Sheah and Abbas Iraqi Journal of Science, 2021, Vol. 62, No. 3, pp: 997-1015
DOI: 10.24996/ijs.2021.62.3.29

ISSN: 0067-2904

Using Multi-Objective Bat Algorithm for Solving Multi-Objective Non-linear Programming Problem

Rajwan Hamood Sheah, Iraq T.Abbas*
Department of Mathematics, College of Science, University of Baghdad, Baghdad, Iraq

Received: 24/7/2020 Accepted: 29/9/2020

Abstract
Human beings are greatly inspired by nature. Nature has the ability to solve very complex problems in its own distinctive way. The problems around us are becoming more and more complex in the real time and at the same instance our mother nature is guiding us to solve these natural problems. Nature gives some of the logical and effective ways to find solutions to these problems. Nature acts as an optimized source for solving the complex problems. Decomposition is a basic strategy in traditional multi-objective optimization. However, it has not yet been widely used in multi-objective evolutionary optimization.
Although computational strategies for taking care of Multi-objective Optimization Problems (MOPs) have been accessible for a long time, the ongoing utilization of Evolutionary Algorithm (EAs) to such issues gives a vehicle to tackle extremely enormous scope MOPs.
MOBATD is a multi-objective bat algorithm that incorporates the dominance concept with the decomposition approach. Whilst decomposition simplifies the MOP by rewriting it as a set of Tchebycheff Approach, solving these problems simultaneously, within the BAT framework, might lead to premature convergence because of the leader selection process which uses the Tchebycheff Approach as a criterion. Dominance plays a major role in building the leaders archive, allowing the selected leaders to cover less dense regions while avoiding local optima and resulting in a more diverse approximated Pareto front. The results from 5 standard MOPs show that the MOBATD outperforms some developmental methods based on decomposition. All the results were achieved by MATLAB (R2017b).

Keywords: Multi-objective problem, Multi-Objective Bat Algorithm, Decomposition Property, Performance Measure.

استخدام خوارزمية الخفاش للدوال المتعددة المستندة الى خاصية تجزئة دوال الهدف

رجوان حمود شياع , عراق طارق عباس*
قسم الرياضيات, كلية العلوم جامعة بغداد, بغداد, العراق

الخلاصه
الإنسان مستوحى بشكل كبير من الطبيعة. الطبيعة لديها القدرة على حل المشكلات المعقدة بطريقتها المميزة الخاصة. أصبحت المشاكل من حولنا أكثر تعقيدًا في الوقت الفعلي وفي نفس الحالة فإن طبيعتنا الأم ترشدنا لحل هذه المشاكل الطبيعية. تعطي الطبيعة بعض الطرق المنطقية والفعالة لإيجاد حل لهذه المشاكل.

*Email: Iraq.t@sc.uobaghdad.edu.iq

تعمل الطبيعة كمحسن لحل المشاكل المعقدة. التحلل هو استراتيجية أساسية في التحسين التقليدي متعدد الأهداف. ومع ذلك ، لم يتم استخدامه على نطاق واسع حتى الآن في التحسين التطوري متعدد لأهداف.

على الرغم من أن التقنيات الحسابية لحل مشكلات التحسين متعددة الأغراض (MOPs) كانت متاحة لسنوات عديدة ، إلا أن التطبيق الأخير للخوارزمية التطورية (EAs) لهذه المشكلات يوفر وسيلة لحل MOPs واسعة النطاق.

MOBATD هي خوارزمية خفاش متعددة الأهداف تدمج مفهوم الهيمنة مع نهج التحلل المقترح. في حين أن التحلل يبسط المشكلة متعددة الأهداف (MOP) بإعادة كتابتها كمجموعة من نهج Tchebyche ، فإن حل هذه المشاكل في وقت واحد ، في إطار BAT ، قد يؤدي إلى التقارب المبكر بسبب عملية اختيار الزعيم التي تستخدم نهج Tchebyche كمعيار . تلعب الهيمنة دورًا رئيسيًا في بناء أرشيف القادة مما يسمح للقادة المختارين بتغطية المناطق الأقل كثافة وتجنب البصريات المحلية وينتج عنها جبهة باريتو تقريبًا أكثر تنوعًا. تظهر النتائج التي تم الحصول عليها من 5 MOPs القياسية أن MOBAT/D يتفوق على بعض الطرق التطويرية القائمة على التحلل. تم تنفيذ جميع النتائج بواسطة (MATLAB (R2017b).

Introduction

An individual might want to augment the opportunity of being sound and well off while as yet having some good times and time for loved ones. A product designer would be keen on finding the least expensive test suite while accomplishing full inclusion (e.g., proclamation inclusion, branch inclusion and choice inclusion). While endorsing radiotherapy to a malignant growth persistent, a specialist would need to adjust the assault on the tumor, the expected effect on the sound organs, and the general state of the patient. These MOPs can be seen in different fields, having a similar problem of simultaneously seeking after a few, frequently interfacing destinations.

In multi-objective enhancement, for the most part, there is no single ideal arrangement, yet rather a lot of Pareto ideal arrangements. Normally, thickness estimation assumes a central job in the developmental procedure of multi-objective enhancement for a calculation to obtain an agent and various guesses of the Pareto front [1, 2].

In multi-objective improvement, it is commonly seen that the interface among closeness and assorted variety necessities is exasperated with the expansion of the quantity of goals [3] and that the Pareto strength loses its electiveness for a high-dimensional space but, however, functions admirably on a low-dimensional space [4]. Enlivened by these two perceptions, bi-objective development changes over a given multi-objective enhancement problem into a bi (objective) advancement problem with respect to closeness and assorted variety. Afterward, it handles the problem utilizing the Pareto strength connection in this bi-objective area.

MOBAT is proposed to discover the Pareto ideal set for multi-objective capacities by differing loads [5]. Additionally, in a previous work [6], the creator present stretched out BAT to take care of multi-objective problems and detail a MOBAT. We will initially approve it against a subset of multi-objective test capacities. At that point, we will apply it to take care of structure enhancement problems in building, for example, bi-objective shaft plan. In crafted by the paper [7] thought about MOBAT as an organic motivated meta-heuristic and have effectively applied it to take care of the difficult floor arranging in VSLI plan. A MOOP was proposed [8] to accomplish both of the referenced destinations. For this reason, another straightforward enhancement calculation known as Bat Algorithm (BAT) in light of WSM was utilized to determine the MOOP. Subsequently, from the literature, we can say here that no examination was performed before that consolidates MOBAT and the deterioration strategy.

A basic problem that regularly emerges in an assortment of fields like example acknowledgment, AI, picture handling, and measurements, is the multi-objective improvement problem, with the end goal that this field is a significant piece of exploratory MOBAT calculation. Numerous calculations exist to overcome this problem, one of which is the Strength Pareto Evolutionary Algorithm II (SPEAII). In any case, it has inadequacy of stalling out in neighborhood optima. To get improved outcomes, we have moved to the utilization of meta-heuristic calculations. Meta-heuristics give the benefit of the investigation and abuse in an inquiry space. This prompts better worldwide and nearby hunting activity. In this paper, we present another calculation dependent on the deterioration meta-heuristic calculation to limit computational endeavors of the field of multi-objective problems.

We start this paper with section 1 that gives the introduction to the work. Section 2 gives the definitions and ideas. Contributions by researchers in the fields of meta-heuristics and optimality problems have been considered. Section 3 gives the exploration procedure used to arrive at the resultant calculation dependent on Bat calculations. Section 4 gives the subtleties of the disintegration property. In section, 5 we introduce a proposed calculation. In section 6, our proposed work is assessed on 2 benchmark datasets. Section 7 presents the outcome and conversation of our work in subtleties. The last section 8 gives the conclusions of our work and its possible future extensions.

Definitions and Basic Concepts

In MOPs, because of the associating idea of destinations, there is normally no single ideal arrangement, but instead a lot of elective arrangements, known as Pareto ideal arrangements. These arrangements are ideal as they have no different arrangements in the quest spaces that are predominant for all destinations considered. Transformative calculations Evolutionary Algorithms (EAs) are a class of stochastic streamlining techniques that mimic the procedure of characteristic advancement. EAs have been perceived to be appropriate for MOPs because of their attributes of 1) low prerequisites on the difficult properties, 2) being fit for taking care of huge and exceptionally complex inquiry spaces, and especially 3) population-based property which can search for a set of solutions in a single optimization run, each representing a particular performance trade-off amongst the objectives.

Definition 1 [9]. A feasible solution $\widetilde{x} \in X$ is an efficient solution if there does not exist any other feasible solution $x \in X$ such that $z(x) \leq z(\widetilde{x})$, then $z(x)$ is called a non-dominated point where $z(x)$ is a set. If $x; x' \in X$ are such that $z(x) \leq z(x')$, we say that x dominates x' and $z(x)$ dominates $z(x')$. If $z(x) = z(x')$, then x and x' are equivalent. Y_N denotes the set of all non-dominated solutions of Y and X_E denotes the set of efficient solutions. Fiq.1 shows the details for the dominance.

