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Near – Rings with Generalized Right ɳ-Derivations

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Abstract
We define a new concept, called " generalized right ɳ -derivation", in near-ring and obtain new essential results in this field. Moreover we improve this paper with examples that show that the assumptions used are necessary.

Keywords: Generalized right derivations, Generalized right n-derivations, Prime near-ring, Right derivations, Right ɳ -derivations.

الحلقات المقتربة مع المشتقات اليمنى المعممة

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الخلاصة
عرفنا مفهوم المشتقات اليمنى المعممة وحصلنا على نتائج اساسية في هذا المجال. اضافة الى ذلك زودنا البحث بالامثلة التي تبين ان الفرضيات التي استخدمت فيه ضرورية.

1. Introduction

A near-ring is defined to be a "set $\mathcal{N}$ with two binary operations $(+)$ and $(\cdot)$ such that (i) $(\mathcal{N},+)$ is a group that is not necessarily abelian: (ii) $(\mathcal{N}, \cdot)$ is a semi group; (iii) $a\cdot(b+c)=a\cdot b+a\cdot c$ for each $a,b,c \in \mathcal{N}$". The elements products, such as a and b in $\mathcal{N}$, will be $a\cdot b$, which is specified by ab. $\mathcal{N}$ is zero-symmetric whenever $0x=0$, for each $x\in\mathcal{N}$ ($x0=0$ yields from left distributivity). The centre of multiplicative of $\mathcal{N}$ will be represented by $\mathcal{Z}$. For each $x,y\in\mathcal{N}$, $[x,y]=xy-yx$ symbolizes the commutator and $(x,y)=x+y-x-y$ is the additive commutator, while $x\circ y$ will denote the well-known Jordan product. N is referred to as prime near-ring in case of $x\mathcal{N}y=\{0\}$, which infers that $y=0$ or $x=0$. "A non-empty subset U of $\mathcal{N}$ is named as semigroup left ideal, resp. semigroup right ideal in case of $\mathcal{N}U\subseteq U$ (resp. $U\mathcal{N}\subseteq U$). But, if U represents both semigroup right and left ideal, then it will be termed as semigroup ideal". For more about near-ring theory and its applications, we make reference to Pilz [1]. In [2], X. Wang defined the derivation as an additive mapping d from $\mathcal{N}$ into itself which satisfies $d(xy)=d(x)y+xd(y)$ for each $x,y\in\mathcal{N}$. Later, the derivation concepts generalization have been achieved through various means according to different authors. "Ashraf and Siddeeque well-defined the concepts of $n-$derivations, generalized $n-$derivations, and $(\sigma,\tau)-n-$derivation in near ring [3 - 6]". Also, various properties of such derivations were examined. In 2015, Abdul Rehman and Enaam defined a new concept, called " right n-derivation", in near-ring and obtained new essential results for researchers in this field [7].

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"An additive mapping d from $\mathcal{N}$ into itself is said to be right derivation of $\mathcal{N}$ if $d(xy) = d(x)y + d(y)x$, for each $x, y \in \mathcal{N}$ and $n-$ additive (i.e. additive in each argument) mapping $d: \underbrace{\mathcal{N} \times \mathcal{N} \times \ldots \times \mathcal{N}}_{n-times} \longrightarrow \mathcal{N}$ is said to be right $n-$derivation of $\mathcal{N}$ if the following equations hold for each $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in \mathcal{N}$":

$$d(x_1 x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_1' + d(x_1', x_2, \ldots, x_n)x_1$$
$$d(x_1, x_2 x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_2' + d(x_1, x_2', \ldots, x_n)x_2$$
$$\vdots$$
$$d(x_1, x_2, \ldots, x_n x_n') = d(x_1, x_2, \ldots, x_n)x_n' + d(x_1, x_2, \ldots, x_n')x_n$$

Motivated by the previous studies, we define here the concepts of generalized right derivation and generalized right n-derivation in near-ring N. After that, we will give new essential results in this field and generalize some results presented in [7]. Finally, we improve this paper with examples that show that the assumptions used are necessary.

Note that we will use the abbreviation C.R to refer to the commutative ring.

Definition 1.1. Let d be a right derivation of $\mathcal{N}$. An additive mapping $\mathcal{G}$ from $\mathcal{N}$ into itself is said to be generalized right derivation of $\mathcal{N}$ connected with d if $\mathcal{G}(xy) = d(x)y + \mathcal{G}(y)x$, for each $x, y \in \mathcal{N}$.

Definition 1.2. If d is a right n-derivation of $\mathcal{N}$ and $\mathcal{G}: \underbrace{\mathcal{N} \times \mathcal{N} \times \ldots \times \mathcal{N}}_{n-\text{times}} \longrightarrow \mathcal{N}$ is an $n-$additive mapping on $\mathcal{N}$, then $\mathcal{G}$ will be called " generalized right $n-$derivation of $\mathcal{N}$ connected with d " if the following equations hold for each $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in \mathcal{N}$.

$$\mathcal{G}(x_1 x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_1' + \mathcal{G}(x_1', x_2, \ldots, x_n)x_1$$
$$\mathcal{G}(x_1, x_2 x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_2' + \mathcal{G}(x_1, x_2', \ldots, x_n)x_2$$
$$\vdots$$
$$\mathcal{G}(x_1, x_2, \ldots, x_n x_n') = d(x_1, x_2, \ldots, x_n)x_n' + \mathcal{G}(x_1, x_2, \ldots, x_n')x_n$$

Example 1.3. If $\mathcal{S}$ be a near-ring and zero symmetric then it is obvious that $\mathcal{N} = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, x, y, 0 \in \mathcal{S} \right\}$ is a near-ring with the addition and multiplication of matrices.

