DOI: 10.24996/ijs.2021.62.12.21

ISSN: 0067-2904

Neutrosophic Simply *b*-Open Set in Neutrosophic Topological Spaces

Suman Das*, Binod Chandra Tripathy
Department Of Mathematics, Tripura University, Agartala, 799022, Tripura, India

Received: 16/7/2020 Accepted: 28/2/2021

Abstract

In this paper, we procure the notions of neutrosophic simply *b*-open set, neutrosophic simply *b*-open cover, and neutrosophic simply *b*-compactness via neutrosophic topological spaces. Then, we establish some remarks, propositions, and theorems on neutrosophic simply
b-compactness. Further, we furnish some counter examples where the result fails.

Keywords: Neutrosophic simply *b*-open, Neutrosophic simply *b*-closed, Neutrosophic *b*-compact, Neutrosophic simply *b*-compact.
2010 AMS Classification No: 03E72; 54A05; 54A40; 54J05

1. Introduction

Smarandache [1] introduced the concept of the neutrosophic set as a generalization of intuitionistic fuzzy set. From then, it became very useful in the areas of decision making, artificial intelligence, etc. In the year 2020, Das *et al*. [2] established a multi-criteria group decision making model using a single-valued neutrosophic set. The notion of neutrosophic topological space was presented by Salama and Alblowi [3] in the year 2012. Salama and Alblowi [4] also studied the generalized neutropsophic set and generalized neutrosophic topological spaces. Thereafter, Arokiarani *et al*. [5] defined the neutrosophic semi-open functions and established some relations between them. Rao and Srinivasa [6] presented the neutrosophic pre-open sets and pre-closed sets in neutrosophic topological spaces. Iswaraya and Bageerathi [7] grounded the notion of the neutrosophic semi-closed sets and neutrosophic semi-open sets. The idea of generalized neutrosophic closed sets in neutrosophic topological spaces was studied by Dhavaseelan and Jafari [8]. Dhavaseelan *et al*. [9] established the neutrosophic α^m-continuity in neutrosophic topological spaces. Later on, Imran *et al*. [10] presented the neutrosophic semi-α-open sets in neutrosophic topological space. Imran *et al*. [11] also defined the notion of neutrosophic generalized alpha generalized continuity via neutrosophic topological spaces. Pushpalatha and Nandhini [12] grounded the concept of neutrosophic generalized closed sets in neutrosophic topological spaces. Later on, Ebenanjar *et al*. [13] introduced the neutrosophic *b*-open sets in neutrosophic topological spaces. In the year 2020, Page and Imran [14] established the neutrosophic generalized homeomorphism via neutrosophic topological spaces. The idea of neutrosophic generalized *b*-closed sets in neutrosophic topological spaces was established by Maheswari *et al*. [15]. Maheswari and Chandrasekar [16] also introduced the neutrosophic *gb*-closed sets and neutrosophic *gb*-continuity in neutrosophic topological spaces. Thereafter, Bageerathi and Puvaneswari [17] defined the neutrosophic feebly connectedness and compactness of neutrosophic topological spaces. In the year 2020, Das and Pramanik [18] introduced the generalized neutrosophic *b*-open sets in neutrosophic topological spaces. Das and Pramanik [19] also grounded the notion

*Email: sumandas18842@gmail.com

of neutrosophic Φ-open sets and neutrosophic Φ-continuous functions. Noori and Yousif [20] presented the soft simply compact space in soft topological spaces. The concept of neutrosophic soft structures was established by Arif *et al*. [21]. Later on, Das and Pramanik [22] introduced the notion of neutrosophic simply soft open set and neutrosophic simply soft compactness in neutrosophic soft topological spaces. Recently, the notion of pairwise neutrosophic-*b*-open set in neutrosophic bitopological spaces was presented by Das and Tripathy [23].

The main aim of this article is to procure the concepts of neutrosophic simply *b*-open set, neutrosophic simply *b*-open cover, and neutrosophic simply *b*-compactness via neutrosophic topological spaces. We establish some remarks, propositions, and theorems on neutrosophic topological spaces. Further, we furnish some illustrative examples.

2. Preliminaries

Definition 2.1: [1]. A neutrosophic set (NS) J over a non-empty fixed set W is defined by: $J=\{(r, T_J(r), I_J(r), F_J(r)): r\in W$ and $T_J(r), I_J(r), F_J(r)\in]^-0,1^+[\}$, where $T_J(r), I_J(r), F_J(r)$ denote, respectively, the degree of truth-membership, indeterminacy-membership, and false-membership of each $r\in W$. There is no restriction on the sum of $T_J(r), I_J(r), F_J(r)$. So $^-0 \leq T_J(r)+I_J(r)+F_J(r) \leq 3^+$, for each $r\in W$.

Definition 2.2: [3]. A collection τ of neutrosophic spaces (NSs) over an universal set W is called a neutrosophic topology (NT) on W if the following axioms hold:

(*i*) $0_N, 1_N \in \tau$;

(*ii*) $J_1, J_2 \in \tau \Rightarrow J_1 \cap J_2 \in \tau$;

(*iii*) $\cup J_i \in \tau$, for every $\{J_i : i \in \Delta\}\subseteq\tau$.