Fig. 1 Dominations in the Pareto sense in a bi-objective space

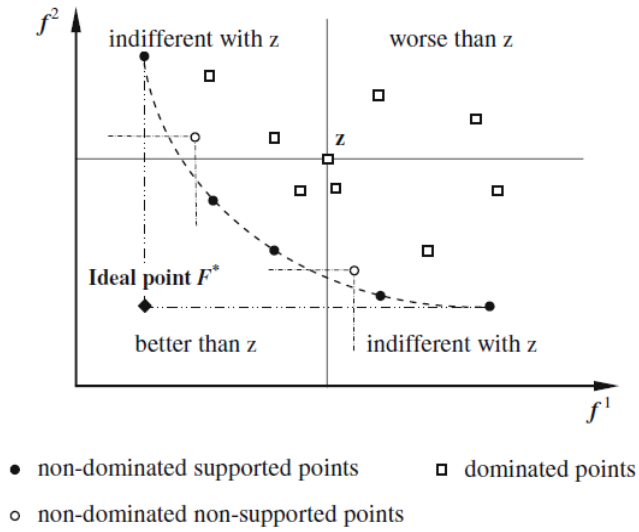

Figure 1- Domination in the Pareto sense in a bi-objective space

Definition 2. [9] A complete set X_E is a set of efficient solutions such that all $x \in X \setminus X_E$ are either dominated by or equivalent to at least one $x' \in X_E$. i.e., for each non-dominated point $y \in Y_N$ there exists at least one $x \in X_E$ such that $z(x) = y$.

In this paper we utilize the task problem with destinations to outline our techniques and we will propose a calculation for the specific arrangement (finding a negligible or maximal complete arrangement) of this problem.

In Pareto optimization, the aim is to find the set of "efficient" solutions in an exact or in an approximate way. Exact methods seek to solve a problem to guarantee optimality but their execution on large real world problems usually requires too much computation time. For practical uses, approximate methods seek to find high quality solutions (not necessarily optimal) within reasonable computation time. We have two classes of approximate methods [10]:

1. Heuristics: which are exceptionally specific and inexact techniques that misuse information on the difficult space.
2. Meta-heuristics: which have been effectively applied to fathom a wide scope of combinatorial streamlining problems, since they are not planned explicitly for a specific problem.

Three choices are then workable for taking care of the multi-objective task problem: either to utilize accurate strategies when it is conceivable [11, 12, 13], to approximate techniques such as those previously depicted [14], or to cross-breed techniques, joining the two points of reference strategies.

Bat Algorithm

Yang [15] proposed another improvement calculation, known as BAT, in view of multitude insights and the conduct of bats. The pieces of the echolocation qualities of a small scale bat can be reenacted utilizing the BAT. BAT is basic, adaptable, and simple to execute. It proficiently takes care of a wide scope of problems, especially profoundly nonlinear problems, and gives promising ideal arrangements. BAT functions admirably with entangled problems and offers the best arrangement inside a brief timeframe. Nevertheless, BAT also shows several accompanying weaknesses. The union rate is quick at the beginning phase and afterward eases back down. It does not play out a numerical investigation to connect the boundaries with the combination rates. Thippa [16] attempted to grow Firefly-BAT (FFBAT) advanced Rule-Based Fuzzy Logic (RBFL) forecast calculation for diabetes. Furthermore, Das and [17] clarified a novel coronary illness expectation dependent on hybridization of OFBAT with RBFL classifier. It, likewise, does not accomplish the best qualities for most applications.

In order to reach the vicinity of the target, each bat is randomly assigned a frequency qi of emitted pulse drawn uniformly from an interval $[Q_{min}, Q_{max}]$ and the frequency can be automatically adjusted within the same range. The pulse emission rate r_i can also be adjusted in an interval [0, 1], where 0 denotes no pulse at all, and 1 denotes the maximum pulse emission rate. Given a virtual bat and a position updating strategy of its position P_i and velocity v_i in a D-dimensional search space, the new solution P_i^t, frequency Q_i , and velocity v_i^t (of each bat in the population) at generation t are generated by the following equations:

$$Q_i = Q_{min} + (Q_{max} - Q_{min})^{\beta} \quad (1)$$
$$v_i^t = v_i^{t-1} + \left(P_i^{t-1} - P_{gbest}^t\right) f_i \quad (2)$$
$$P_i^t = P_i^{t-1} + v_i^t \quad (3)$$

where the estimation of β is an irregular number inside the scope of [0,1], Q_i is the recurrence of the ith bat that controls the range and speed of development of the bats, vi and Pi mean the speed and position of i_{th} bat, separately, and P_{gbest}^t represents the current worldwide best situation at time step t . So as to upgrade the decent variety of the potential arrangements, a neighborhood search approach is applied to those arrangements that meet a specific condition in the bat calculation. In the event that the arrangement meets the condition, an irregular walk (Eq. (4)) is utilized to create another arrangement:

$$P_{new} = P_{old} + \theta \, \mathrm{A}^{\mathrm{t}} \quad (4)$$

in which $\theta \in [-1,1]$ is an arbitrary number that endeavors to the force and course of the irregular walk and A^t indicates the normal uproar of all bats up until now.

The loudness A_i and the beat rate r_i must be refreshed in every cycle. The commotion ordinarily diminishes when a bat discover its prey while the beat rate increments. The clamor A_i and heartbeat rater_i are refreshed as follows:

$$A_i^{t+1} = \varepsilon A_i^t \quad (5)$$
$$r_i^{t+1} = r_i^0 \, [1 - exp(-\mu t)] \quad (6)$$

in which ε and μ are constant values, both are equal to 0.9. The loudness and pulse rate are updated only if the new solution is accepted.

Multi-Objective Bat Algorithm

Bats are well evolved creatures with wings and echolocation capacity. Around 996 distinctive bat species have been distinguished around the world, and they represent roughly 20% of all vertebrate species [18]. In a previous report [19], another improvement calculation known as BAT is proposed based on swarm insight and bat perception. One can reproduce the pieces of the echolocation qualities of a miniaturized scale bat by utilizing the BAT. The benefits of this calculation incorporate straightforwardness, adaptability and simple execution. Moreover, the calculation effectively takes care of a wide scope of problems, for example, profoundly nonlinear ones [16]. BAT likewise rapidly gives promising ideal arrangements and functions admirably with convoluted problems. The inconveniences of this calculation are that the union occurs rapidly at the beginning phases a reduction takes place in the intermingling. Moreover, no numerical examination connects the boundaries with the intermingling rates. The most reasonable qualities for most applications are likewise muddled [20].

Decomposition Property

As the soonest multi-objective improvement method that can be followed back to the middle of the last century [21], the deterioration based methodology can be a decent option in managing MOPs. Rather than scanning the whole quest space for Pareto ideal arrangements, decay based calculations break down a MOP into a lot of scalar improvement sub-problems by many weight vectors and the accomplishment of scalar punch work via an Achievement Scalar Function (ASF). Regularly utilized ASFs incorporate weighted total, Tchebycheff, vector edge separation scaling, and limit crossing point [7, 22]. In the deterioration based methodology, since the ideal point related with each search course (weight vector) is focused on, adequate determination pressure advances can be given and furthermore a decent dispersion among arrangements can be kept up in a high-dimensional space. As per the predefined different focuses on, the disintegration based methodology can be additionally separated into search headings-based and reference focuses-based calculations [8]. The test results have checked the adequacy of the proposed system in adjusting nearness and decent variety. Then again, scientists have additionally structured a scope of deterioration-based calculations, particularly for multi-objective improvement. Hughes [23] utilized various single Pareto, inspecting Multiple Single Objective Pareto Sampling (MSOPS) to address MOPs. In MSOPS, a lot of T weight vectors are utilized to assess every arrangement, utilizing a weighted min-max technique, which is opposed to the Multi-Objective Evolutionary Algorithm with Decomposition MOEA/D where an answer compares to only one weight vector. MSOPS has been found to perform better than the Non-Sorting Genetic Algorithm II (NSGA-II) in a few MOPs [24]. Later on, Hughes [25] gave MSOPS-II two augmentations to MSOPS. The principal augmentation is a strategy that utilizes the current populace as contribution to produce a lot of objective vectors, and the subsequent one is to diminish the time unpredictability of wellness task of the first calculation. As related to the latter, the author consolidated the collection technique from MSOPS with the coordinated line search, dependent on approximated nearby inclination [7]. The proposed calculation has shown its seriousness on an obliged work with an inward Pareto front having up to 20 goals.

Przybylski [26] introduced the idea of summed up decay. Summed up disintegration furnishes a structure with which the DM can direct the pursuit calculation toward the Pareto front with the ideal conveyance of ideal arrangements. This methodology permits disintegration put-together calculations to center with respect to just the closeness to the Pareto front (nearness, consistency and extensity). Joined with the cross-entropy technique, the proposed approach has appeared to perform better than MOEA/D [27].

The Proposed Algorithm (MOBAT/D)

In this section, we first present a couple of known definitions. Then, we present the arrangement of the proposed figuring. Next, we portray the health task process. Finally, the systems for mating and regular decision methods are presented .

