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A study of m-N-extremally Disconnected Spaces With Respect toτ, Maximum m_X-N-open Sets

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Abstract
The aim of this research is to prove the idea of maximum m_X-N-open set, m-N-extremally disconnected with respect to τ and provide some definitions by utilizing the idea of m_X-N-open sets. Some properties of these sets are studied.

Keywords: minimal structure, maximum m_X-N-open set, m-N-extremally disconnected respect toτ, mixed space.

دراسة الفضاء m–N–extremally disconnected بالنسبة لفضاء البنيةτ، الحد الاعلى لل m_X–N–open من ناحية عدد عناصرها

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الخلاصة:
الهدف من هذا البحث تقديم مفهوم الحد الاعلى لل m_X–N–open من ناحية عدد عناصرها، مفهوم (m–N–extremally disconnected respect toτ) وبعض المفاهيم باستعمال المفهوم m_X–N– المفتوحة. بعض خصائص هذه المجموعات تم دراستها والتحقيق فيها

Introduction

The notions of minimal open sets and maximal open sets in topological spaces were presented and investigated by Nakaoka and Oda [5, 6, 7]. Roohi *et al.* [8] showed the idea of maximal m_X-open set in m-space. They obtained numerous portrayals of maximal m_X-open set in m-space. Al-Omari *et al.* [1] presented an alteration of extremally disconnected spaces which is said to be m-extremally disconnected. Furthermore, they obtained numerous portrayals of m-extremally disconnected spaces. The motivation behind this paper is to present and test the idea of maximal m_X-N-open sets in m-spaces, called maximal m_X-N-open set. We also presented the idea of m-N-extremally disconnected with respect toτ. Few outcomes about the presence of maximal m_X-N-open sets and m-N-extremally disconnected spaces with respect to τ are given. We have upheld our outcomes by models and counterexamples.

1: PRELIMINARIES

Definition (1.1) [3]

Let X be a non-vacuous set and P(X) be the power set of X. A subfamily m_X of P(X) is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_X$ and $X \in m_X$. By (X, m_X), we indicate a non-

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vacuous set X with an m-structure m_X on X, which is called an m-space. Every individual from m_X is nominated to be m_X-open and the complement of an m_X-open set is nominated to be m_X-closed se

Definition (1.2) [4]

A subset B of m-space X is called m_XN-open set if every single $x \in U$, then there be found an m_X-open set V that comprises x to such an extent that V/U is finite and the complement of an m_XN-open set is designated to be m_XN-closed set. The family of all m_XN-open sets in X is designated to be m_N.

Definition (1.3) [8]

Let (X, m_X) be an m-space. A non-vacuous proper m_X-open subcategory U of X is designated to be maximal m_X-open if any m_X-open set which comprises U is X or U. We indicate the set of all maximal m_X-open sets of an m-space (X, m_X) by max (X, m_X).

Definition (1.4)[1]

A topological space (X, τ) with an m-structure m_X on X is designated to be mixed space and is indicated by (X, τ, m_X).

Definition (1.5) [2]

A space X is extremally disconnected if the closure of every open subset of X is open.

Definition (1.6) [1]

A mixed space (X, τ, m_X) is nominated to be m-extremally disconnected if m_X-cl(U) $\in \tau$, for each $U \in \tau$.

Definition (1.7)

Let X be a non-vacuous set and m_X an m-structure on X. For a subcategory U of X, the m_X-N-closure of U and the m_X-N-interior of U are acquainted as pursues:

i. m_X-N-int(U) = $\cup$ {V: V $\subseteq$ U, V is an m_X-N-open}.

ii. m_X-N-cl(U) = $\cap$ {F: U $\subseteq$ F, F is an m_X-N-closed}.