The pair (W,τ) is called a neutrosophic topological space (NTS). Each element of τ is called a neutrosophic open set (NOS) in (W,τ). The complement of a NOS is called a neutrosophic closed set (NCS) in (W,τ).

Remark 2.3: [3]. The collection of all NOSs and NCSs in a NTS (W,τ) may be denoted by NOS(W) and NCS(W), respectively.

Definition 2.4: [3]. Assume that (W,τ) is a NTS and J is a NS over W. Then, the neutrosophic interior (N_{int}) and neutrosophic closure (N_{cl}) of J are defined by

$N_{int}(J) = \cup\{Q : Q$ is a NOS in W and $Q\subseteq J\}$

and $N_{cl}(J) = \cap\{R : R$ is a NCS in W and $J\subseteq R\}$.

Definition 2.5: [5]. Let (W,τ) be a NTS and J be a NS over W. Then, J is called a neutrosophic α-open (Nα-O) set if and only if $J\subseteq N_{int}(N_{cl}(N_{int}(J)))$.

Definition 2.6: [7]. Let (W,τ) be a NTS and J be a NS over W. Then, J is called a neutrosophic semi-open (NSO) set if and only if $J\subseteq N_{cl}(N_{int}(J))$.

Definition 2.7: [5]. Let (W,τ) be a NTS and J be a NS over W. Then, J is called a neutrosophic pre-open (NPO) set if and only if $J\subseteq N_{int}(N_{cl}(J))$.

Definition 2.8: [19]. Let (W,τ) be a NTS and J be a NS over W. Then, J is called a neutrosophic *b*-open (N-*b*O) set if and only if $J\subseteq N_{int}(N_{cl}(J))\cup N_{cl}(N_{int}(J))$.

Remark 2.9: The collection of all neutrosophic α-open, neutrosophic semi-open, neutrosophic pre-open, neutrosophic *b*-open, and neutrosophic *b*-closed sets in a neutrosophic topological space (W,τ) may be denoted by Nα-O(W), NSO(W), NPO(W), N-*b*O(W), and N-*b*C(W), respectively. Clearly, NOS(W)⊆N-*b*O(W) and NCS(W)⊆ N-*b*C(W).

Definition 2.10: [13]. Let J be a NS over W and (W,τ) be a NTS. Then, the neutrosophic *b*-interior (N_{bint}) and neutrosophic *b*-closure (N_{bcl}) of J are defined by

(*i*) $N_{bint}(J)= \cup\{Q :Q$ is a N-*b*O set in W and $Q\subseteq J\}$;

(*ii*) $N_{bcl}(J)= \cap\{R :R$ is a N-*b*C set in W and $J\subseteq R\}$.

Definition 2.11: [5]. Let ξ be a function from a NTS (W,τ_1) to another NTS (M,τ_2). Then, ξ is called as
(*i*) neutrosophic open function if $\xi(K)$ is a NOS in M, whenever K is a NOS in W;
(*ii*) neutrosophic α-open function if $\xi(K)$ is a Nα-O set in M, whenever K is a NOS in W;
(*iii*) neutrosophic pre-open function if $\xi(K)$ is a NPO set in M, whenever K is a NOS in W;
(*iv*) neutrosophic semi-open function if $\xi(K)$ is a NSO set in M, whenever K is a NOS in W;
(*v*) neutrosophic *b*-open function if $\xi(K)$ is a N-*b*O set in M, whenever K is a NOS in W;
(*vi*) neutrosophic continuous function if $\xi^{-1}(K)$ is a NOS in W, whenever K is a NOS in M;
(*vii*) neutrosophic *b*-continuous function if $\xi^{-1}(K)$ is a N-*b*O set in W, whenever K is a NOS in M.

3. Main Results

Definition 3.1: A family $\{Z_\alpha: \alpha\in\Delta\}$, where Δ is an index set and Z_α is a N-*b*O set in (W,τ), for each $\alpha\in\Delta$, is said to be a neutrosophic *b*-open cover of a neutrosophic set Z if $Z \subseteq \cup\{Z_\alpha: \alpha\in\Delta\}$.
Definition 3.2: A NTS (W,τ) is said to be a neutrosophic *b*-compact space if each neutrosophic *b*-open cover of W has a finite sub-cover.
Definition 3.3: A neutrosophic subset B of a NTS (W,τ) is said to be a neutrosophic *b*-compact relative to W if every neutrosophic *b*-open cover of B has a finite sub-cover.
Proposition 3.4: Every neutrosophic *b*-compact space is a neutrosophic compact space.
Proof: Let (W,τ) be a neutrosophic *b*-compact space. Therefore, every neutrosophic *b*-open cover of (W,τ) has a finite sub-cover. Suppose that (W,τ) may not be a neutrosophic compact space. Then, there exists a neutrosophic open cover $\mathcal{H}$ (say) of W, which has no finite sub-cover. Since every neutrosophic open set is a neutrosophic *b*-open set, so we have a neutrosophic *b*-open cover $\mathcal{H}$ of W, which has no finite sub-cover. This contradicts our assumption that (W,τ) is a neutrosophic *b*-compact space. Hence, (W,τ) is a neutrosophic compact space.
Definition 3.5: A neutrosophic set Z over a non-empty fixed set W is called a neutrosophic simply *b*-open (N^s-*b*O) set iff it is a neutrosophic *b*-open set and $N_{int}N_{cl}(Z)\subseteq N_{cl}N_{int}(Z)$.
The complement of a N^s-*b*O set is called a neutrosophic simply *b*-closed (N^s-*b*C) set. The family of all N^s-*b*O and N^s-*b*C sets may be denoted as N^s-*b*O(W) and N^s-*b*C(W), respectively.