Let $d, \mathcal{G}: \mathcal{N} \rightarrow \mathcal{N}$ and $d_1, \mathcal{G}_1: \underbrace{\mathcal{N} \times \mathcal{N} \times \ldots \times \mathcal{N}}_{n-\text{times}} \longrightarrow \mathcal{N}$ defined by:

$$d\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \mathcal{G}\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & y & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d_1\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & x_1 x_2 \ldots x_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{G}_1\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & y_1 y_2 \cdots y_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Simply can check that $\mathcal{G}$ is a generalized right derivation connected with right derivation d of $\mathcal{N}$ and $\mathcal{G}_1$ is a nonzero generalized right $n-$derivation connected with right $n-$derivation d_1 of $\mathcal{N}$.

2. Preliminaries

The next lemmas are fundamental to develop the proofs of our work.

Lemma 2.1 [8]. "Let N be a near-ring. If there is an element $z \neq 0$ of $\mathcal{Z}$ such that $z + z \in \mathcal{Z}$, then $(\mathcal{N}, +)$ is abelian".

Lemma 2.2 [9] "Let N be a prime near-ring. If $z \in \mathcal{Z} \setminus \{0\}$ and x is an element of $\mathcal{N}$ such that $xz \in Z$ or $zx \in \mathcal{Z}$, then $x \in \mathcal{Z}$".

Lemma 2.3 [9]. "Let N be a prime near-ring and Z contains a nonzero semigroup left ideal or nonzero semigroup left ideal, then N is a C.R".

Lemma 2.4 [6] "Let N be a prime near-ring, d is a nonzero n-derivation of N and $x \in N$. If $d(N, N, \ldots, N)x = \{0\}$, then $x = 0$".

Lemma 2.5 [7] "Let N be a prime near-ring and let $d \neq 0$ be a right $n-$derivation d. If $d([x, y], x_2, \ldots, x_n) = 0$ for each $x, y, \ldots, x_n \in N$, then N is a C.R".

Lemma 2.6 "Let N be a near-ring, then N is zero symmetric if and only if N admits a right n −derivation d".

Proof. If N is zero symmetric then $d = 0$ is a right n-derivation on N.

Now, if N admits right derivation d, then $0 = d(x0, x_2, \dots, x_n) = d(x, x_2, \dots, x_n)\, 0 + d(0, x_2, \dots, x_n)x = 0x$. Hence, N is zero symmetric near-ring.

We would like to point out that we will consider N in the rest of this article as a prime near-ring.

Lemma 2.7 Let $g \neq 0$ is a generalized right n −derivation of N connected with right n–derivation d, and $a \in N$ s.t $g(\mathcal{N}, \mathcal{N}, \dots, \mathcal{N})a = \{0\}$, then $a = 0$.

Proof . From assumption

$$g(x_1, x_2, \dots, x_n)a = 0, \text{ for each } x_1, x_2, \dots, x_n \in \mathcal{N}. \quad (1)$$

By putting ax_1 in place of x_1 in relation (1), we get

$$\begin{aligned} 0 &= g(ax_1, x_2, \dots, x_n)a \\ &= (d(a, x_2, \dots, x_n)\, x_1 + g(x_1, x_2, \dots, x_n)\, a)a \\ &= d(a, x_2, \dots, x_n)x_1 a \end{aligned}$$

So, we get $d(a, x_2, \dots, x_n)\, \mathcal{N}\, a = \{0\}$ for each $x_2, \dots, x_n \in \mathcal{N}$. It follows that either $a = 0$ or $d(a, x_2, \dots, x_n) = 0$ for each $x_2, \dots, x_n \in \mathcal{N}$ according to primeness of $\mathcal{N}$. If $d(a, x_2, \dots, x_n) = 0$ for each $x_2, \dots, x_n \in \mathcal{N}$ (2)

and since $g(x(ay), x_2, \dots, x_n) = g((xa)y, x_2, \dots, x_n)$ for each $x, y, x_2, \dots, x_n \in N$, we get

$d(x, x_2, \dots, x_n)ay + g(ay, x_2, \dots, x_n)x = d(xa, x_2, \dots, x_n)y + g(y, x_2, \dots, x_n)xa$, i.e.

$$d(x, x_2, \dots, x_n)ay + (d(a, x_2, \dots, x_n)y + g(y, x_2, \dots, x_n)a)\, x = (d(x, x_2, \dots, x_n)a + d(a, x_2, \dots, x_n)x)y + g(y, x_2, \dots, x_n)xa$$

Use (1) and (2) in last expression to get $g(y, x_2, \dots, x_n)\, N\, a = \{0\}$ for each $y, x_2, \dots, x_n \in N$. Since N is prime and $g \neq 0$, we conclude $a = 0$.

The following lemmas deduce directly from Lemma 2.7.

