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A study on Soft Pre-Open Sets using γ Operation

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Abstract

The concept of strong soft γ pre-open set was initiated by Biswas and Parsanann.We utilize this notion to study several characterizations and properties of this set. We investigate the relationships between this set and other types of soft open sets. Moreover, the properties of the strong soft γ pre-interior and closure are discussed. Furthermore, we define a new concept by using strong soft γ pre-closed that we denote as locally strong soft γ pre-closed, in which several results are obtained. We establish a new type of soft pre-open set, namely soft γ pre-open. Also, we continue to study pre- γ soft open set and discuss the relationships among all these sets. Some counter examples are given to show some relationships obtained in this work.

Keywords: soft γ regular space**,** soft γ pre open**,** strong soft γ pre closure**,** locally strong soft γ pre closed**,** soft open.

دراسة حول المجموعات الناعمة المفتوحة الأولياً بأستخدام عامل التشغيل γ

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الخلاصة

عرف كل من Biswas و Parasnnan مفهوم المجموعة القوية الناعمة ذات معامل γ المفتوحة اوليا في هذا العمل أعطينا تمييزات عدة ودرسنا خواص هذه المجموعة حيث قمنا بتحقيق العلاقات بين هذه المجموعة مع انواع اخرى من المجموعات الناعمة المفتوحة اوليا بالأضافة الى ذلك تمت مناقشة خواص الأنغلاق والداخلية لمجموعة القوية الناعمة ذات معامل γ المفتوحة اوليا. كذلك استخدمنا مفهوم القوية الناعمة ذات معامل γ المغلقة اوليا لتعريف القوية الناعمة ذات معامل γ المغلقة اوليا محليا حيث حصلنا على عدة نتائج حولها . تمكنا من تقديم نوع جديد من المجموعات الناعمة المفتوحة اوليا وذلك بأستخدام المعامل γ أسميناها المجموعة الناعمة ذات ذات معامل γ المفتوحة اوليا وأستمرينا المجموعة المفتوحة اوليا ذات معامل تشغيل γ الناعمة. بالأضافة الى ذلك ناقشنا العلاقات بين هذه المجموعات المختلفة واعطاء الأمثلة المضادة لتلك العلاقات

1. Introduction

Moldstov [1] investigated the soft set theory, as a new approach for uncertainties, and the vague set theory. Also, he presented many uses of soft sets in some directions, such as game theory, Perron integration, and probability.

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Kasahara [2] initiated the notion of γ operation on topological space and explored several important properties. Later, Maji and others [3] studied several operations on soft sets in more details and proved many propositions about these operations. Jamil [4] established new open sets by using the idea of operation and provided several characterizations

Moreover, Ogata [5] utilized the idea of operation to introduce new types of open sets, called γ open and pre-γ open sets. Also, he mentioned several properties of each set.

Recently, Biswas and Prasannan [6] introduced and discussed various types of soft open sets by using the notions $\tau_\gamma int$ and $\tau_\gamma cl$. In this paper, we continue studying such many forms of soft pre-open sets involving γ operation.

Finally, Al-shami [7-11] studied a new type of soft open sets and investigated the properties of its separation axioms.

2. Preliminaries and basic results

Definition 2.1 [1]. Let X be the set of the universe and A be a set of parameters, then the pair (F,A) is named a soft set over space X such that F is mapping from A to the family of all subsets of X, which is denoted by $F: A \to P(X)$.

Definition 2.2 [3]. Let (N,A) and (M,A) be two soft sets over a soft space X on a common parameter A, then $(N,A) \tilde{\subseteq} (M,A)$, if $N(e) \subseteq M(e)$ for each $e \in A$.

Definition 2.3 [3]. Let (N,A) and (M,A) be two soft sets over a space X on a common parameter A. $(N,A) \tilde{\subseteq} (M,A)$ and $(M,A) \tilde{\subseteq} (N,A)$, if $N(e) \subseteq M(e)$ and $M(e) \subseteq N(e)$ for each $e \in A$. Hence (N,A) is equal to (M,A).

Definition 2.4 [3]. A soft set (F,A) of X is named a null soft set, which is represented by $\tilde{\phi}$, if for any $e \in A$, $F(e) = \phi$.

Definition 2.5 [3]. A soft set (F,A) of X is named an absolute soft set that is represented by $\tilde{X}$ if for any $e \in A$, $F(e) = X$.

Definition 2.6 [1]. Let (N,A) and (M,A) be two soft sets over X, then 1) $(S,A) = (N,A) \bigcup (M,A)$ is determined by $S(e) = N(e) \cup M(e)$ for each $e \in A$

2) $(T,A) = (N,A) \bigcap (M,A)$ is determined by $T(e) = N(e) \bigcap M(e)$ for each $e \in A$.

Definition 2.7 [10]. Relative complement for any soft set (F,A) is defined by $(F,A)^c = (F^c, A)$, such that $F^c: A \to P(X)$ is a function given by $F^c(e) = X - F(e)$ for each $e \in A$.

Definition 2.8 [12]. The family τ of soft sets is in the universe set X, and A is a set of parameters, then τ is called soft topology on X if the following conditions hold

1) $\tilde{\phi}$, $\tilde{X}$ belong to τ,

2) If $\{(N_i, A): i \in I\} \in \tau$, then $\bigcup_{i\in I}(N_i, A) \in \tau$,

3) If (N,A) , $(M,A) \in \tau$, then $(N,A) \bigcap (M,A) \in \tau$.

