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Oscillatory Flow MHD of Jeffrey Fluid with Temperature-Dependent Viscosity (TDV) in a Saturated Porous Channel

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Abstract

In this research, we studied the impact of Magnetohydrodynamic (MHD) on Jeffrey fluid with porous channel saturated with temperature-dependent viscosity (TDV). It is obtained on the movement of fluid flow equations by using the method of perturbation technique in terms of number Weissenberg ($We \ll 1$) to get clear formulas for the field of velocity. All the solutions of physical parameters of the Reynolds number (Re), Magnetic parameter (M), Darcy parameter (Da), Peclet number (Pr) and are discussed under the different values, as shown in the plots.

Keywords: Jeffrey fluid; Porous channel; Magnetohydrodynamic (MHD); Perturbation Technique.

التدفق التذبذبي MHD لمائع جفري مع اللزوجة المعتمدة على درجة الحرارة (TDV) في قناة مسامية

مشبعة

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الخلاصة

في هذا البحث، درسنا تأثير الهيدروديناميك المغناطيسية (MHD) على مائع جفري مع قناة مسامية مشبعة بلزوجة تعتمد على درجة الحرارة (TDV). تم الحصول على معادلات الحركة لتدفق السوائل باستخدام طريقة تقنية الاضطراب حيث استخدم عدد وزنبرك ($We \ll 1$) للحصول على صيغ واضحة لمجال السرعة. تم ايجاد جميع حلول المعلمات الفيزيائية لرقم رينولدز (Re)، المعلمة المغناطيسية (M)، معلمة دراسي (Da)، رقم بيكلت (Pr)، وتم مناقشة المعلمات باستخدام قيم مختلفة لها كما موضح في الرسوم.

Introduction

In many biological streams and engineering requests, with arterial blood flow, Magnetohydrodynamic generators, geothermal liveliness withdrawal, petroleum engineering, atomic reactors and much more, -MHD- flux investigation of viscous liquid in a permeable channel that is full with insignificant permeable media is insignificant. The rheological fluid in the ducts is of fundamental importance for particular natural and scientific presentations of transit flux (or oscillation).

For example, the semi-intermittent flow inside the heart background container been represented by pressure (regularity component) and flow frequency beats. A lot of infections accompanying with

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the vascular method are due to a trouble in the native flow of blood vessels. In many generalized applications such as photogravure from inkjet printers, a quick switch is necessary between "non-flow" and "flow" for a non-Newtonian fluid.

Nadeem et al. [1] premeditated the impact of Jeffrey fluid with pressure-dependent viscosity (PDV) and there is other interesting articles studied that the effect of -PDV- in different fluids, including (Carreau fluid [2] and Williamson fluid [3]). Ajibade [4] obtainable that influence of -suction /injection- on (MHD).

There are few authors extended the investigation to the impact of temperature-dependent viscosity Falade et al. [5], and Abbas et al. [6] on oscillatory flow of Casson fluid with porous channel. There are same researchers extended the problem by presenting -injection/suction- [7] and time-dependent boundary conditions (TDB) [8].

This study aims to employ a chain of perturbation method to fix the issue of an elevated medium with variable viscosity for the impact magneto hydrodynamics of temperature-dependent viscosity on Jeffrey fluid in a saturated porous channel.

Preparation of the problem

Consider the incompressible unstable flow for Magneto hydrodynamic of Jeffrey fluid with -TDV- in a saturated permeable channel with variable viscosity and at height (L), see (Fig.1). We choose the Cartesian coordinates system (x, y) is velocity vector in which (u) is the (x -component) of velocity and (y) is perpendicular to (x -axis).