Lemma 2.8 [7 : Lemma 2.5]. "If $d \neq 0$ is a right $n-$ derivation of N, and $a \in N$ s.t $d(N, N, \dots, N)a = \{0\}$, then $a = 0$".

Lemma 2.9 "If $g \neq 0$ is a generalized right derivation of N connected with right derivation d, and a ϵ N s.t $g(N)a = \{0\}$, then $a = 0$".

3. Main Results

Theorem 3.1 If $g \neq 0$ is a generalized right n −derivation of N associated with right n-derivation d $\neq 0$, s.t $g([x, y], x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$, then N is a C.R .

Proof. By assumption

$$g([x, y], x_2, \dots, x_n) = 0 \text{ for each } x, y, x_2, \dots, x_n \in N \quad (3)$$

Replace y by xy in (3) to get

$g([x, xy], x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$,

which implies that $g(x[x, y], x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$.

Therefore,

$d(x, x_2, \dots, x_n)[x, y] + g([x, y], x_2, \dots, x_n)x = 0$ for each $x, y, x_2, \dots, x_n \in N$.

Using (3) in the previous equation, we get

$d(x, x_2, \dots, x_n)[x, y] = 0$ for each $x, y, x_2, \dots, x_n \in N$, or equivalently

$$d(x, x_2, \dots, x_n)xy = d(x, x_2, \dots, x_n)yx \text{ for each } x, y, x_2, \dots, x_n \in N. \quad (4)$$

Replacing y by yz in (4) and using it again, we get

$d(x, x_2, \dots, x_n)y[x, z] = 0$ for each $x, y, z, x_2, \dots, x_n \in N$.

Hence, we get

$$d(x, x_2, \dots, x_n)\, N\, [x, z] = \{0\} \text{ for each } x, z, x_2, \dots, x_n \in N \quad (5)$$

We arrive at, for any $x \in N$

either d(x, $x_2, \dots, x_n$) = 0 for each $x_2, \dots, x_n \in N$ or $x \in Z$ (6)

If $d(x, x_2, \dots, x_n) = 0$ for each $x_2, \dots, x_n \in N$ and for each $x \in N$. then $d = 0$ and this contradicts assumption. Therefore, there is $x_1, x_2, \dots, x_n \in N$, such that d($x_1, x_2, \dots, x_n$) $\neq 0$ and $x_1 \in$ Z.

Since $x_1 \in Z$, we conclude that $[x_1 y, n] = x_1\, [y, n]$, where $y, n \in N$. By hypothesis we get $g([x_1 y, n]\, x_2, \dots, x_n) = 0$, which implies that

$$\begin{aligned} 0 &= g(x_1\, [y, n], x_2, \dots, x_n) \\ &= d(x_1, x_2, \dots, x_n)[y, n] + g([y, n], x_2, \dots, x_n)\, x_1 \\ &= d(x_1, x_2, \dots, x_n)[y, n] \quad \text{for each } y, n, x_2, \dots, x_n \in N. \end{aligned}$$

Therefore,
$d(x_1, x_2, \ldots, x_n)yn = d(x_1, x_2, \ldots, x_n)ny$ for each $y, n, x_2, \ldots, x_n \in N$.
Replace n by nt, where $t \in N$, in the last equation to get $d(x_1, x_2, \ldots, x_n)n[y, t] = 0$ for each $y, t, n, x_2, \ldots, x_n \in N$, i.e. $d(x_1, x_2, \ldots, x_n) N[y, t] = \{0\}$ for each $y, t, x_2, \ldots, x_n \in N$. But $d(x_1, x_2, \ldots, x_n) \neq 0$ and N is prime, so using Lemma 2.3 implies the required result.

Corollary 3.2 "[7,Theorem 3.11]. Let $d \neq 0$ be a right $n-$derivation d, s.t $d([x, y], x_2, \ldots, x_n) = 0$ for each $x, y, x_2, \ldots, x_n \in N$, then N is a C.R".

Corollary 3.3 "Let $g \neq 0$ be a generalized right derivation of N connected with the right derivation $d \neq 0$ s.t $g[x, y] = 0$ for each $x, y \in N$, then N is a C.R".

Theorem 3.4 If $g \neq 0$ be a generalized right n-derivation of N connected with right n-derivation $d \neq 0$ s.t $g(N, N, \ldots, N) \subseteq \mathcal{Z}$, then N is a C.R .

Proof. Because of $g \neq 0$, there exist $x_1, x_2, \ldots, x_n \in N$ all of them nonzero such that $g(x_1, x_2, \ldots, x_n) \in \mathcal{Z} \backslash \{0\}$. We have
$g(x_1 + x_1, x_2, \ldots, x_n) = g(x_1, x_2, \ldots, x_n) + g(x_1, x_2, \ldots, x_n) \in \mathcal{Z}$, which implies that $(N, +)$ is an abelain by Lemma 2.1. Therefore, $d([x, y], x_2, \ldots, x_n) = 0$ for each $x, y, x_2, \ldots, x_n \in N$ and, finally using Lemma 2.5 complete the proof.

Corollary 3.5 [7, Theorem 3.1] "If $d \neq 0$ be a right $n-$derivation of Ns.t $d(N, N, \ldots, N) \subseteq Z$, then N is a C.R".