That is, (X,τ,A) being a soft topological space.

The family of all soft open sets is stated by $S^*O(X)$.

Definition 2.9 [13]. The soft closure of a soft subset (F,A) over a space (X,τ,A) is the intersection of each soft closed supersets of (F,A) which is stated by $cl(F,A)$.

Definition 2.10 [13]. The soft interior of a soft set of (F,A) in space (X,τ,A) is the union of all soft open subsets of (F,A), which is stated by $int(F,A)$.

Definition 2.11 [2]. A soft space (X,τ,A). An operation γ is a function from soft topology τ into $P(X)$, that is $(K,A) \tilde{\subseteq} (K,A)^\gamma$ for all $(K,A) \in \tau$, in which $(K,A)^\gamma$ represents the value of γ at (K,A).

For simplicity, we use the notation $\tilde{X}_\gamma$ to indicate the soft space (X,τ,A) in which γ is an operation defined on τ.

Definition 2.12 [6]. A soft set (U,A) in $\tilde{X}_\gamma$ is named as a γ soft open set, if for any $e_F \in (U,A)$, there is a soft open set (H,A) containing e_F such that $(H,A)^\gamma \tilde{\subseteq} (U,A)$. The family of all γ soft open sets is stated by $S^*_\gamma O(\tilde{X}_\gamma)$.

Definition 2.13 [13]. A space $\tilde{X}_\gamma$ is named a soft γ regular space if, for any $e_F \in (X,A)$ and for all soft open sets (K,A), there exists a soft open set (H,A) containing e_F in which $(K,A)^\gamma \tilde{\subseteq} (H,A)$.

Definition 2.14 [13]. The soft γ interior of (H,A) over $\tilde{X}_\gamma$ is identified as the union of every γ soft open set subset of (F,A), which is stated by $\tau_\gamma int(H,A)$, $\tau_\gamma int(H,A) = \bigcup\{(U,A): (U,A) \text{ is } \gamma \text{ soft open set }, (U,A) \tilde{\subseteq} (H,A)\}$.

Definition 2.15 [13]. The soft γ closure of (H,A) over $\tilde{X}_\gamma$ is identified as the intersection of every γ soft closed super-set of (H,A) which is stated by $\tau_\gamma cl(H,A)$, $\tau_\gamma cl(H,A) = \bigcap\{(U,A): (U,A) \text{ is a } \gamma \text{ soft closed set }, (U,A) \tilde{\subseteq} (H,A)\}$

Definition 2.16 [14]. A soft set (F,A) in soft space (X,τ,A) is named pre-soft open if $(F,A) \tilde{\subseteq} int\ cl\ (F,A)$.

Definition 2.17 [15]. A soft set (H,A) over $\tilde{X}_\gamma$ is called soft γ semi-open, if there is γ soft open set (U,A) in which $(U,A) \tilde{\subseteq} (H,A) \tilde{\subseteq} \tau_\gamma cl(U,A)$.

Definition 2.18 [13]. An operation γ associated with soft topology τ is named soft regular, if for any soft open (K,A) and (H,A) and any $e_F \in (X,A)$, there is a soft open (T,A) containing e_F in which $(T,A)^\gamma \tilde{\subseteq} (K,A)^\gamma \bigcup (H,A)^\gamma$.

Proposition 2.19 [12]. Let (Y,τ_Y,A) be soft subspace of (X,τ,A), then (K,A) is soft open over Y if and only if $(K,A) = Y \bigcap (G,A)$ for soft open set (G,A).

Proposition 2.20 [12]. Let (Y,τ_Y,A) be a soft subspace of soft topological space (X,τ,A), then (F,A) is soft closed set over Y if and only if $(F,A) = Y \bigcap (H,A)$ for soft closed set (H,A).

Proposition 2.21 [6]. Each γ soft open set is soft open.

Proposition 2.22 [6]. If (N,A) is soft open and (M,A) is soft subset of (X,τ,A) , then $cl[(N,A)\bigcap(M,A)] = cl(N,A)\bigcap cl(M,A)$.

Proposition 2.23 [13]. Let (U,A)be a soft subset of $\tilde{X}_\gamma$, then (U,A) is a soft γ semi-open, if and only if $(U,A) \tilde{\subseteq} \tau_\gamma cl\ \tau_\gamma int(U,A)$.

Proposition 2.24 [6]. Let (U,A) be soft set in $\tilde{X}_\gamma$, then (U,A) is soft γ open if and only if $\tau_\gamma int(U,A) = (U,A)$.

Proposition 2.25 [6]. Let (N,A) be soft set over $\tilde{X}_\gamma$, then

1) $cl(N,A) \tilde{\subseteq} \tau_\gamma cl(N,A)$,

2) $\tau_\gamma int(N,A) \tilde{\subseteq} int(N,A)$.

Proposition 2.26 [6]. Let (N,A) be any soft set over soft γ regular space $\tilde{X}_\gamma$, then $\tau_\gamma int(N,A) = int(N,A)$.

Proposition 2.27. Let γ be soft regular-operation defined over soft topology τ, the intersection of two soft γ open over $\tilde{X}_\gamma$ is soft γ open set.