Theorem 3.6: In a NTS (W,τ), every NOS is a N^s-*b*O set.
Proof: Let J be a NOS in a NTS (W,τ). Therefore, $N_{int}(J)=J$. Since every NOS is a N-*b*O set, so J is a N-*b*O set in (W,τ). It is known that $J\subseteq N_{cl}(J)$. This implies that $J\subseteq N_{cl}N_{int}(J)$.
Now, $J\subseteq N_{cl}N_{int}(J)$
$\Rightarrow N_{cl}(J)\subseteq N_{cl}N_{cl}N_{int}(J)$
$=N_{cl}N_{int}(J)$ [since $N_{cl}N_{int}(J)$ is a NCS in (W,τ)] (1)
Further, we have $N_{int}N_{cl}(J)\subseteq N_{cl}(J)$ (2)
From (1) and (2), we get $N_{int}N_{cl}(J)\subseteq N_{cl}N_{int}(J)$.
Hence, J is a N-*b*O set in (W,τ) and $N_{int}N_{cl}(J)\subseteq N_{cl}N_{int}(J)$. Therefore, J is a N^s-*b*O set in (W,τ).
Remark 3.7: The converse of the above theorem may not be true in general, which follows from the following example.
Example 3.8: Let us consider a NT $\tau=\{0_N, 1_N, \{(x, 0.2, 0.4, 0.6), (y, 0.3, 0.5, 0.7)\}, \{(x, 0.4, 0.2, 0.4), (y, 0.5, 0.3, 0.5)\}, \{(x, 0.6, 0, 0.2), (y, 0.7, 0.1, 0.3)\}\}$ on a non-empty set $W=\{x, y\}$. Let $B=\{(x, 0.2, 0.3, 0.5), (y, 0.5, 0.4, 0.6)\}$ be a NS over W. Clearly, B is a N-*b*O set in (W,τ) and $N_{int}N_{cl}(B)\subseteq N_{cl}N_{int}(B)$. Hence, B is a N^s-*b*O set in (W,τ). But it is not a NOS in (W,τ).
Proposition 3.9: In a NTS (W,τ), every NSO set is a N^s-*b*O set.

Proof: Assume that Q is a neutrosophic semi-open set in a NTS (W,τ). Therefore, $Q\subseteq N_{cl}N_{int}(Q)$. Since every NSO set is a N-*b*O set, so Q is a N-*b*O set in (W,τ).
Now, $Q\subseteq N_{cl}N_{int}(Q)$
$\Longrightarrow N_{cl}(Q)\subseteq N_{cl}N_{cl}N_{int}(Q)$
$=N_{cl}N_{int}(Q)$ [Since $N_{cl}N_{int}(Q)$ is a NCS in (W,τ)]
$\Longrightarrow N_{cl}(Q)\subseteq N_{cl}N_{int}(Q)$ (3)
We have $N_{int}N_{cl}(Q)\subseteq N_{cl}(Q)$ (4)
From (3) and (4), we have $N_{int}N_{cl}(Q)\subseteq N_{cl}N_{int}(Q)$. Therefore, Q is a N-*b*O set in (W,τ) and $N_{int}N_{cl}(Q)\subseteq N_{cl}N_{int}(Q)$. Hence, Q is a N^s-*b*O set.

Remark 3.10: The converse of the above proposition may not be true in general. This follows from the following example.

Example 3.11: Let us consider a NT $\tau=\{0_N, 1_N, \{(d_1, 0.3, 0.7, 0.5), (d_2, 0.2, 0.6, 0.4)\}, \{(d_1, 0.5, 0.5, 0.3), (d_2, 0.4, 0.4, 0.2)\}, \{(d_1, 0.7, 0.3, 0.1), (d_2, 0.6, 0.2, 0)\}\}$ on $W=\{d_1, d_2\}$. Let $Q=\{(d_1, 0.5, 0.6, 0.4), (d_2, 0.2, 0.5, 0.3)\}$ be a NS over W. It is clear that Q is a N-*b*O set in (W,τ) and $N_{int}N_{cl}(Q)\subseteq N_{cl}N_{int}(Q)$. Therefore, Q is a N^s-*b*O set in (W,τ). But $Q\not\subseteq N_{cl}N_{int}(Q)$. Hence, Q is not a NSO set in (W,τ).

We formulate the following result in view of Definition 3.4.

Lemma 3.12: Let (W,τ) be a NTS. Then, every N^s-*b*O set is a N-*b*O set.

Remark 3.13: The converse of Lemma 3.12. may not be true in general. This follows from the following example.

