Ekram and Majeed *Iraqi Journal of Science, 2021, Vol. 62, No. 8, pp: 2676-2684*
DOI: 10.24996/ijs.2021.62.8.20

ISSN: 0067-2904

Soft Continuous Mappings in Soft Closure Spaces

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Received: 5/8/2020 Accepted: 9/11/2020

Abstract
Soft closure spaces are a new structure that was introduced very recently. These new spaces are based on the notion of soft closure operators. This work aims to provide applications of soft closure operators. We introduce the concept of soft continuous mappings and soft closed (resp. open) mappings, support them with examples, and investigate some of their properties.

Keywords: Soft closure operator, soft closure space, closed soft set, product of soft closure spaces, soft continuous mapping, soft closed mapping, soft projection mapping.

التطبيقات المستمرة الناعمة في فضاءات الاغلاق الناعمة

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الخلاصة
فضاءات الاغلاق الناعمة هي بنية جديدة تم تقديمها مؤخرا. تستند هذه الفضاءات الجديدة على مفهوم مؤثرات الاغلاق الناعمة. يهدف هذا البحث الى إعطاء تطبيقات لمؤثرات الاغلاق الناعمة. قدمنا مفهوم التطبيقات المستمرة الناعمة والتطبيقات المغلقة (المفتوحة على التوالي) الناعمة، وتم دعم هذه التطبيقات بالامثلة، و البحث في بعض خصائصها.

1.Introduction

The concept of soft sets was first introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with uncertain objects. Soft set theory has been applied in many directions, e.g., stability and regularization [1], game theory and operations research [1], soft analysis [1], group theory [2], general topology [3], etc. Moreover, in the structure of closure spaces, Gowri and Jegadeesan [4] and Krishnaveni and Sekar [5] introduced and studied $\check{C}$ech soft closure spaces. In the classical soft set theory, because of the fuzzy existence of the parameters, a condition can be complicated in the real world. In this respect, classical $\check{C}$ech soft closure spaces were expanded to $\check{C}$ech soft closure spaces [6, 7, 8]. Recently, Ekram and Majeed introduced the notion of soft closure spaces [9] as an expansion to this concept in the ordinary case of the set theory that was introduced by $\check{C}$ech [10].

Continuity is an important notion in general topology, soft topology, and closure spaces as well as all branches of mathematics and quantum physics. Kharal and Ahmad [11] presented the concept of a mapping on the classes of soft sets that is a central notion for the advancement of every new field of mathematical science. An idea of soft mapping was presented and some of its properties were studied

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in [12]. Boonpok [13] defined and studied the concept of continuity in closure spaces. Our work in the present paper is dedicated to presenting the concept of soft continuity in soft closure spaces. In Section 3, we introduce the concept of soft continuous and study some of their properties. Also, the notion of soft closed (resp. open) mappings is introduced. In Section 4, several properties and characterizations related to soft projection mappings, closed (resp. open) soft sets, and soft continuous (resp. soft closed) mappings in the product of soft closure spaces are discussed.

2.Preliminaries

In this section, we introduce the basic definitions and results of soft set theory and soft closure spaces that will be needed in the sequel.

Definition 2.1[1]. A soft set $\mathcal{F}_R = (\mathcal{F}, R)$ over the universe set $\mathcal{M}$ is defined by a mapping $\mathcal{F}: R \to P(\mathcal{M})$. Then, $\mathcal{F}_R$ can be represented by the set $\mathcal{F}_R = \{(r, \mathcal{F}(r)): r \in R \text{ and } \mathcal{F}(r) \in P(\mathcal{M})\}$. We denote the family of all soft sets over $\mathcal{M}$ by $\mathcal{SS}(\mathcal{M}, R)$.

Definition 2.2 [14]. A null soft set, denoted by $\widetilde{\Phi}_R$, is a soft set $\mathcal{F}_R$ over $\mathcal{M}$ such that for all $r \in R$, $\mathcal{F}(R) = \emptyset$ (empty set).

Definition 2.3 [14]. An absolute soft set, denoted by $\widetilde{\mathcal{M}}$, is a soft set $\mathcal{F}_R$ over $\mathcal{M}$ such that for all $r \in R, \mathcal{F}(r) = \mathcal{M}$.

Definition 2.4 [15]. Let $\mathcal{F}_R$ and G_R be two soft sets over $\mathcal{M}$. Then, $\mathcal{F}_R$ is called a soft subset of G_R, denoted by $\mathcal{F}_R \sqsubseteq G_R$, if $\mathcal{F}(r) \subseteq G(r)$ for all $r \in R$. $\mathcal{F}_R$ equals G_R, denoted by $\mathcal{F}_R = G_R$, if $\mathcal{F}_R \sqsubseteq G_R$ and $G_R \sqsubseteq \mathcal{F}_R$.

Definition 2.5 [14]. The union of two soft sets $\mathcal{F}_R$ and G_R over $\mathcal{M}$ is the soft set $\mathcal{H}_R$ defined as b $\mathcal{H}(r) = \mathcal{F}(r) \cup G(r)$ for all $r \in R$. This is denoted by $\mathcal{F}_R \sqcup G_R$. Also, the soft intersection of $\mathcal{F}_R$ and G_R is the soft set $\mathcal{H}_R$ given by $\mathcal{H}(r) = \mathcal{F}(r) \cap G(r)$ for all $r \in R$ and denoted by $\mathcal{F}_R \sqcap G_R$.

Definition 2.6 [3]. Let $\mathcal{F}_R$ and G_R be two soft sets over $\mathcal{M}$, the difference $\mathcal{H}_R$ of $\mathcal{F}_R$ and G_R is denoted by $\mathcal{F}_R - G_R$, and defined as $\mathcal{H}(r) = \mathcal{F}(r) - G(r)$ for all $r \in R$.

Definition 2.7 [3]. The relative complement of a soft set $\mathcal{F}_R$ is denoted by $\mathcal{F}_R^c$, where $\mathcal{F}^c: R \longrightarrow P(\mathcal{M})$ defined as $\mathcal{F}^c(r) = \mathcal{M} - \mathcal{F}(r)$, for all $r \in R$. Clearly, $\mathcal{F}_R^c = \widetilde{\mathcal{M}} - \mathcal{F}_R$.

Definition 2.8 [16]. The soft set $\mathcal{F}_R \in \mathcal{SS}(\mathcal{M}, R)$ is called soft point in $\mathcal{M}$, denoted by x_r, if for the element $r \in R$, we have $\mathcal{F}(r) = \{x\}$ and $\mathcal{F}(r') = \emptyset$ for every $r' \in R - \{r\}$.

Definition 2.9 [16]. The soft point x_r is said to be in the soft set G_R, denoted by $x_r \widetilde{\in} G_R$, if for the element $r \in R$, we have $\{x\} \subseteq G(r)$.