Bats are vertebrates with wings and echolocation limit [23]. In light of multitude understanding and bat recognition, an earlier work [28] proposed another improvement computation known as BAT. One can reenact the bits of the echolocation characteristics of a littler scope bat by using the BAT. The upsides of this figuring are its ease, versatility, and basic execution. Additionally, the estimation capably deals with a wide extent of problems, for instance, particularly nonlinear problems. BAT, moreover, gives promising perfect courses of action quickly and works decently with puzzling problems. Disadvantages of this computation are that the blend happens quickly at the starting phases while slowing down at the intermixing rate. Besides, no logical assessment exists that interfaces the boundaries with the intermixing rates. To procure a superior perfect strategy for multi-objective limits using BAT, specialists developed a count called MOBAT by introducing two new sections, which are the document and pioneer, as found in the MOPSO computation [27]. The narrative is liable for saving and restoring the most significant non-overpowered and non-controllable Pareto perfect game plans that have been known to date. The narrative in this manner shows an essential unit, which is the control unit of the document. This unit controls the amount of no controlling courses of action when new no controlling plans exist . Simultaneously, the chronicle size is finished. During the procedure of replication, the non-commanded arrangements acquired against the chronicle populace are looked at. Thus, four distinct circumstances will be acquired:

1. The new part is signed into the document if an individual from the file is in charge, in which the client is permitted access to the chronicle.
2. The new arrangement overwhelms the arrangement of at least one of the others in the file. For this situation, the arrangement or the prevailing arrangements in the chronicle must be erased. The new arrangement will have the option to get to the document .
3. If neither the new solution nor the archive member dominates each other, a new archive solution must be added.
4. If the file is full, the system component is run first to repartition the objective space, decide the busiest part, and erase one of the current arrangements. The new arrangement along these lines ought to be consolidated into a less jam-packed opening in the framework to improve the last enhancement of Pareto's rough arrangement.

Expanding the likelihood of erasing an answer is relative for arrangements in a hyper cup (fragment). A unique case exists, in which an answer is embedded by hypercube. For this situation, all sections are reached out to cover new arrangements. Hence, different arrangements can likewise be changed. The subsequent instrument is choosing a pioneer, where pioneer coordinates are chosen for an individual inside the examination region. In MOBAT calculation, the most appropriate arrangement acquired is utilized. This pioneer guides individuals inside the exploration territory to acquire an answer near the most reasonable arrangement. However, solutions cannot be in a multi-objective research space compared with Pareto's ideal concepts. The pioneer determination system is intended to deal with the problem. A file is created that contains the most appropriate non-prevailing arrangements acquired. The pioneer chooses the section from the jam-packed fragments of the space arrangement and offers one of the non-prevailing arrangements. Choice is performed through the roulette wheel with the accompanying opportunities for each hypercube:

$$P_i = \frac{c}{N_i} \tag{7}$$

where c is a steady number higher than 1 and N_i is the quantity of acquired Pareto ideal arrangements in the ith portion. The condition demonstrates that the absence of blockage in the hypercube shows a high likelihood in the proposition of another pioneer.

MOBAT/D Procedure with Decomposition

Set $k := 0$ and velocity =0, μ=0.1, $r0 = 0.5, A = 0.6$.
Randomly initialize Point P_i $for\ n.population$;
Calculate the fitness values of initial Population: $f\ (P\ )$;
Find the non-dominated solutions and initialize the archive with them
WHILE (the termination conditions are not met)
1) BAT Steps
$\mathrm{Q = Qmin + (Qmin - Qmax) * rand}$ (equation 1)
$\mathrm{P_{leader1}} = \text{Select Leader (archive)}$
$\mathrm{V_{(t+1)} = V_{(t)} + \left(P_{leader1} - P_{(t)}\right) * Q}$ (equation 2)
$\mathrm{P_{new} = P_{(t)} + V_{(t+1)}}$ (equation 3)
If rand > r , r=1
$\mathrm{P_{leader2}} = \text{SelectLeader(archive)}$
$\mathrm{P_{new}=P_{(t)}+rand * \left(P_{leader2} - P_{(t)}\right)}$
End
$\boldsymbol{if}\ P_{new}\ dominated\ on\ P_{(t)}$ & $(rand < A), A = 0$
$\mathrm{P_{(t)}=P_{new}}$
End

If rand $<(\frac{\mathbf{1-(k-1)}}{\boldsymbol{Max\ iteration-1}})^{\mathbf{1/\mu}}$
S =Mutation($P_{(t)}$) , i.e. reach the Max iteration.
$\boldsymbol{if}\ P_{new}\ dominated\ on\ P_{(t)}$ & $(rand < A)$
$P_{(t)}$=S
End
End
Find the non-dominated solutions
Update the archive with respect to the obtained non-dominated solutions
If the archive is full
Run the grid mechanism to omit one of the current archive members
Add the new solution to the archive
end if
If any of the new added solutions to the archive is located outside the hyper cubes
Update the **grids to** cover the new solution(s)
end if
$\boldsymbol{Increase}$ **r** $\boldsymbol{and\ reduce\ A}$
$Set\ k := k + 1;$
END WHILE

Simulation Experiment and Analysis
Performance Measures

Both quantitative and subjective examinations are made to approve the EPMOPSO calculation against different MOPSOs. For subjective examination, the plots of definite Pareto fronts are introduced. Concerning the quantitative examination, intermingling metric gravitational separation, Inverted gravitational separation, and hyper volume [1] are utilized, as appeared in Equations (8), (9) and (10).

Generational distance (GD) in deciding if the arrangements of Q can be incorporated with the arrangement of P* or not. The utilization of the GD metric is fitting in light of the fact that it evaluates the normal separations of the arrangement sets of Q from P*, as follows:

$$GD = \frac{\sqrt[p]{\sum_{i=1}^{R}(d_i^p)}}{R} \tag{8}$$

$$IGD = \frac{1}{R}\ (\sum_{i=1}^{R} \min(\sqrt[p]{\sum_{i=1}^{R}(d_i^p)}\ )) \tag{9}$$

where R is the culpability allocated to the P^* set, which is otherwise called the IGD metric and measures the consistency of circulation of the got arrangements regarding scattering and expansion. The normal separation is determined for each purpose of the real PF.

Hyper volume pointer (Hyper volume) gauges the volume of the objective space that is feebly ruled by a Pareto Front (PF) estimation (A). Hyper volume utilizes a reference point $v*$ which signifies an upper bound over all goals. $v*$ is characterized as the most exceedingly terrible objective esteems found in A ; for example, v* is commanded by all arrangements in A. Utilizing the Lebesgue measure (Λ), which is a Cantor set of zero Lebesgue measure for all real nonzero A, where the spectral measures are purely singular continuous, the hyper volume is characterized as:

$$HV(A) = \Lambda(\cup \{x\ |a \prec x \prec v^*, a \in A) \tag{10}$$

Table-1 shows the consequences of applying IGD. Table 2 shows the outcomes of utilizing hyper volume. The last column presents the $p - value$ of two followed combined t-test between the MOPSOD and different strategies, where the intense textual style demonstrates a factually huge contrast. A portray of the PF valid and the PF approximated for the five calculations under scrutiny can be seen in Figure-1.

Test Functions

To show the proficiency of the proposed MOBAT/D calculation benchmark problems are chosen, i.e., multi-objective or test functions (ZDT1) [28].

The ZDT suite comprises six bi-objective test problems, with ZDT1, DT4 and ZDT5 having a raised Pareto front, ZDT2 and ZDT6 having an inward Pareto front, and ZDT3 having a detached Pareto front. All the problems are distinct as in the Pareto ideal set and can be acquired by streamlining every choice variable independently. ZDT4 and ZDT6 are multi-modular (i.e. they have various neighborhood Pareto fronts) and ZDT6 likewise has a non-uniform planning.

Not that the test function ZDT5 was not mentioned in any paper related to multi-objective functions [28].

Results and Discussion

This section is dedicated to the presentation of the results of the proposed calculations. The test results revealed the adequacy of the proposed technique in adjusting nearness and assorted variety. Specialists have likewise planned a scope of disintegration-based calculations, particularly for multi-objective enhancement. In order to know how competitive MOBAT-D was, we contrasted it to two multi-objective PSO calculations that are illustrative of the cutting edge. These two calculations are MOPSO [29] and MOEAD [30]. Every calculation is run multiple times to accomplish the metrics of IGD, GD and HV for each test work. The mean qualities and standard deviation of the outcomes are gathered in Table-1.