Example 3.14: Let us consider a NT $\tau=\{0_N, 1_N, \{(x, 0.2, 0.4, 0.6), (y, 0.3, 0.5, 0.7)\}, \{(x, 0.4, 0.2, 0.4), (y, 0.5, 0.3, 0.5)\}, \{(x, 0.6, 0, 0.2), (y, 0.7, 0.1, 0.3)\}\}$ on $W=\{x, y\}$. Let $A=\{(x, 0.3, 0.5, 0.7), (y, 0.2, 0.4, 0.6)\}$ be a NS over W. It is clear that $N_{cl}N_{int}(A)=0_N$ and $N_{int}N_{cl}(A)=1_N$. Therefore, $N_{int}N_{cl}(A)\cup N_{cl}N_{int}(A)=1_N$. This implies, $A\subseteq N_{int}N_{cl}(A)\cup N_{cl}N_{int}(A)$ i.e., A is a N-*b*O set. But $N_{int}N_{cl}(A)\not\subseteq N_{cl}N_{int}(A)$. Hence, A is not a N^s-*b*O set in (W,τ).

Proposition 3.15: If A is both neutrosophic pre-open set and neutrosophic simply *b*-open set in a NTS (W,τ), then it is a neutrosophic semi-open set in (W,τ).

Proof: Assume that Q_1 is both neutrosophic pre-open set and neutrosophic simply *b*-open set in a NTS (W,τ). Since Q_1 is neutrosophic pre-open set, so $Q_1\subseteq N_{int}N_{cl}(Q_1)$. Again, since Q_1 is neutrosophic simply *b*-open set, so Q_1 is a neutrosophic *b*-open set and $N_{int}N_{cl}(Q_1)\subseteq N_{cl}N_{int}(Q_1)$. This implies that $Q_1\subseteq N_{cl}N_{int}(Q_1)$. Therefore, Q_1 is a neutrosophic semi-open set.
By figure 1, we connect all the relations among NOS, NSO, N-*b*O, and N^S-*b*O sets.