Proposition (1.8): Let U, V be a subcategory of m-space X and U $\subseteq$ V, thereafter:

i. m_X-int(U) $\subseteq$ m_X-N-int(U).
ii. m_X-N-cl(U) $\subseteq$ m_X-cl(U).
iii. m_X-N-cl(U) $\subseteq$ m_X-N-cl(V).
iv. m_X-N-int(U) $\subseteq$ m_X-N-int(V)
v. m_X-N-int(X) = X and m_X-N-int(Ø) = Ø.
vi. m_X-N-cl(X) = X and m_X-N-cl(Ø) = Ø.
vii. m_X-N-int(U) $\subseteq$ U and U $\subseteq$ m_X-N-cl(U)
viii. m_X-N-int(m-N-int(U)) = m_X-N-int(U)
ix. m_X-N-cl(m-N-cl(U)) = m_X-N-cl(U)
x. m_X-N-cl(U^C) = (m_X-N-int(U))^C .
xi. m_X-N-int(U^C) = (m_X-N-cl(U))^C.

2: Maximal m_x-N-Open Sets

Definition (2.1)

A non-vacuous proper m_X-N-open subset U of m-space X is called maximal m_X-N-open if any m_X-N-open set which comprises U is X or U. We indicate the arrangement of all maximal m_X-N-open sets of an m-space (X, m_X) by max-N-(X, m_X).

Proposition (2.2): Let (X, m_X) be an m-space and let U, V $\in$ max-N-(X, m_X) and K be an m_X-N-open set. Then:

i. U $\cup$ K = X or K $\subseteq$ U.
ii. U $\cup$ V = X or U= V.

***Proof*:**

i. Assume that, K is an m_X-N-open set in which U $\cup$ K $\neq$ X. Since U is an maximal m_X-N-open set, U $\subseteq$ U $\cup$ K, we conclude that U $\cup$ K= U. Therefore K $\subseteq$ U.
ii. Let U $\cup$ V $\neq$ X. Since U, V $\in$ max-N-(X, m_X), by (i) we conclude that U $\subseteq$ V and V $\subseteq$ U. Hence U = V.

Remark (2.3): This proposition may not be true when U, V $\in$ max (X, m_X).

Example (2.4): Let X = {1,2,3,4}, U = {1,2}, V = {2,3} and K = {3}. Put $m_{X=}$ {Ø, U, V, K}, then distinctly U, V $\in$ max(X, m_X) and K is an m_X-open set. We note the following:

i. U $\cup$ K $\neq$ X and K $\not\subset$ U.
ii. U $\cup$ V $\neq$ X and U $\neq$ V.

Proposition (2.5): Let (X, m_X) be an m-space. $U_1, U_2, U_3 \in$ max-N-(X, m_X), in which $U_1 \neq U_2$. In the event that $U_1 \cap U_2 \subseteq U_3$, then $U_1 = U_3$ or $U_2 = U_3$.

Proof: Since $U_1 \cap U_3 = U_1 \cap (U_3 \cap X)$

$= U_1 \cap (U_3 \cap (U_1 \cup U_2))$ (Proposition 2.2)

$= U_1 \cap ((U_3 \cap U_1) \cup (U_3 \cap U_2))$

$=(U_1 \cap U_3) \cup (U_1 \cap U_3 \cap U_2)$

$=(U_1 \cap U_3) \cup (U_1 \cap U_2)$ (since $U_1 \cap U_2 \subseteq U_3$)

$= U_1 \cap (U_2 \cup U_3)$,

subsequently, $U_1 \cap U_3 = U_1 \cap (U_2 \cup U_3)$. In the event that $U_3 \neq U_2$ (Proposition 2.2), we have $U_2 \cup U_3 = X$. Henceforth we can say that $U_1 \cap U_3 = U_1$, which implies that $U_1 \subseteq U_3$. Since U_1 and U_3 are maximal m-N-open sets, we get $U_1 = U_3$.