Corollary 3.6 if $g \neq 0$ is a generalized right derivation connected with right derivation $d \neq 0$ of N s.t $g(N) \subseteq Z$, then N is a C.R.

Theorem 3.7 If g_1 and g_2 are nonzero generalized right $n-$derivations conneced with nonzero right $n-$derivations d_1 and d_2,respectively s.t $[g_1(N, N, \ldots, N), g_2(N, N, \ldots, N)] = \{0\}$, then N is a C.R.

Proof. If z and $z + z$ both commute with $g_2(N, N, \ldots, N)$, hence, for each $x_1, x_2, \ldots, x_n \in$ N, we have

$$zg_2(x_1, x_2, \ldots, x_n) = g_2(x_1, x_2, \ldots, x_n)z \quad (7)$$

and

$$(z + z)\, g_2(x_1, x_2, \ldots, x_n) = g_2(x_1, x_2, \ldots, x_n)(z + z) \quad (8)$$

Substituting $x_1 + x_1'$ instead of x_1 in (8), we get
$(z + z)g_2(x_1 + x_1', x_2, \ldots, x_n) = g_2(x_1 + x_1', x_2, \ldots, x_n)(z + z)$ for each $x_1, x_1', x_2, \ldots, x_n \in N$.
From (7) and (8), the previous equation can be reduced to
$zg_2(x_1 + x_1' - x_1 - x_1', x_2, \ldots, x_n) = 0$ for each $x_1, x_1', x_2, \ldots, x_n \in$ N ., i.e.;
$zg_2((x_1, x_1'), x_2, \ldots, x_n) = 0$ for each $x_1, x_1', x_2, \ldots, x_n \in$ N .
Put $z = g_1(y_1, y_2, \ldots, y_n)$ to get
$g_1(y_1, y_2, \ldots, y_n)\, g_2((x_1, x_1'), x_2, \ldots, x_n) = 0$ for each $x_1, x_1', x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$.
Use Lemma 2.6 to find that

$$g_2((x_1, x_1'), x_2, \ldots, x_n) = 0 \text{ for each } x_1, x_1', x_2, \ldots, x_n \in N \quad (9)$$

For each $w \in$ N, $w(x_1, x_1') = w(x_1 + x_1' - x_1 - x_1') = wx_1 + wx_1' - wx_1 - wx_1' = (wx_1, wx_1')$ which is again an additive commutator, we put $w(x_1, x_1')$ instead of (x_1, x_1') in (9) to get
$g_2(w(x_1, x_1'), x_2, \ldots, x_n) = 0$ for each $w, x_1, x_1', x_2, \ldots, x_n \in$ N . i.e.;
$d_2(w, x_2, \ldots, x_n)(x_1, x_1') + g_2((x_1, x_1'), x_2, \ldots, x_n)\, w = 0$. Using (9) in previous equation yields $d_2(w, x_2, \ldots, x_n)(x_1, x_1') = 0$. Using Lemma 2.8, we conclude that $(x_1, x_1') = 0$ for each $x_1, x_1' \in$ N. Hence, $(N, +)$ is an abelain group. Therefore, $d_1([x, y], x_2, \ldots, x_n) = 0$ for each $x, y, x_2, \ldots, x_n \in N$ and, using Lemma 2.5, we finally obtain that N is a C.R.

Corollary 3.8 If d_1 and d_2 are nonzero right n-derivations of N , s.t $[d_1(N, N, \ldots, N), d_2(N, N, \ldots, N)] = \{0\}$, then N is a C.R .

Corollary 3.9 If g_1 and g_2 are nonzero generalized right derivations of N connected with the nonzero right derivations d_1, d_2, respectively, s.t $[g_1(N), g_2(N)] = \{0\}$, then N is a C.R.

Theorem 3.10 if g_1 and g_2 are nonzero right generalized n-derivations of N connected with the nonzero right n-derivations d_1 and d_2, respectively, s.t $g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)\, g_1(y_1, y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in$ N, then N is a C.R.

Proof. From hypothesis,

$$g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)\, g_1(y_1, y_2, \ldots, y_n) = 0$$
$$\text{for each } x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in \text{N} . \quad (10)$$

Substitute $y_1 + y_1'$ in place of y_1 in (10) to get
$g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1 + y_1', y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)\, g_1(y_1 + y_1', y_1, y_2, \ldots, y_n) = 0$

for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_1, y_2, \ldots, y_n \epsilon$ N .

Therefore,

$g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1, y_2, \ldots, y_n) + g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1', y_2, \ldots, y_n) +$
$g_2(x_1, x_2, \ldots, x_n)\, g_1(y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)\, g_1(y_1', y_2, \ldots, y_n) = 0$
for each $x_1, x_2, \ldots, x_n, y_1', y_1, y_2, \ldots, y_n \epsilon$ N .

By using (10) again in the preceding equation, we get

$g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1, y_2, \ldots, y_n) + g_1(x_1, x_2, \ldots, x_n)\, g_2(y_1', y_2, \ldots, y_n) +$
$g_1(x_1, x_2, \ldots, x_n)\, g_2(-y_1, y_2, \ldots, y_n) + g_1(x_1, x_2, \ldots, x_n)\, g_2(-y_1', y_2, \ldots, y_n) = 0$
for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \epsilon$ N .