Proof: Consider (U,A) and (V,A) are γ soft open sets over space X. Let $e_F \in (U,A)\bigcap(V,A)$, then $e_F \in (U,A)$ and $e_F \in (V,A)$. It follows that there exist two soft open (H_1,A) and (H_2,A) containinge_F, in which $e_F \in (H_1,A)^\gamma \tilde{\subseteq} (U,A)$ and $e_F \in (H_2,A)^\gamma \tilde{\subseteq} (V,A)$. Hence, $e_F \in (H_1,A)^\gamma \bigcap (H_2,A)^\gamma \tilde{\subseteq} (U,A)\bigcap(V,A)$. By definition 2.18, there is soft open (H_3,A) containing e_F, in which $(H_3,A)^\gamma \tilde{\subseteq} (H_1,A)^\gamma \bigcap (H_2,A)^\gamma \tilde{\subseteq} (U,A)\bigcap(V,A)$. Hence, $(U,A)\bigcap(V,A)$ is γ soft open set.

Proposition 2.28. Let (N,A) and (M,A) be any soft subsets over $\tilde{X}_\gamma$, then the following statements are true

1) If $(N,A) \tilde{\subseteq} (M,A)$, then $\tau_\gamma cl(N,A) \tilde{\subseteq} \tau_\gamma cl(M,A)$

2) $\tau_\gamma cl(N,A)\bigcup\tau_\gamma cl(M,A) \tilde{\subseteq} \tau_\gamma cl[\ (N,A)\bigcup\ (M,A)]$

3) $\tau_\gamma cl[\ (N,A)\bigcap\ (M,A)] \tilde{\subseteq} \tau_\gamma cl(N,A)\bigcap\tau_\gamma cl(M,A)$

4) For soft regular -operation γ, $\tau_\gamma cl(N,A)\bigcup\tau_\gamma cl(M,A) = \tau_\gamma cl[\ (N,A)\bigcup\ (M,A)]$.

Proof: Obvious.

Proposition 2.29. If $\tilde{Y}_\gamma$ is a soft subspace of $\tilde{X}_\gamma$ and $(F,A) \tilde{\subseteq} \tilde{Y}$, then $\tau_\gamma cl_Y(F,A) = \tau_\gamma cl_X(F,A)\bigcap\tilde{Y}$.

Proof: Consider $\tau_\gamma cl_Y(F,A) = \bigcap\{(S,A): (S,A) \text{ is } \gamma \text{ soft closed in } Y \text{ and } (F,A) \tilde{\subseteq} (S,A)\} = \bigcap\{(L,A)\bigcap Y: (L,A) \text{ is } \gamma \text{ soft closed in } X, (F,A) \tilde{\subseteq} (L,A)\bigcap Y \tilde{\subseteq} (L,A)\} = [\bigcap\{(L,A): (L,A) \text{ is } \gamma \text{ soft closed in } X, (F,A) \tilde{\subseteq} (L,A)\}]\bigcap Y = \tau_\gamma cl_X(F,A)\bigcap Y$.

Proposition 2.30. Let γ be soft- regular operation set to $\tilde{\tau}$. If (K,A) is any soft set and (L,A) is γ soft open set over $\tilde{X}_\gamma$, then $\tau_\gamma cl(K,A)\bigcap(L,A) \tilde{\subseteq} \tau_\gamma cl\big((K,A)\bigcap(L,A)\big)$.

Proof: Let $e_F \in \tau_\gamma cl(K,A) \cap (L,A)$, then $e_F \in \tau_\gamma cl(K,A)$ and $e_F \in (L,A)$. It follows that for every γ soft open set (H,A) containing e_F, $(K,A) \cap (H,A) \neq \tilde{\phi}$, and by proposition 2.27, $(K,A) \cap (H,A)$ is soft γ open set containing e_F. But $e_F \in \tau_\gamma cl(K,A)$, therefore $(K,A) \cap (L,A) \cap (H,A) \neq \tilde{\phi}$ which implies that $e_F \in \tau_\gamma cl\big((K,A) \cap (L,A)\big)$.

3. Some properties of strong soft γ pre open set

Definition 3.1 [6]. A soft set (K,A) in $\tilde{X}_\gamma$ is named strong soft γ pre-open set if $(K,A) \tilde{\subseteq} int\, \tau_\gamma cl(K,A)$.

The family of all strong soft γ pre-open sets over $\tilde{X}_\gamma$ is stated by $SS^*_\gamma P(\tilde{X}_\gamma)$.

Example 3.2. Consider $X = \{h_1, h_2\}$ and $= \{a_1, a_2\}$. Let $\tau = \{\phi, X, (K_1,A), (K_2,A), (K_3,A)\}$ where $K_1(a_1) = \{h_1\}$, $K_2(a_1) = \{h_2\}$, $K_3(a_1) = \phi$, $K_1(a_2) = \{h_2\}$, $K_2(a_2) = X$, $K_3(a_2) = \{h_2\}$.

Assign an operation $\gamma: \tau \longrightarrow P(X)$ as $\gamma(U,A) = (U,A) \cap (K_1,A)$ for every $(U,A) \in \tau$, then $\tau_\gamma = \{\phi, X, (K_1,A)\}$ where (K_1,A) is strong soft γ pre-open set.