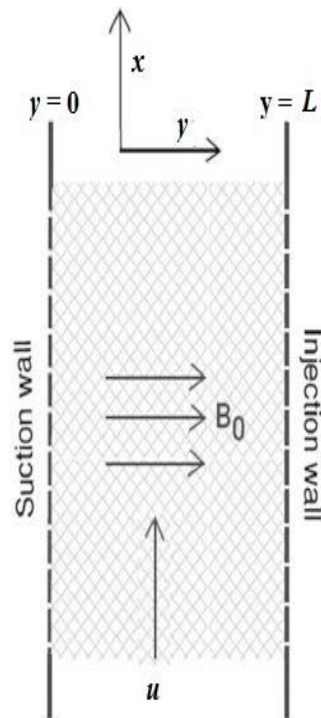


Figure 1-Construction of the Problem

The Jeffrey fluid model, the constitutive equation below [9]:

$$\bar{\tau} = \frac{\mu(T)}{1+\lambda_1} (\dot{\gamma} + \ddot{\gamma}\lambda_2) \tag{1}$$

where $\mu(T)$ is temperature-dependent viscosity, $\dot{\gamma}$ is the shear rate, λ_1 is the ratio of relaxation to retardation time and λ_2 is the retardation time.

The velocity field and the heat field of the present problem are:

$$V = [u(y, t), -v_0, 0], \quad T = T(y, t) \tag{2}$$

The Jeffrey fluid model equations for flow with -TDV- are given by:

$$\rho \frac{\partial \bar{u}}{\partial t} - \rho v_0 \frac{\partial \bar{u}}{\partial y} = -\frac{\partial \bar{p}}{\partial x} + \frac{\mu(T)}{1+\lambda_1} \frac{\partial^2 \bar{u}}{\partial y^2} + \rho g \beta (T - T_0) - \sigma B_0^2 \bar{u} - \frac{\mu(T)}{\rho K} \bar{u} \tag{3}$$

$$\rho \frac{\partial T}{\partial \bar{t}} - \frac{k}{c_p} \frac{\partial^2 T}{\partial \bar{y}^2} - v_0 \frac{\partial T}{\partial \bar{y}} = \frac{4b^2}{c_p} (T - T_0) \tag{4}$$

with associated boundary conditions

$$\left. \begin{aligned} \bar{u} &= \frac{\sqrt{K}}{\alpha_s} \frac{\partial \bar{u}}{\partial \bar{y}}, \quad T = T_0 \quad \text{at} \quad \bar{y} = 0 \\ \bar{u} &= 0, \quad T = T_1 \quad \text{at} \quad \bar{y} = L \end{aligned} \right\} \tag{5}$$

In which \bar{u} is the axial velocity, g is an accelerating due to importance, k is a penetrability, ρ is a fluid density, c_p is a specific heat at constant pressure, β is a measurement of volume intensification due to heat, q is a radioactive heat flux, σ is a conductivity of the fluid, B_0 is a attractive field and K is a thermal conductivity.

Non-dimensional parameters are given by (Wissam et al., 2019) [10]:

$$\left. \begin{aligned} u &= \frac{\bar{u}\rho L}{\mu_0}, x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{L}, \theta = \frac{T-T_0}{T_1-T_0}, t = \frac{\bar{t}V}{L}, p = \frac{\bar{p}h}{\mu_0 V}, M^2 = \frac{\sigma B_0^2 h^2}{\mu_0} \\ Re &= \frac{\rho LV}{\mu_0}, Pr = \frac{\rho LV c_p}{K}, N^2 = \frac{4b^2 L^2}{K}, \mu(T) = \mu_0 \mu(\theta) \\ Gr &= \frac{\rho g \beta L^2 (T-T_0)}{\mu_0 u}, S = \frac{v_0 L \mu_0}{\rho}, Da = \frac{k}{L^2}, \eta = \frac{\sqrt{K}}{\alpha_s L^2}, \dot{\gamma} = \frac{L}{u} \bar{\gamma}, \end{aligned} \right\} \tag{6}$$

where U is the mean flow velocity, (α_s) stagnation speed, (k) Porous permeability, (η) Slip parameter, Grashof number (Gr), radiation parameter (N), and (v_0) Constant horizontal velocity..