Which means that

$g_1(x_1, x_2, \ldots, x_n)\, g_2((y_1, y_1'), y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \epsilon$ N. By Lemma 2.6, we obtain $g_2((y_1, y_1'), y_2, \ldots, y_n) = 0$ for each $y_1, y_1', y_2, \ldots, y_n \epsilon$ N. Now, by putting $w(y_1, y_1')$ instead of (y_1, y_1'), where $w \epsilon N$, in the previous equation, we get $g_2(w(y_1, y_1'), y_2, \ldots, y_n) = 0$ for each $y_1, y_1', y_2, \ldots, y_n \epsilon$ N. So, we have $d_2(w, y_2, \ldots, y_n)(y_1, y_1') = 0$. Byusing Lemma 2.8, we conclude that $(y_1, y_1') = 0$ for each $y_1, y_1' \epsilon N$ i.e., $(N, +)$ is an abelain group. Therefore, $d_1([x, y], x_2, \ldots, x_n) = 0$ for each $x, y, x_2, \ldots, x_n \epsilon N$ and, using Lemma 2.5, we finally obtain that *N* is a C.R.

Corollary 3.11 If d_1 and d_2 are right n-derivations of *N* s.t $d_1(x_1, x_2, \ldots, x_n)\, d_2(y_1, y_2, \ldots, y_n) + d_2(x_1, x_2, \ldots, x_n) d_1(y_1, y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \epsilon$ N, then *N* is a C.R.

Corollary 3.12 If g_1 and g_2 are nonzero generalized right derivations of *N* connected with the nonzero right derivations d_1 and d_2, respectively s.t $g_1(x)g_2(y) + g_2(x)g_1(y) = 0$ for each $x, y \epsilon N$, then *N* is a C.R.

Theorem 3.13 If g_1 is a nonzero generalized right n-derivation of *N* connected with the nonzero right n-derivation d_1, and g_2 is a nonzero generalized n-derivation of *N* associated with the nonzero n-derivation d_2 .

(i) If $g_1(x_1, x_2, \ldots, x_n)g_2(y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)g_1(y_1, y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \epsilon N$, then *N* is a C.R.

(ii) If $g_2(x_1, x_2, \ldots, x_n)g_1(y_1, y_2, \ldots, y_n) + g_1(x_1, x_2, \ldots, x_n)g_2(y_1, y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \epsilon N$, then *N* is a C.R.

Proof.

(i) By hypothesis,

$$g_1(x_1, x_2, \ldots, x_n)g_2(y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)\, g_1(y_1, y_2, \ldots, y_n) = 0$$
for each $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \epsilon N$. (11)

Substituting $y_1 + y_1'$, where $y_1' \epsilon$ N, for y_1 in (11), we get

$g_1(x_1, x_2, \ldots, x_n)g_2(y_1 + y_1', y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)g_1(y_1 + y_1', y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \epsilon$ N. x_1 ,

So, we have

$g_1(x_1, x_2, \ldots, x_n)g_2(y_1, y_2, \ldots, y_n) + g_1(x_1, x_2, \ldots, x_n)g_2(y_1', y_2, \ldots, y_n) +$
$g_2(x_1, x_2, \ldots, x_n)g_1(y_1, y_2, \ldots, y_n) + g_2(x_1, x_2, \ldots, x_n)g_1(y_1', y_2, \ldots, y_n) = 0$
for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \epsilon$ N.

Using (11) in the previous equation implies that

$g_1(x_1, x_2, \ldots, x_n)g_2(y_1, y_2, \ldots, y_n) + g_1(x_1, x_2, \ldots, x_n)g_2(y_1', y_2, \ldots, y_n) +$
$g_1(x_1, x_2, \ldots, x_n)g_2(-y_1, y2, \ldots, yn) + g_1(x_1, x_2, \ldots, x_n)g_2(-y_1', y_2, \ldots, y_n) = 0$
for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \epsilon$ N,

which means that

$g_1(x_1, x_2, \ldots, x_n)g_2((y_1, y_1'), y_2, \ldots, y_n) = 0$ for each $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \epsilon N$.

Now, using Lemma 2.7, we conclude that

$$g_2((y_1, y_1'), y_2, \ldots, y_n) = 0 \text{ for each } y_1, y_1', y_2, \ldots, y_n \epsilon N \quad (12)$$

Now putting $w(y_1, y_1')$ instead of (y_1, y_1'), in (12) and usw it again to get $d_2(w, y_2, \ldots, y_n)(y_1, y_1') = 0$ for each w, $y_1, y_1', y_2, \ldots, y_n \epsilon N$, use Lemma 2.4 lastly to conclude that $(y_1, y_1') = 0$ for each $y_1, y_1' \epsilon$ N. Thus, $(N, +)$ is an abelain, hence $d_1([x, y], x_2, \ldots, x_n) = 0$ for each $x, y, x_2, \ldots, x_n, \epsilon$ N and, finally, we obtain that *N* is a C.R by Lemma 2.5.

By using same arguments as in (i), we can proof (ii)

Corollary 3.14 Let $d_1 \neq 0$ be a right n-derivation of N, and $d_2 \neq 0$ be an n-derivation of N.