Table 1-Comparative between algorithms by using IGD when (M=2(No. of objective)) and (N=100 (No. of variables)), D = No. of Diminutions

Problem	D	MOEAD	NSGAII	MOPSO	SPEA2	MOBATD
ZDT1	30	1.6911e-1 (6.74e-2) +	1.7321e-1 (1.14e-1) +	3.8729e+1 (1.02e+1) -	4.1127e-1 (7.52e-1)-	1.5444e-1 (8.38e-2)+
ZDT2	30	3.5107e-1 (2.02e-1) =	5.0010e-1 (1.48e-1) =	4.2262e+1 (9.23e+0) -	5.2527e-1 (1.19e-1) =	6.2694e-1 (1.43e+0)=
ZDT3	30	2.1093e-1 (1.32e-1) -	1.4308e-1 (8.31e-2) =	3.9992e+1 (1.06e+1) -	1.8784e-1 (2.54e-1)=	1.3432e-1 (7.95e-2) +
ZDT4	10	5.2515e-1 (1.44e-1) +	3.2019e-1 (1.83e-1) +	1.7544e+1 (8.39e+0) -	1.9078e-1 (1.26e-1) +	1.0697e+1 (5.11e+0)=
ZDT6	10	8.9041e-2 (3.05e-2) -	9.8414e-2 (4.39e-2) -	2.6425e+0 (3.74e+0) -	1.0295e-1 (5.11e-2) -	3.7708e-3 (2.24e-3)+
+/-/=		2/2/1	2/1/2	0/5/0	1/2/2	**3/0/2**

Table 3-Comparative between algorithms by using Hyper Volume (M=2(No. of objective)) and (N=100 (No. of variables)), D= No. of Diminutions

Problem	D	MOEAD	NSGAII	MOPSO	SPEA2	MOBATD
ZDT1	30	5.0564e-1 (8.38e-2) -	5.6080e-1 (7.54e-2) +	0.0000e+0 (0.00e+0) -	5.6455e-1 (5.63e-1) -	5.0741e-1 (2.87e-2)+
ZDT2	30	1.5340e-1 (8.58e-2) =	7.0277e-2 (6.29e-2) -	0.0000e+0 (0.00e+0) -	5.7971e-2 (4.63e-2) -	1.9041e-1 (1.71e-1)+
ZDT3	30	4.8563e-1 (1.21e-1) =	6.2903e-1 (9.04e-2) +	0.0000e+0 (0.00e+0) -	6.1536e-1 (8.03e-1) -	5.0968e-1 (1.67e-2)+
ZDT4	10	1.6777e-1 (1.05e-1) +	4.3244e-1 (1.65e-1) +	0.0000e+0 (0.00e+0) =	5.1645e-1 (1.20e-1) +	0.0000e+0 (0.00e+0)=
ZDT6	10	2.7826e-1 (2.60e-2) -	2.6937e-1 (4.75e-2) -	2.2235e-1 (1.88e-1) -	2.6466e-1 (5.25e-2) -	3.8763e-1 (2.26e-3)+
+/-/=		1/2/2	3/2/0	0/4/1	1/4/0	**4/0/1**

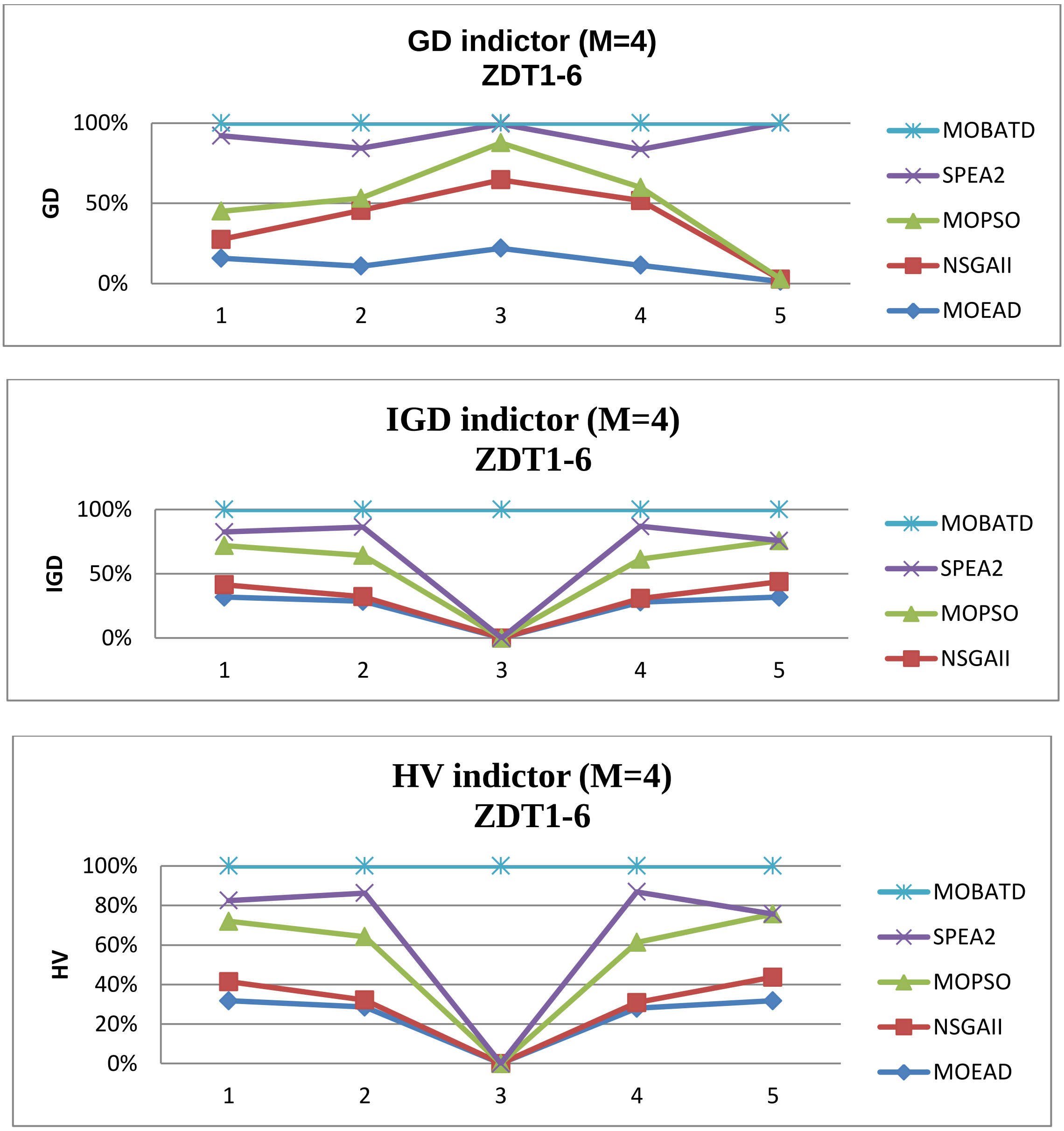

Figure 2-Shows the Priority of the Proposed Method (MOBAT/D) with Others

This graphs above shows that the MOBAT/D algorithm performs better than the set of algorithms. It can be seen that MOBAT/D has a fixed value that does not change over time.

