delta\text{-}\beta C\Sigma(Y)$*)* $\Longrightarrow A \in E\Sigma(Y)$ ***(****resp.* $\delta\text{-}\beta\Sigma(Y)$*),* and for all $x \in A$ there exists a closed set $F \in Y$ *(s. t)* $x \in F \subseteq A$. Since Y is clopen so that Y is $E\Sigma(X)$ ***(****resp.* $\delta\text{-}\beta\Sigma(X)$*)* and since $A \in E\Sigma(Y)$ ***(****resp.* $\delta\text{-}\beta\Sigma(Y)$*),* then via Lemma-(2.13), $A \in E\Sigma(X)$ ***(****resp.* $\delta\text{-}\beta\Sigma(X)$*).* Moreover, since Y is clopen $\Longrightarrow Y$ is closed in X and since F is closed in Y, hence via Theorem-(2.15) F is closed set in X. So $A \in EC\Sigma(X)$ ***(****resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*).*

Corollary 3.22: Let A and Y be any sub-sets of a space X (s. t) $A \subseteq Y \subseteq X$ and Y is clopen set. Then $A \in EC\Sigma(Y)$ ***(****resp.* $A \in \delta\text{-}\beta C\Sigma(Y)$*) iff* $A \in EC\Sigma(X)$ ***(****resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*)*

Proof: The proof follows from Theorems (3.20) and (3.21).

Corollary 3.23: Let A and Y be any sub-sets of a space X. if $A \in EC\Sigma(X)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*)* and Y is δ-clopen subset of X. Then $A \cap Y \in EC\Sigma(Y)$(resp. $A \cap Y \in \delta\text{-}\beta C\Sigma(Y)$).

Proof: Let $A \in EC\Sigma(X)$ *(resp.* $\delta\text{-}\beta C\Sigma(X)$*)* and Y is δ-clopen of $X \Longrightarrow A \in E\Sigma(X)$ *(resp.* $\delta\text{-}\beta\Sigma(X)$*)* and Y is δ-open and δ-closed of X, so via Lemma-(2.12), $A \cap Y \in E\Sigma(X)$(resp. $A \cap Y \in \delta\text{-}\beta\Sigma(X)$).Since $A \in EC\Sigma(X)$ *(resp.* $\delta\text{-}\beta C\Sigma(X)$*)* $\Longrightarrow \forall x \in A \ \exists$ a closed set F in X (s. t) $x \in F \subseteq A$ thus, $x \in F \cap Y \subseteq A \cap Y \Longrightarrow A \cap Y \in EC\Sigma(X)$(resp. $A \cap Y \in \delta\text{-}\beta C\Sigma(X)$) (s. t), $A \cap Y \subseteq Y$. Hence via Theorem (3.20), $A \cap Y \in EC\Sigma(Y)$(resp. $A \cap Y \in \delta\text{-}\beta C\Sigma(Y)$).

Theorem 3.24: Let (X, T_X) and (Y, T_Y) be Top. SP and $X \times Y$ be the product Top. If $A \in EC\Sigma(X)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(X)$*)* and $B \in EC\Sigma(Y)$ *(resp.* $A \in \delta\text{-}\beta C\Sigma(Y)$*)* then, $A \times B \in EC\Sigma(X \times Y)$ *(resp.* $A \times B \in \delta\text{-}\beta C\Sigma(X \times Y)$*)*

Proof: suppose that$(x, y) \in A \times B \Longrightarrow x \in A$ and $y \in B$. Since $A \in EC\Sigma(X)$ *(resp.* $\delta\text{-}\beta C\Sigma(X)$*)* $\Longrightarrow$ $A \in (X)$ *(resp.* $\delta\text{-}\beta\Sigma(X)$*)* $\Longrightarrow \forall x \in A \ \exists$ a closed set F in X (s. t) $x \in F \subseteq A$.

Since $B \in EC\Sigma(Y)$ *(resp.* $\delta\text{-}\beta C\Sigma(Y)$*)* $\Longrightarrow B \in (Y)$ *(resp.* $\delta\text{-}\beta\Sigma(Y)$*)* $\Longrightarrow \forall y \in B \ \exists$ a closed set E in Y (s. t) $y \in E \subseteq B$, so $(x, y) \in F \times E \subseteq A \times B$, and via Lemma (2.14) $A \times B \in E\Sigma(X \times Y)$ *(resp.*

$A \times B \in \delta$-$\beta\Sigma(X \times Y)$).Since F *and* E are closed in sp. X and Y respectively, we get $F \times E$ is closed in $X \times Y$. Thus, $A \times B \in EC\Sigma(X \times Y)$ (resp. $A \times B \in \delta$-$\beta C\Sigma(X \times Y)$)

4. CHARACTERIZATIONS OF E_c-(δ-ß$_c$)-CONTINUOUS MAPPINGS

In this section, we introduce and investigate new classes of continuous mappings that are called E_c-(δ-ß$_c$)-continuous mappings via new generalized open sets. Some characterizations and several properties concerning these types of continuous mappings are obtained. Moreover, the relationships among E_c-(δ-ß$_c$)- continuous mapping and other well-known forms of generalized continuous mappings are discussed.