Definition 2.10 [11]. Let $\mathcal{SS}(\mathcal{M}, R)$ and $\mathcal{SS}(\mathcal{N}, \mathcal{K})$ be families of soft sets, where $\psi: \mathcal{M} \longrightarrow \mathcal{N}$ and $\ell: R \longrightarrow \mathcal{K}$ are mappings. The mapping $\psi_\ell: \mathcal{SS}(\mathcal{M}, R) \longrightarrow \mathcal{SS}(\mathcal{N}, \mathcal{K})$ is defined as:

1- If $\mathcal{F}_R \in \mathcal{SS}(\mathcal{M}, R)$, then the image of $\mathcal{F}_R$ under ψ_ℓ, written as $\psi_\ell(\mathcal{F}_R) = \left(\psi_\ell(\mathcal{F}_R), \ell(R)\right)$, is a soft set in $\mathcal{SS}(\mathcal{N}, \mathcal{K})$ such that:

$$\psi_\ell(\mathcal{F}_R)(k) = \begin{cases} \psi(\cup_{r \in \ell^{-1}(k) \cap R} \mathcal{F}(r)), & if\ r \in \ell^{-1}(k) \cap R \neq \emptyset; \\ \emptyset, & \text{otherwise.} \end{cases}$$

2- If $G_\mathcal{K} \in \mathcal{SS}(\mathcal{N}, \mathcal{K})$, then the pre-image of $G_\mathcal{K}$ under ψ_ℓ, written as $\psi_\ell^{-1}(G_\mathcal{K}) = \left(\psi_\ell^{-1}(G_\mathcal{K}), \ell^{-1}(\mathcal{K})\right)$, is a soft set in $\mathcal{SS}(\mathcal{M}, R)$ such that

$$\psi_\ell^{-1}(G_\mathcal{K})(r) = \begin{cases} \psi^{-1}\left(G(\ell(r))\right), & if\ \ell(r) \in \mathcal{K}; \\ \emptyset, & otherwise. \end{cases}$$

Definition 2.11 [17]. Let $\mathcal{F}_R \in \mathcal{SS}(\mathcal{M}, R)$ and $G_\mathcal{K} \in \mathcal{SS}(\mathcal{N}, \mathcal{K})$. The Cartesian product $\mathcal{F}_R \times G_\mathcal{K}$ is defined by $(\mathcal{F} \times G)_{R \times \mathcal{K}}$ where

$(\mathcal{F} \times G)_{R \times \mathcal{K}}(r, k) = \mathcal{F}(r) \times G(k)$, for all $(r, k) \in R \times \mathcal{K}$.

From this definition, the soft set $\mathcal{F}_R \times G_\mathcal{K}$ is a soft set over $\mathcal{M} \times \mathcal{N}$ and its universe parameter is $R \times \mathcal{K}$.

The pairs of projections $p_\mathcal{M}: \mathcal{M} \times \mathcal{N} \longrightarrow \mathcal{M}$, $q_R: R \times \mathcal{K} \longrightarrow R$ and $p_\mathcal{N}: \mathcal{M} \times \mathcal{N} \longrightarrow \mathcal{N}$, $q_S: R \times \mathcal{K} \longrightarrow \mathcal{K}$ determine, respectively, the morphisms $(p_\mathcal{M}, q_R)$ from $\mathcal{M} \times \mathcal{N}$ to $\mathcal{M}$ and $(p_\mathcal{N}, q_\mathcal{K})$ from $\mathcal{M} \times \mathcal{N}$ to $\mathcal{N}$, where

$(p_\mathcal{M}, q_R)(\mathcal{F}_R \times G_\mathcal{K}) = p_\mathcal{M}(\mathcal{F} \times G)_{q_R(R \times \mathcal{K})}$ and

$(p_\mathcal{N}, q_\mathcal{K})(\mathcal{F}_R \times G_\mathcal{K}) = p_\mathcal{N}(\mathcal{F} \times G)_{q_S(R \times \mathcal{K})}$ [18].

Definition 2.12 [9]. An operator $\tilde{u}: \mathcal{SS}(\mathcal{M}, R) \longrightarrow \mathcal{SS}(\mathcal{M}, R)$ is called a soft closure operator (soft- co, for short) on $\mathcal{M}$, if for all $\mathcal{F}_R, G_R \in \mathcal{SS}(\mathcal{M}, R)$ the following axioms are satisfied:
($\mathcal{C}1$) $\widetilde{\Phi}_R = \tilde{u}(\widetilde{\Phi}_R)$;
($\mathcal{C}2$) $\mathcal{F}_R \sqsubseteq \tilde{u}(\mathcal{F}_R)$;
($\mathcal{C}3$) $\mathcal{F}_R \sqsubseteq G_R \Longrightarrow \tilde{u}(\mathcal{F}_R) \sqsubseteq \tilde{u}(G_R)$.
The triple $(\mathcal{M}, \tilde{u}, R)$ is called a soft closure space (soft-cs, for short). A soft subset $\mathcal{F}_R$ over $\mathcal{M}$ is said to be a closed soft set, if $\mathcal{F}_R = \tilde{u}(\mathcal{F}_R)$. A soft subset G_R over $\mathcal{M}$ is called an open soft set if it is a relative complement $\widetilde{\mathcal{M}} - \mathcal{F}_R$ and is a closed soft set.
Definition 2.13 [9]. Let $(\mathcal{M}, \tilde{u}, R)$ be a soft-cs and let $\mathcal{Y} \subseteq \mathcal{M}$. Let $\tilde{u}_{\mathcal{Y}}: \mathcal{SS}(\mathcal{Y}, R) \longrightarrow \mathcal{SS}(\mathcal{Y}, R)$ defined by $\tilde{u}_{\mathcal{Y}}(\mathcal{F}_R) = \widetilde{\mathcal{Y}} \sqcap \tilde{u}(\mathcal{F}_R)$. Then, $\tilde{u}_{\mathcal{Y}}$ is called the relative soft closure operator on $\mathcal{Y}$ induced by $\tilde{u}$. The triple $(\mathcal{Y}, \tilde{u}_{\mathcal{Y}}, R)$ is called a soft closure subspace (soft-c.subsp, for short) of $(\mathcal{M}, \tilde{u}, R)$.
Theorem 2.14 [9]. Let $\{(\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha): \alpha \in \mathcal{I}\}$ be a family of soft-cs's. If $G_{\prod_{\alpha\in\mathcal{I}} R_\alpha}$ is an open soft set in the product soft-cs $(\prod_{\alpha\in\mathcal{I}} \mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}} R_\alpha)$, then $\left(p_{\mathcal{M}_\alpha}, q_{R_\alpha}\right)\left(G_{\prod_{\alpha\in\mathcal{I}} R_\alpha}\right)$ is an open soft set in $(\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha)$ for all $\alpha \in \mathcal{I}$.

3.Soft continuous mappings

In this section, we introduce the concept of soft continuous (resp. soft closed) mappings between soft closure spaces, with some examples to explain these notions. Also, some properties related to these concepts are given.

Definition 3.1. Let $(\mathcal{M}, \tilde{u}, R)$ and $(\mathcal{N}, \tilde{v}, \mathcal{K})$ be soft-cs's. A soft mapping $\psi_\ell: (\mathcal{M}, \tilde{u}, R) \to (\mathcal{N}, \tilde{v}, \mathcal{K})$ is said to be soft continuous, if $\psi_\ell(\tilde{u}(\mathcal{F}_R)) \sqsubseteq \tilde{v}\ (\psi_\ell(\mathcal{F}_R))$ for every soft set $\mathcal{F}_R \in \mathcal{SS}(\mathcal{M}, R)$.
In the next paragraphs, two examples are introduced to explain Definition 3.1.

Example 3.2. Let $\mathcal{M} = \{a, b, c\}, R = \{r_1, r_2, r_3\}$ and $= \{x, y, z\}$, $\mathcal{K} = \{k_1, k_2\}$. Let $\tilde{u}: \mathcal{SS}(\mathcal{M}, R) \longrightarrow \mathcal{SS}(\mathcal{M}, R)$ and $\tilde{v}: \mathcal{SS}(\mathcal{N}, \mathcal{K}) \longrightarrow \mathcal{SS}(\mathcal{N}, \mathcal{K})$ be soft-co's defined as follows:

$$\tilde{u}(\mathcal{F}_R) = \begin{cases} \widetilde{\Phi}_R & if\ \mathcal{F}_R \sqsubseteq \widetilde{\Phi}_R, \\ \{(r_1, \{a\}), (r_2, \{b\})\} & if\ \mathcal{F}_R \sqsubseteq \{(r_1, \{a\}, (r_2, \{b\})\}, \\ \widetilde{\mathcal{M}} & otherwise. \end{cases}$$

$$\tilde{v}(G_{\mathcal{K}}) = \begin{cases} \widetilde{\Phi}_{\mathcal{K}} & if G_{\mathcal{K}} \sqsubseteq \widetilde{\Phi}_{\mathcal{K}}, \\ \{(k_1, \{x, y\}), (k_2, \{x, y\})\} & if\ G_{\mathcal{K}} \sqsubseteq \{(k_1, \{x, y\}), (k_2, \{x, y\})\}, \\ \widetilde{\mathcal{N}} & otherwise. \end{cases}$$

Clearly, $(\mathcal{M}, \tilde{u}, R)$ and $(\mathcal{N}, \tilde{v}, \mathcal{K})$ are soft-cs's. Then, the soft mapping $\psi_\ell: (\mathcal{M}, \tilde{u}, R) \to (\mathcal{N}, \tilde{v}, \mathcal{K})$ defined by $\psi(a) = x, \psi(b) = y,\ \psi(c) = z$ and $\ell(r_1) = k_1$, $\ell(r_2) = \ell(r_3) = k_2$ is a soft continuous mapping.