Below the above expectations and with the assistance of Eqs. (1-6):

$$Re \frac{\partial u}{\partial t} - S \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\mu(\theta)}{1+\lambda_1} \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{\mu(\theta)}{Da} \right) u + Gr\theta \tag{7}$$

$$\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \tag{8}$$

and Eq. (5) becomes

$$\left. \begin{aligned} u &= \eta \frac{\partial y}{\partial y}, \theta = 0, \text{ on } y = 0 \\ u &= 0, \theta = 1, \text{ on } y = 1 \end{aligned} \right\} \tag{9}$$

To solve the temperature equation (8), let:

$$\theta(y, t) = \theta_f(y) e^{i\omega t} \tag{10}$$

Substituting the Eq. (10) into the Eq. (8), we have:

$$\frac{\partial^2 \theta_f}{\partial y^2} + S \times Pr \frac{\partial \theta_f}{\partial y} + (N^2 - i\omega) Pr \times \theta_f = 0 \tag{11}$$

The exact solution of Eq. (11) and Eq. (9) is given by:

$$\theta_f(y) = \frac{1}{-1+e^{\sqrt{A^2-4B}}} (e^{\frac{1}{2}(-A-\sqrt{A^2-4B})y} \left(-e^{\frac{A}{2}+\frac{1}{2}\sqrt{A^2-4B}} \right) + e^{\frac{1}{2}(-A+\sqrt{A^2-4B})y} \left(e^{\frac{A}{2}+\frac{1}{2}\sqrt{A^2-4B}} \right)),$$

where $A = S \times Pr, B = (N^2 - i\omega) \times Pe$. Therefore

$$\theta(y, t) = \frac{1}{-1+e^{\sqrt{A^2-4B}}} (e^{\frac{1}{2}(-A-\sqrt{A^2-4B})y} \left(-e^{\frac{A}{2}+\frac{1}{2}\sqrt{A^2-4B}} \right) + e^{\frac{1}{2}(-A+\sqrt{A^2-4B})y} \left(e^{\frac{A}{2}+\frac{1}{2}\sqrt{A^2-4B}} \right)) e^{i\omega t} \tag{12}$$

Solution of the problem

To resolve Eq. (7), with the boundary conditions (9), let:

$$-\frac{dp}{dx} = \lambda e^{i\omega t} \tag{13}$$

$$u(y, t) = u_f(y) e^{i\omega t} \tag{14}$$

By substituting equations (10), (13), and (14) into Eq.(7), It follows that:

$$Rei\omega \frac{\partial u_f}{\partial t} - S \frac{\partial u_f}{\partial y} = \lambda + \frac{\mu(\theta)}{1+\lambda_1} \frac{\partial^2 u_f}{\partial y^2} - \left(M^2 + \frac{\mu(\theta)}{Da} \right) u_f + Gr\theta_f \tag{15}$$

Reynold's perfect and distinction of viscosity with heat is distinct by [11]:

$$\mu(\theta) = e^{-\epsilon\theta} \tag{16}$$

by Maclaurin series:

$$\mu(\theta) = 1 - \epsilon\theta, \epsilon \ll 1 \tag{17}$$

By substituting Eq. (17) into Eq. (7), have got:

$$Rei\omega \frac{\partial u_f}{\partial t} - S \frac{\partial u_f}{\partial y} = \lambda + \frac{(1-\epsilon\theta)}{1+\lambda_1} \frac{\partial^2 u_f}{\partial y^2} - \left(M^2 + \frac{(1-\epsilon\theta)}{Da} \right) u_f + Gr\theta_f \tag{18}$$