Example (2.6): Let $X = \{1,2,3,4\}$, $U_1 = \{1,2,3\}$, $U_2 = \{1,2\}$ and $U_3 = \{1,2,4\}$. Let $m_X = \{Ø, U_1, U_2, U_3, X\}$, so $U_1 \cap U_2 \subseteq U_3$ while $U_1 \neq U_3$ and $U_2 \neq U_3$. This shows that Proposition 2.5 may not hold when one of the sets $\{U_1, U_2, U_3\}$ do not have a place with the maximal m_X-N-open sets.

Example (2.7): Let $X = \{1, 2, 3, 4\}$ and $m_X = \{Ø, \{1,2\}, \{2,3\}, X\}$. Let $U_1 = \{1,2\}$, $U_2 = U_3 = \{2,3\}$. Obviously, $U_1 \neq U_2$, $U_1 \cap U_2 \subseteq U_3$ and $U_2 = U_3$. We note that Proposition 2.5 is achieved when the sets $\{U_1, U_2, U_3\}$ do not have a place with the maximal m_X-N-open sets.

Proposition (2.8): Let (X, m_X) be an m-space, $U_1, U_2, U_3 \in$ max-N-(X, m_X) which are dissimilar from any others. Then $U_A \cap U_B \not\subset U_A \cap U_C$, where $\{A, B, C\} = \{1,2,3\}$.

Proof: Suppose that $U_A \cap U_B \subseteq U_A \cap U_C$, so $(U_A \cap U_B) \cup (U_B \cap U_C) \subseteq (U_A \cap U_C) \cup (U_B \cap U_C)$. Hence $U_B \cap (U_A \cup U_C) \subseteq (U_A \cup U_B) \cap U_C$. Since $U_A \cup U_B = U_A \cup U_C = X$, we get $U_B \subseteq U_C$. Presently, it pursues from Definition 2.1 that $U_B = U_C$, which repudiates our supposition.

Example (2.9): Let $X = \{1,2,3,4\}$, $U_1 = \{1,2,4\}$, $U_2 = \{3,4\}$, $U_3 = \{1,3,4\}$. Put $m_X = \{Ø, U_1, U_2, U_3, X\}$ then $U_1 \cap U_2 \subseteq U_1 \cap U_3$. This shows that proposition 2.8 may not hold when one of the sets $\{U_1, U_2, U_3\}$ do not have a place with the maximal m_X-N-open sets.

Example (2.10): Let $X = \{1,2,3,4\}$, $U_1 = \{1,2\}$, $U_2 = \{2,3\}$, $U_3 = \{1,3\}$. Put $m_X = \{Ø, U_1, U_2, U_3, X\}$. It is easy to see that $U_A \cap U_B \not\subset U_A \cap U_C$, where $\{A, B, C\} = \{1,2,3\}$. This shows that it is conceivable that (proposition 2.8) holds when the sets $\{U_1, U_2, U_3\}$ do not have a place with the maximal m_X-N-open sets.

Proposition (2.11): Let (X, m_X) be an m-space, $U \in$ max-N-(X, m_X), and $x \in U$. Then $U = \cup \{W: W\}$ is an m_X-N-open neighborhood of x in which $U \cup W \neq X$.

Proof: Since the maximal m_X-N-open set U is an m_X-N-open neighborhood of x, so $U \subseteq \cup\{W: W\}$ is an m_X-N-open neighborhood of x in which $U \cup W \neq X$. Now, whenever $U \cup W \neq X$, then by proposition 2.2, we have that $U \subseteq \cup\{W: W\}$ is an m_X-N-open neighborhood of x in which $U \cup \{W \neq X\} \subseteq U$. Hence $U = \cup\{W: W\}$ is an m_X-N-open neighborhood of x in which $U \cup W \neq X$.

Proposition (2.12): Let (X, m_X) and (X, m_X^*) be m-spaces in which $m_X \subseteq m_X^*$. Then max-N-$(X, m_X^*) \cap m_X \subseteq \max(X, m_X)$.