Example 3.3. Let $\mathcal{M} = \{a, b, c\}, R = \{r_1, r_2\}$ and $= \{x, y, z\}$, $\mathcal{K} = \{k_1, k_2\}$. Let $\tilde{u}: \mathcal{SS}(\mathcal{M}, R) \longrightarrow \mathcal{SS}(\mathcal{M}, R)$ and $\tilde{v}: \mathcal{SS}(\mathcal{N}, \mathcal{K}) \longrightarrow \mathcal{SS}(\mathcal{N}, \mathcal{K})$ be soft-co's defined as follows:

$$\tilde{u}(\mathcal{F}_R) = \begin{cases} \widetilde{\Phi}_R & if\ \mathcal{F}_R = \widetilde{\Phi}_R, \\ \{(r_1, \{b\})\} & if\ \mathcal{F}_R = \{(r_1, \{b\})\}, \\ \widetilde{\mathcal{M}} & otherwise. \end{cases}$$

$$\tilde{v}(G_{\mathcal{K}}) = \begin{cases} \widetilde{\Phi}_{\mathcal{K}} & if G_{\mathcal{K}} = \widetilde{\Phi}_{\mathcal{K}}, \\ \{(k_1, \{x, y\})\} & if\ G_{\mathcal{K}} = \{(k_1, \{x\})\}, \\ \{(k_1, \{y\})\} & if\ G_{\mathcal{K}} = \{(k_1, \{y\})\}, \\ \widetilde{\mathcal{N}} & otherwise. \end{cases}$$

Clearly, $(\mathcal{M}, \tilde{u}, R)$ and $(\mathcal{N}, \tilde{v}, \mathcal{K})$ are soft-cs's. Then, the soft mapping $\psi_\ell: (\mathcal{M}, \tilde{u}, R) \to (\mathcal{N}, \tilde{v}, \mathcal{K})$ defined by $\psi(a) = x, \psi(b) = z,\ \psi(c) = y$ and $\ell(r_1) = k_2$, $\ell(r_2) = k_1$ is not soft continuous mapping. Since there exists a soft set $\mathcal{F}_R = \{(r_2, \{c\})\} \in \mathcal{SS}(\mathcal{M}, R)$, such that

$\psi_\ell(\tilde{u}(\mathcal{F}_R)) = \widetilde{\mathcal{N}} \not\sqsubseteq \{(k_1,\{y\})\} = \tilde{v}(\psi_\ell(\mathcal{F}_R))$.

Proposition 3.4. Let $(\mathcal{M},\tilde{u},R)$ and $(\mathcal{N},\tilde{v},\mathcal{K})$ be soft-cs's. If $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ is soft continuous mapping, then $\tilde{u}(\psi_\ell^{-1}(G_\mathcal{K})) \sqsubseteq \psi_\ell^{-1}(\tilde{v}(G_\mathcal{K}))$ for every soft set $G_\mathcal{K} \in \mathcal{SS}(\mathcal{N},\mathcal{K})$.

Proof. Let $G_\mathcal{K} \in \mathcal{SS}(\mathcal{N},\mathcal{K})$. Then, $\psi_\ell^{-1}(G_\mathcal{K}) \in \mathcal{SS}(\mathcal{M},R)$. From the hypothesis, we obtain $\psi_\ell(\tilde{u}(\psi_\ell^{-1}(G_\mathcal{K})) \sqsubseteq \tilde{v}(\psi_\ell(\psi_\ell^{-1}(G_\mathcal{K}))) \sqsubseteq \tilde{v}(G_\mathcal{K})$. Consequently, by taking the inverse image we get, $\psi_\ell^{-1}(\psi_\ell(\tilde{u}(\psi_\ell^{-1}(G_\mathcal{K}))) \sqsubseteq \psi_\ell^{-1}(\tilde{v}(G_\mathcal{K}))$. Hence, $\tilde{u}(\psi_\ell^{-1}(G_\mathcal{K})) \sqsubseteq \psi_\ell^{-1}(\tilde{v}(G_\mathcal{K}))$. □

Proposition 3.5. If $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ is a soft continuous mapping, then $\psi_\ell^{-1}(G_\mathcal{K})$ is a closed soft set of $(\mathcal{M},\tilde{u},R)$ for every closed soft set $G_\mathcal{K}$ of $(\mathcal{N},\tilde{v},\mathcal{K})$.

Proof: Let $G_\mathcal{K}$ be a closed soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$. From ψ_ℓ is a soft continuous mapping and by Proposition 3.4, we have $\tilde{u}\left(\psi_\ell^{-1}(G_\mathcal{K})\right) \sqsubseteq \psi_\ell^{-1}(\tilde{v}(G_\mathcal{K}))$. Since $G_\mathcal{K}$ is a closed soft set, $\tilde{u}\left(\psi_\ell^{-1}(G_\mathcal{K})\right) \sqsubseteq \psi_\ell^{-1}(G_\mathcal{K})$ and from $(C2)$ of Definition 2.12, we obtain $\tilde{u}\left(\psi_\ell^{-1}(G_\mathcal{K})\right) = \psi_\ell^{-1}(G_\mathcal{K})$. Therefore, $\psi_\ell^{-1}(G_\mathcal{K})$ is a closed soft set of $(\mathcal{M},\tilde{u},R)$.

Corollary 3.6. If $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ is a soft continuous mapping, then $\psi_\ell^{-1}(G_\mathcal{K})$ is an open soft set of $(\mathcal{M},\tilde{u},R)$ for every open soft set $G_\mathcal{K}$of $(\mathcal{N},\tilde{v},\mathcal{K})$.

Remark 3.7. The converse of Proposition 3.5 and Corollary 3.6 may not be true.