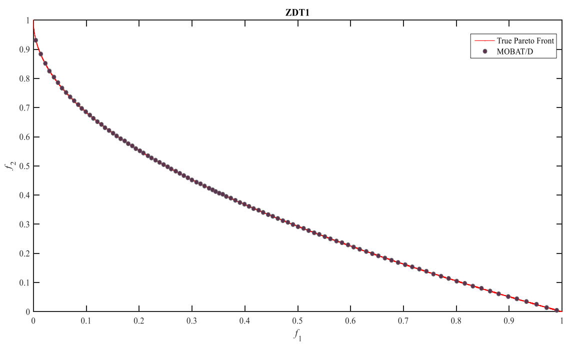

Figure 3-Plot of Pareto front for ZDT1 by MOBAT/D

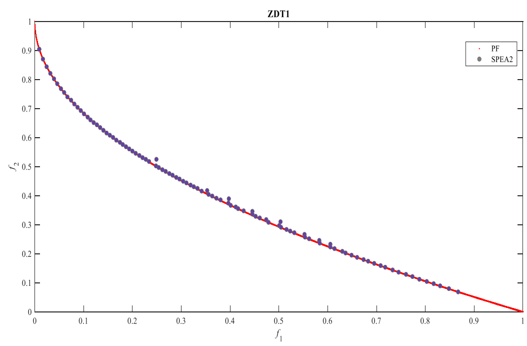

Figure 4-Plot of Pareto front for ZDT1 by MOEAD

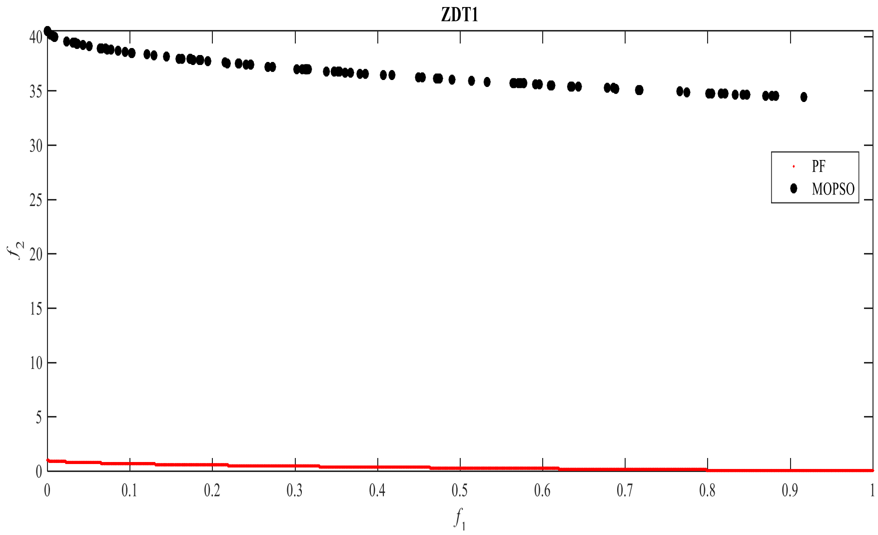

Figure 5-Plot of Pareto front for ZDT1 by MOPSO

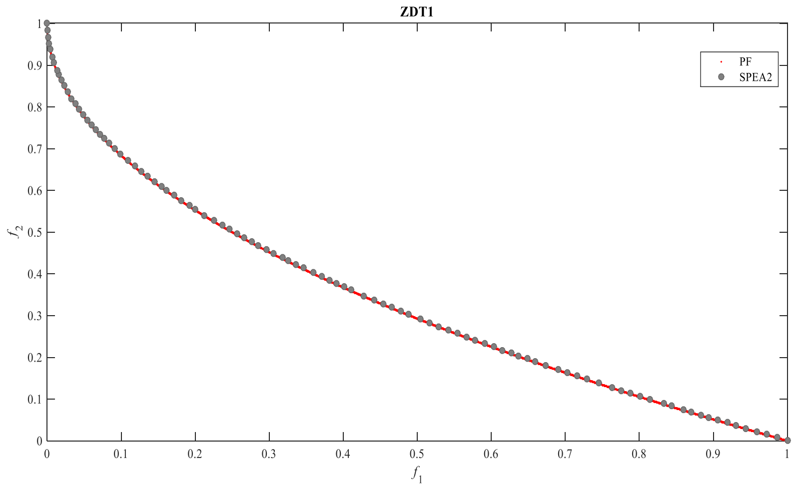

Figure 6-Plot of Pareto front for ZDT1 by NSGAII

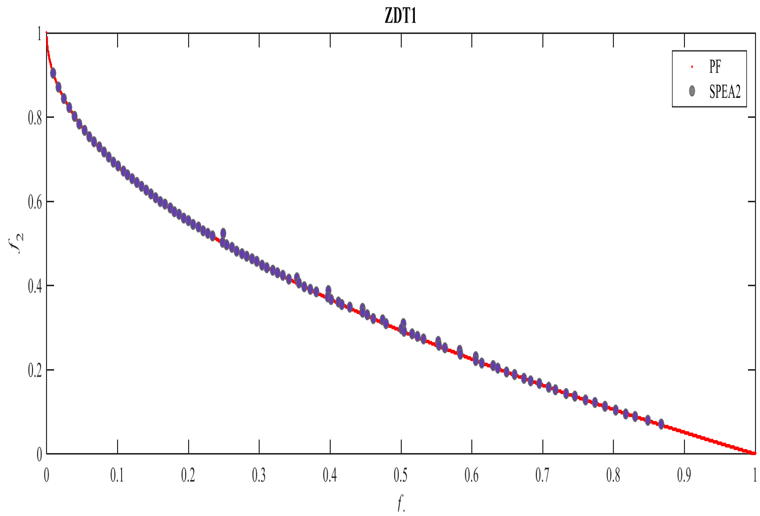

Figure 7-Plot of Pareto front for ZDT1 by SPEAII

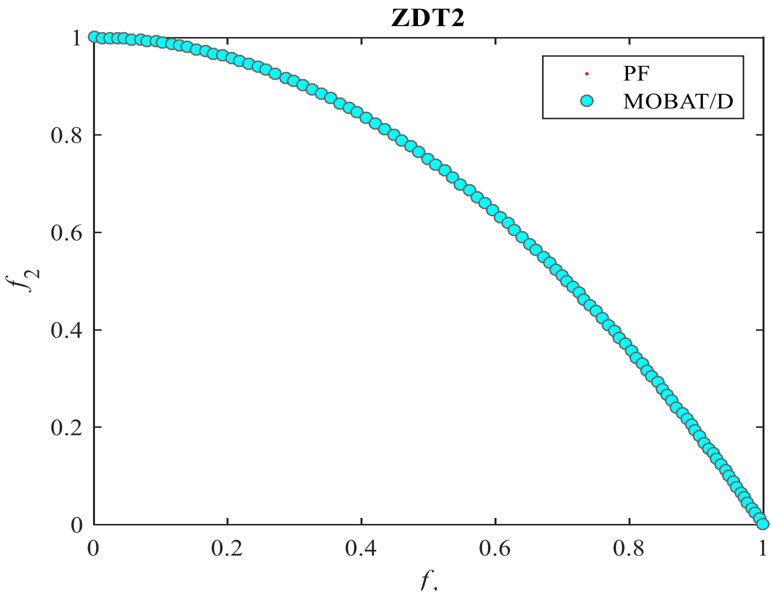

Figure 8- Plot of Pareto front for ZDT2 by MOBAT/D

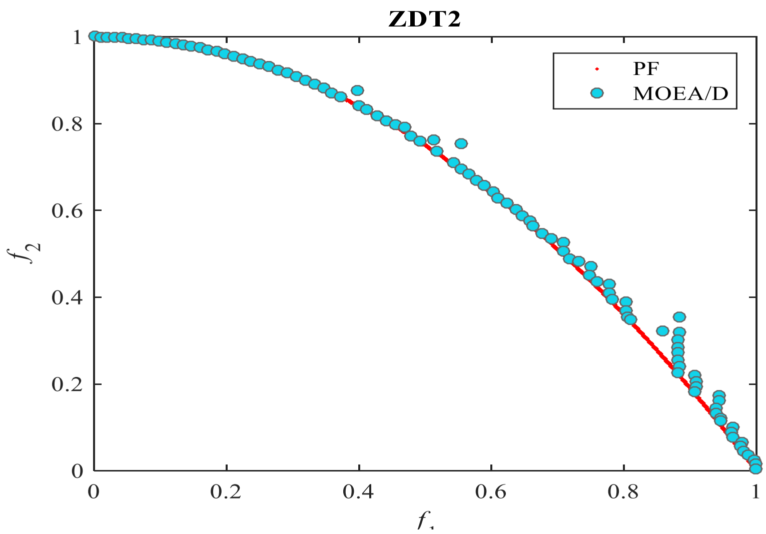

Figure 9- Plot of Pareto front for ZDT2 by MOEAD

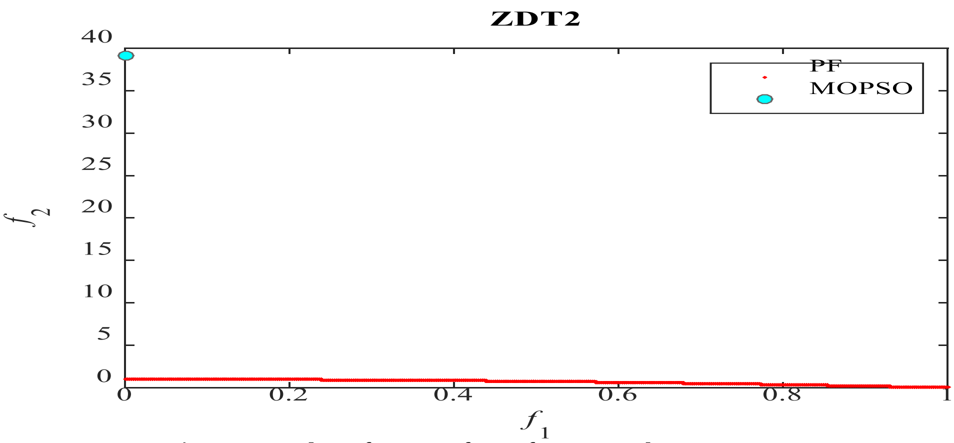

Figure 10-Plot of Pareto front for ZDT2 by MOPSO

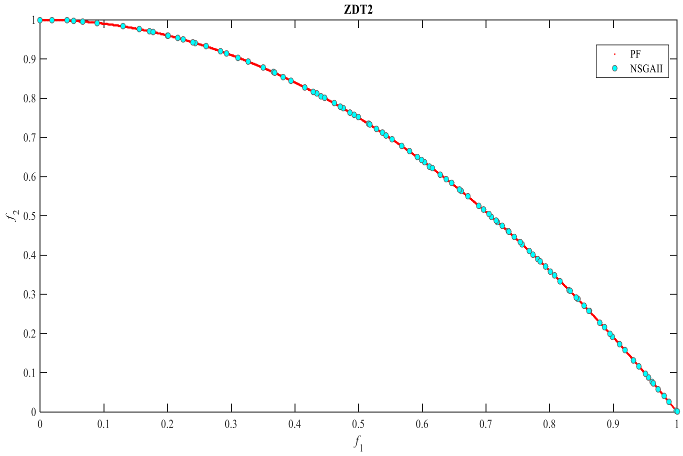

Figure 11-Plot of Pareto front for ZDT2 by MOPSO

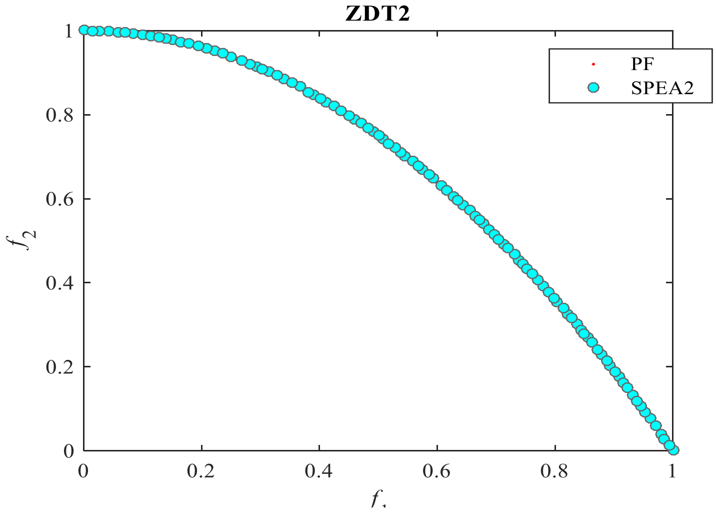

Figure 12-Plot of Pareto front for ZDT2 by SPEA2

To test the validity of the new research strategy, the MOBAT/D algorithm was compared to the results of the algorithms [MOEA/D, MOPSO, NSGAII and SPEA2]. The algorithms are drawn with turquoise points and the ideal Pareto fronts with red dots, to evaluate the results of our new method, the MOBATD algorithm, we took experimental results on the functions ZDT1 and ZDT2. The MOBAT/D algorithm approximated the solution with the curve of Pareto fronts. The MOBAT/D algorithm demonstrated its promising superiority over the other algorithms, as reflected by the results.