Proof:

If max-N-$(X, m_X^*) \cap m_X = Ø$, so the investigation ends. Let $U \in$ max-N-$(X, m_X^*) \cap m_X$ and assume that $U \notin \max(X, m_X)$, then there be found $K \in m_X$ with the end goal that $U \subset K \subset X$, which is a logical inconsistency, since $U \in$ max-N-(X, m_X^*) and K is an m_X-N-open set.

Example (2.13): Let $X = \{1,2,3\}$ and $m_X = \{Ø, \{1\}, \{2\}, \{2,3\}, X\}$, $m_X^* = \{Ø, \{1\}, \{2\}, \{1,2\}, \{2,3\}, X\}$. We note that max-N-$(X, m_X^*) = \{\{1,2\}, \{1,3\}, \{2,3\}\}$ and $\max(X, m_X) = \{\{1\}, \{2,3\}\}$, and therefore max-N-$(X, m_X^*) \cap m_X \subseteq \max(X, m_X)$.

Proposition (2.14): Let (X, m_X) be an m-space. Then max-N-$(X, m_X) \cap m_X \subseteq \max(X, m_X)$.

Proof: If max-N-$(X, m_X) \cap m_X = Ø$, so the investigation ends. Let $U \in$ max-N-$(X, m_X) \cap m_X$ and assume that $U \notin \max(X, m_X)$, then there be found $K \in m_X$ in which $U \subset K \subset X$, which is a logical inconsistency, since $U \in$ max-N-(X, m_X) and K is an m_X-N-open set.

Example (2.15): Let $X = \{1,2,3\}$ and $m_X = \{Ø, \{1\}, \{2\}, \{2,3\}, X\}$.We note that max-N-$(X, m_X) = \{\{1,2\}, \{1,3\}, \{2,3\}\}$ and $\max(X, m_X) = \{\{1\}, \{2,3\}\}$, and therefore max-N-$(X, m_X) \cap m_X \subseteq \max(X, m_X)$.

3: m-N-extremally disconnected with **respect to τ**

Definition (3.1):

A mixed space (X, τ, m_X) is nominated to be m-N-extremally disconnected with respect to τ if m_X-N-cl(U) $\in \tau$ for each $U \in \tau$.

Example (3.2): Let X= {1,2,3}, τ = {Ø, {1}, {2}, {1,2}, X} and m_X = {Ø, {1}, {2}, X}. Then the topological space (X, τ) is not extremally disconnected and the mixed space (X, τ, m_X) is not m-extremally disconnected, but a mixed space (X, τ, m_X) is an m-N-extremally disconnected with respect to τ.

Definition (3.3):

A subcategory U of a mixed space (X, τ, m_X) is nominated to be:

i. m_X-Nα*-open if U $\subseteq$ int(m_X-N-cl(int(U))).
ii. m_X-Ns*-open if U $\subseteq$ m_X-N-cl(int(U)).
iii. m_X-Np*-open if U $\subseteq$ int(m_X-N-cl(U)).
iv. m_X-NB*-open if U $\subseteq$ m_X-N-cl(int(m_X-N-cl(U))).
v. m_X-Ns**-open if U $\subseteq$ cl(m_X-N-int(U)).

Theorem (3.4): Let (X, τ, m_X) be a mixed space, then the accompanying properties are identical:

i. X is an m-N-extremally disconnected with respect to τ..
ii. m_X-N-int(U) is closed for each closed subcategory U of X.
iii. m_X-N-cl(int(U)) $\subseteq$ int(m_X-N-cl(U)) for each subcategory U of X.
iv. Every m_X-Ns*-open set is m_X-Np*-open.
v. m_X-N-cl(U) $\in \tau$ for each m_X-NB*-open set U.
vi. Every m_X-NB*-open set is m_X-Np*-open.
vii. U is an m_X-Nα*-open if and only if it is m_X-Ns*-open for each U$\subseteq$ X.

Proof: (i) → (ii): Let U be a closed set in X. Then X / U is open. By (i), m_X-N-cl(X / U) = X / (m_X-N-int(U)) is open. Subsequently, m_X-N-int(U) is closed.