(i) If $d_1(x_1, x_2, \dots, x_n)d_2(y_1, y_2, \dots, y_n) + d_2(x_1, x_2, \dots, x_n)\, d_1(y_1, y_2, \dots, y_n) = 0$ for each $x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n \in N$, then N is a C.R.

(ii) If $d_2(x_1, x_2, \dots, x_n)\, d_1(y_1, y_2, \dots, y_n) + d_1(x_1, x_2, \dots, x_n)\, d_2(y_1, y_2, \dots, y_n) = 0$ for each $x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n \in N$, then N is a C.R.

Corollary 3.15 Let g_1 be a nonzero generalized right derivation of N associated with the nonzero right derivation d_1, and g_2 be a nonzero generalized derivation of N associated with the nonzero derivation d_2.

(i) If $g_1(x)d_2(y) + g_2(x)\, d_1(y) = 0$ for each $x, y \in N$, then N is a C.R.

(ii) If $g_2(x)\, d_1(y) + g_1(x)d_2(y) = 0$ for each $x, y \in N$ then N is a C.R.

Theorem 3.16. If N is two-torsion free, then N admits no nonzero generalized right n-derivation g associated with the nonzero right derivation d, such that $g(x \diamond y, x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$.

Proof. Assume that

$$g(x \diamond y, x_2, \dots, x_n) = 0 \text{ for each } x, y, x_2, \dots, x_n \in N. \quad (13)$$

put xy in place of y in (13) to get $g(x \diamond xy, x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$, which implies that $g(x(x \diamond y), x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$. It follows that $d(x, x_2, \dots, x_n)(x \diamond y) + g(x \diamond y, x_2, \dots, x_n)x = 0$ for each $x, y, x_2, \dots, x_n \in N$, use (13) in the last equation to get $d(x, x_2, \dots, x_n)(x \diamond y) = 0$ for each $x, y, x_2, \dots, x_n \in N$, or equivalently

$$d(x, x_2, \dots, x_n)yx = -d(x, x_2, \dots, x_n)xy \text{ for each } x, y, x_2, \dots, x_n \in N \quad (14)$$

Replacing y by yz, where $z \in N$, in (14), we get

$$\begin{aligned} d(x, x_2, \dots, x_n)yzx &= -d(x, x_2, \dots, x_n)xyz \\ &= d(x, x_2, \dots, x_n)xy(-z) \\ &= d(x, x_2, \dots, x_n)y(-x)(-z) \text{ for each } x, y, z, x_2, \dots, x_n \in N. \end{aligned}$$

In the last equation, using the fact

$-d(x, x_2, \dots, x_n)yzx = d(x, x_2, \dots, x_n)yz(-x)$ for each $x, y, z, x_2, \dots, x_n \in N$ implies that $d(x, x_2, \dots, x_n)yz(-x) - d(x, x_2, \dots, x_n)y(-x)z = 0$ for each $x, y, z, x_2, \dots, x_n \in N$, which implies that

$d(x, x_2, \dots, x_n)y[-x, z] = 0$ for each $x, y, z, x_2, \dots, x_n \in N$.

Replacing x by $-x$ in the previous equation, we get

$d(-x, x_2, \dots, x_n)y[x, z] = 0$ for each $x, y, z, x_2, \dots, x_n \in N$.

Hence, we get

$$d(-x, x_2, \dots, x_n)N[x, z] = \{0\} \text{ for each } x, z, x_2, \dots, x_n \in N. \quad (15)$$

By primeness we find that

For each $x \in N$, either $d(-x, x_2, \dots, x_n) = 0$ for each $x_2, \dots, x_n \in N$ or $x \in Z$.

Since $(x, x_2, \dots, x_n) = -d(-x, x_2, \dots, x_n) = 0$, we get:

for each fixed $x \in N$, either $d(x, x_2, \dots, x_n) = 0$ for each $x_2, \dots, x_n \in N$ or $x \in Z$.

If $d(x, x_2, \dots, x_n) = 0$ for each $x_2, \dots, x_n \in N$ and for each $x \in N$, we get $d = 0$, and this contradicts assumption. Therefore, there exist $x_1, x_2, \dots, x_n \in N$, all being nonzero, such that $d(x_1, x_2, \dots, x_n) \neq 0$ and $x_1 \in Z$. Since $x_1 \in Z$, we conclude that $(x_1 y \diamond z) = x_1(y \diamond z)$, *where* $y, z \in N$ and $g(x_1 y \diamond z, x_2, \dots, x_n) = 0$ for each $x_1, y, z, x_2, \dots, x_n \in N$.