Definition 4.1: Let (X, T) and (Y, T^*) be two Top-Sp The mapping f: $X \to Y$ is called E_c-(δ-ß$_c$)- continuous mapping at a point $x \in X$ if for all open set V of Y containing $f(x)$, there exists an E_c-(*resp.* δ-ß$_c$)-open set U of X containing x (s. t) $f(U) \subseteq V$. If f is E_c-(δ-ß$_c$)-continuous at each point $x \in X$, then it is called E_c-(δ-ß$_c$)- continuous.

Definition 4.2: A mapping f: $(X, T) \to (Y, T^*)$ is said to be:

i) E- Continuous 11], if $f^{-1}(V)$ is E-open in X for every open subset V of Y.
ii) δ-ß- Continuous [17], if $f^{-1}(V)$ is δ-ß-open in X for every open subset V of Y.
iii) Perfectly (clopen)- continuous [21],if $f^{-1}(V)$ is clopen in X for every open subset V of Y.
iv) Contra-Continuous [22], if $f^{-1}(V)$ is closed in X for every open subset V of Y.
v) Strongly θ-Continuous [23], if $f^{-1}(V)$ is θ-open in X for every open subset V of Y.
vi) RC- Continuous [24], if $f^{-1}(V)$ is regular closed in X for every open subset V of Y.
vii) Semi- Continuous [25], if $f^{-1}(V)$ is semi-open in X for every open subset V of Y.
viii) θS-Continuous [26], if $f^{-1}(V)$ is θ-semi open in X for every open subset V of Y.
ix) S_c- Continuous [4], if $f^{-1}(V)$ is S_c-open in X for every open subset V of Y.
x) P_c- Continuous [5], if $f^{-1}(V)$ is P_c-open in X for every open subset V of Y.
xi) ß$_c$- Continuous [6], if $f^{-1}(V)$ is ß$_c$-open in X for every open subset V of Y.
xii) ß- Continuous [27], if $f^{-1}(V)$ is ß-open X for every open subset V of Y.

Theorem 4.3: A mapping f: $(X, T) \to (Y, T^*)$ is E_c-(δ-ß$_c$)- Continuous *iff* the inverse image of every open set in Y is E_c-(δ-ß$_c$)-open set in X.

***Proof*:** ($\Longrightarrow$) Let f be E_c-(δ-ß$_c$)- Continuous mapping and V be an open set in Y. there are two cases: First if $f^{-1}(V) = \varphi \Longrightarrow f^{-1}(V)$ is E_c-(δ-ß$_c$)-open set in X. Second if $f^{-1}(V) \neq \varphi \Longrightarrow$ for all $x \in f^{-1}(V) \Longrightarrow f(x) \in V$. Thus, by E_c-(δ-ß$_c$)-continuity, there exists E_c(δ-ß$_c$)-open set $U \subseteq X$ containing x (s. t), $f(U) \subseteq V$. So, $x \in U \subseteq f^{-1}(V)$. Hence, via Theorem (3.10), we get $f^{-1}(V)$ is E_c-(δ-ß$_c$)-open set in X.

($\Longleftarrow$) Suppose that the inverse image of every open sub-set in in Y is E_c-(δ-ß$_c$)-open set in X and let V be an open set in Y containing $f(x)$. Then, $x \in f^{-1}(V)$ which is E_c-(δ-ß$_c$)-open set via our supposition and $f(f^{-1}(V)) \subseteq V$. Therefore, f is E_c-(δ-ß$_c$)- Continuous mappings.

Corollaries 4.4: Let (X, T) be a Top- sp. Then the following are hold:

i) Every P_c Continuous mapping - is E_c-Continuous.
ii) Every E_c- Continuous mapping is E-Continuous.
iii) Every E- Continuous mapping is δ-ß-Continuous.
iv) Every E_c- Continuous mapping is δ-ß$_c$-Continuous.
v) Every S_c- Continuous mapping is ß$_c$-Continuous.
vi) Every ß$_c$- Continuous mapping is ß-Continuous.
vii) Every ß$_c$- Continuous mapping is δ-ß$_c$-Continuous.
viii) Every δ-ß$_c$- Continuous mapping is δ-ß-Continuous.

***Proof*:** The proof of the above corollaries is obvious, and it is followed from their respective definitions .

Remark 4.5: From the respective definitions, the among E_c-(δ-ß$_c$)- Continuous mappings and various other well-known forms of generalized continuous mapping are shown in the following figure:

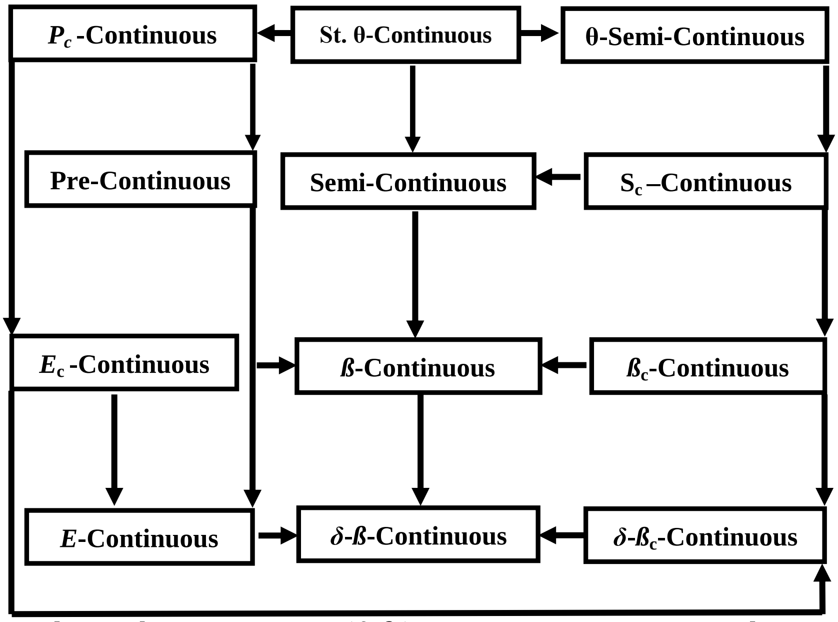

Figure 4.1-The relationships among E_c-(δ-$ß_c$)-Continuous mapping and various other well-known forms of generalized continuous mappings

However, none of these implications is reversible as shown via examples of [4, 5, 6] and the following examples:

Example 4.6: Let $X = Y=$ {a, b, c, d}, define a topology $T = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$ on X and a topology $T^* = \{\varphi, Y, \{c\}, \{d\}, \{c, d\}\}$ on Y and let f: $(X, T) \rightarrow (Y, T^*)$ be mapping defined as follows: $f(a) = d, f(b) = c, f(c) = b$ and $f(d) = a$. Then

1) f is E- Continuous mapping but not E_c- Continuous mapping because $\{c\}$ is an open set in Y but $f^{-1}(\{c\}) = \{b\}$ which is not E_c-open set in X.

2) f is δ-ß- Continuous mapping but not δ-$ß_c$- Continuous mapping because $\{d\}$ is an open set in Y but $f^{-1}(\{d\}) = \{a\}$ which is not δ-$ß_c$-open in X.

Example 4.7: Let $X = \{a, b, c\}$ and define a topology $T = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$ on X. Then the identity mapping f: $(X, T) \rightarrow (X, T)$ *is* δ-ß-Continuous mapping which is not E_c-Continuous because $\{b\}$ is an open set in X but $f^{-1}(\{b\}) = \{b\}$ which is not E_c-open set in X.

Example 4.8: Let $X = \{x, y, w, z\}$ and $Y = \{a, b, c, d\}$, define a topology $T = \{\varphi, X, \{x\}, \{w\}, \{x,, y\},\{x,, w\},\{x,, y, w\},\{x,, w, z\}\}$ on X and a topology $T^* = \{\varphi, Y, \{a, b\}\}$ on Y and f: $(X, T) \rightarrow (Y, T^*)$ be mapping defined as follows: $f(x) = c, f(y) = a, f(w) = d$ *and* $f(z) = b$. Then

1) f is δ-ß- Continuous mapping which is not E- Continuous mapping because $\{a, b\}$ is an open set-in Y but $f^{-1}(\{a, b\}) = \{y, z\}$ which is not E-open set in X.

2) f is δ-$ß_c$- Continuous mapping which is not E_c- Continuous mapping because $\{a, b\}$ is an open set in Y but $f^{-1}(\{a, b\}) = \{y, z\}$ which is not E_c-open set in X.

3) f is δ-$ß_c$- Continuous mapping which is not $ß_c$- Continuous mapping because $\{a, b\}$ is an open set in Y but $f^{-1}(\{a, b\}) = \{y, z\}$ which is not $ß_c$-open set in X.

Example 4.9: Let $X = \{a, b, c\}$ and $Y = \{x,, y, z\}$, define a topology $T = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$ on X and a topology $T^* = \{\varphi, Y, \{x,, y\}\}$ on Y and let f: $(X, T) \rightarrow (Y, T^*)$ be a mapping defined as follows: $f(a) = z, f(b) = x$, and $f(c) = y$.

Then f is E_c- Continuous mapping but not P_c- Continuous mapping because $\{x,, y\}$ is an open set in Y but $f^{-1}(\{, y\}) = \{b, c\}$ which is not P_c-open in X.

Theorem 4.10: Let f: $(X, T) \to (Y, T^*)$ be a mapping whenever X is T_1 space. Then, f is E_c-(δ-$ß_c$)- Continuous mapping if and only if f is E(δ-$ß$)- Continuous mapping.

***Proof*:** ($\Longrightarrow$) Suppose that f is E-(δ-$ß$)- Continuous mapping whenever X is T_1-Sp, then for any open set V of Y we have, $f^{-1}(V)$ is E (*resp.* δ-$ß$)-open set in X, and via Theorem (3.7), $f^{-1}(V)$ is E_c(δ-$ß_c$)-open set. Therefore, f is E_c-(δ-$ß_c$)- Continuous mapping.

($\Longleftarrow$) Assume that f is E_c(δ-$ß_c$)-Continuous mapping, therefore via Corollary *(4.4-part(ii) and (viii))*, we have f is E-(δ-$ß$)-Continuous mapping.