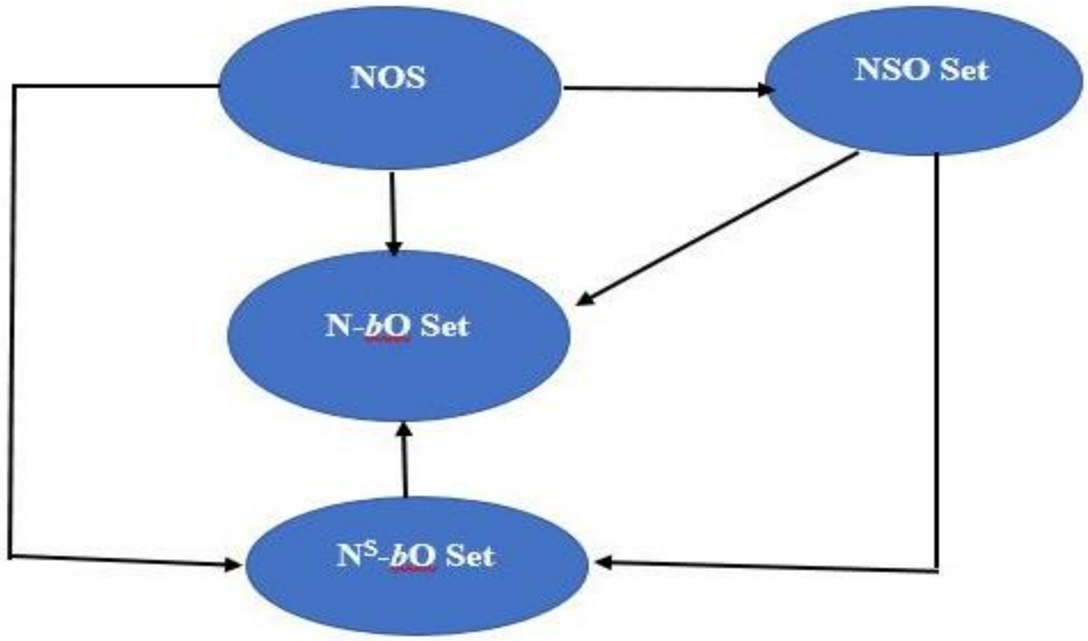

Figure 1- Relations among NOS, NSO, N-*b*O and N^S-*b*O sets

Remark 3.16: If $Z_1, Z_2 \in N^s\text{-}bO(W)$, then $Z_1 \cap Z_2$ may not belong to $N^s\text{-}bO(W)$.
Proof: Assume that Z_1, Z_2 are two N^s-*b*O sets in a NTS (W,τ). Therefore, Z_1 and Z_2 are N-*b*O sets in (W,τ) and $N_{int}N_{cl}(Z_1) \subseteq N_{cl}N_{int}(Z_1)$, $N_{int}N_{cl}(Z_2) \subseteq N_{cl}N_{int}(Z_2)$. It is known that, the intersection of two N-*b*O sets may not be a N-*b*O set in (W,τ). This implies that $Z_1 \cap Z_2$ may not be a N-*b*O set in (W,τ). Therefore, $Z_1 \cap Z_2$ is not a N^s-*b*O set in (W,τ).
Definition 3.17: A function $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ is called a neutrosophic simply *b*-continuous function if for each neutrosophic open set Z in M, $\xi^{-1}(Z)$ is a N^s-*b*O set in W.
Definition 3.18: A function $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ is said to be a neutrosophic simply *b*-open function if $\xi(K)$ is a N^s-*b*O set in M, whenever K is a NOS in W.
Definition 3.19: A family $\{Z_\alpha: \alpha \in \Delta$ and Z_α is a N^s-*b*O set in $(W,\tau)\}$, where Δ is an index set, is called a neutrosophic *b*-open cover of a neutrosophic set Z if $Z \subseteq \cup_{\alpha \in \Delta} Z_\alpha$.
Definition 3.20: A NTS (W,τ) is called a neutrosophic simply *b*-compact space if each neutrosophic simply *b*-open cover of W has a finite sub-cover.
Definition 3.21: A neutrosophic subset K of (W,τ) is said to be a neutrosophic simply *b*-compact set relative to W if every neutrosophic simply *b*-open cover of K has a finite sub-cover.
Theorem 3.22: Every neutrosophic simply *b*-closed subset of a neutrosophic simply *b*-compact space (W,τ) is neutrosophic simply *b*-compact relative to W.
Proof: Let (W,τ) be a neutrosophic simply *b*-compact space and K be a neutrosophic simply *b*-closed set in (W,τ). Therefore, K^c is a neutrosophic simply *b*-open set in (W,τ). Let $U=\{U_i: i \in \Delta$ and $U_i \in N^s\text{-}bO(W)\}$ be a simply *b*-open cover of K. Then, $\mathcal{H}=\{K^c\} \cup U$ is a neutrosophic simply *b*-open cover of X. Since X is a neutrosophic simply *b*-compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \ldots, H_n, K^c\}$. This implies that $\{H_1, H_2, H_3, \ldots, H_n\}$ is a finite neutrosophic simply *b*-open cover of K. Hence, K is a neutrosophic simply *b*-compact set relative to W.
Theorem 3.23:
(*i*) Every neutrosophic *b*-compact space is a neutrosophic simply *b*-compact space.

(*ii*) Every neutrosophic simply *b*-compact space is a neutrosophic compact space.

Proof:

(*i*) Let (W,τ) be a neutrosophic *b*-compact space. Suppose that (W,τ) is not a neutrosophic simply *b*-compact space. Then, there exists a neutrosophic simply *b*-open cover $\mathcal{H}$(say) of *W*, which has no finite sub-cover. Since every neutrosophic simply *b*-open set is a neutrosophic *b*-open set, so we have a neutrosophic *b*-open cover $\mathcal{H}$ of *W*, which has no finite sub-cover. This contradicts our assumption. Hence, (W,τ) is a neutrosophic simply *b*-compact space.

(*ii*) Let (W,τ) be a neutrosophic simply *b*-compact space. Suppose that (W,τ) is not a neutrosophic compact space. Then, there exists a neutrosophic open cover $\Re$(say) of *W*, which has no finite sub-cover. Since every neutrosophic open set is a neutrosophic simply *b*-open set, so we have a neutrosophic simply *b*-open cover $\Re$ of *W*, which has no finite sub-cover. This contradicts our assumption. Hence, (W,τ) is a neutrosophic compact space.

By figure 2, we connect all the relations among the neutrosophic *b*-compact space, neutrosophic simply *b*-compact space, and neutrosophic compact space.

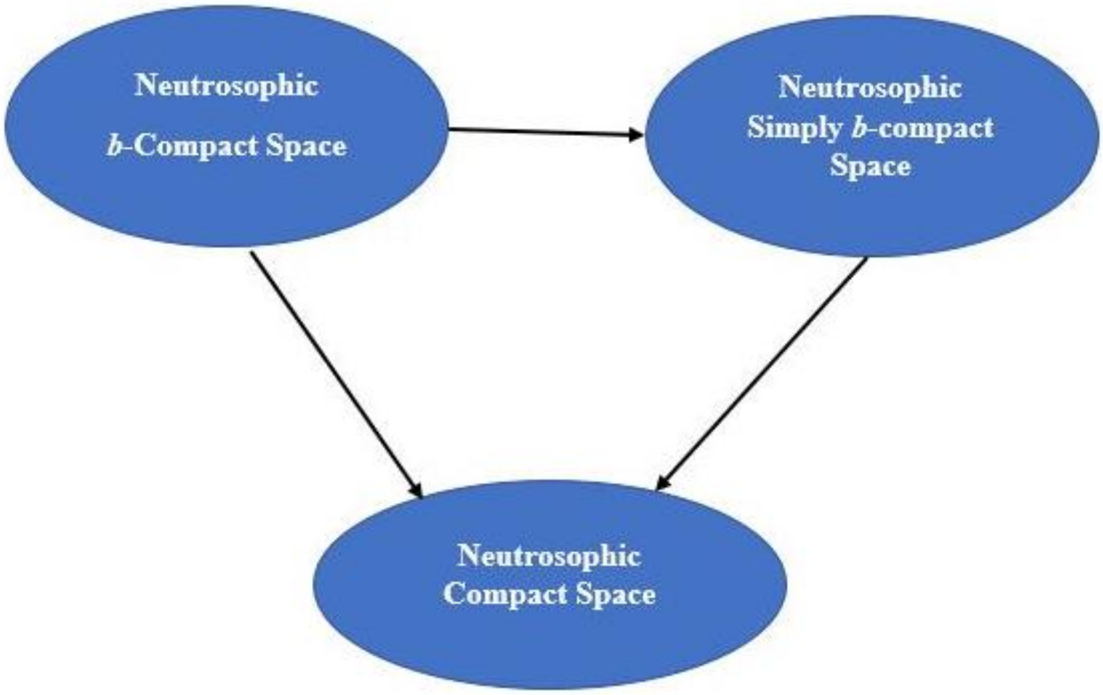

Figure2-Relations among neutrosophic compact space, neutrosophic *b*-compact space and neutrosophic simply *b*-compact space.