We use the -Perturbation method- to solve the equation (18), as follows [12]:

$$u_f = u_{00} + Weu_{01} + O(We^2) \tag{19}$$

By substituting Eq. (19) into Eq. (18), which equalizes the powers of (We) , we obtain:

i. Zeros-order system (We^0)

$$\frac{\partial^2 u_{00}}{\partial y^2} - S \frac{\partial u_{00}}{\partial y} - (1 + \lambda_1) \left(M^2 + iwRe + \frac{1}{Da} \right) u_{00} = -(1 + \lambda_1)(\lambda + Gr\theta_f) \quad (20)$$

$$u_{00} = \eta \frac{\partial u_{00}}{\partial y} \quad \text{on } y = 0 \quad \text{and} \quad u_{00} = 0 \quad \text{on } y = 1 \quad (21)$$

ii. First-order system (We^1)

$$\frac{\partial^2 u_{01}}{\partial y^2} - S \frac{\partial u_{01}}{\partial y} - (1 + \lambda_1) \left(M^2 + iwRe + \frac{1}{Da} \right) u_{01} = \left(\frac{\partial^2 u_{00}}{\partial y^2} - \frac{(1 + \lambda_1)}{Da} u_{00} \right) \theta_f \quad (22)$$

$$u_{01} = \eta \frac{\partial u_{01}}{\partial y} \quad \text{on } y = 0 \quad \text{and} \quad u_{01} = 0 \quad \text{on } y = 1 \quad (23)$$

Results and discussion

In this part of the paper, magneto hydrodynamics oscillatory flow problematic of Jeffrey fluid with -TDV- in a saturated permeable channel is discoursed. The perturbation method is comprised in the direction of solving the motion equations, while the meticulous solution of temperature equation is attained. The numerical calculations were have been performed using (Mathematical ver.11) using with the set of values: $Re = 1, \omega = \pi, Pr = 1, S = 1, M = 1, N = 1, Gr = 1, Da = 0.8, \lambda = 1, We = 0.05, \lambda_1 = 0.3, \epsilon = 0.02, \eta = 0.1, t = 0.5$.

From Figure-2, the cumulative frequency of the oscillation ω, θ during the canal decreases is clarified. The significances payable to variation in S on θ are shown in Figure-.3. From this figure, it is initially distinguished that in the absenteeism absence of (injection/suction), the fluid temperature is linearly dispersed during the channel. Conversely, the temperature of the fluid increases in the channel by increasing the injection on the heated bowl.

The linearity inspected at $S = 0$ provides an approach to concave circulation. By increasing S , the concavity is a important due to the pathway of the flow of temperature which is absorbed each time from the animated superficial to the cold superficial. From Figure-4, it is distinguished that θ increases by increasing the values of N because the temperature is communicated from the animated wall near the fluid.

Figure-5 displays that velocity distribution increases, with increasing the thoughts Da . Figure-6 illustrates the effect of Gr on the velocity field. The figure demonstrates that after increasing Gr the velocity field u increased. Figure-7 illustrations the effects of the parameter of λ . By increasing λ , velocity distribution u is illustrates. Figure-8 shows that the velocity distribution u is increased with increasing λ_1 . Figure-9 shows the influence M on the velocity field function u . By increasing M the velocity distribution is decreased.

Figure-10 illustrates that by increasing N velocity distribution u is increased. Figure-11 displays that increasing Pr , leads to an increase in velocity distribution u . Figure-12 it is displays that velocity distribution u is decreased with increasing Re . Figure-13 Shows that increasing S results in an increase in the velocity field is increased. Figure-14 Shows that velocity θ is decreased with increasing ω .

Conclusion

The oscillatory magneto hydrodynamics flow problem of Jeffrey fluid with -TDV- in a saturated porous channel is discussed. It is established that the velocity field and temperature, examined by exhausting the perturbation method are satisfactory to resolve the problem. Different sets of values have been employed to tackle the problem. We conclude that, by increasing the $\lambda_1, \lambda, Da, Pr, Gr, S, N$ and Pr , the velocity distribution is increased. While increasing the M, ω and Re leads to a decrease in the velocity field. It is also shown that by increasing N and S causes an increase in the temperature but, however, increasing ω leads to a decrease in the temperature θ .