Corollary 4.11: Let f: $(X, T) \to (Y, T^*)$ be a continuous mapping whenever X is T_1-Sp. Then, f is E_c-(δ-$ß_c$)- Continuous mapping.

***Proof*:** The proof is obvious, it is directly followed by Remark-(3.11).

Theorem 4.12: A mapping f: $(X, T) \to (Y, T^*)$ is E_c-(δ-$ß_c$)-Continuous if and only if f is E(δ-$ß$)- Continuous mapping and for each $x \in X$ and each open set V of Y (s. t) $f(x) \in V$, there exists a closed set $F \subseteq X$ containing x (s. t) $f(F) \subseteq V$.

***Proof*:** ($\Longrightarrow$) Let f: $(X, T) \to (Y, T^*)$ be E_c-(δ-$ß_c$)-Continuous mapping, then f is E(δ-$ß$)-Continuous, and we have $x \in X$, and V is any open set of Y containing $f(x)$. Then via E_c(δ-$ß_c$)-continuity, there exists E_c-(δ-$ß_c$)-open set $U \subseteq X$ containing x (s. t) $f(U) \subseteq V$. Since U is E_c-(δ-$ß_c$)-open set. Then for all $x \in U$, there exists a closed set F of X (s. t) $x \in F \subseteq U$. Therefore, $f(F) \subseteq f(U) \subseteq V$. Hence, we get $f(F) \subseteq V$, and E_c-(δ-$ß_c$)- Continuous implies E-(δ-$ß$)- Continuous.

($\Longleftarrow$) Let V be any open set of Y. We have to show that $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X. by hypothesis since f is E-(δ-$ß$)- Continuous, then $f^{-1}(V)$ E(δ-$ß$)-open set in X and for any $x \in f^{-1}(V)$ we have, $f(x) \in V$. By hypothesis, there exists a closed set F of X containing x such that $f(F) \subseteq V \Longrightarrow x \in F \subseteq f^{-1}(V)$.Therefore, via definition (3.2), $f^{-1}(V)$ is E_c(δ-$ß_c$)-open set in X . Thus, via Theorem (4.3), f is E_c-(δ-$ß_c$)-Continuous mapping.

Some characterizations concerning the E_c-(*resp.* δ-$ß_c$)-Continuous mapping are explained in the following Theorem .

Theorem 4.13: The following characterizations are equivalent for a mapping f: $(X, T) \to (Y, T^*)$:

i) f is E_c-(δ-$ß_c$)- Continuous mapping.

ii) $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X, for every open set V *of* Y.

iii) $f^{-1}(F)$ is E_c-(δ-$ß_c$)-closed set in X, for every closed set F of Y.

iv) $f(E_c\text{-}Cl(A))$ (*resp.* δ-$ß_c$-$Cl(A)$)) $\subseteq Cl(f(A))$(*resp.* $Cl(f(A))$), for every sub-set A of X.

v) $E_c\text{-}Cl(f^{-1}(B))$ (*resp.* δ-$ß_c$-$Cl(f^{-1}(B))$) $\subseteq f^{-1}(Cl(B))$ (*resp.* $f^{-1}(Cl(B))$), for every sub-set B of Y.

vi) $f^{-1}(Int(B)) \subseteq E_c\text{-}Int(f^{-1}(B))$ (*resp.* δ-$ß_c$-$Int(f^{-1}(B))$) for every sub-set B of Y.

vii) $Int(f(A)) \subseteq f(E_c\text{-}Int(A))$ (*resp.* δ-$ß_c$- $Int(A)$)) for every sub-set A *of* X.

***Proof*:** It is straightforward and obvious. Thus, it is omitted .

Theorem 4.14: If f: $(X, T) \to (Y, T^*)$ is a strongly θ-Continuous mapping, then f is E_c-(δ-$ß_c$)-Continuous mapping.

***Proof*:** Suppose that f is a strongly θ-Continuous mapping and V an open subset of Y, then from Definition 4.2-part (*v*), we have $f^{-1}(V)$ is θ-open set. Since every θ-open is E_c-[(δ-$ß_c$)-open set, hence via Theorem (3.12),we have $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X. Therefore, via Theorem-(4.3), f is E_c-(δ-$ß_c$)- Continuous mapping.

Theorem 4.15: If f: $(X, T) \to (Y, T^*)$ is contra continuous and E-(δ-$ß$)-Continuous mapping, then f is E_c-(δ-$ß_c$)- Continuous mapping.

***Proof*:** Assume that f is contra continuous and E(δ-$ß$)-Continuous mapping and consider V as open subset of Y, then by Definition 4.2-part (*i, ii, iv*), we have $f^{-1}(V)$ is closed and E(δ-$ß$)-

open set in X which implies, $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set ,thus f is E_c-(δ-$ß_c$)- Continuous mapping.

Theorem 4.16: Let f: $(X, T) \rightarrow (Y, T^*)$ be a perfectly (clopen)-Continuous mapping, then f is E_c-(δ-$ß_c$)-Continuous mapping.