Example 3.8. Let $\mathcal{M} = \{a,b,c\}, R = \{r_1,r_2\}$ and $= \{x,y,z\}$, $\mathcal{K} = \{k_1,k_2\}$. Let $\tilde{u}:\mathcal{SS}(\mathcal{M},R) \longrightarrow \mathcal{SS}(\mathcal{M},R)$ and $\tilde{v}:\mathcal{SS}(\mathcal{N},\mathcal{K}) \longrightarrow \mathcal{SS}(\mathcal{N},\mathcal{K})$ be soft-co's defined as follows:

$$\tilde{u}(\mathcal{F}_R) = \begin{cases} \widetilde{\Phi}_R & if\ \mathcal{F}_R = \widetilde{\Phi}_R, \\ \{(r_{1,}\{c\})\} & if\ \mathcal{F}_R = \{(r_{1,}\{c\}\}, \\ \{(r_{2,}\{a,b\})\} & if\ \mathcal{F}_R = \{(r_{2,}\{a\})\}, \\ \widetilde{\mathcal{M}} & otherwise. \end{cases}$$

$$\tilde{v}(G_\mathcal{K}) = \begin{cases} \widetilde{\Phi}_\mathcal{K} & if G_\mathcal{K} = \widetilde{\Phi}_\mathcal{K}, \\ \{(k_1,\{y\})\} & if\ G_\mathcal{K} = \{(k_1,\{y\})\}, \\ \{(k_2,\{x,y\})\} & if\ G_\mathcal{K} = \{(k_2,\{x\})\}, \\ \widetilde{\mathcal{N}} & otherwise. \end{cases}$$

Clearly, $(\mathcal{M},\tilde{u},R)$ and $(\mathcal{N},\tilde{v},\mathcal{K})$ are soft-cs's. Let $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ be a soft mapping defined by $\psi(a) = x, \psi(b) = z,\ \psi(c) = y$ and $\ell(r_1) = k_1$, $\ell(r_2) = k_2$. Then, it is clear that, for any closed soft set $G_\mathcal{K}$ of $(\mathcal{N},\tilde{v},\mathcal{K})$, $\psi_\ell^{-1}(G_K)$ is a closed soft set in $(\mathcal{M},\tilde{u},R)$. However, there exists a soft set $\mathcal{F}_R = \{(r_{2,}\{a\})\} \in \mathcal{SS}(\mathcal{M},R)$ such that $\psi_\ell(\tilde{u}(\mathcal{F}_R)) = \{(k_2,\{x,z\})\} \not\sqsubseteq \{(k_2,\{x,y\})\} = \tilde{v}(\psi_\ell(\mathcal{F}_R))$. Hence, ψ_ℓ is not soft continuous.

Proposition 3.9. Let $(\mathcal{M},\tilde{u},R)$, $(\mathcal{N},\tilde{v},\mathcal{K})$, and $(\mathcal{Z},\widetilde{w},\mathcal{Q})$ be soft-cs's. If $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ and $\varphi_q:(\mathcal{N},\tilde{v},\mathcal{K}) \to (\mathcal{Z},\widetilde{w},\mathcal{Q})$ are soft continuous mappings, then $\varphi_q o\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{Z},\widetilde{w},\mathcal{Q})$ is a soft continuous mapping.

Proof: Let $\mathcal{F}_R \in \mathcal{SS}(\mathcal{M},R)$. By the definition of composition $\varphi_q o\psi_\ell(\tilde{u}(\mathcal{F}_R)) = \varphi_q(\psi_\ell(\tilde{u}(\mathcal{F}_R))$ and since ψ_ℓ is soft continuous, then it follows that $\varphi_q(\psi_\ell((\tilde{u}(\mathcal{F}_R)) \sqsubseteq \varphi_q(\tilde{v}(\psi_\ell(\mathcal{F}_R))$. As φ_q is soft continuous, we get $\varphi_q(\tilde{v}(\psi_\ell(\mathcal{F}_R)) \sqsubseteq \widetilde{w}(\varphi_q(\psi_\ell(\mathcal{F}_R))$. Consequently, $\varphi_q o\psi_\ell(\tilde{u}(\mathcal{F}_R)) \sqsubseteq \widetilde{w}(\varphi_q o\psi_\ell(\mathcal{F}_R))$.

Hence, $\varphi_q o\psi_\ell$ is a soft continuous mapping. □

Proposition 3.10. Let $(\mathcal{M},\tilde{u},R)$ and $(\mathcal{N},\tilde{v},\mathcal{K})$ be soft-cs's and let $(\mathcal{Y},\tilde{u}_\mathcal{Y},R)$ be a closed soft-c.subsp of $(\mathcal{M},\tilde{u},R)$. If $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ is soft continuous mapping, then the restriction mapping $\psi_\ell|_\mathcal{Y}:(\mathcal{Y},\tilde{u}_\mathcal{Y},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ is soft continuous mapping.

Proof: We must prove that, for all $\mathcal{F}_R \in \mathcal{SS}(\mathcal{Y},\tilde{u}_\mathcal{Y},R)$, $\psi_\ell|_\mathcal{Y}(\tilde{u}_\mathcal{Y}(\mathcal{F}_R)) \sqsubseteq \tilde{v}(\psi_\ell|_\mathcal{Y}(\mathcal{F}_R))$. Now,

$\psi_\ell|_\mathcal{Y}\left(\tilde{u}_\mathcal{Y}(\mathcal{F}_R)\right) = \psi_\ell|_\mathcal{Y}\left(\widetilde{\mathcal{Y}} \sqcap \tilde{u}(\mathcal{F}_R)\right)$ (by definition of $\tilde{u}_\mathcal{Y}$)

$= \psi_\ell|_\mathcal{Y}(\tilde{u}(\mathcal{F}_R))$ (by $(C2\ )\mathcal{F}_R \sqsubseteq \widetilde{\mathcal{Y}} \Rightarrow \tilde{u}(\mathcal{F}_R) \sqsubseteq \tilde{u}(\widetilde{\mathcal{Y}}) = \widetilde{\mathcal{Y}}$)

$= \psi_\ell(\tilde{u}(\mathcal{F}_R))$ (by $\tilde{u}(\mathcal{F}_R) \sqsubseteq \widetilde{\mathcal{Y}}$)

$= \tilde{v}(\psi_\ell(G_R))$ (by ψ_ℓ is soft continuous).

Hence, $\psi_\ell|_{\mathcal{Y}}$ is a soft continuous mapping. □

Definition 3.11. Let $(\mathcal{M},\tilde{u},R)$ and $(\mathcal{N},\tilde{v},\mathcal{K})$ be soft-cs's. A soft mapping $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ is said to be soft closed (resp. open), if $\psi_\ell(\mathcal{F}_R)$ is closed (resp. open) soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$ whenever $\mathcal{F}_R$ is closed (resp. open) soft set of $(\mathcal{M},\tilde{u},R)$.

Example 3.12 Let $\mathcal{M} = \{a,b,c\}, R = \{r_1,r_2\}$ and $\mathcal{N} = \{x,y,z\}, \mathcal{K} = \{k_1,k_2\}$. Let $\tilde{u}: \mathcal{SS}(\mathcal{M},R) \longrightarrow \mathcal{SS}(\mathcal{M},R)$ and $\tilde{v}: \mathcal{SS}(\mathcal{N},\mathcal{K}) \longrightarrow \mathcal{SS}(\mathcal{N},\mathcal{K})$ be soft-co's defined as follows:

$$\tilde{u}(\mathcal{F}_R) = \begin{cases} \widetilde{\Phi}_R & if\ \mathcal{F}_R = \widetilde{\Phi}_R, \\ \{(r_1,\{b,c\}),(r_2,\{b,c\})\} & if\ \mathcal{F}_R \sqsubseteq \{(r_1,\{b,c\}),(r_2,\{b,c\})\}, \\ \widetilde{\mathcal{M}} & otherwise. \end{cases}$$

$$\tilde{v}(G_{\mathcal{K}}) = \begin{cases} \widetilde{\Phi}_{\mathcal{K}} & if\, G_{\mathcal{K}} = \widetilde{\Phi}_{\mathcal{K}}, \\ \{(k_1,\{y,z\}),(k_2,\{y,z\})\} & if\ G_{\mathcal{K}} \sqsubseteq \{(k_1,\{y,z\}),(k_2,\{y,z\})\}, \\ \widetilde{\mathcal{N}} & otherwise. \end{cases}$$

Clearly, $(\mathcal{M},\tilde{u},R)$ and $(\mathcal{N},\tilde{v},\mathcal{K})$ are soft-cs's. Then, the soft mapping $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ defined by $\psi(a) = \psi(b) = \psi(c) = x$ and $\ell(r_1) = k_1$, $\ell(r_2) = k_2$ is a soft open mapping. But it is not soft closed mapping since there exists a closed soft set $\mathcal{F}_R = \{(r_1,\{b,c\}),(r_2,\{b,c\})\}$, but $\psi_\ell(\mathcal{F}_R) = \{(k_1,\{x\}),(k_2,\{x\})\}$ is not closed soft set in $(\mathcal{N},\tilde{v},\mathcal{K})$.