(ii) →(iii): Let U be any set of X. Then X / int(U) is closed and by (ii), m_X-N-int[X / int(U)] is closed. Therefore, m_X-N-cl(int(U)) is open and subsequently m_X-N-cl(int(U)) $\subseteq$ int(m_X-N-cl(U)).

(iii) →(iv): Let U be m_X-Ns*-open set. By (iii), we conclude that U$\subseteq$ m_X-N-cl(int(U)) $\subseteq$ int(m_X-N-cl(U)). Thus, U is an m_X-Np*-open set.

(iv) → (v): Let U be an m_X-NB*-open set. Then m_X-N-cl(U) is an m_X-Ns*-open. By (iv), m_X-N-cl(U) is m_X-Np*-open. Subsequently, m_X-N-cl(U) $\subseteq$ int(m_X-N-cl(U)) and hence m_X-N-cl(U) is open.

(v) → (vi): Let U be an m_X-NB*-open. By (v), m_X-N-cl(U) = int(m_X-N-cl(U)). Subsequently, U $\subseteq$ m_X-N-cl(U) = int(m_X-N-cl(U)) and hence U is an m_X-Np*-open.

(vi) → (vii): Let U be an m_X-Ns*-open set. Since every m_X-Ns*-open set is an m_X-NB*-open, then by (vi), it is an m_X-Np*-open. Since U is an m_X-Ns*-open and an m_X-Np*-open, then it is an m_X-Nα*-open.

(vii) → (i): Let U be an open set of X. Then m_X-N-cl(U) is m_X-Ns*-open and by (vii), m_X-N-cl(U) is m_X-Nα*-open. Thusly, m_X-N-cl(U) $\subseteq$ int(m_X-N-cl(int(m_X-N-cl(U)))) $\subseteq$ int(m_X-N-cl(U)) and subsequently m_X-N-cl(U) = int(m_X-N-cl(U)), and finally, m_X-N-cl(U) is open and X is m-N-extremally disconnected with respect to τ.

Corollary (3.5): Let (X, τ, m_X) be a mixed space. Then, the accompanying properties are identical:

i. X is m-N-extremally disconnected with respect to τ.
ii. m_X-N-cl(U)$\in \tau$ for each m_X-Nα*-open set U of X.
iii. m_X-N-cl(U) $\in \tau$ for each m_X-Ns*-open set U of X.
iv. m_X-N-cl(U) $\in \tau$ for each m_X-Np*-open set U of X.

Proof:

i→ii: Let U be an m_X-Nα*-open set of X, then U $\subseteq$ int(m_X-N-cl(int(U)), so m_X-N-cl(U) $\subseteq$ m_X-N-cl(int(m_X-N-cl(int(U))) $\subseteq$ m_X-N-cl(int(U))). Hence, m_X-N-cl(U) = m_X-N-cl(int(U)), and by (i), we conclude that m_X-N-cl(U)$\in \tau$.

ii →iii: By Theorem 3.4 (vii), the verification ends.

iii → iv: Let U be an m_X-Np*-open set of X, then $U \subseteq$ int(m_X-N-cl(U)), so m_X-N-cl(U) $\subseteq$ m_X-N-cl(int(m_X-N-cl(U))). Subsequently, m_X-N-cl(U) is an m_X-Ns*-open set, and by (iii), m_X-N-cl(U) $\in_\tau$.
iv →i: Since every open set is an m_X-Np*-open set, then the verification ends.

Theorem (3.6): Let (X, τ, m_X) be a mixed space. Then, the accompanying properties are identical:

i. X is m-N-extremally disconnected with respect to τ.
ii. For any U $\in_\tau$ and K is an m_X-N-open set in which U∩K= Ø, there be found a disjoint of an m_X-N-closed set U_1 and a closed set K_1 in which U⊆ U_I and K⊆ K_I.
iii. m_X-N-cl(U)∩cl(K) = Ø for each U$\in_\tau$ and K is an m_X-N-open set with U∩K=Ø.
iv. m_X-N-cl[int(m_X-N-cl(U))] ∩ cl(K) = Ø for each U⊆ X and K is an m_X-N-open set with U∩K=Ø.