***Proof*:** Assume that f is perfectly Continuous mapping and let V be an open subset of Y, then from Definition 4.2-part (*iii*), we have $f^{-1}(V)$ is clopen set in X which implies that $f^{-1}(V)$ is open set and so, E(δ-$ß$)-open set and $f^{-1}(V)$ is closed set. Thus, $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X and so, f is E_c-(δ-$ß_c$)-Continuous mapping.

Theorem 4.17: Let (X, T) *be* an Alexandroff. Sp. Then, a mapping f: $(X, T) \rightarrow (Y, T^*)$ *is* a contra continuous and E-(δ-$ß$)-Continuous mapping *iff* f is E_c-(δ-$ß_c$)-Continuous mapping.

***Proof*:** (⟹) Assume that (X, T) *is* an Alexandroff. Sp. and let f: $(X, T) \rightarrow (Y, T^*)$ is E_c-(δ-$ß_c$)-Continuous mapping. Then $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X, for every open set V of Y, which implies $f^{-1}(V)$ is E(δ-$ß$)-open set in X, and $f^{-1}(V)$ is a union of closed sets which is closed in an Alexandroff. Sp. (X, T). Therefore, f is E(δ-$ß$)-Continuous and contra continuous mapping.
(⟸) The converse part is immediately followed from Theorem (4.15).

Theorem 4.18: Let f: $(X, T) \rightarrow (Y, T^*)$ be RC-Continuous mapping, then f is (δ-$ß_c$)-continuous.

***Proof*:** suppose that f is RC- Continuous mapping and let V be an open subset of Y, then via definition 4.2-part (*vi*), we have $f^{-1}(V)$ is regular closed in X and via Theorem (3.14), we have $f^{-1}(V)$ is (δ-$ß_c$)-open set. So f is (δ-$ß_c$)-Continuous mapping.

Theorem 4.19: Let f: $(X, T) \rightarrow (Y, T^*)$ be a continuous mapping, (s. t) (X, T) is a regular-Sp. Then, f is E_c-(δ-$ß_c$)-Continuous mapping.

***Proof*:** Let f be Continuous mapping and (X, T) is regular space. Then $f^{-1}(V)$ is open set in a regular sp. X for any open sub-set V of Y and therefore, via Theorem (3.16), $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X. Hence f is E_c-(δ-$ß_c$)-Continuous mapping.

5. FUNDAMENTAL PROPERTIES OF E_c(δ-$ß_c$)-CONTINUOUS MAPPINGS

In this section, we recall several fundamental properties concerning the E_c-(*resp*. δ-$ß_c$)-Continuous mappings.

Theorem 5.1: Let f: $(X, T) \rightarrow (Y, T^*)$ be E_c-(δ-$ß_c$)-Continuous mapping (s. t),$Y \subseteq Z$. If (Y, T^*) is an open sub-space of the Top-Sp (Z, T^{**}), then f: $(X, T) \rightarrow (Z, T^{**})$ is E_c-(δ-$ß_c$)- Continuous mapping.

***Proof*:** let V be an open sub-set of Z. Then $V \cap Y$ is open sub-set of Y. Since f: $(X, T) \rightarrow (Y, T^*)$ is E_c-(δ-$ß_c$)-continuous, from Theorem (4.3), $f^{-1}(V \cap Y)$ is E_c-(δ-$ß_c$)-open sub-set of X. Since $f(x) \in Y$ for all $x \in X$, hence $f^{-1}(V) = f^{-1}(V \cap Y)$ is E_c-(δ-$ß_c$)-open sub-set of X. So via Theorem (4.3), f: $(X, T) \rightarrow (Z, T^{**})$ is E_c-(δ-$ß_c$)-Continuous mapping.

Theorem 5.2: The following properties are equivalent for a mapping f: $(X, T) \rightarrow (Y, T^*)$***:***

i) f is clopen- Continuous mapping.

ii) f is E_c-(δ-$ß_c$)- Continuous contra- Continuous mapping.

iii) f is E-(δ-$ß$)- Continuous contra-Continuous mapping.

***Proof*:** The proof followed from the Theorem (3.19) immediately**.**

In the following results, we recall some conditions in which the restrictions of E_c-(δ-$ß_c$)-continuous mappings on sub-Sp are E_c-(δ-$ß_c$)-Continuous.

Theorem 5.3: Let f: $(X, T) \rightarrow (Y, T^*)$ be E_c-(δ-$ß_c$)- Continuous mapping. If A is a clopen sub-set of X, then $f|A$: $A \rightarrow Y$ is E_c-(δ-$ß_c$)-Continuous the sub-space A.

***Proof*:** Let V be an open sub-set of Y. Then from Theorem-(4.3), we have $f^{-1}(V)$ is E_c-(δ-$ß_c$)-open sub-set of X. Since A is clopen subset of X, then via Corollary (3.23), $(f|A)^{-1}(V) = f^{-1}(V) \cap A$ is E_c(δ-$ß_c$)-open subset in the sub-Sp of A. This shows that, $f|A$: $A \rightarrow Y$ is E_c-(δ-$ß_c$)-Continuous the subspace A.