Proposition 3.13. Let $(\mathcal{M},\tilde{u},R)$ and $(\mathcal{N},\tilde{v},\mathcal{K})$ be soft-cs's. Let ψ_ℓ be a soft mapping from $(\mathcal{M},\tilde{u},R)$ onto $(\mathcal{N},\tilde{v},\mathcal{K})$. Then ψ_ℓ is soft closed if and only if, for each soft set $\mathcal{F}_{\mathcal{K}} \in \mathcal{SS}(\mathcal{N},\mathcal{K})$ and each open soft set G_R of $(\mathcal{M},\tilde{u},R)$ containing $\psi_\ell^{-1}(\mathcal{F}_{\mathcal{K}})$, there is an open soft set $H_{\mathcal{K}}$ of $(\mathcal{N},\tilde{v},\mathcal{K})$ such that $\mathcal{F}_{\mathcal{K}} \sqsubseteq H_{\mathcal{K}}$ and $\psi_\ell^{-1}(H_{\mathcal{K}}) \sqsubseteq G_R$.

Proof: Let ψ_ℓ be a soft closed mapping, $\mathcal{F}_{\mathcal{K}} \in \mathcal{SS}(\mathcal{N},\mathcal{K})$, and let G_R be an open soft set of $(\mathcal{M},\tilde{u},R)$ such that $\psi_\ell^{-1}(\mathcal{F}_{\mathcal{K}}) \sqsubseteq G_R$. Then, $\psi_\ell(\widetilde{\mathcal{M}} - G_R)$ is a closed soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$. Let $H_{\mathcal{K}} = \widetilde{\mathcal{N}} - \psi_\ell(\widetilde{\mathcal{M}} - G_R)$. Then, $H_{\mathcal{K}}$ is an open soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$ and $\psi_\ell^{-1}(H_{\mathcal{K}}) = \psi_\ell^{-1}\left(\widetilde{\mathcal{N}} - \psi_\ell(\widetilde{\mathcal{M}} - G_R)\right) = \widetilde{\mathcal{M}} - \psi_\ell^{-1}\left(\psi_\ell(\widetilde{\mathcal{M}} - G_R)\right) \sqsubseteq \widetilde{\mathcal{M}} - (\widetilde{\mathcal{M}} - G_R) = G_R$. Thus, $H_{\mathcal{K}}$ is an open soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$ containing $\mathcal{F}_{\mathcal{K}}$ such that $\psi_\ell^{-1}(\mathcal{F}_{\mathcal{K}}) \sqsubseteq G_R$.

Conversely, let $\mathcal{F}_R$ be a closed soft set of $(\mathcal{M},\tilde{u},R)$. Then, $\psi_\ell^{-1}\left(\widetilde{\mathcal{N}} - \psi_\ell(\mathcal{F}_R)\right) \sqsubseteq \widetilde{\mathcal{M}} - \mathcal{F}_R$ and $\widetilde{\mathcal{M}} - \mathcal{F}_R$ is an open soft of $(\mathcal{M},\tilde{u},R)$. From the hypothesis, there is an open soft set $H_{\mathcal{K}}$ of $(\mathcal{N},\tilde{v},\mathcal{K})$ such that $\widetilde{\mathcal{N}} - \psi_\ell(\mathcal{F}_R) \sqsubseteq H_{\mathcal{K}}$ and $\psi_\ell^{-1}(H_{\mathcal{K}}) \sqsubseteq \widetilde{\mathcal{M}} - \mathcal{F}_R$. Therefore, $\mathcal{F}_R \sqsubseteq \widetilde{\mathcal{M}} - \psi_\ell^{-1}(H_{\mathcal{K}})$. Therefore, $\widetilde{\mathcal{N}} - H_{\mathcal{K}} \sqsubseteq \psi_\ell(\mathcal{F}_R) \sqsubseteq \psi_\ell(\widetilde{\mathcal{M}} - \psi_\ell^{-1}(H_{\mathcal{K}})) \sqsubseteq \widetilde{\mathcal{N}} - H_{\mathcal{K}}$, which yields $\psi_\ell(\mathcal{F}_R) = \widetilde{\mathcal{N}} - H_{\mathcal{K}}$. Therefore, $\psi_\ell(\mathcal{F}_R)$ is a closed soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$. Hence, ψ_ℓ is a soft closed mapping. □

Proposition 3.14. Let $(\mathcal{M},\tilde{u},R)$, $(\mathcal{N},\tilde{v},\mathcal{K})$ and $(\mathcal{Z},\widetilde{w},Q)$ be soft-cs's and let $\psi_\ell:(\mathcal{M},\tilde{u},R) \to (\mathcal{N},\tilde{v},\mathcal{K})$ and $\varphi_q:(\mathcal{N},\tilde{v},\mathcal{K}) \to (\mathcal{Z},\widetilde{w},Q)$ be soft mappings. Then

i- if ψ_ℓ and φ_q are soft closed, then so is $\varphi_q o \psi_\ell$.

ii- if $\varphi_q o \psi_\ell$ is soft closed and ψ_ℓ is soft continuous and surjection, then φ_q is a soft closed mapping.

iii- if $\varphi_q o \psi_\ell$ is soft closed and φ_q is soft continuos and injection, then ψ_ℓ is a soft closed mapping.

Proof

i- Let $\mathcal{F}_R$ be a closed soft set of $(\mathcal{M},\tilde{u},R)$. By ψ_ℓ is soft closed mapping, we get $\psi_\ell(\mathcal{F}_R)$ closed soft set in $(\mathcal{N},\tilde{v},\mathcal{K})$. Since φ_q is closed soft, then $\varphi_q(\psi_\ell(\mathcal{F}_R))$ is closed soft set in $(\mathcal{Z},\widetilde{w},Q)$. Hence, $\varphi_q o \psi_\ell$ is soft closed mapping.

ii- Let $G_{\mathcal{K}}$ be a soft closed set of $(\mathcal{N},\tilde{v},\mathcal{K})$. Since ψ_ℓ is a soft continuous, then $\psi_\ell^{-1}(G_{\mathcal{K}})$ is soft closed in $(\mathcal{M},\tilde{u},R)$. Since $\varphi_q o \psi_\ell$ is soft closed, then $\varphi_q o \psi_\ell\left(\psi_\ell^{-1}(G_{\mathcal{K}})\right) = \varphi_q(\psi_\ell\left(\psi_\ell^{-1}(G_{\mathcal{K}})\right))$ is a soft closed set in $(\mathcal{Z},\widetilde{w},Q)$. But ψ_ℓ is surjection, then $\varphi_q o \psi_\ell\left(\psi_\ell^{-1}(G_{\mathcal{K}})\right) = \varphi_q\left(\psi_\ell\left(\psi_\ell^{-1}(G_{\mathcal{K}})\right)\right) = \varphi_q(G_{\mathcal{K}})$. Consequently, $\varphi_q(G_{\mathcal{K}})$ is a soft closed set in $(\mathcal{Z},\widetilde{w},Q)$. Hence, φ_q is soft closed mapping.

iii- Let $\mathcal{F}_R$ be closed in $(\mathcal{M},\tilde{u},R)$ to prove that $\psi_\ell(\mathcal{F}_R)$ closed in $(\mathcal{N},\tilde{v},\mathcal{K})$. Since $\varphi_q o\psi_\ell$ is soft closed, then $(\varphi_q o\psi_\ell)(\mathcal{F}_R)$ is closed in $(\mathcal{Z},\tilde{w},\mathcal{Q})$ and since φ_q is soft continuous, then $\varphi_q^{-1}((\varphi_q o\psi_\ell)(\mathcal{F}_R))$ is closed of $(\mathcal{N},\tilde{v},\mathcal{K})$. This implies that $\varphi_q^{-1}(\varphi_q(\psi_\ell(\mathcal{F}_R))$ is a closed soft set of $(\mathcal{N},\tilde{v},\mathcal{K})$. Since φ_q is one to one, then $\psi_\ell(\mathcal{F}_R)$ is closed soft in $(\mathcal{N},\tilde{v},\mathcal{K})$.