Proof:

(i) →(ii): Let X be an m-N-extremally disconnected with respect to τ. Let U and K be two disjoint open and m_X-N-open sets, respectively. Then m_X-N-cl(U) and X / (m_X-N-cl(U)) are disjoint m_X-N-closed and closed sets comprising U and K, respectively.
(ii) →(iii): Let U$\in_\tau$ and K is an m_X-N-open set with U∩K=Ø. By (ii), there be found a disjoint of an m_X-N-closed set U_I and a closed set K_I in which U⊆ U_I and K⊆ K_I. In this manner, m_X-N-cl(U) ∩cl(K) ⊆ U_I ∩ K_I = Ø. Accordingly, m_X-N-cl(U) ∩cl(K) = Ø.
(iii) →(iv): Let U⊆ X and K is an m_X-N-open set with U∩K= Ø. Since int(m_X-N-cl(U)) $\in$ τ and int(m_X-N-cl(U))∩K = Ø, then by (iii), m_X-N-cl[int(m_X-N-cl(U))] ∩cl(K) = Ø.
(iv) →(i): Let U be any open set. Then [X / (m_X-N-cl(U))] ∩U= Ø. As X / m_X-N-cl(U) is an m_X-N-open set and by (iv), m_X-N-cl(int(m_X-N-cl(U)))∩cl(X / m_X-N-cl(U)) = Ø. Since U$\in_\tau$, therefore m_X-N-cl(U)∩[X / int(m_X-N-cl(U))] = Ø.
In this manner, m_X-N-cl(U) ⊆ int(m_X-N-cl(U)) and m_X-N-cl(U) is open. Accordingly, X is m-N-extremally disconnected with respect to τ.

Definition (3.7): A subcategory U of a mixed space (X, τ, m_X) is called an m_X-NR*-open set if U = int(m-N-cl(U)). The complement of an m_X-NR*-open set is said to be m_X-NR*-closed.

Theorem (3.8): Let (X, τ, m_X) be a mixed space. Then, the accompanying properties are identical:

i. X is an m-N-extremally disconnected with respect to τ.
ii. Every m_X-NR*-open set of X is m_X-N-closed in X.
iii. Every m_X-NR*-closed set of X is m_X-N-open in X.

Proof:

(i) →(ii): Let X be an m-N-extremally disconnected with respect to τ. Let U be an m_X-NR*-open set of X, then U = int(m_X-N-cl(U)). Since U is an open set, then m_X -N-cl(U) is open. Subsequently, U = int(m_X-N-cl(U)) = m_X-N-cl(U) and consequently U is m_X-N-closed.
(ii) →(i): Assume that every m_X-NR*-open subcategory of X is m_X-N-closed in X. Let U be an open subcategory of X. Since int(m_X-N-cl(U)) is m_X-NR*-open, then it is m_X-N-closed. This leads to m_X-N-cl(U) ⊆ m_X-N-cl(int(m_X-N-cl(U))) = int(m_X-N-cl(U)). Thus, m_X-N-cl(U) is open and subsequently X is an m-N-extremally disconnected with respect to τ.
(ii) →(iii): Let U be m_X-NR*-closed, then U^C is m_X-NR*-open. Then, by (ii), U^C is m_X-N-closed and in this manner U is an m_X-N-open set in X.
(iii) →(i): Clear.

Theorem (3.9): Let (X, τ, m_X) be a mixed space. Then the accompanying properties are identical:

i. X is m-N-extremally disconnected with respect to τ.
ii. m_X-N-cl(U) $\in_\tau$ for each m_X-NR*-open set U of X.