Proposition 3.3. A strong soft γ is pre-open (K,A) over $\tilde{X}_\gamma$ if and only if there is soft open set (N,A) in which $(K,A) \tilde{\subseteq} (N,A) \tilde{\subseteq} \tau_\gamma cl(K,A)$.

Proof: As (K,A) is a strong soft γ pre-open. Set$(N,A) = int\, \tau_\gamma cl(K,A)$, then (N,A) is soft open over X in which $(K,A) \tilde{\subseteq} (N,A) = int\, \tau_\gamma cl(K,A) \tilde{\subseteq} \tau_\gamma cl(K,A)$.

Conversely, since $(N,A) \tilde{\subseteq} \tau_\gamma cl(K,A)$, then $(N,A) \tilde{\subseteq} int\, \tau_\gamma cl(K,A)$. It follows that (K,A) is strong soft γ pre-open set.

Remark 3.4. Every soft open over $\tilde{X}_\gamma$ is strong soft γ pre open-set.

Proof: assume that (K,A) is soft open over $\tilde{X}_\gamma$, then $(K,A) \tilde{\subseteq} int(K,A) \tilde{\subseteq} int\, \tau_\gamma cl(K,A)$.

Note that the intersection of finite strong soft γ pre-open sets may not be a strong soft γ pre-open set, as shown in the next example.

Example 3.6. Consider $X = \{h_1, h_2, h_3\}$ and $= \{a_1, a_2\}$. Let $\tau = \{\tilde{\phi}, \tilde{X}, (K_1,A), (K_2,A), (K_3,A), (K_4,A)\}$ where $K_1(a_1) = \{h_1\}$, $K_2(a_1) = \{h_3\}$, $K_3(a_1) = \{h_1, h_2\}$, $K_4(a_1) = \{h_1, h_3\}$

$K_1(a_2) = \{h_1\}$, $K_2(a_2) = \{h_3\}$, $K_3(a_2) = \{h_1, h_2\}$, $K_4(a_2) = \{h_1, h_3\}$.

Assign operation $\gamma: \tau \longrightarrow P(X)$ as $\gamma(K,A) = \begin{cases} (K,A) & if\ h_2 \in (K,A) \\ cl(K,A) & if\ h_2 \notin (K,A) \end{cases}$ for every$(K,A) \in \tau$, then $\tau_\gamma = \{\tilde{\phi}, \tilde{X}, (K_2,A), (K_4,A)\}$. Clearly, (K_3,A) and (K_5,A), represented as $K_5(a_1) = \{h_2, h_3\}$ and $K_5(a_2) = \{h_2, h_3\}$, respectively, are strong soft γ pre-open sets, but the intersection of (K_3,A) and (K_5,A) is (F_6,A) such that $K_6(a_1) = \{h_2\}$ and $K_6(a_2) = \{h_2\}$, which is not strong soft γ pre-open.

Proposition 3.7. If $(K,A) \in S^*_\gamma O(\tilde{X}_\gamma)$ and $(H,A) \in SS^*_\gamma P(\tilde{X}_\gamma)$, then $(K,A) \cap (H,A) \in SS^*_\gamma P(\tilde{X}_\gamma)$.

Proof: As $(H,A) \in SS^*_\gamma P(\tilde{X}_\gamma)$, then by Proposition 3.3, there exists a soft open $(G,A) \in S^*O(X)$ in which $(H,A) \tilde{\subseteq} (G,A) \tilde{\subseteq} \tau_\gamma cl(H,A)$. And so, $(K,A) \cap (V,A) \tilde{\subseteq} (K,A) \cap (G,A) \tilde{\subseteq} (K,A) \cap \tau_\gamma cl(H,A) \tilde{\subseteq} \tau_\gamma cl\big((K,A) \cap (H,A)\big)$. Also, since $(U,A) \cap (G,A) \in S^*O(X)$, hence $(K,A) \cap (H,A) \in SS^*_\gamma P(\tilde{X}_\gamma)$.

Proposition 3.8. If $\{(K_i,A): i \in I\}$ is a collection of strong soft γ pre-open sets over $\tilde{X}_\gamma$, then $\bigcup_{i\in I}(K_i,A)$ is strong soft γ pre-open set.

Proof: Let $(K_i,A) \tilde{\subseteq} int\, \tau_\gamma cl\big((K_i,A)\big)$, then $\bigcup_{i\in I}(K_i,A) \tilde{\subseteq} \bigcup_{i\in I} int\, \tau_\gamma cl\big((K_i,A)\big)$ and so, $\bigcup_{i\in I}(K_i,A) \tilde{\subseteq} int \bigcup_{i\in I} \tau_\gamma cl\big((K_i,A)\big) \tilde{\subseteq} int\, \tau_\gamma cl(\bigcup_{i\in I}(K_i,A))$. Hence $\bigcup_{i\in I}(K_i,A)$ is strong soft γ pre-open.

Proposition 3.9. For $\tilde{Y}_\gamma$ is a soft subspace of$\tilde{X}_\gamma$, if $(K,A) \in SS^*_\gamma P(\tilde{Y}_\gamma)$, then $(K,A) \in SS^*_\gamma P(\tilde{X}_\gamma)$.