4. Soft continuous mappings between product soft closure spaces

In this section, we study some properties of soft continuous mappings in the product soft closure space. First, we show that the soft projection map is soft closed and continuous.

Theorem 4.1. Let $\{(\mathcal{M}_\alpha,\tilde{u}_\alpha,R_\alpha):\alpha\in\mathcal{I}\}$ be a family of soft-cs's and let $v\in\mathcal{I}$. Then, the soft projection mapping $(p_{\mathcal{M}_v},q_{R_v}):(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)\longrightarrow(\mathcal{M}_v,\tilde{u}_v,R_v)$ is a soft closed and soft continuous mapping.

Proof. First, we prove that for all $v\in\mathcal{I}$, the soft projection mapping $(p_{\mathcal{M}_v},q_{R_v})$ is soft closed. Let $\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha}$ be a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. Then, $\widetilde{\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha}-\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha}$ is an open soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. From Theorem 2.15, we have $(p_{\mathcal{M}_v},q_{R_v})(\widetilde{\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha}-\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$ is an open soft set of $(\mathcal{M}_v,\tilde{u}_v,R_v)$. But, $(p_{\mathcal{M}_v},q_{R_v})(\widetilde{\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha}-\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})=\widetilde{\mathcal{M}_v}-(p_{\mathcal{M}_v},q_{R_v})(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$, which is an open soft set in $(\mathcal{M}_v,\tilde{u}_v,R_v)$. Hence, $(p_{\mathcal{M}_v},q_{R_v})(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$ is a closed soft set in $(\mathcal{M}_v,\tilde{u}_v,R_v)$. Thus, the soft projection map $(p_{\mathcal{M}_v},q_{R_v})$ is a soft closed mapping.

Now, we shall show that $(p_{\mathcal{M}_v},q_{R_v})$ is soft continuous mapping for all $v\in\mathcal{I}$. Let $G_{\prod_{\alpha\in\mathcal{I}}R_\alpha}\sqsubseteq\widetilde{\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha}$ to prove that $(p_{\mathcal{M}_v},q_{R_v})(\otimes\tilde{u}(G_{\prod_{\alpha\in\mathcal{I}}R_\alpha}))\sqsubseteq\tilde{u}_v((p_{\mathcal{M}_v},q_{R_v})(G_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$. From the definition of $\otimes\tilde{u}$, we have

$$(p_{\mathcal{M}_v},q_{R_v})(\otimes\tilde{u}(G_{\prod_{\alpha\in\mathcal{I}}R_\alpha})=(p_{\mathcal{M}_v},q_{R_v})(\prod_{\alpha\in\mathcal{I}}\tilde{u}_\alpha\left((p_{\mathcal{M}_\alpha},q_{R_\alpha})(G_{\prod_{\alpha\in\mathcal{I}}R_\alpha})\right)$$
$$=\tilde{u}_v((p_{\mathcal{M}_v},q_{R_v})(G_{\prod_{\alpha\in\mathcal{I}}R_\alpha}).$$

Therefore, $(p_{\mathcal{M}_v},q_{R_v})$ is soft continuous mapping.

Theorem 4.2. Let $\{(\mathcal{M}_\alpha,\tilde{u}_\alpha,R_\alpha):\alpha\in\mathcal{I}\}$ be a family of soft-cs's and let $v\in\mathcal{I}$. Then, $\mathcal{F}_{R_v}$ is a closed set of $(\mathcal{M}_v,\tilde{u}_v,R_v)$ if and only if $\mathcal{F}_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ is a closed soft set in $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$.

Proof. Let $v\in\mathcal{I}$ and let $\mathcal{F}_{R_v}$ be a soft closed set of $(\mathcal{M}_v,\tilde{u}_v,R_v)$. Then, from Theorem 4.1, $(p_{\mathcal{M}_v},q_{R_v})$ is soft continuous and, by Proposition 3.5, we have $(p_{\mathcal{M}_v},q_{R_v})^{-1}(\mathcal{F}_{R_v})$ is a soft closed set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. But, $(p_{\mathcal{M}_v},q_{R_v})^{-1}(\mathcal{F}_{R_v})=\mathcal{F}_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$. This implies that $\mathcal{F}_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ is a soft closed set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$.

Conversely, $\mathcal{F}_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ is a soft closed set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. From Theorem 4.1, this implies that, for all $v\in\mathcal{I}$, the soft projection $(p_{\mathcal{M}_v},q_{R_v})$ is soft closed mapping. This implies that $(p_{\mathcal{M}_v},q_{R_v})(\mathcal{F}_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha})=\mathcal{F}_{R_v}$ is a soft closed set of $(\mathcal{M}_v,\tilde{u}_v,R_v)$. □

Theorem 4.3. Let $\{(\mathcal{M}_\alpha,\tilde{u}_\alpha,R_\alpha):\alpha\in\mathcal{I}\}$ be a family of soft-cs's and let $v\in\mathcal{I}$. Then, G_{R_v} is an open soft set of $(\mathcal{M}_v,\tilde{u}_v,R_v)$ if and only if $G_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ is an open soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$.

Proof: Let $v\in\mathcal{I}$ and let G_{R_v} be an open soft set of $(\mathcal{M}_v,\tilde{u}_v,R_v)$. From Theorem 4.1, $(p_{\mathcal{M}_v},q_{R_v})$ is soft continuous mapping, then $(p_{\mathcal{M}_v},q_{R_v})^{-1}(G_{R_v})$ is an open soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. But, $(p_{\mathcal{M}_v},q_{R_v})^{-1}(G_{R_v})=G_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$, therefore, $G_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ is an open soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$.

Conversely, let $G_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ be an open soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. Then, $\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}-(G_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha})$ is a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. But, $\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}-(G_{R_v}\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha})=(\widetilde{\mathcal{M}_v}-G_{R_v})\times\widetilde{\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}}\mathcal{M}_\alpha}$ is a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha,\otimes\tilde{u},\prod_{\alpha\in\mathcal{I}}R_\alpha)$. By Theorem

4.2, $(\widetilde{\mathcal{M}_v} - G_{R_v}) \times \prod_{\substack{\alpha\neq v \\ \alpha\in\mathcal{I}}} \widetilde{\mathcal{M}_\alpha}$ is a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha)$. Consequently, G_{R_v} is an open soft set of $(\mathcal{M}_v, \tilde{u}_v, R_v)$. □