Theorem 5.4: A mapping f: $(X, T) \rightarrow (Y, T^*)$ is E_c-(δ-$ß_c$)-continuous. If for all $x \in X$, there exists a clopen set A of X containing x such that $f|A$: $A \rightarrow Y$ is E_c-(δ-$ß_c$)-continuous.

***Proof*:** suppose that for all $x \in X$, there exists a clopen set A of X containing x where $f|A: A \to Y$ is E_c-(δ-ß$_c$)- Continuous mapping. Let V be any open sub-set of Y containing $f(x)$, then there exists a E_c-(δ-ß$_c$)-open set U in A containing x (s. t), $f|A(U) \subseteq V$. Since A is clopen set, via Theorem (3.21), U is E_c-(δ-ß$_c$)-open set in X and hence $f(U) \subseteq V$. This shows that $f: (X, T) \to (Y, T^*)$ is E_c-(δ-ß$_c$)-continuous.

Theorem 5.5: Let $f: (X, T) \to (Y, T^*)$ be a mapping where $X = A \cup B$ (s. t) both A and B are clopen sets. If $f|A: A \to Y$ and $f|B: B \to Y$ are E_c-(δ-ß$_c$)- Continuous mappings, then f is E_c-(δ-ß$_c$)- Continuous mapping.

***Proof*:** Suppose that V is an open sub-set of Y. then $f^{-1}(V) = (f|A)^{-1}(V) \cup (f|B)^{-1}(V)$. Since $f|A$ and $f|B$ are E_c-(δ-ß$_c$)-continuous, so via Theorem-(4.3) $(f|A)^{-1}(V)$ and $(f|B)^{-1}(V)$ are E_c-(δ-ß$_c$)- open sets in A and B respectively. Since A and B are clopen sets in X, so via Theorem (3.21), $(f|A)^{-1}(V)$ and $(f|B)^{-1}(V)$ are E_c-(δ-ß$_c$)-open sets in X. Since the union of E_c-(δ-ß$_c$)-open sets is E_c-(δ-ß$_c$)-open set, thus $f^{-1}(V)$ is E_c-(δ-ß$_c$)-open set in X. Therefore, via Theorem-(4.3), f is E_c-(δ-ß$_c$)- Continuous mapping.

Theorem 5.6: Let $f, g: (X, T) \to (Y, T^*)$ be two mapping where (Y, T^*) is Hausdorff-sp. If f is E_c-(δ-ß$_c$)-Continuous g is perfectly (clopen) continuous, then the set $E = \{x \in X: f(x) = g(x)\}$ is E_c-(δ-ß$_c$)-closed in X.

***Proof*:** Let $x \notin E$. Then $f(x) \neq g(x)$, since (Y, T^*) is Hausdorff, so there exist disjoint open sets V_1 and V_2 of Y (s. t) $f(x) \in V_1$ and $g(x) \in V_2$. Since f is E_c-(δ-ß$_c$)-continuous, there exists E_c-(δ-ß$_c$)-open sets U_1 of X containing x such that $f(U_1) \subseteq V_1$. Since g is perfectly continuous, there exists clopen set U_2 of X containing x such that $g(U_2) \subseteq V_2$. Put $U = U_1 \cap U_2$ is E_c-(δ-ß$_c$)-open set of X containing x via Theorem (3.17). It follows that $U \cap E = \varphi$. So that, $U \subseteq X|E$ and hence, $X|E$ is E_c-(δ-ß$_c$)-open set Therefore E is E_c-(δ-ß$_c$)- closed set in X.

Corollary 5.7: Let $f, g: (X, T) \to (Y, T^*)$ be two mappings where (Y, T^*) is Urysohn space. If f is E_c-(δ-ß$_c$)-continuous and g is perfectly (clopen) continuous, then the set $E = \{x \in X: f(x) = g(x)\}$ is E_c-(δ-ß$_c$)-closed in X.

***Proof*:** The proof is obvious it is immediately followed from Proposition (2.17) and Theorem-(5.6).

Theorem 5.8: Let $f: (X_1, T) \to (Y, T^*)$ and $g: (X_2, T) \to (Y, T^*)$ be two E_c-(δ-ß$_c$)-Continuous mapping. If (Y, T^*) is Hausdorff space , then the set $E = \{(x_1, x_2) \in X_1 \times X_2: f(x_1) = g(x_2)\}$ is E_c-(δ-ß$_c$)-closed in $X_1 \times X_2$.

***Proof*:** Let $(x_1, x_2) \notin$ E. Then $f(x_1) \neq g(x_2)$, since (Y, T^*) is Hausdorff, so there exist disjoint open sets V_1 and V_2 of Y (s. t) $f(\mathbf{x}_1) \in V_1$ and $g(x_2) \in V_2$. Since f and g are E_c-(δ-ß$_c$)-Continuous mapping, so there exists E_c-(δ-ß$_c$)-open set U_1 and U_2 of X_1 and X_2 containing x_1 and x_2 (s. t) $f(U_1) \subseteq V_1$ and $f(U_2) \subseteq V_2$ respectively.