Proof: Let $(K,A) \in SS^*_\gamma P(\tilde{Y}_\gamma)$, then there exists $(U,A) \in S^*O(Y)$ in which $(K,A) \tilde{\subseteq} (U,A) \tilde{\subseteq} \tau_\gamma cl_Y(K,A)$. According to Proposition 2.19, there exists $(H,A) \in S^*O(X)$ in which $(K,A) \tilde{\subseteq} (H,A) \cap \tilde{Y} \tilde{\subseteq} \tau_\gamma cl_X(K,A) \cap \tilde{Y}$. It follows by Proposition 2.29 that $(K,A) \tilde{\subseteq} (H,A) \tilde{\subseteq} \tau_\gamma cl_X(K,A)$. Hence $(K,A) \in SS^*_\gamma P(\tilde{X}_\gamma)$.

Proposition 3.10. Let $\tilde{Y}_\gamma$ be soft subspace of $\tilde{X}_\gamma$ such that $(K,A) \tilde{\subseteq} \tilde{Y}$. If $(K,A) \in SS_\gamma^* P(\tilde{X}_\gamma)$, then $(K,A) \in SS_\gamma^* P(\tilde{Y}_\gamma)$.

Proof: Let $(K,A) \in SS_\gamma^* P(\tilde{X}_\gamma)$, there is $(V,A) \in S^*O(X)$ in which $(K,A) \tilde{\subseteq} (V,A) \tilde{\subseteq} \tau_\gamma cl_X(K,A)$, and since $(K,A) \tilde{\subseteq} \tilde{Y}$, then $(K,A) \tilde{\subseteq} (H,A) \tilde{\subseteq} \tau_\gamma cl_X(K,A) \cap Y$ and $(H,A) \in S^*O(Y)$. By Proposition 2.29, we get $(K,A) \subseteq (H,A) \subseteq \tau_\gamma cl_Y(K,A)$. Hence $(K,A) \in SS_\gamma^* P(\tilde{Y}_\gamma)$.

Definition 3.11. A soft set (K,A) in $\tilde{X}_\gamma$ is called strong soft γ pre-closed, if $\tilde{X} - (K,A)$ is strong soft γ pre-open set, equivalently, $cl\, \tau_\gamma int(K,A) \subseteq (K,A)$. The family of all strong soft γ pre-closed is stated by $SS_\gamma^* PC(\tilde{X}_\gamma)$.

Proposition 3.12. If $\{(K_i,A): i \in I\}$ is a family of strong soft γ pre-closed sets over $\tilde{X}_\gamma$, then $\bigcap_{i\in I}(K_i,A)$ is strong soft γ pre-closed.

Proof: Follows from Proposition 3.8.

Proposition 3.13. If $(K,A) \in S_\gamma^* C(\tilde{X}_\gamma)$ and $(H,A) \in SS_\gamma^* PC(\tilde{X}_\gamma)$ are closed sets over $\tilde{X}_\gamma$, then $(K,A) \cap (H,A) \in SS_\gamma^* PC(\tilde{X}_\gamma)$.

Proposition 3.14. Let $\tilde{Y}_\gamma$ be a soft subspace of $\tilde{X}_\gamma$ and let $(K,A) \tilde{\subseteq} \tilde{Y}$. If $(K,A) \in SS_\gamma^* PC(\tilde{Y}_\gamma)$, then $(K,A) \in SS_\gamma^* PC(\tilde{X}_\gamma)$.

Proposition 3.15. Let $\tilde{Y}_\gamma$ be a soft subspace of $\tilde{X}_\gamma$ such that $(K,A) \tilde{\subseteq} Y$. If $(K,A) \in SS_\gamma^* PC(\tilde{X}_\gamma)$, then $(K,A) \in SS_\gamma^* PC(\tilde{Y}_\gamma)$.

Proposition 3.16. A soft subset (K,A) of space $\tilde{X}_\gamma$, then (K,A) is strong soft γ -pre-closed if and only if $cl(K,A) - (K,A) \tilde{\subseteq} int\left(\tilde{X} - \tau_\gamma int(K,A)\right) - \left(\tilde{X} - cl(K,A)\right)$.

Proof: since $cl(K,A) - (K,A) \tilde{\subseteq} int\left(\tilde{X} - \tau_\gamma int(K,A)\right) - \left(\tilde{X} - cl(K,A)\right) \Leftrightarrow cl(K,A) - (K,A) \tilde{\subseteq} \left(\tilde{X} - cl\, \tau_\gamma int(K,A)\right) - \left(\tilde{X} - cl(K,A)\right) \Leftrightarrow$
$cl(K,A) - (K,A) \tilde{\subseteq} \left(\tilde{X} - cl\, \tau_\gamma int(K,A)\right) \cap cl(K,A) \Leftrightarrow cl(K,A) - (K,A) \tilde{\subseteq} \left(\tilde{X} \cap cl(K,A)\right) - \left(cl\, \tau_\gamma int(K,A) \cap cl(K,A)\right) \Leftrightarrow cl(K,A) - (K,A) \tilde{\subseteq} cl(K,A) - cl\, \tau_\gamma int(K,A) \Leftrightarrow$
$cl\, \tau_\gamma int(K,A) \tilde{\subseteq} (K,A)$.

Definition 3.17. A soft element $e_F \in P(X)$ is said to be strong soft γ pre-interior element of $(K,A) \tilde{\subseteq} \tilde{X}_\gamma$ if there is strong soft γ pre-open set (G,A) in which $e_F \in (G,A) \subseteq (K,A)$. The set of each strong soft γ pre-interior element of (K,A) is named strong soft γ pre-interior set of (K,A) which is stated by $spint_\gamma(K,A)$.