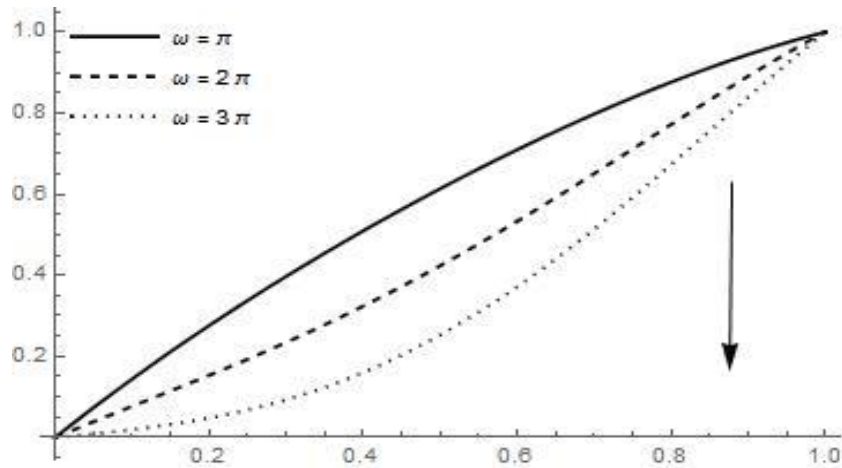


Figure 2-Impact of ω on Temperature field.

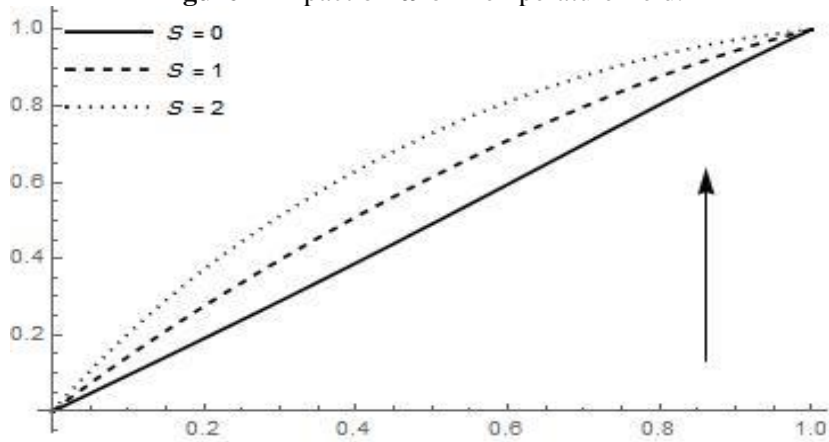


Figure 3-Impact of S on Temperature field.

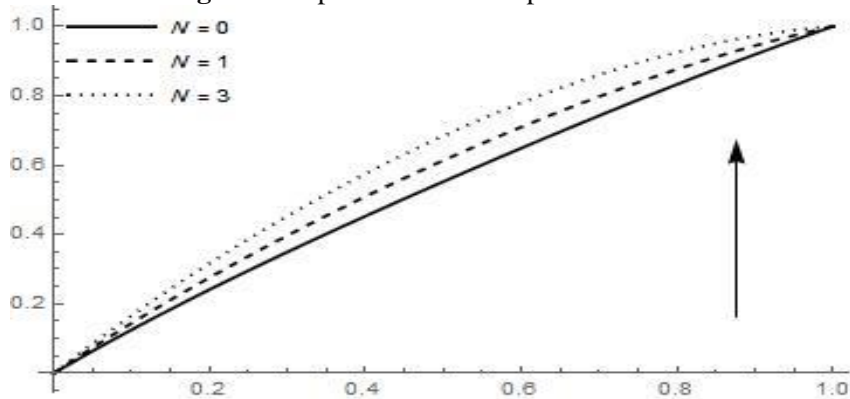


Figure 4-Impact of N on Temperature field.