Theorem 4.4. Let $(\mathcal{N}, \widetilde{\omega}, \mathcal{K})$ be a soft-cs, $\{(\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha): \alpha \in \mathcal{I}\}$ be a family of soft-cs's, and $\psi_\ell: (\mathcal{N}, \widetilde{\omega}, \mathcal{K}) \longrightarrow (\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha)$ be soft mapping. Then, ψ_ℓ is a soft closed mapping if and only if $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ is a soft closed mapping for each $\alpha \in \mathcal{I}$.
Proof. Let ψ_ℓ be a soft closed mapping. From Theorem 4.1, $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})$ is soft closed mapping and by Proposition 3.14 part (1), we get that $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ is a soft closed mapping for each $\alpha \in \mathcal{I}$.
Conversely, let $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ be a soft closed mapping for each $\alpha \in \mathcal{I}$. Suppose that ψ_ℓ is not a soft closed mapping. Then, there exists a closed soft set $\mathcal{F}_\mathcal{K}$ of $(\mathcal{N}, \widetilde{\omega}, \mathcal{K})$ such that $\psi_\ell(\mathcal{F}_\mathcal{K})$ is not a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha)$, i.e., $\otimes\tilde{u}(\psi_\ell(\mathcal{F}_\mathcal{K})) \neq \psi_\ell(\mathcal{F}_\mathcal{K})$, which implies that $\otimes\tilde{u}(\psi_\ell(\mathcal{F}_\mathcal{K})) \not\sqsubseteq \psi_\ell(\mathcal{F}_\mathcal{K})$. It follows that $\prod_{\alpha\in\mathcal{I}} \tilde{u}_\alpha(p_{\mathcal{M}_\alpha}, q_{R_\alpha})(\psi_\ell(\mathcal{F}_\mathcal{K}))) \not\sqsubseteq \psi_\ell(\mathcal{F}_\mathcal{K})$. This implies that there exists $v \in \mathcal{I}$ such that $\tilde{u}_v((p_{\mathcal{M}_v}, q_{R_v}))(\psi_\ell(\mathcal{F}_\mathcal{K})) \not\sqsubseteq (p_{\mathcal{M}_\alpha}, q_{R_\alpha})(\psi_\ell(\mathcal{F}_\mathcal{K}))$. Since $\mathcal{F}_\mathcal{K}$ is a closed soft set of $(\mathcal{N}, \widetilde{\omega}, \mathcal{K})$ and $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ is a soft closed map for each $\alpha \in \mathcal{I}$, then $((p_{\mathcal{M}_v}, q_{R_v})o\psi_\ell)(\mathcal{F}_\mathcal{K})$ is a closed soft set of $(\mathcal{M}_v, \tilde{u}_v, R_v)$. This implies that $(p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(\mathcal{F}_\mathcal{K}))$ is a closed soft set of $(\mathcal{M}_v, \tilde{u}_v, R_v)$. Consequently, $\tilde{u}_v\left((p_{\mathcal{M}_v}, q_{R_v})\right)(\psi_\ell(\mathcal{F}_\mathcal{K})) = (p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(\mathcal{F}_\mathcal{K}))$ and this is a contradiction.
Theorem 4.5. Let $\{(\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha): \alpha \in \mathcal{I}\}$ and $\{(\mathcal{N}_\alpha, \widetilde{\omega}_\alpha, \mathcal{K}_\alpha): \alpha \in \mathcal{I}\}$ be families of soft-cs's. For each $\alpha \in \mathcal{I}$, let $(\psi_\ell)_\alpha: (\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha) \longrightarrow (\mathcal{N}_\alpha, \widetilde{\omega}_\alpha, \mathcal{K}_\alpha)$ be a surjection and let $\psi_\ell: (\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha) \longrightarrow (\prod_{\alpha\in\mathcal{I}}\mathcal{N}_\alpha, , \otimes\widetilde{\omega}, \prod_{\alpha\in\mathcal{I}}\mathcal{K}_\alpha)$, defined by $\psi_\ell((x_{\alpha_{r_\alpha}})_{\alpha\in\mathcal{I}}) = ((\psi_\ell)_\alpha(x_{\alpha_{r_\alpha}}))_{\alpha\in\mathcal{I}}$. Then, ψ_ℓ is soft closed if and only if $(\psi_\ell)_\alpha$ is soft closed mapping for each $\alpha \in \mathcal{I}$.
Proof. Let ψ_ℓ is soft closed mapping. Let $v \in \mathcal{I}$ and let $\mathcal{F}_{R_v}$ be a closed soft set of $(\mathcal{M}_v, \tilde{u}_v, R_v)$. Then, by Theorem 4.2, $\mathcal{F}_{R_v} \times \prod_{\substack{\alpha\neq v \\ \alpha\in\mathcal{I}}} \widetilde{\mathcal{M}_\alpha}$ is a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha)$. Since ψ_ℓ is a soft closed mapping, then $\psi_\ell(\mathcal{F}_{R_v} \times \prod_{\substack{\alpha\neq v \\ \alpha\in\mathcal{I}}} \widetilde{\mathcal{M}_\alpha})$ is a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{N}_\alpha, , \otimes\widetilde{\omega}, \prod_{\alpha\in\mathcal{I}}\mathcal{K}_\alpha)$. But $\psi_\ell(\mathcal{F}_{R_v} \times \prod_{\substack{\alpha\neq v \\ \alpha\in\mathcal{I}}} \widetilde{\mathcal{M}_\alpha}) = (\psi_\ell)_v(\mathcal{F}_{R_v}) \times \prod_{\substack{\alpha\neq v \\ \alpha\in\mathcal{I}}} \widetilde{\mathcal{N}_\alpha}$, hence $\psi_\ell(\mathcal{F}_{R_v}) \times \prod_{\substack{\alpha\neq v \\ \alpha\in\mathcal{I}}} \widetilde{\mathcal{N}_\alpha}$ is a closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{N}_\alpha, , \otimes\widetilde{\omega}, \prod_{\alpha\in\mathcal{I}}\mathcal{K}_\alpha)$. By Theorem 4.2, $\psi_\ell(\mathcal{F}_{R_v})$ is a closed soft set of $(\mathcal{N}_v, \widetilde{\omega}_v, \mathcal{K}_v)$. Hence, $(\psi_\ell)_v$ is soft closed mapping for each $\alpha \in \mathcal{I}$.
Conversely, let $(\psi_\ell)_\alpha$ be a soft closed mapping for each $\alpha \in \mathcal{I}$. Now, we shall prove that ψ_ℓ is a soft closed mapping. Suppose that ψ_ℓ is not a soft closed mapping. Then, there exists a closed soft set $\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha}$ of $(\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha)$ such that $\psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$ is not closed soft set of $(\prod_{\alpha\in\mathcal{I}}\mathcal{N}_\alpha, , \otimes\widetilde{\omega}, \prod_{\alpha\in\mathcal{I}}\mathcal{K}_\alpha)$. This implies that $\otimes\widetilde{\omega}(\psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})) \not\sqsubseteq \psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$. From the definition of $\otimes\widetilde{\omega}$, we get that $\prod_{\alpha\in\mathcal{I}}\widetilde{\omega}_\alpha((p_{\mathcal{N}_\alpha}, q_{\mathcal{K}_\alpha})(\psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})))) \not\sqsubseteq \psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$, which implies that there exists $v \in \mathcal{I}$ such that $\widetilde{\omega}_v(p_{\mathcal{N}_v}, q_{\mathcal{K}_v})(\psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha}) \sqsubseteq (p_{\mathcal{N}_v}, q_{\mathcal{K}_v})(\psi_\ell(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$. Therefore, $\widetilde{\omega}_v((\psi_\ell)_v((p_{\mathcal{M}_v}, q_{R_v})(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})) \not\sqsubseteq (\psi_\ell)_v(p_{\mathcal{M}_v}, q_{R_v})(\mathcal{F}_{\prod_{\alpha\in\mathcal{I}}R_\alpha})$.
Theorem 4.6. Let $(\mathcal{N}, \widetilde{\omega}, \mathcal{K})$ be a soft-cs, $\{(\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha): \alpha \in \mathcal{I}\}$ be a family of soft-cs's, and $\psi_\ell: (\mathcal{N}, \widetilde{\omega}, \mathcal{K}) \longrightarrow (\prod_{\alpha\in\mathcal{I}}\mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}}R_\alpha)$ is a soft mapping. Then, ψ_ℓ is soft continuous if and only if $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ is soft continuous for each $\alpha \in \mathcal{I}$.
Proof. Let ψ_ℓ be soft continuous. From Theorem 4.1, $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})$ is soft continuous for each $\alpha \in \mathcal{I}$ and by Proposition 3.9, $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ is soft continuous for each $\alpha \in \mathcal{I}$.
Conversely, let $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ be soft continuous for each $\alpha \in \mathcal{I}$. We must prove that ψ_ℓ is soft continuous. Suppose that ψ_ℓ is not soft continuous. Then, there exists a soft set $H_\mathcal{K} \in \mathcal{SS}(\mathcal{N}, \mathcal{K})$ such that $\psi_\ell(\widetilde{\omega}(H_\mathcal{K})) \not\sqsubseteq \otimes\tilde{u}(\psi_\ell(H_\mathcal{K}))$, which implies that $\psi_\ell(\widetilde{\omega}(H_\mathcal{K})) \not\sqsubseteq \prod_{\alpha\in\mathcal{I}}(\tilde{u}_\alpha((p_{\mathcal{M}_\alpha}, q_{R_\alpha})(\psi_\ell(H_\mathcal{K}))$. Then, there exists $v \in \mathcal{I}$ such that $(p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(\widetilde{\omega}(H_\mathcal{K})) \not\sqsubseteq \tilde{u}_v((p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(H_\mathcal{K}))$. Since, for all $\alpha \in \mathcal{I}$, $(p_{\mathcal{M}_\alpha}, q_{R_\alpha})o\psi_\ell$ is soft continuous, then $((p_{\mathcal{M}_v}, q_{R_v})o\psi_\ell)(\widetilde{\omega}(H_\mathcal{K}))) \sqsubseteq \tilde{u}_v((p_{\mathcal{M}_v}, q_{R_v})o\psi_\ell)(H_\mathcal{K}))$. Then, $(p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(\widetilde{\omega}(H_\mathcal{K})) \sqsubseteq \tilde{u}_v((p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(H_\mathcal{K}))$, which is a contraction. Consequently, ψ_ℓ is soft continuous mapping.