Proposition 3.18. For any soft subsets (K,A) and $(H,A)$of $\tilde{X}_\gamma$, then the following hold

1) $spint_\gamma(\tilde{\phi}) = \tilde{\phi}$
2) $spint_\gamma(\tilde{X}) = \tilde{X}$
3) If $(K,A) \tilde{\subseteq} (H,A)$, then $spint_\gamma(K,A) \tilde{\subseteq} spint_\gamma(H,A)$
4) $spint_\gamma(K,A) \cup spint_\gamma(H,A) \tilde{\subseteq} spint_\gamma\left((K,A) \cup (H,A)\right)$
5) $spint_\gamma\left((K,A) \cap (H,A)\right) \tilde{\subseteq} spint_\gamma(K,A) \cap spint_\gamma(H,A)$
6) $\tau_\gamma int(K,A) \cap spint_\gamma(H,A) \tilde{\subseteq} spint_\gamma\left((K,A) \cap (H,A)\right)$

Proposition 3.19. Let (K,A) be a soft subset of $\tilde{X}_\gamma$, then $\tau_\gamma int(K,A) \tilde{\subseteq} int(K,A) \tilde{\subseteq} spint_\gamma(K,A)$.

Definition 3.20. The strong soft γ pre-closure of soft subset (K,A) over $\tilde{X}_\gamma$ is the intersection of all strong soft γ pre-closed sets containing (K,A) which is stated by $spcl_\gamma(K,A)$.

Proposition 3.21. Let (K,A) be soft set in $\tilde{X}_\gamma$, then $e_F \in spcl_\gamma(K,A)$ if and only if, for any $(H,A) \in SS_\gamma^* PC(\tilde{X}_\gamma)$, $(H,A) \cap (K,A) \neq \tilde{\phi}$.

Proposition 3.22. Let(K,A) be a soft subset of $\tilde{X}_\gamma$, then $spcl_\gamma(K,A) \tilde{\subseteq} cl(K,A) \tilde{\subseteq} \tau_\gamma cl(K,A)$.

Proposition 3.23. For any (K,A) and (H,A) soft sets in $\tilde{X}_\gamma$, then the following hold

1) $spcl_\gamma(\tilde{\phi}) = \tilde{\phi}$
2) $spcl_\gamma(\tilde{X}) = \tilde{X}$
3) If $(K,A) \tilde{\subseteq} (H,A)$, then $spcl_\gamma(K,A) \tilde{\subseteq} spcl_\gamma(H,A)$