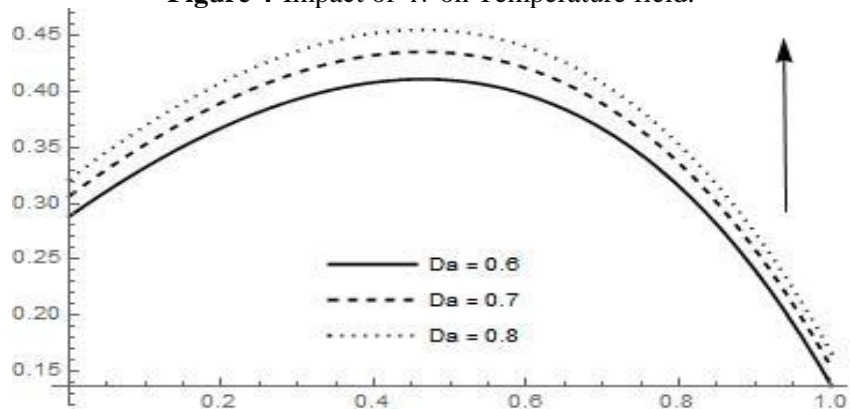


Figure 5-Impact of Da on Velocity distribution.

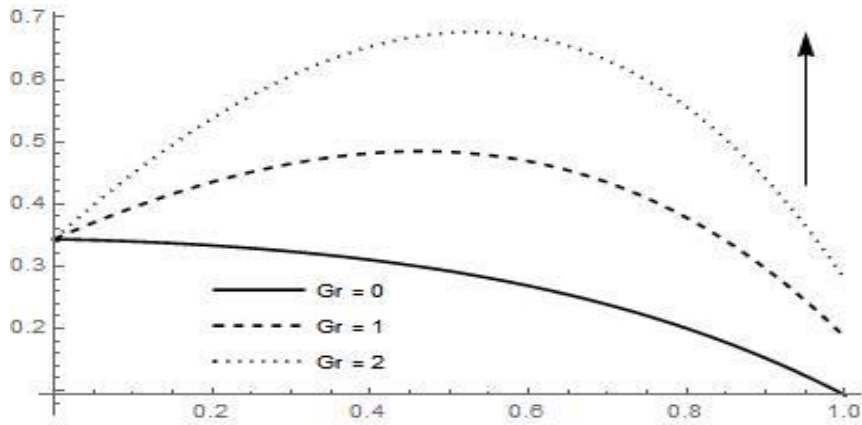


Figure 6-Impact of Gr on Velocity distribution.

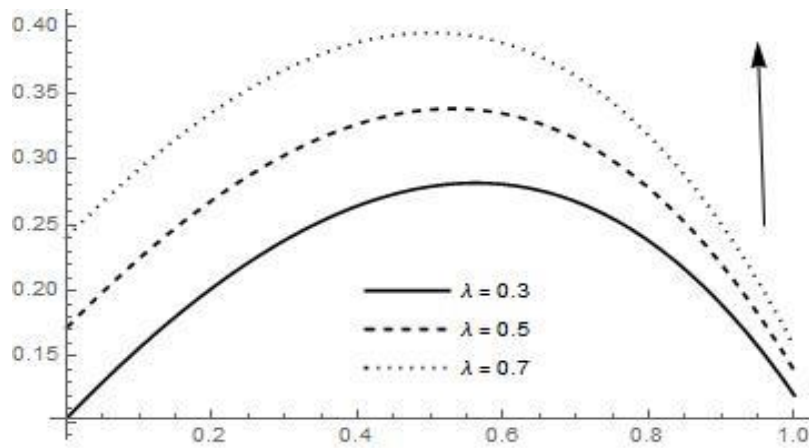


Figure 7-Impact of λ on Velocity distribution.