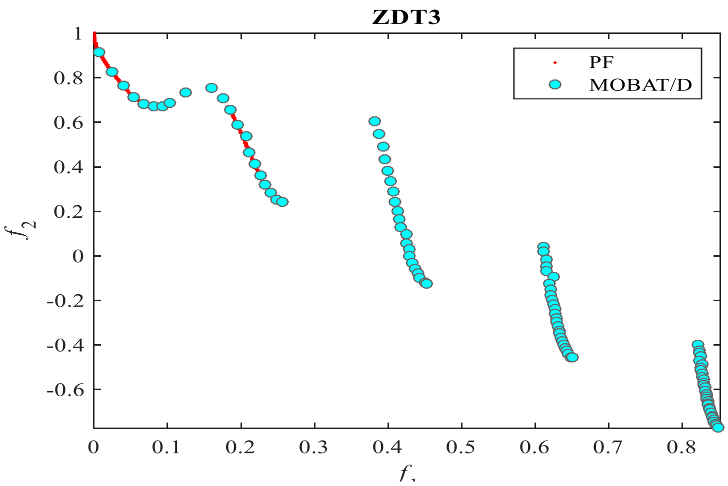

Figure 13- Plot of Pareto front for ZDT3 by MOBAT/D

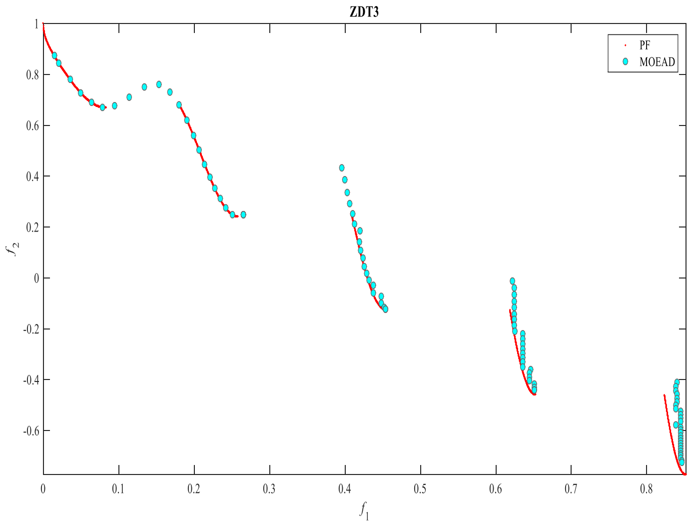

Figure 14-Plot of Pareto front for ZDT3 by MOEAD

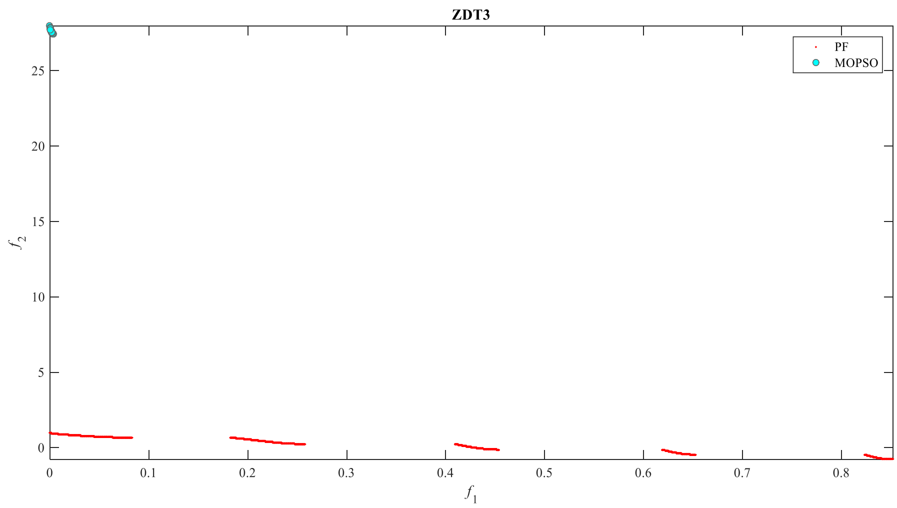

Figure 15-Plot of Pareto front for ZDT3 by MOPSO

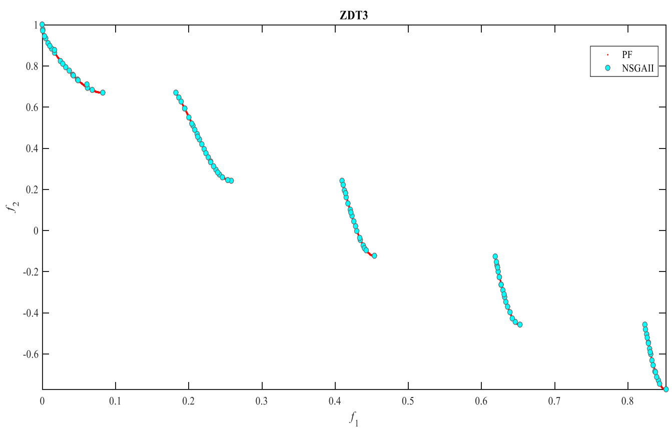

Figure 16-Plot of Pareto front for ZDT3 by NSGAII

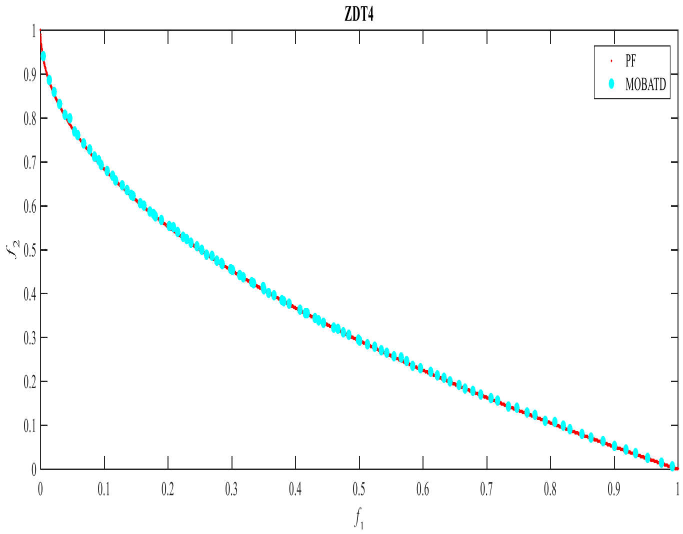

Figure 17-Plot of Pareto front for ZDT3 by SPEA2

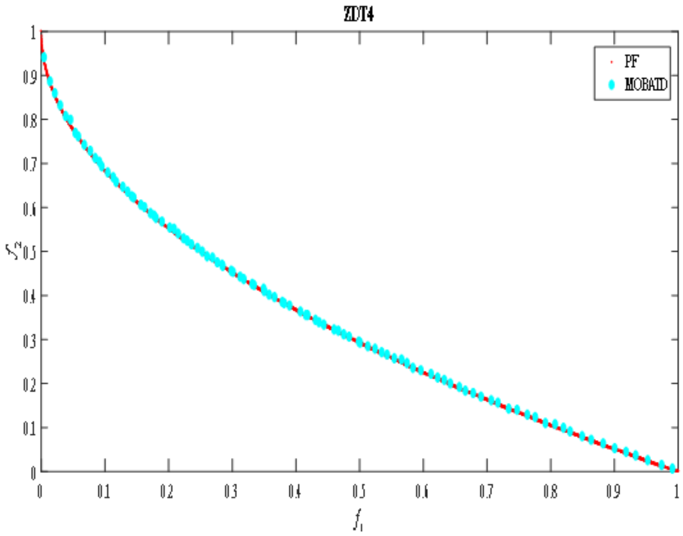

Figure 18- Plot of Pareto front for ZDT4 by MOBAT/D

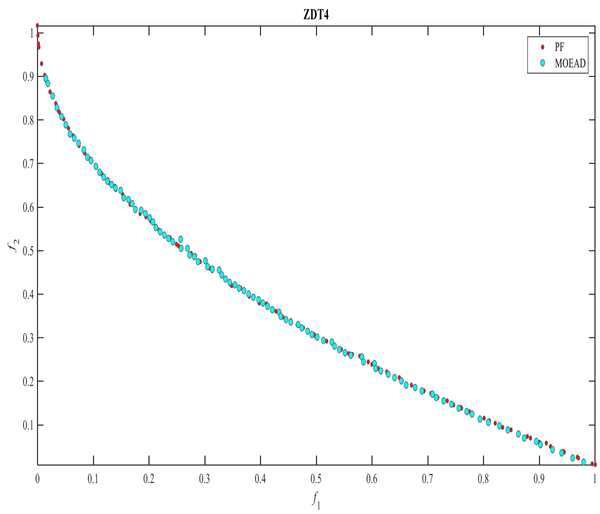

Figure 19-Plot of Pareto front for ZDT4 by MOEAD

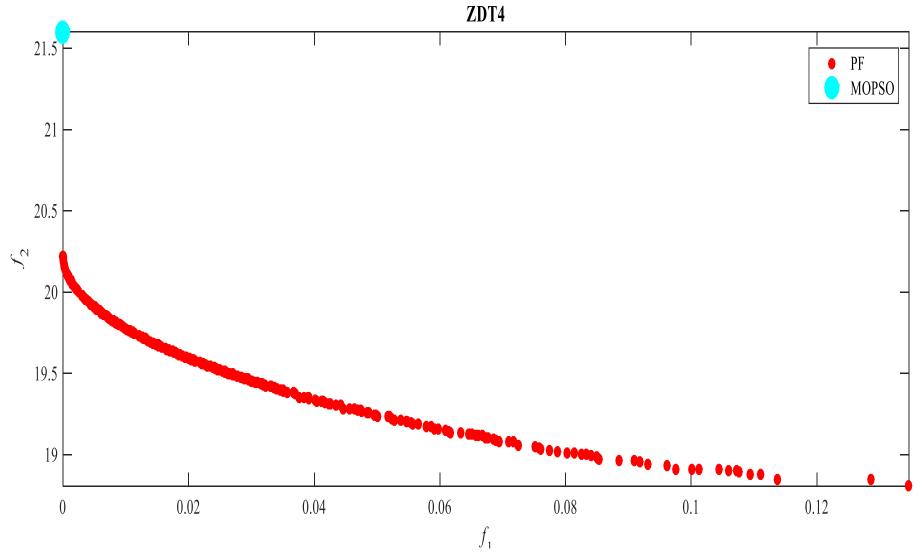

Figure 20-Plot of Pareto front for ZDT4 by MOPSO

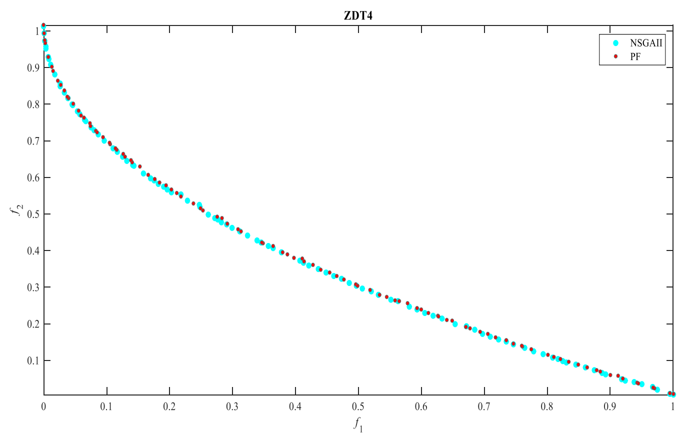

Figure 21-Plot of Pareto front for ZDT4 by NSGAII

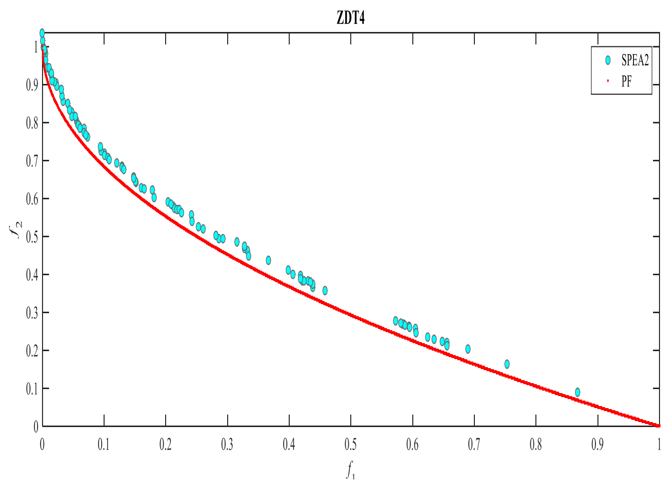

Figure 22-Plot of Pareto front for ZDT4 by SPEA2

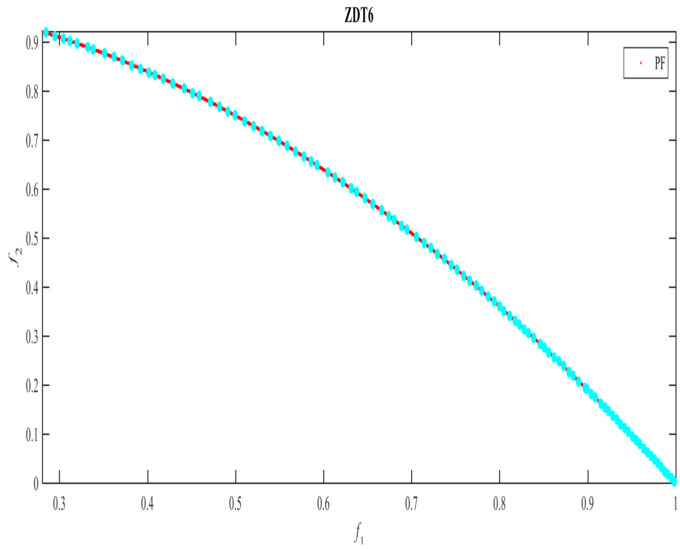

Figure 23-Plot of Pareto front for ZDT6 by MOBAT/D

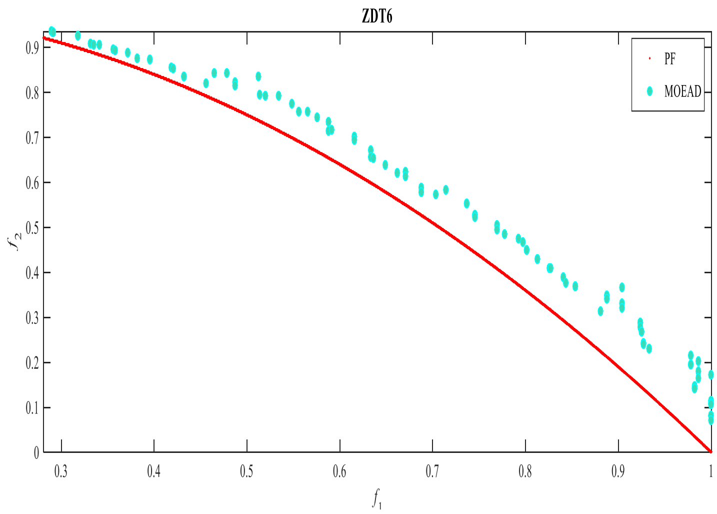

Figure 24-Plot of Pareto front for ZDT6 by MOEAD

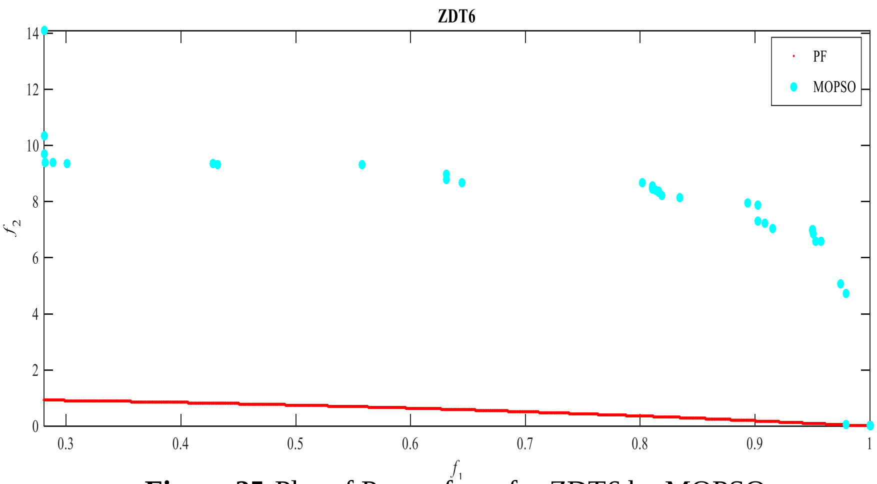

Figure 25-Plot of Pareto front for ZDT6 by MOPSO

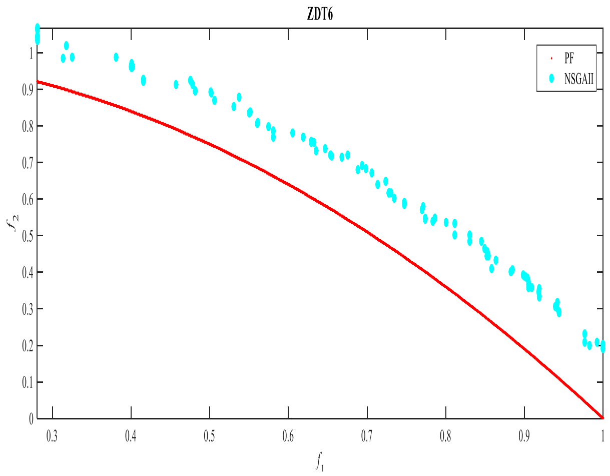

Figure 26-Plot of Pareto front for ZDT6 by MOPSO

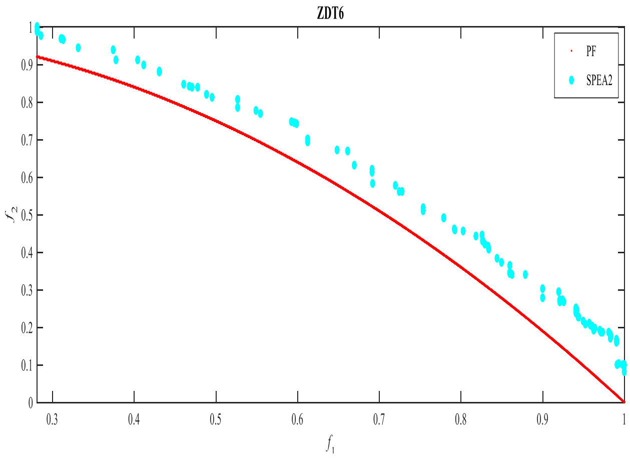

Figure 27- Plot of Pareto front for ZDT6 by MOPSO

It can be observed that evolution has some multi-purpose problems and that nature has ways to solve it. We created the new MOBATD algorithm and measured its efficiency with a set of algorithms (MOEAD, MOPSO, NSGAII and SPEA2). We found that those with the functions ZDT3, ZDT4 and ZDT6 have a high efficiency that surpasses all other algorithms.

Note that the test function ZDT5 is not mentioned in this paper because all variables inside it belong to the open interval, and that the function $f(x_1) = 1 + u(x_1)$ depends on the function $u(x_1)$ which implies time to get the approximate or to go the optimal solution; It is very difficult to use this function inside the new algorithm and, therefore, it was not included here [30].

Conclusions and Future Work

This paper the proposes a MOBAT based on decomposition strategy (MOBAT/D), in which MOPs is decomposed into a number of scalar optimization sub-problems, and each sub-problem is optimized by only using information from its several neighboring sub-problems in a single run. The three execution estimations (GD, IGD and HV) evidently show that MOBAT/D is significantly genuine and even outmaneuvers the selected MOBATs. The figures of Pareto fronts also show that MOBAT/D has the ability to produce relatively better-distributed Pareto fronts compared with the selected MOBATs.