4) (K,A) is strong soft γ preclosed if and only if $(K,A) = spcl_\gamma(K,A)$
5) $spcl_\gamma\big((K,A)\cap(H,A)\big) \tilde{\subseteq} spcl_\gamma(K,A)\cap spcl_\gamma(H,A)$
6) $spcl_\gamma\big((K,A)\cup(H,A)\big) = spcl_\gamma(K,A)\cup spcl_\gamma(H,A)$
7) $\tilde{X} - spcl_\gamma(K,A) = spcl\left(\tilde{X} - (K,A)\right)$
Proof: we are going to prove only (7).
Let $e_F \in \tilde{X} - spcl_\gamma(K,A) \Leftrightarrow e_F \notin spcl_\gamma(K,A) \Leftrightarrow \exists\ (U,A) \in SS^*_\gamma P\big(\tilde{X}_\gamma\big)$ in which $(U,A)\cap(K,A) = \tilde{\phi} \Leftrightarrow e_F \in (U,A) \tilde{\subseteq} \tilde{X} - (K,A) \Leftrightarrow e_F \in spint(K,A)$.
Definition 3.24. A soft set (K,A) over $\tilde{X}_\gamma$ is called locally strong soft pre-closed set γ, if $(K,A) = (U,A)\cap(H,A)$ for (U,A) is soft open and (H,A) is strong soft pre-closed γ set.
Proposition 3.25. Let (K,A) be a soft subset of $\tilde{X}_\gamma$, then (K,A) is locally strong soft γ pre-closed if and only if $(K,A) = (U,A)\cap spcl_\gamma(K,A)$, for soft open (U,A) over $\tilde{X}_\gamma$.
Proof: since (K,A) is locally strong soft γ pre-closed, then there exists $(U,A) \in S^*O\big(\tilde{X}_\gamma\big)$ in which $(K,A) = (U,A)\cap(S,A)$ where $(S,A) \in SS^*_\gamma PC\big(\tilde{X}_\gamma\big)$. That is $(K,A) \tilde{\subseteq} spcl_\gamma(K,A) \tilde{\subseteq} spcl_\gamma(S,A) = (S,A)$. It follows that $(K,A) \tilde{\subseteq} (U,A)\cap spcl_\gamma(K,A) \tilde{\subseteq} (U,A)\cap(S,A) = (K,A)$, thus $(K,A) = (U,A)\cap spcl_\gamma(K,A)$.
Conversely, assume that $(K,A) = (U,A)\cap spcl_\gamma(K,A)$. Note that $(U,A) \in S^*O\big(\tilde{X}_\gamma\big)$ and since $spcl_\gamma(K,A)$ is strong soft γ pre-closed, so (K,A) is locally strong soft γ pre-closed.
Proposition 3.26. Every soft open set is locally strong soft γ pre-closed.
Proof: given that $(K,A) = (K,A)\cap X$ in which $(K,A) \in S^*O(X)$ and since $\tilde{X}_\gamma$ is strong soft γ pre-closed, then (K,A) is locally strong soft γ pre-closed set.
Remark 3.27. Every strong soft γ pre-closed is locally strong soft γ pre-closed.
Proposition 3.28. Let (K,A) be a soft subset of $\tilde{X}_\gamma$. If (K,A) is locally strong soft γ pre-closed over$\tilde{X}_\gamma$, then $(K,A)\cup\left(X - spcl_\gamma(K,A)\right)$ is strong soft γ pre-closed.
Proof: Given that (K,A) is locally strong soft γ pre-closed, $\exists\ (U,A) \in S^*O\big(\tilde{X}_\gamma\big)$ in which $(K,A) = (U,A)\cap\ spcl_\gamma(K,A)$. It follows that $(K,A)\cup\left(X - spcl_\gamma(K,A)\right) = \left((U,A) \cap spcl_\gamma(K,A)\right) \cup \left(X - spcl_\gamma(K,A)\right) = \left((U,A) \cup \left(X - spcl_\gamma(K,A)\right)\right) \cap \left(spcl_\gamma(K,A) \cup \left(X - spcl_\gamma(K,A)\right)\right) =$
$(U,A) \cup \left(X - spcl_\gamma(K,A)\right)$ is strong soft γ pre-closed set.
Proposition 3.29. If (K,A) is locally strong soft γ pre-closed and (H,A) is soft open set over $\tilde{X}_\gamma$, then $(K,A) \cap(H,A)$ is locally strong soft γ pre-closed
Proof: Given that (K,A) is a locally strong soft γ pre-closed set, then there is $(U,A) \in S^*O\big(\tilde{X}_\gamma\big)$ in which $(K,A) = (U,A)\cap(S,A)$ where $(S,A) \in SS^*_\gamma PC\big(\tilde{X}_\gamma\big)$. It follows that $(K,A) \cap(H,A) = (U,A)\cap(S,A) \cap(H,A)$ where $(U,A)\cap(H,A) \in S^*O\big(\tilde{X}_\gamma\big)$, and so $(K,A) \cap(H,A)$ is locally strong soft γ pre-closed.
Proposition 3.30. A soft set (K,A) over $\tilde{X}_\gamma$ is locally strong soft γ pre-closed set if and only if $\tilde{X} - (K,A)$ is the union of the soft closed set and strong soft γ pre-open
Definition 3.31. A soft set (K,A) over $\tilde{X}_\gamma$ is named strong soft γ pre-dense whenever $spcl_\gamma(K,A) = \tilde{X}$.
Proposition 3.32. A soft space (X,τ,A) such that γ be an operation defined on soft topology τ is named strong soft γ pre-submaximal, if every strong soft γ pre-dense is strong soft γ pre-open.
Proposition 3.33. If any soft set over $\tilde{X}_\gamma$ is locally strong soft γ pre-closed, then $\tilde{X}_\gamma$ is strong soft γ pre-submaximal.
Proof: Assume that (K,A) is locally strong soft γ pre-closed over X.It follows $(K,A) \cup \left(X - spcl_\gamma(K,A)\right) = (K,A) \cup \phi = (K,A)$. But $(K,A) \cup \left(X - spcl_\gamma(K,A)\right)$ is strong soft γ pre-open, therefore (K,A) is strong soft γ pre-open.

4- Pre γ soft open sets and soft γ preopen sets

Definition 4.1. A soft set (K,A) is named soft γ pre-open over $\tilde{X}_\gamma$, if $(K,A) \tilde{\subseteq} \tau_\gamma int\ \tau_\gamma cl(K,A)$.

Definition 4.2 [9]. A soft set (K,A) is named soft γ pre-open over $\tilde{X}_\gamma$ and is called pre γ soft open, if $(K,A) \tilde{\subseteq} \tau_\gamma int\ cl(K,A)$

Proposition 4.3. Let (K,A) be a soft subset of $\tilde{X}_\gamma$. Then (K,A) is soft γ pre-open (respectively pre γ soft open) if and only if there is $(V,A) \in S^*_\gamma O(\tilde{X}_\gamma)$ in which $(K,A) \subseteq (V,A) \subseteq \tau_\gamma cl(K,A)$(respectively $(K,A) \subseteq (V,A) \subseteq cl(K,A)$).

Proposition 4.4. Every pre γ soft open is soft p-open set.

Remark 4.5 The reverse of the above proposition may not be true.

Recall Example 3.6. We have $X = \{h_1,h_2,h_3\}$ and $= \{a_1,a_2\}$. Let $\tau = \{\tilde{\phi},\tilde{X},(K_1,A),(K_2,A),(K_3,A),(K_4,A)\}$ where $K_1(a_1) = \{h_1\}$, $K_2(a_1) = \{h_3\}$, $K_3(a_1) = \{h_1,h_2\}$, $K_4(a_1) = \{h_1,h_3\}$

$K_1(a_2) = \{h_1\}$, $K_2(a_2) = \{h_3\}$, $K_3(a_2) = \{h_1,h_2\}$, $K_4(a_2) = \{h_1,h_3\}$.