Theorem 4.7 Let $\{(\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha): \alpha \in \mathcal{I}\}$ and $\{(\mathcal{N}_\alpha, \tilde{\omega}_\alpha, \mathcal{K}_\alpha): \alpha \in \mathcal{I}\}$ be families of soft-cs's. For each $\alpha \in \mathcal{I}$, let $(\psi_\ell)_\alpha: (\mathcal{M}_\alpha, \tilde{u}_\alpha, R_\alpha) \longrightarrow (\mathcal{N}_\alpha, \tilde{\omega}_\alpha, \mathcal{K}_\alpha)$ be a soft mapping and let $\psi_\ell: (\prod_{\alpha\in\mathcal{I}} \mathcal{M}_\alpha, \otimes\tilde{u}, \prod_{\alpha\in\mathcal{I}} R_\alpha) \longrightarrow (\prod_{\alpha\in\mathcal{I}} \mathcal{N}_\alpha, , \otimes\tilde{\omega}, \prod_{\alpha\in\mathcal{I}} \mathcal{K}_\alpha)$, defined by $\psi_\ell((x_{\alpha_{r_\alpha}})_{\alpha\in\mathcal{I}}) = ((\psi_\ell)_\alpha(x_{\alpha_{r_\alpha}}))_{\alpha\in\mathcal{I}}$. Then, ψ_ℓ is soft continuous if and only if $(\psi_\ell)_\alpha$ is soft continuous for each $\alpha \in \mathcal{I}$.

Proof. Let ψ_ℓ be a soft continuous, let $v \in \mathcal{I}$, and let $H_{R_v} \sqsubseteq \widetilde{\mathcal{M}_v}$. Then,

$$
\begin{aligned}
(\psi_\ell)_v\left(\tilde{u}_v(H_{R_v})\right) &= (p_{\mathcal{M}_v}, q_{R_v})((\psi_\ell)_v\left(\tilde{u}_v(H_{R_v})\right) \times \textstyle\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}} (\psi_\ell)_\alpha(\tilde{u}_\alpha(\widetilde{\mathcal{M}_\alpha})) \\
&= (p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(\left(\tilde{u}_v(H_{R_v})\right) \times \textstyle\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}} (\tilde{u}_\alpha(\widetilde{\mathcal{M}_\alpha})) \\
&= (p_{\mathcal{M}_v}, q_{R_v})(\psi_\ell(\otimes\tilde{u}(H_{R_v} \times \widetilde{\textstyle\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}} \mathcal{M}_\alpha})) \\
&\sqsubseteq (p_{\mathcal{M}_v}, q_{R_v})(\otimes\tilde{\omega}(\psi_\ell(H_{R_v} \times \widetilde{\textstyle\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}} \mathcal{M}_\alpha})) \text{ (} \psi_\ell \text{ is soft continuous)} \\
&= (p_{\mathcal{M}_v}, q_{R_v})(\otimes\tilde{\omega}((\psi_\ell)_v(H_{R_v})) \times \textstyle\prod_{\substack{\alpha\neq v\\ \alpha\in J}} (\psi_\ell)_\alpha(\widetilde{\mathcal{M}_\alpha})) \\
&= (p_{\mathcal{M}_v}, q_{R_v})(\tilde{\omega}_v\left((\psi_\ell)_v(H_{R_v})\right) \times \textstyle\prod_{\substack{\alpha\neq v\\ \alpha\in\mathcal{I}}} \tilde{\omega}_\alpha(((\psi_\ell)_\alpha(\widetilde{\mathcal{M}_\alpha})) \\
&= \tilde{\omega}_v((\psi_\ell)_v(H_{R_v}).
\end{aligned}
$$

Thus, $(\psi_\ell)_v\left(\tilde{u}_v(H_{R_v})\right) \sqsubseteq \tilde{\omega}_v((\psi_\ell)_v(H_{R_v})$. Consequently, $(\psi_\ell)_v$ is soft continuous for all $v \in \mathcal{I}$.

Conversely, let $(\psi_\ell)_\alpha$ be soft continuous for each $\alpha \in \mathcal{I}$ and let $H_{\prod_{\alpha\in\mathcal{I}} R_\alpha} \sqsubseteq \widetilde{\prod_{\alpha\in\mathcal{I}} \mathcal{M}_\alpha}$. Then,

$$
\begin{aligned}
\psi_\ell\left(\otimes\tilde{u}(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha})\right) &= \psi_\ell(\textstyle\prod_{\alpha\in\mathcal{I}} \tilde{u}_\alpha((p_{\mathcal{M}_\alpha}, q_{R_\alpha})(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha})) \\
&= \textstyle\prod_{\alpha\in\mathcal{I}} (\psi_\ell)_\alpha(\prod_{\alpha\in\mathcal{I}} \tilde{u}_\alpha((p_{\mathcal{M}_\alpha}, q_{R_\alpha})(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha})) \\
&= \textstyle\prod_{\alpha\in\mathcal{I}} (\psi_\ell)_\alpha(\tilde{u}_\alpha((p_{\mathcal{M}_\alpha}, q_{R_\alpha})(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha})) \\
&\sqsubseteq \textstyle\prod_{\alpha\in\mathcal{I}} \tilde{\omega}_\alpha((\psi_\ell)_\alpha((p_{\mathcal{M}_\alpha}, q_{R_\alpha})(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha})) \\
&= \textstyle\prod_{\alpha\in\mathcal{I}} \tilde{\omega}_\alpha((p_{\mathcal{M}_\alpha}, q_{R_\alpha})((\psi_\ell(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha}))) \\
&= \otimes\tilde{\omega}(\psi_\ell(H_{\prod_{\alpha\in\mathcal{I}} R_\alpha})
\end{aligned}
$$

Therefore, ψ_ℓ is soft continuous.

Conclusions

Soft closure spaces are a very new concept and an important topic for investigators because it is more general as compared to the concept of soft topological spaces. The notions of soft continuous and soft closed (resp. open) mappings were introduced in this paper, and some related properties and theorems were developed. To explain our notions, we put forward some examples.

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