Additional tests and assessments of the proposed approach are especially required. For future work, we focus on the parametric assessments for an increasingly broad extent of test problems, including discrete and mixed sort of progress problems. We endeavor to test the various assortment of the Pareto front to recognize the ways to improve this computation to suit a contrasting extent of problems. There are a few profitable techniques to achieve the arranged Pareto fronts, where a mixture of these systems may further improve MOBAT/D fundamentally. Further investigations can underline the display connection of this estimation with other notable procedures for multi-objective improvement. In addition, hybridization with various figuring may produce more sufficient results.

References

1. P. A. N. Bosman and D. Thierens. **2003**. The balance between proximity and diversity in multi-objective evolutionary algorithms. *IEEE Transactions on Evolutionary Computation*, **7**(2): 174-188.
2. M. Laumanns, E. Zitzler, and L. Thiele. **2001**.On the e_ects of archiving, elitism, and density based selection in evolutionary multi-objective optimization. In *Evolutionary Multi-Criterion Optimization* (EMO): 181-196.
3. R. C. Purshouse and P. J. Fleming. **2007**. On the evolutionary optimization of many connecting objectives. *IEEE Transactions on Evolutionary Computation*, **11**(6): 770-784.
4. D. W. Corne and J. D. Knowles. **2007**. Techniques for highly multi-objective optimization: some non-dominated points are better than others. In Proceedings of the 9th Annual Conference on Genetic and Evolutionary Computation (GECCO): 773-780.
5. Erfani T. and Utyuzhnikov S. **2011**. Directed search domain: a method for even generation of Pareto frontier in multi-objective optimization, Engineering Optimization, (in press).
6. Yang, X. S. **2011**. Bat algorithm for multi-objective optimization. *International Journal of Bio-Inspired Computation*, **3**(5): 267-274.
7. Laudis, L. L., Shyam, S., Jemila, C., & Suresh, V. **2018**. MOBAT: Multi Objective Bat Algorithm for Combinatorial Optimization in VLSI. *Procedia Computer Science*, **125**: 840- 846.
8. Remha, S., Chettih, S., & Arif, S. (2018). A Novel Multi-Objective Bat Algorithm for Optimal Placement and Sizing of Distributed Generation in Radial Distributed ystems. *Advances in Electrical and Electronic Engineering*, **15**(5): 736-746.
9. Karimi-Nasab M, Ghomi SMTF. **2012**. Multi-objective production scheduling with controllable processing times and sequencedependent setups for deteriorating items. *Int J Prod Res*, **50**(24): 7378–7400.
10. Desale, S., Rasool, A., Andhale, S., & Rane, P. **2015**. Heuristic and meta-heuristic algorithms and their relevance to the real world: a survey. *Int. J. Comput. Eng. Res. Trends*, **351**(5): 2349-7084.
11. Ramli MR, Abas ZA, Desa MI, Abidin, ZZ, Alazzam MB. **2018**. Enhanced convergence of Bat Algorithm based on dimensional and inertia weight factor. *J King Saud Univ Comput Inf Sci.* https://doi.org/10.1016/j.jksuci.2018.03.010.
12. H. W. Kuhn and A. W. Tucker. **1951**. Nonlinear programming. In the Second Berkeley *Symposium on Mathematical Statistics and Probability*, pp: 481-492. Berkeley, CA, University of California Press., 1951.