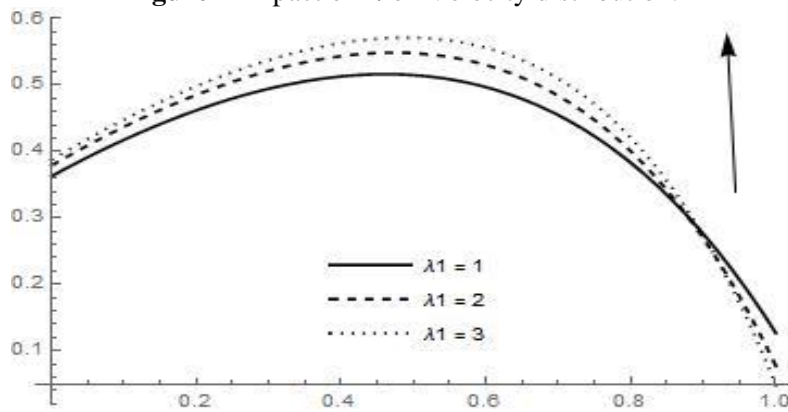


Figure 8-Impact of λ_1 on Velocity distribution.

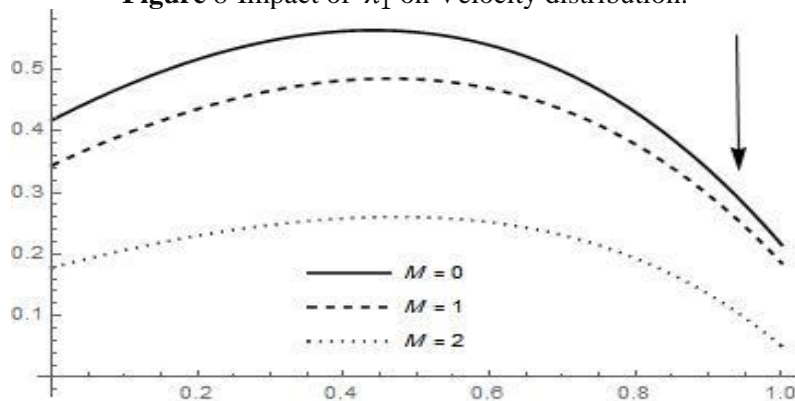


Figure 9-Impact of M on Velocity distribution.

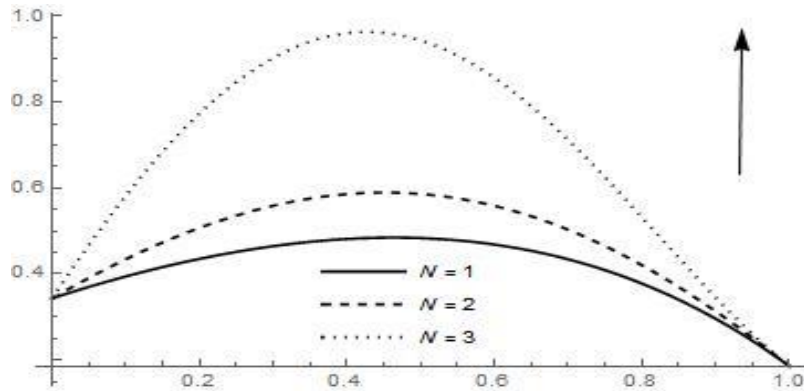


Figure 10-Impact of N on Velocity distribution.

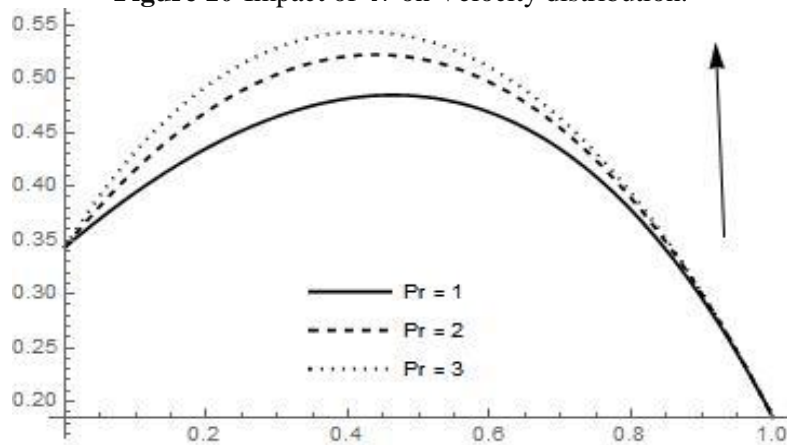


Figure 11-Impact of Pr on Velocity distribution.