Theorem 3.24:

(*i*) If $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$ is a neutrosophic *b*-open function and (M,τ_2) is a neutrosophic *b*-compact space, then (W,τ_1) is also a neutrosophic compact space.

(*ii*) If $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$ is a neutrosophic simply *b*-open function and (M,τ_2) is a neutrosophic simply *b*-compact space, then (W,τ_1) is also a neutrosophic *b*-compact space.

Proof:

(*i*) Let $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$ be a neutrosophic *b*-open function and (M,τ_2) be a neutrosophic *b*-compact space. Let $\mathcal{H}=\{H_i: i\in\Delta \text{ and } H_i\in \text{N-}b\text{O}(W)\}$ be a neutrosophic open cover of *W*. Therefore, $\xi(\mathcal{H})=\{\xi(H_i): i\in\Delta \text{ and } \xi(H_i)\in \text{N-}b\text{O}(M)\}$ is a neutrosophic *b*-open cover of *M*.

Since (M,τ_2) is a neutrosophic *b*-compact space, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \ldots\ldots, \xi(H_n)\}$ such that $M \subseteq \cup\{\xi(H_i): i=1, 2, \ldots, n\}$. This implies that $\{H_1, H_2, \ldots, H_n\}$ is a finite sub-cover for *W*. Hence, $(W,\tau_1)$is a neutrosophic compact space.

(*ii*) Let $\xi:(W,\tau_1)\to(M,\tau_2)$ be a neutrosophic simply *b*-open function and (M,τ_2) is a neutrosophic simply *b*-compact space. Let $\mathcal{H}=\{K_i: i\in\Delta$ and $K_i\in$N-*b*O$(W)\}$ be a neutrosophic *b*-open cover of *W*. Therefore, $\xi(\mathcal{H})=\{\xi(K_i): i\in\Delta$ and $\xi(K_i)\in$N-*b*O$(M)\}$ is a neutrosophic simply *b*-open cover of *M*. Since (M,τ_2) is a neutrosophic simply *b*-compact space, so there exists a finite sub-cover say $\{\xi(K_1), \xi(K_2), \ldots\ldots, \xi(K_n)\}$ such that $M \subseteq\cup\{\xi(K_i): i=1, 2, \ldots, n\}$. Therefore, $\{K_1, K_2, \ldots, K_n\}$ is a finite sub-cover for *W*. Hence, (W,τ_1) is a neutrosophic *b*-compact space.

Theorem 3.25:

(*i*) If $\xi:(W,\tau_1)\to(M,\tau_2)$ is a neutrosophic *b*-continuous function, then for each neutrosophic *b*-compact set *Q* relative to *W*, $\xi(Q)$ is a neutrosophic simply *b*-compact set in *M*.

(*ii*) If $\xi: (W,\tau_1)\to(M,\tau_2)$ is a neutrosophic *b*-continuous function, then for each neutrosophic *b*-compact set *Z* relative to *W*, $\xi(Z)$ is a neutrosophic compact set in *M*.

Proof:

(*i*) Assume that $\xi:(W,\tau_1)\to(M,\tau_2)$ be a neutrosophic *b*-continuous function and *Q* be a neutrosophic *b*-compact set relative to *W*. Let $\mathcal{H}=\{H_i: i\in\Delta$ and $H_i\in N^s$-*b*O$(M)\}$ be a neutrosophic simply *b*-open cover of $\xi(Q)$. Since every N^s-*b*O set is a N-*b*O set, so $\mathcal{H}=\{H_i: i\in\Delta$ and $H_i\in$N-*b*O$(M)\}$ is a neutrosophic b-open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(\mathcal{H})= \{\xi^{-1}(H_i): i\in\Delta$ and $\xi^{-1}(H_i)\in$N-*b*O$(M)\}$ is a neutrosophic *b*-open cover of $\xi^{-1}(\xi(Q))=Q$. Since *Q* is a neutrosophic *b*-compact set relative to *W*, so there exists a finite sub-cover of *Q*, say $\{H_1, H_2, H_3, \ldots.., H_n\}$, such that $Q\subseteq\cup_i\{H_i:i=1,2,\ldots., n\}$.

Now $Q\subseteq\cup_i\{H_i:i=1,2,\ldots., n\}$

$\Rightarrow\xi(Q)\subseteq\cup_i\{\xi(H_i):i=1,2,\ldots., n\}$

Therefore there exists a finite sub-cover $\{\xi(H_1), \xi(H_2), \xi(H_3),\ldots.,\xi(H_n)\}$of $\xi(Q)$ such that $\xi(Q)\subseteq\cup_i\{\xi(H_i):i=1,2,\ldots., n\}$. Hence, $\xi(Q)$ is a neutrosophic simply *b*-compact set relative to *M*.