Therefore,

$$\begin{aligned} 0 &= g(x_1(y \diamond z), x_2, \dots, x_n) \\ &= d(x_1, x_2, \dots, x_n)(y \diamond z) + g(y \diamond z, x_2, \dots, x_n)x_1 \\ &= d(x_1, x_2, \dots, x_n)(yoz) \text{ for each } y, z \in N. \end{aligned}$$

which implies that

$d(x_1, x_2, \dots, x_n)yz = -d(x_1, x_2, \dots, x_n)zy$ for each $y, z \in N$. Replace z by zt, where $t \in N$, in the last equation and use it to get $d(x_1, x_2, \dots, x_n)N[y, t] = \{0\}$ for each $y, z, t \in N$. Since $d(x_1, x_2, \dots, x_n) \neq 0$ and N is prime, we conclude that N is a C.R in view of Lemma 2.3. Now, return to (13) to get $2d(xy, x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$, it follows that $d(xy, x_2, \dots, x_n) = 0$ for each $x, y, x_2, \dots, x_n \in N$ by two torsion freeness of N, and this get $d(x, x_2, \dots, x_n)y + d(y, x_2, \dots, x_n)x = 0$ for each $x, y, x_2, \dots, x_n \in N$. if we replace x by zx, where $z \in N$, in the previous equation we find that $d(zx, x_2, \dots, x_n)y + d(y, x_2, \dots, x_n)zx = 0$ for each

$x, y, z, x_2, \ldots, x_n \in N$, it follows that $d(y, x_2, \ldots, x_n)Nx = \{0\}$ for each $x, y, x_2, \ldots, x_n \in N$. Since $d \neq 0$, primeness of N forces that x = 0 for each $x \in N$: a contradiction.

Theorem 3.17 If $g \neq 0$ is a generalized n-derivation of N connected with the right n-derivation $d \neq 0$ s.t $[g(x, x_2, \ldots, x_n), y] \in Z$ for each $x, y, x_2, \ldots, x_n \in N$, then N is a C.R .

Proof. By assumption

$$[g(x, x_2, \ldots, x_n), y] \in Z \text{ for each } x, y, x_2, \ldots, x_n \in N \quad (16)$$

Hence

$$[[g(x, x_2, \ldots, x_n), y], t] = 0 \text{ for each } x, y, t, x_2, \ldots, x_n \in N. \quad (17)$$

Replacing y by $[g(x, x_2, \ldots, x_n), y]$ in (17), we get

$$[[g(x, x_2, \ldots, x_n), [g(x, x_2, \ldots, x_n), y]], t] = 0 \text{ for each } x, y, t, x_2, \ldots, x_n \in N \quad (18)$$

In view of (16), equation (18) assures that

$$[g(x, x_2, \ldots, x_n, y] N [g(x, x_2, \ldots, x_n), t] = \{0\} \text{ for each } x, y, t, x_2, \ldots, x_n \in N \quad (19)$$

The primeness of N implies that $[g(x, x_2, \ldots, x_n), y] = 0$ for each $x, y, x_2, \ldots, x_n \in N$ and hence $g(N, N, \ldots, N) \subseteq Z$. The application of Theorem 3.4 assures that N is a C.R.

Corollary 3.18 [7, Theorem 3.15] If $d \neq 0$ is a right n-derivation of Ns.t $[d(x, x_2, \ldots, x_n), y] \in Z$ for each $x, y, x_2, \ldots, x_n \in N$, then N is a C.R.

Corollary 3.19 If $g \neq 0$ is a generalized right derivation of N connected with the right derivation $d \neq 0$ s.t $[g(x), y] \in Z$ for each $x, y \in N$, then N is a C.R .

Theorem 3.20 If $g \neq 0$ is a generalized right n-derivation of N connected with the right n-derivation $d \neq 0$ s.t $g(x, x_2, \ldots, x_n) \diamond y \in Z$ for each $x, y, x_2, \ldots, x_n \in N$, then N is a C.R.

Proof. By assumption

$$g(x, x_2, \ldots, x_n) \diamond y \in Z \text{ for each } x, y, x_2, \ldots, x_n \in N \quad (20)$$

(a) If $Z = 0$, from (20) we get

$$yg(x, x_2, \ldots, x_n) = -(g(x, x_2, \ldots, x_n)y)$$
$$= g(x, x_2, \ldots, x_n)(-y) \text{ for each } x, y, x_2, \ldots, x_n \in N \quad (21)$$

Substituting zy for y in (21) and using it again ,we obtain

$$zyg(x, x_2, \ldots, x_n) = -(g(x, x_2, \ldots, x_n)zy)$$
$$= g(x, x_2, \ldots, x_n)z(-y)$$
$$= zg(-x, x_2, \ldots, x_n))(-y) \text{ for each } x, y, z, x_2, \ldots, x_n \in N.$$

Using the fact that $-zyg(x, x_2, \ldots, x_n) = zyg(-x, x_2, \ldots, x_n)$ in the previous equation implies that

$$zyg(-x, x_2, \ldots, x_n) = zg(-x, x_2, \ldots, x_n))y \text{ for each } x, y, z, x_2, \ldots, x_n \in N.$$

This implies that

$$z(yg(-x, xx_2, \ldots, x_n) - g(-x, x2, \ldots, xn)y) = 0 \text{ for each } x, y, z, x_2, \ldots, x_n \in N. \quad (22)$$

Taking $-x$ instead of x in (22), we get

$zN(yg(x, x_2, \ldots, x_n) - g(x, x_2, \ldots, x_n)y) = \{0\}$for each $x, y, z, x_2, \ldots, x_n \in N$. It follows that $g(N, N, \ldots, N) \subseteq Z$ because of primeness of N, and using Theorem 3.4 assures that N is a C.R.