Put $U = U_1 \times U_2$, Thus via Theorem (3.24), $(x_1, x_2) \in U$ where U is E_c-(δ-ß$_c$)-open set in $X_1 \times X_2$ and It follows that $U \cap E = \varphi$. Therefore we obtain $U \subseteq (X_1 \times X_2) \setminus E$ and hence, $(X_1 \times X_2) \setminus E$ is E_c(δ-ß$_c$)-open set. Therefore E is E_c-(δ-ß$_c$)-closed set in $X_1 \times X_2$.

Corollary 5.9: Let $f: (X_1, T) \to (Y, T^*)$ and $g: (X_2, T) \to (Y, T^*)$ be two E_c-(δ-ß$_c$)- continuous mapping. If (Y, T^*) is Uresohn-Sp, then the set $E = \{(x_1, x_2) \in X_1 \times X_2: f(x_1) = g(x_2)\}$ is E_c-(δ-ß$_c$)-closed in $X_1 \times X_2$.

The proof is obvious, it is immediately followed from Proposition (2.17) and Theorem-(5.8).

Theorem 5.10: If $f_i: (X_i, T) \to (Y_i, T^*)$ is E_c-(δ-ß$_c$)-Continuous mapping for $i = 1,2$, and $f: X_1 \times X_2 \to Y_1 \times Y_2$ is a mapping defend as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f is E_c-(δ-ß$_c$)-continuous.

***Proof*:** Suppose that $R_1 \times R_2 \subseteq Y_1 \times Y_2$, (s. t) R_i is open sets in Y_i for $i = 1, 2$. Then,
$f^{-1}(R_1 \times R_2) = f_1^{-1}(R_1) \times f_2^{-1}(R_2)$. Since f_i is E_c-(δ-ß$_c$)- Continuous mappings for $i = 1,2$. Thus By Theorem (4. 3) and Theorem (3.24), we get $f^{-1}(R_1 \times R_2)$ is E_c-(δ-ß$_c$)-open set in $X_1 \times X_2$ Therefore f is E_c-(δ-ß$_c$)-Continuous mappings.

Definition 5.11: A mapping $f: (X, T) \to (Y, T^*)$ is said to be:

i) E_c-irresolute, if $f^{-1}(K)$ is E_c-open in X for every E_c-open subset K *of* Y.
ii) δ-$ß_c$-irresolute, if $f^{-1}(K)$ is δ-$ß_c$-open in X for every δ-$ß_c$-open subset K *of* Y.
iii) E_c-open, if, $f(K)$ is E_c-open Y for every open subset K *of* Y.
iv) δ-$ß_c$-open, if, $f(K)$ is δ-$ß_c$-open of Y for every open subset K *of* Y.

Theorem 5.12: The following properties hold for a mapping f: $(X, T) \to (Y, T^*)$ and g: $(Y, T^*) \to (Z, T^{**})$:
i) If f is E_c-(δ-$ß_c$)- Continuous g is continuous, then $g\ o\ f$ is E_c-(δ-$ß_c$)-Continuous.
ii) If f is continuous and g is perfectly (clopen) continuous, then $g\ o\ f$ is E_c-(δ-$ß_c$)-continuous.
iii) If f is E_c-(δ-$ß_c$)-irresolute and g is E_c-(δ-$ß_c$)-Continuous, then $g\ o\ f$ is E_c-(δ-$ß_c$)-Continuous.

***Proof*: (i) -** Assume that V is an open set in Z. Then $g^{-1}(V)$ is open in Y via continuity of g. Since f is E_c-(δ-$ß_c$)-Continuous, $f^{-1}(g^{-1}(V))$ is E_c-(δ-$ß_c$)-open set in X. Thus, $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is E_c-(δ-$ß_c$)-open set in X. Therefore, $g\ o\ f$ is E_c-(δ-$ß_c$)-Continuous.

(ii) - suppose that V is an open set in Z. Then $g^{-1}(V)$ is clopen in Y via perfect continuity of g. Since f is continuous, $f^{-1}(g^{-1}(V))$ is clopen in X . Thus, E_c-(δ-$ß_c$)-open via Theorem-(3.18), therefore $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is E_c-(δ-$ß_c$)-open in X. Hence, $g\ o\ f$ is E_c-(δ-$ß_c$)-Continuous.

(iii) – Let V be an open set in Z. Then $g^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in Y via E_c-(δ-$ß_c$)-continuity of g.
Since f is E_c-(δ-$ß_c$)-irresolute, $f^{-1}(g^{-1}(V))$ is E_c-(δ-$ß_c$)-open set in X and thus $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is E_c-(δ-$ß_c$)-open set in X. Hence, $g\ o\ f$ is E_c-(δ-$ß_c$)-Continuous.

Theorem 5.13: Let f: $(X, T) \to (Y, T^*)$ *and* g: $(Y, T^*) \to (Z, T^{**})$ be two mappings. If f is E_c-(δ-$ß_c$)-open surjective mapping and $g\ o\ f$ is E_c-(δ-$ß_c$)-Continuous, then g is E_c-(δ-$ß_c$)-Continuous.