(*ii*) Assume that $\xi:(W,\tau_1)\to(M,\tau_2)$ is a neutrosophic *b*-continuous function and *Z* is a neutrosophic *b*-compact set relative to *W*. Let $\mathcal{H}=\{H_i: i\in\Delta$ and $H_i\in$N-*b*O$(M)\}$ be a neutrosophic open cover of $\xi(Z)$. By hypothesis, $\xi^{-1}(\mathcal{H})=\{\xi^{-1}(H_i): i\in\Delta$ and $\xi^{-1}(H_i)\in$N-*b*O$(M)\}$ is a neutrosophic *b*-open cover of $\xi^{-1}(\xi(Z))$=Z. Since *Z* is a neutrosophic *b*-compact set relative to *W*, so there exists a finite sub-cover of *Z* say $\{H_1, H_2, H_3, \ldots.., H_n\}$ such that $Z\subseteq \cup_i\{H_i:i=1,2,\ldots., n\}$.

Now, $Z\subseteq\cup_i\{H_i:i=1,2,\ldots., n\}$

$\Rightarrow\xi(Z)\subseteq\cup_i\{\xi(H_i):i=1,2,\ldots., n\}$

Therefore, there exists a finite sub-cover $\{\xi(H_1), \xi(H_2), \xi(H_3),\ldots.,\xi(H_n)\}$ of $\xi(Z)$ such that $\xi(Z)\subseteq\cup_i\{\xi(H_i):i=1,2,\ldots., n\}$. Hence, $\xi(Z)$ is a neutrosophic compact set relative to *M*.

Theorem 3.26: Every neutrosophic continuous function from a NTS (W,τ_1) to a NTS (M,τ_2) is a neutrosophic simply *b*-continuous function.

Proof: Let $\xi:(W,\tau_1)\to(M,\tau_2)$ be a neutrosophic continuous function. Let *Q* be any arbitrary NOS in (M,τ_2). By hypothesis, $\xi^{-1}(Q)$ is a NOS in (W,τ_1). Since every NOS is a N^s-*b*O set, so $\xi^{-1}(Q)$ is a N^s-*b*O set in (W,τ_2). Therefore, $\xi^{-1}(Q)$ is a N^s-*b*O set in (W,τ_2), whenever *Q* is a NOS in (M,τ_2). Hence, $\xi:(W,\tau_1)\to(M,\tau_2)$ is a neutrosophic simply *b*-continuous function.

Theorem 3.27: Every neutrosophic simply *b*-continuous function from a NTS (W,τ_1) to a NTS (M,τ_2) is a neutrosophic *b*-continuous function.

Proof: Let $\xi:(W,\tau_1)\to(M,\tau_2)$ be a neutrosophic simply *b*-continuous function. Let *Q* be any arbitrary NOS in (M,τ_2). By hypothesis, $\xi^{-1}(Q)$ is a N^s-*b*O set in (W,τ_1). Since every N^s-*b*O set is a N-*b*O set, so $\xi^{-1}(Q)$ is a N-*b*O set in (W,τ_2).Therefore, $\xi^{-1}(Q)$ is a N-*b*O set in (W,τ_2), whenever Q is a NOS in (M,τ_2). Hence, $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$ is a neutrosophic *b*-continuous function.

Theorem 3.28: If $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$ is a neutrosophic simply *b*-continuous mapping and $\gamma:(M,\tau_2)\rightarrow(L,\tau_3)$ is a neutrosophic continuous mapping, then the composition mapping $\gamma\circ\xi: (W,\tau_1)\rightarrow(L,\tau_3)$ is a neutrosophic simply *b*-continuous mapping.

Proof: Let Q be a neutrosophic open set in (L,τ_3). Since $\gamma:(M,\tau_2)\rightarrow(L,\tau_3)$ is a neutrosophic continuous mapping, so $\gamma^{-1}(Q)$ is a neutrosophic open set in (M,τ_2). Again, since $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$ is a neutrosophic simply *b*-continuous mapping, so $\xi^{-1}(\gamma^{-1}(Q))= (\gamma\circ\xi)^{-1}(Q)$ is a neutrosophic simply *b*-open set in (W,τ_1). Hence, $(\gamma\circ\xi)^{-1}(Q)$ is a neutrosophic simply *b*-open set in (W,τ_1), whenever Q is a neutrosophic open set in (L,τ_3). Therefore, $\gamma\circ\xi: (W,\tau_1)\rightarrow(L,\tau_3)$ is a neutrosophic simply *b*-continuous mapping.

4. Conclusions

In this paper, we present the concepts of neutrosophic *b*-open cover, neutrosophic *b*-compactness, neutrosophic simply *b*-open cover, and neutrosophic simply *b*-compactness in neutrosophic topological spaces. By defining the neutrosophic *b*-open cover, neutrosophic *b*-compactness, neutrosophic simply *b*-open cover, and neutrosophic simply *b*-compactness, we prove some remarks and theorems on neutrosophic *b*-compactness and neutrosophic simply *b*-compactness and give some illustrative examples.