Proof:

(i) →(ii): Let U be m_X-NR*-open set of X. Then U is open and by (i), m_X-N-cl(U) $\in_\tau$.
(ii) → (i): Assume that m_X-N-cl(U) $\in_\tau$ for each m_X-NR*-open set U of X. Let K be any open set of X. Thereafter, int(m_X-N-cl(K)) is m_X-NR*-open set and m_X-N-cl(K) = m_X-N-cl(int(m_X-N-cl(K))) $\in_\tau$. Consequently m_X-N-cl(K) $\in_\tau$ and therefore X is m-N-extremally disconnected with respect to τ.

Theorem (3.10): Let (X, τ, m_X) be a mixed space, then the accompanying properties are identical:

i. X is m-N-extremally disconnected with respect to τ.
ii. If U is m_X-Ns*-open, K is m_X-Ns**-open and U∩K=Ø, then m_X-N-cl(U) ∩cl(K) = Ø.

Proof:

(i) → (ii): Let U be m_X-Ns*-open, K is an m_X-Ns**-open, and U∩K=Ø. By Corollary (3.5), m_X-N-cl(U) is open, and since m_X-N-cl(U) ∩ m_X-N-int(K) = Ø, m_X-N-cl(U) ∩ cl(m_X-N-int(K)) = Ø. Since K is m_X-Ns**-open, cl(K) = cl(m_X-N-int(K)), and therefore m_X-N-cl(U) ∩cl(K) = Ø.

(ii) →(i): Let U be an m_X-Ns*-open set. Since U and X / m_X-N-cl(U) are disjoint m_X-Ns*-open and m_X-Ns**-open, respectively, then by (ii), we deduce that m_X-N-cl(U)∩cl[X / m_X-N-cl(U)] = Ø. This leads to the result that m_X-N-cl(U) $\subseteq$ int(m_X-N-cl(U)). Thus m_X-N-cl(U) is open. Thereafter, by Corollary (3.5), X is an m_X-N-extremally disconnected with respect to τ.

Theorem (3.11): Let (X, τ, m_X) be a mixed space. Thereafter, X is m-N-extremally disconnected with respect to τ if and only if, for each open set U and every m_X-N-closed K with U $\subseteq$ K, there be found an open set U_I and an m_X-N-closed set K_I in which U $\subseteq K_I \subseteq U_I \subseteq$ K.

Proof:

Assume that X is m-N-extremally disconnected with respect to τ. Let U be an open set and K m_X-N-closed in which U $\subseteq$ K. Then U∩K^C = Ø. Then by Theorem (3.6), m_X-N-cl(U) ∩ cl(K^C) = Ø. Thereafter, m_X-N-cl(U) $\subseteq$ int(K) $\subseteq$ K. Suppose that m_X-N-cl(U) = K_I, int(K) = U_I, then we get U $\subseteq K_I \subseteq U_I \subseteq$ K.

Conversely, let U be an open set and K^C be m_X-N-open in which U∩K^C = Ø. Then, U$\subseteq$ K and K is an m_X-N-closed. So, there be found an open set U_I and an m_X-N-closed set K_I in which U $\subseteq K_I \subseteq U_I \subseteq$ K. This indicates that m_X-N-cl(U) ∩$[int(K)]^C$ = Ø. But $[int(K)]^C$ = cl(K^C). Thereafter, m_X-N-cl(U)∩cl(K^C) = Ø and by Theorem (3.6), X is m-N-extremally disconnected with respect to τ.

Remark (3.12): The above theorem may not be true when K and K_I are m_X-closed sets.

Example:

Let X={1,2,3}, τ ={Ø, {1}, X}, and m_X ={Ø,{1},{2}, X}. Evidently, X is m-N-extremally disconnected with respect to τ and U= {1} $\subseteq$ {1, 3} = K. But there be not found an open set U_I and an m_X-closed set K_I in which U $\subseteq K_I \subseteq U_I \subseteq$ K.

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