***Proof*:** Let V be an open sub-set of Z. Since $g\ o\ f$ is E_c-(δ-$ß_c$)- Continuous, so $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is E_c-(δ-$ß_c$)-open set in X. Since f is E_c-(δ-$ß_c$)-open set and surjective, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in Y. Thus, g is E_c-(δ-$ß_c$)- Continuous.

Corollary 5.14: Let f: $(X, T) \to (Y, T^*)$ be E_c-(δ-$ß_c$)-irresolute and E_c-(δ-$ß_c$)-open set surjective mapping and let g: $(Y, T^*) \to (Z, T^{**})$ be mapping. Then $g\ o\ f$: $(X, T) \to (Z, T^{**})$ is E_c-(δ-$ß_c$)- Continuous *iff* g is E_c-(δ-$ß_c$)-Continuous.

***Proof*:** The proof follows from Theorems (5.12-part-3) and (5.13) immediately .

Theorem 5.15: Let f: $(X, T) \to (Y, T^*)$ be a mapping and let $G: X \to X\times Y$ be the graph mapping of f defined via, $G(x) = (x, f(x))$ where $G(f) = \{(x, f(x)): x \in X\}$. Then, G is E_c-(δ-$ß_c$)-Continuous *iff* f is E_c-(δ-$ß_c$)-Continuous.

***Proof*:** (⟹) Suppose that G is E_c-(δ-$ß_c$)- Continuous V is an open sub-set of Y, then $X\times V$ is open set in $X\times Y$. Since G is E_c-(δ-$ß_c$)- Continuous, so $G^{-1}(X\times V)$ is E_c-(δ-$ß_c$)-open in X. Since $G^{-1}(X\times V) = \{x \in X: G(x) = (x, f(x)) \in X\times V\} = \{x \in X: f(x) \in V\} = f^{-1}(V)$ is E_c-(δ-$ß_c$)-open set in X. Hence, via Theorem (4. 3), we have f is E_c-(δ-$ß_c$)- Continuous.

(⟸) Assume that f is E_c-(δ-$ß_c$)-Continuous let $x \in X$ where H is an open sub-set of $X\times Y$ containing $G(x)$. Since via Theorem (2.16), $\{x\} \times Y$ is homeomorphic to Y and $H \cap (\{x\}\times Y)$ is open in the sub-Sp $\{x\}\times Y$ containing $G(x)$, $\{y \in Y: (x, y) \in H\}$ is open sub-set Y. Since f is E_c-(δ-$ß_c$)-Continuous, so $f^{-1}(\{y: (x, y) \in H\})$ is E_c-(δ-$ß_c$)-open set in X. Since $f^{-1}(\{y: (x, y) \in H\}) = \bigcup\{ f^{-1}(y) : (x, y) \in H\}$ is E_c-(δ-$ß_c$)-open set in X and $x \in \bigcup \{ f^{-1}(y) : (x, y) \in H\} \subseteq G^{-1}(H)$. Therefore, $G^{-1}(H)$ is E_c-(δ-$ß_c$)-open set in X. Hence, by Theorem (4. 3), we have G is E_c-(δ-$ß_c$)-Continuous mapping.

Theorem 5.16: Let f: $(X, T) \to (Y, T^*)$ be E_c-(δ-$ß_c$)- Continuous mappings (Y, T^*) is Hausdorff-sp., then $G(f) = \{(x, f(x)): x \in X\}$ is E_c-(δ-$ß_c$)-closed set in $X\times Y$.

***Proof*:** Let $(x, y) \notin G(f)$.Then $y \neq f(x)$, since (Y, T^*) is Hausdorff, so there exist disjoint open sets V and H of Y such that $f(x) \in H$ and $y \in V$. Since f is E_c-(δ-$ß_c$)-Continuous mappings, so there exists E_c-(δ-$ß_c$)-open set U in X containing x where $f(U) \subseteq H$. Thus $(x, y) \in U\times V \subseteq X\times Y$ (s. t) $U\times V$ is E_c(δ-$ß_c$)-open set because U is E_c(δ-$ß_c$)-open and V is open set in Hausdorff and thus, via Theorem-(3.7), V is E_c-(δ-$ß_c$)-open set. So $X\times Y \setminus G(f)$ is E_c-(δ-$ß_c$)-open set and hence, $G(f)$ is E_c-(δ-$ß_c$)- closed set in $X\times Y$.

CONCLUSIONS

It is well-known that the branch of mathematics called topology is related to all questions directly or indirectly concerned with continuity. Therefore, the generalization of continuity is one of the most important subjects in topology. One of the most important subjects in studying topology and physics is continuity, which has been researched and investigated by many mathematicians and quantum physicists . One can observe the influence made in the realms of applied research by general topological spaces, properties and structures. In digital topology, information systems, particle physic, computational topology for geometric design and molecular design. Thus we study new notions of generalized continuous mappings, called E_c-Continuous and δ-$ß_c$- continuous mappings which may have very important applications in quantum particle physics and theoretical Physics, particularly in connections with string theory. As well as the fuzzy topological version of the concepts and results are introduced in this paper. They are very important due to the work of El-Naschie . He has shown that the notion of fuzzy topology has very important applications in quantum particle physics, especially in relation to both string and ε^{∞} theory .

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