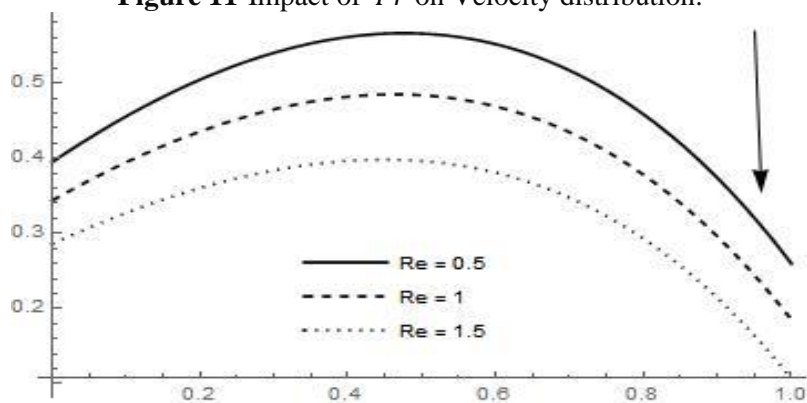


Figure 12-Impact of Re on Velocity distribution.

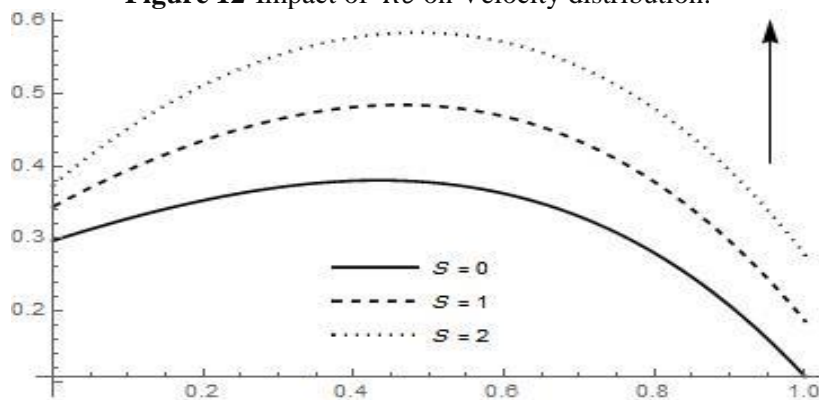


Figure 13-Impact of S on Velocity distribution.

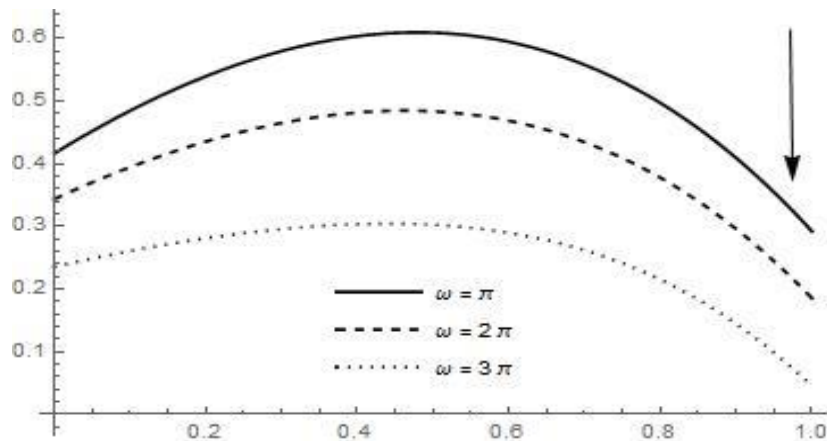


Figure 14 Impact of ω on Velocity distribution.

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