13. Q. Zhang and H. Li. **2007**. MOEA/D: A multi-objective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation*, **11**(6): 712-731.
14. E. J. Hughes. **2011**. Many-objective directed evolutionary line search. In Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation (GECCO), pp: 761-768.
15. X.-S. Yang. **2010**. A new meta-heuristic bat-inspired algorithm, in: Nature Inspired Cooperative Strategies for Optimization (NISCO 2010), Springer, 2010, pp.65–74.
16. Thippa Reddy, G., and Khare, N. **2016**. FFBAT-optimized rule based fuzzy logic Classifier.
17. Das and J. E. Dennis. **1998**. “Normal-boundary intersection: A new method for generating Pareto optimal points in multi-criteria optimization problems,” *SIAM J. Optim*., **8**(3): 631–657.
18. Messac, A. Ismail-Yahaya, and C. Mattson. **2003**. “The normalized normal constraint method for generating the Pareto frontier,” *Struct Multidisc. Optim*., **25**: 86–98, 2003.
19. Coello, C. C., & Lechuga, M. S. **2002**. MOPSO: A proposal for multiple objective particle swarm optimization. In Proceedings of the 2002 Congress on Evolutionary Computation. CEC'02 (Cat. No. 02TH8600) (**2**: 1051-1056). IEEE.
20. Qi, Y., Ma, X., Liu, F., Jiao, L., Sun, J., & Wu, J. **2014**. MOEA/D with adaptive weight adjustment. *Evolutionary computation*, **22**(2): 231-264.

21. Prakash, R., Lokeshgupta, B., & Sivasubramani, S. **2018**. Multi-objective bat algorithm for optimal placement and sizing of DG. In 2018 20th National Power Systems Conference (NPSC) (pp. 1-6). IEEE.

22. Yang, X. S. **2012**. Bat algorithm for multi-objective optimization. arXiv preprint arXiv:1203.6571.

23. P. Hansen, Bicriterion path problems, in: G. Fandel, T. Gal (Eds.). **1979**. Multiple Criteria Decision Making Theory and Application, in: Lecture Notes in Economics and Mathematical Systems, vol. 177, Springer Verlag, Berlin, 1979, pp. 109_127.

24. E. J. dw. MSOPS-II: A general-purpose many-objective optimizer. In IEEE Congress on Evolutionary Computation (CEC), pages 3944-3951, 2007.

25. Bhatia, H.L., Malhotra, R., Puri, M.C.: Bi-criteria assignment problem. Oper. Res. 19(2), 84–96 (1982).

26. Przybylski A., Gandibleux, X., Ehrgott, M.: Two phase algorithm for the bi-objective Assignment problem. Eur. J. Oper. Res. 185(2), 509–533 (2008).

27. Ulungu, E.L., Teghem, J.: The two phases method : an efficient procedure to solve bi-objective combinatorial optimization problems. Found Computer decision Science 20(2), 149– 165 (1994).

28. Peng G (2016) Multi-objective particle optimization algorithm based on sharing learning and dynamic crowding distance. Opt Int. J Light Electron Opt 127(12):5013–5020.

29. E. J. Hughes. Multiple single objective Pareto sampling. In IEEE Congress on Evolutionary Computation (CEC), volume 4, pages 2678-2684, 2003.

30. I. Giagkiozis, R. C. Purshouse, and P. J. Fleming. Generalized decomposition and cross entropy methods for many-objective optimization. Information Sciences, 282:363{387, 2014.