Assign the operation $\gamma: \tau \longrightarrow P(X)$ as $\gamma(K,A) = \begin{cases} (K,A) & if\ h_2 \in (K,A) \\ cl(K,A) & if\ h_2 \notin (K,A) \end{cases}$ for any $(K,A) \in \tau$, then (K_3,A) is soft pre- open but it is not pre γ soft open set.

Proposition 4.6. Each pre γ soft open set is soft γ pre-open.

Remark 4.7. The reverse of the previous proposition needs not to be true, as shown in the next example.

Example 4.8. Given that $X = \{h_1,h_2,h_3,h_4\}$ and $= \{a_1,a_2\}$, consider $\tau = \{\tilde{\phi},\tilde{X},(K_1,A),(K_2,A),(K_3,A)\}$ such that $K_1(a_1) = \{h_1\}$, $K_2(a_1) = \{h_2,h_3\}$, $K_3(a_1) = \{h_1,h_2,h_3\}$, $K_1(a_2) = \{h_1\}$, $K_2(a_2) = \{h_2,h_3\}$, $K_3(a_2) = \{h_1,h_2,h_3\}$. Assign $\gamma(K,A) = \begin{cases} int\ cl(K,A) & if\ (K,A) = \{h_1\} \\ cl(K,A) & if\ (K,A) \neq \{h_1\} \end{cases}$

That is, $\tau_\gamma = \{\tilde{\phi},\tilde{X},(K_2,A),(K_4,A)\}$ where $K_4(a_1) = \{h_1,h_4\}$ and $K_4(a_2) = \{h_1,h_4\}$ is soft γ pre-open set. However, it is not pre γ soft open set.

Proposition 4.9. Every soft γ pre-open is strong soft γ pre-open.

Remark 4.10. The reverse of Proposition 4.9 may not be true, as shown in the following example.

In Example 3.6 , we have $X = \{h_1,h_2,h_3\}$ and $A = \{a_1,a_2\}$. Let $\tau = \{\tilde{\phi},\tilde{X},(K_1,A),(K_2,A),(K_3,A),(K_4,A)\}$ where $K_1(a_1) = \{h_1\}$, $K_2(a_1) = \{h_3\}$, $K_3(a_1) = \{h_1,h_2\}$, $K_4(a_1) = \{h_1,h_3\}$.

$K_1(a_2) = \{h_1\}$, $K_2(a_2) = \{h_3\}$, $K_3(a_2) = \{h_1,h_2\}$, $K_4(a_2) = \{h_1,h_3\}$.

Assign an operation $\gamma: \tau \longrightarrow P(X)$ as $\gamma(K,A) = \begin{cases} (K,A) & if\ h_2 \in (K,A) \\ cl(K,A) & if\ h_2 \notin (K,A) \end{cases}$ for every $(K,A) \in \tau$, (K_3,A) is strong soft γ pre-open but it is not soft γ pre-open.

Proposition 4.11. Let γ be an operation defined on soft topology τ and let $\tilde{X}$ be a soft γ regular topological space, then the followings are equivalents

1) Soft pre open

2) Pre γ soft open

3) Strong soft γ pre open

4) Soft γ pre-open

Proof: Straightforward from Proposition 2.20 and Proposition 3.2.

Proposition 4.12. If (K,A) is γ soft open set over $\tilde{X}_\gamma$, then (K,A) is soft γ pre-open set.

. Proposition 4.13. If (K,A) is γ soft open set in $\tilde{X}_\gamma$, then (K,A) is pre γ soft open

Definition 4.14. A space $\tilde{X}_\gamma$ is extremally γ soft disconnected if soft γ closure of every soft γ open set is soft γ open.

Proposition 4.15. A space $\tilde{X}_\gamma$ is extremally γ soft disconnected if and only if each soft γ semi-open is soft γ pre-open set.

Proof: Let $\tilde{X}_\gamma$ be extremally γ soft disconnected, then for every soft γ semi-open (K,A), we get $(K,A) \subseteq \tau_\gamma cl\, \tau_\gamma int(K,A) = \tau_\gamma int\, \tau_\gamma cl\, \tau_\gamma int(K,A) \subseteq \tau_\gamma int\, \tau_\gamma cl(K,A)$. Hence (K,A) is soft γ pre-open set.

Conversely, let (K,A) be soft γ open, so $\tau_\gamma cl(K,A) = \tau_\gamma cl\, \tau_\gamma int(K,A)$. Since $\tau_\gamma cl\, \tau_\gamma int(K,A)$ is soft γ semi-open, hence by hypothesis, $\tau_\gamma cl\, \tau_\gamma int(K,A)$ is soft γ pre-open. It follows that $\tau_\gamma cl(K,A) \subseteq \tau_\gamma int\, \tau_\gamma cl(K,A)$, therefore $\tau_\gamma cl(K,A)$ is soft γ open set.

5- Conclusions

Soft sets were initiated by Molodtstove in 1999 and, since then, many researchers defined and investigated several types of soft sets. Some of these studies have real applications such as solving problems for medical diagnosis, determining educational obstacles, and taking right decisions about them. In this paper, we defined and studied new soft sets and named them soft γ pre-open sets. Also, we provided several properties and characterizations about pre γ soft open and strong soft γ pre-open sets. Also, the relationships among these sets were discussed.

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