Now, if $Z \neq 0$, then there exists $0 \neq z \in Z$ and from (20) we have $g(x, x_2, \ldots, x_n) \diamond z \in Z$ for each $x, x_2, \ldots, x_n \in N$, it follows that $g(x, x_2, \ldots, x_n)z + zg(x, x_2, \ldots, x_n) \in Z$ for each $x, x_2, \ldots, x_n \in N$. Since $z \in Z$, we get $z(g(x, x_2, \ldots, x_n) + g(x, x_2, \ldots, x_n)) \in Z$ for each $x, x_2, \ldots, x_n \in N$. By Lemma 2.2, we conclude that

$$g(x, x_2, \ldots, x_n) + g(x, x_2, \ldots, x_n) \in Z \text{ for each } x, x_2, \ldots, x_n \in N \quad (23)$$

By (20), we get

$$g(x + x, x_2, \ldots, x_n)y + yg(x + x, x_2, \ldots, x_n) \in Z \text{ for all } x, y, x_2, \ldots, x_n \in N. \quad (24)$$

Using equation (23) in (24), we conclude that

$$y(g(x + x, x_2, \ldots, x_n) + g(x + x, x_2, \ldots, x_n)) \in Z \text{ for all } x, y, x_2, \ldots, x_n \in N. \quad (25)$$

For each $x, y, t, x_2, \ldots, x_n \in N$, we get

$$ty(g(x + x, x_2, \ldots, x_n) + g(x + x, x_2, \ldots, x_n))$$
$$= y(g(x + x, x_2, \ldots, x_n) + g(x + x, x_2, \ldots, x_n))t$$
$$= (g(x + x, x_2, \ldots, x_n) + g(x + x, x_2, \ldots, x_n))yt.$$

This implies that

$$(g(x + x, x_2, \ldots, x_n) + g(x + x, x_2, \ldots, x_n))N[t, y] = \{0\}$$
$$\text{for each } x, y, t, x_2, \ldots, x_n \in N \quad (28)$$

The primeness of N implies that either $g(x+x,x_2,\dots,x_n)+g(x+x,x_2,\dots,x_n)=0$ and thus $g=0$ "which is a contradiction" or $N=Z$, hence $g(N,N,\dots,N)\subseteq Z$ and using Theorem 3.4 assures that N is a C.R

Corollary 3.21 [7, Theorem 3.17]. Let d be nonzero right n-derivation of N. If $d(x,x_2,\dots,x_n)\diamond y\epsilon Z$ for each $x,y,x_2,\dots,x_n\epsilon N$, then N is a C.R.

Corollary 3.22. Let g be generalized right derivation of N associated with the nonzero right derivation d. If $g(x)\diamond y\,\epsilon\, Z$ for each $x,y\epsilon N$, then N is a C.R.

Primeness assumption is necessary in our results and the following example will show that:

Example 3.23. Let $\mathcal{S}$ be a zero-symmetric and two-torsion free near-ring. It is obvious that $\mathcal{M}=\left\{\begin{pmatrix}0 & x & y\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, x, y, 0\in\mathcal{S}\right\}$ is zero symmetric near-ring " not prime" with addition and multiplication of matrices.

Define $d_1,g_1,d_2,g_2:\underbrace{\mathcal{M}\times\mathcal{M}\times\ldots\times\mathcal{M}}_{\text{n-times}}\longrightarrow\mathcal{M}$ such that

$$d_1\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & x_1x_2\dots x_n & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$g_1\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & y_1y_2\dots y_n & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$d_2\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & 0 & x_1x_2\dots x_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$g_2\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & 0 & y_1y_2\dots y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

It is easy to check that g_1 and g_2 are nonzero right generalized n-derivations of $\mathcal{M}$ associated with the right n-derivations d_1, d_2, respectively and

(i) Let $A\epsilon\mathcal{M}$, $A=\begin{pmatrix}0 & x & y\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$ such that $x,y\neq 0$, then we can see that $g_1(\mathcal{M},\mathcal{M},\dots,\mathcal{M})A=0$. But $A\neq 0$ and

(ii) $g_1(\mathcal{M},\mathcal{M},\dots,\mathcal{M})\subseteq Z$;

(iii) $[g_1(\mathcal{M},\mathcal{M},\dots,\mathcal{M}), g_2(\mathcal{M},\mathcal{M},\dots,\mathcal{M})]=\{0\}$;

(iv) $g_1(A_1,A_2,\dots,A_n)\,g_2(B_1,B_2,\dots,B_n)+g_2(A_1,A_2,\dots,A_n)\,g_1(B_1,B_2,\dots,B_n)=0$ for each $A_1,A_2,\dots,A_n,B_1,B_2,\dots,B_n\epsilon\mathcal{M}$;

(v) $A_1\,g_1(B_1,B_2,\dots,B_n)=g_1(A_1,A_2,\dots,A_n)B_1$ for each $A_1,A_2,\dots,A_n,B_1,B_2,\dots,B_n\epsilon\mathcal{M}$;

(vi) $g_1([A,B],A_2,\dots,A_n)=0$ for each $A,B,A_2,\dots,A_n\epsilon\mathcal{M}$;

(vii) $g_1(A\diamond B,A_2,\dots,A_n)=0$ for each $A,B,A_2,\dots,A_n\epsilon\mathcal{M}$.

But $\mathcal{M}$ is not a C.R.

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