It is hoped that the notion of neutrosophic simply *b*-compactness in neutrosophic topological spaces can be applied in neutrosophic bi-topological spaces and by researcher working in other areas of research.

Funding

The work of the first author is financially supported by the University Grants Commission Fellowship F.No. 16-6(DEC.2018)/2019(NET/CSIR).

Conflict of interest

Authors declare that they have no conflict of interest.

References

[1] F. Smarandache, "*A unifying field in logics, neutrosophy: neutrosophic probability, set and logic*," *Rehoboth: American Research Press*.

[2] S. Das, R. Das and B. C. Tripathy, "Multi-criteria group decision making model using single-valued neutrosophic set," *LogForum*, vol. 16, no. 3, pp. 421-429, 2020.

[3] A. A. Salama and S. A. Alblowi, "Neutrosophic set and neutrosophic topological space," *ISOR Journal of Mathematics*, vol. 3, no. 4, pp. 31-35, 2012.

[4] A. A. Salama and S. A. Alblowi, "Generalized neutrosophic set and generalized neutrosophic topological space," *Computer Science and Engineering*, vol. 2, no. 7, pp. 129-132, 2012.

[5] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, "On some new notations and functions in neutrosophic topological spaces," *Neutrosophic Sets and Systems*, vol. 16, pp. 16-19, 2017.

[6] V. V. Rao and R. Srinivasa, "Neutrosophic pre-open sets and pre-closed sets in neutrosophic topology," *International Journal of Chem Tech Research*, vol. 10, no. 10, pp. 449-458.

[7] P. Iswarya and K. Bageerathi, "On neutrosophic semi-open sets in neutrosophic topological spaces," *International Journal of Mathematical Trends and Technology*, vol. 37, no. 3, pp. 214-223, 2016.

[8] R. Dhavaseelan and S. Jafari, "Generalized neutrosophic closed sets," *New trends in neutrosophic theory and applications*, vol-2, pp. 261-273, 2018.

[9] R. Dhavaseelan, R. Narmada Devi, S. Jafari and Q. H. Imran, "Neutrosophic α^m-continuity," *Neutrosophic Sets and Systems*, vol. 27, pp. 171-179, 2019.

[10] Q. H. Imran, F. Smarandache, R. K. Al-Hamido and R. Dhavaseelan, "On neutrosophic semi-α-open sets," *Neutrosophic Sets and Systems*, vol. 18, pp. 37-42, 2017.

[11] Q. H. Imran, R. Dhavaseelan, A. H. M. Al-Obaidi and M. H. Page, "On neutrosophic generalized alpha generalized continuity," *Neutrosophic Sets and Systems*, vol. 35, pp. 511-521, 2020.

[12] A. Pushpalatha and T. Nandhini, "Generalized closed sets via neutrosophic topological spaces," *Malaya Journal of Matematik*, vol. 7, no. 1, pp. 50-54, 2019.

[13] E. Ebenanjar, J. Immaculate and C. B. Wilfred, "On neutrosophic *b*-open sets in neutrosophic topological space," *Journal of Physics Conference Series*, vol. 1139, no. 1, pp. 012062, 2018.

[14] M. H. Page and Q. H. Imran, "Neutrosophic generalized homeomorphism," *Neutrosophic Sets and Systems*, vol. 35, pp. 340-346, 2020.

[15] C. Maheswari, M. Sathyabama and S. Chandrasekar, "Neutrosophic generalized b-closed sets in neutrosophic topological spaces," *Journal of Physics Conference Series*, vol. 1139, no. 1, pp. 012065.

[16] C. Maheswari and S. Chandrasekar, "Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity," *Neutrosophic Sets and Systems*, vol. 29, pp. 89-100, 2019.

[17] K. Bageerathi and P. Jeya Puvaneswari, "Neutrosophic feebly connectedness and compactness," *IOSR Journal of Polymer and Textile Engineering*, vol. 6, no. 3, pp. 07-13, 2019.

[18] S. Das and S. Pramanik, "Generalized neutrosophic *b*-open sets in neutrosophic topological space," *Neutrosophic Sets and Systems*, vol. 35, pp. 522-530, 2020.

[19] S. Das and S. Pramanik, "Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions," *Neutrosophic Sets and Systems*, vol. 38, pp. 355-367, 2020.

[20] S. Noori and Y. Y. Yousif, "Soft Simply Compact Space," *Iraqi Journal of Science*, Special Issue, pp. 108-113, 2020.

[21] M. Arif, W. Ullah, S. Broumi, M. I. Khan, H. Qureshi, M. I. Abbas, H. Kalsoom and F. Nadeem, "Neutrosophic Soft Structures," *Neutrosophic Sets and Systems*, vol. 33, pp. 23-58, 2020.

[22] S. Das and S. Pramanik, "Neutrosophic simply soft open set in neutrosophic soft topological space," *Neutrosophic Sets and Systems*, vol. 38, pp. 235-243, 2020.

[23] S. Das and B. C. Tripathy, "Pairwise neutrosophic-*b*-open set in neutrosophic bitopological spaces," *Neutrosophic Sets and Systems*, vol. 38, pp. 135-144, 2020.