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## Efficient Iterative Methods for Solving the SIR Epidemic Model

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### Abstract

In this article, the numerical and approximate solutions for the nonlinear differential equation systems, represented by the epidemic SIR model, are determined. The effective iterative methods, namely the Daftardar-Jafari method (DJM), Temimi-Ansari method (TAM), and the Banach contraction method (BCM), are used to obtain the approximate solutions. The results showed many advantages over other iterative methods, such as Adomian decomposition method (ADM) and the variation iteration method (VIM) which were applied to the non-linear terms of the Adomian polynomial and the Lagrange multiplier, respectively. Furthermore, numerical solutions were obtained by using the fourth-order Runge-Kutta (RK4), where the maximum remaining errors showed that the methods are reliable. In addition, the fixed point theorem was used to show the convergence of the proposed methods. Our calculation was carried out with MATHEMATICA®10 to evaluate the terms of the approximate solutions.

**Keywords:** SIR epidemic model; Semi-analytical method; Daftardar-Jafari method; Temimi-Ansari method; Banach contraction method; Maximum error remainder.

### طرق تكرارية فعالة لحل نموذج SIR

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### الخلاصة

في هذه المقالة ، يتم تحديد الحلول العددية والتقريبية لأنظمة المعادلات التفاضلية غير الخطية ، حيث مثلها نموذج SIR الوبائي. الطرق التكرارية الفعالة هي: طريقة Daftardar-Jafari (DJM)، طريقة Temimi-Ansari (TAM) وطريقة Banach contraction (BCM) المستخدمة للحصول على الحلول التقريبية. أظهرت النتائج العديد من المزايا للطرق ، مثل: خالي من طريقة التحلل الأدمي (ADM) وطريقة تكرار التباين (VIM) التي تم تطبيقها على الحدود غير الخطية من متعددة احوود ادمين ومضاعف Lagrange ، على التوالي. يتم أيضًا الحصول على الحلول العددية باستخدام طريقة رانج-كوتا (RK4) من الرتبة الرابعة ، اظهر حساب أقصى الأخطاء المتبقية أن الطرق موثوقة. بالإضافة إلى ذلك ، تم استخدام نظرية النقطة الصادة لإظهار تقارب الطرق المقترحة. تمت الحسابات باستخدام MATHEMATICA®10 لحساب الحدود للحلول التقريبية.

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**1. Introduction**

In the SIR epidemic model, individuals are categorized into three groups; S is the fraction of the population that is susceptible to disease; I is the fraction of the population that is infectious at any given time; and R is the fraction of the population that has recovered (removed) after infection. Numerous methods were studied by many researchers to solve the epidemic SIR model, as reviewed before [1].

The differential equations play a prominent role in the disciplines of engineering, physics, economics and biology [2- 6]. The nonlinear problems are difficult to solve analytically and efficient methods must be used to obtain either approximate or numerical solutions. Mathematical methods in epidemiology were introduced at the beginning of the 20th century by Ronald Ross [7]. However, the most influential contribution in this field of research is apparently that made by Anderson Gray McKendrick in 1927. It was an ideal model for many infectious diseases. The model includes three types of persons: those at risk of infection, infected individuals with an infection, and those recovering from the disease [8].

In this paper, three iterative methods will be used to solve the epidemic SIR model and obtain a new approximate solution. The first method, namely the DJM, was proposed by Daftardar-Gejji and Jafari in 2006[9- 10]. This method has been used to solve various linear and nonlinear differential equations [11] and the solution of nonlinear ODEs of second order in physics [12]. The second iterative method, namely the TAM, was proposed by Temimi-Ansari in 2011 [13] and used to solve different types of nonlinear ODEs [14], PDEs with the KdV equations [15], and differential algebraic equations (DAEs) [16- 17]. The third iterative method, namely the BCM, was proposed by Daftardar-Gejji and Bhalekar in 2009 [18], which provided the required solution for various types of nonlinear equations.

We have organized this paper as follows: The SIR mathematical model for epidemic diseases is shown in section 2. In section 3, the basic ideas of the three iterative methods are given. The convergence of the proposed methods is presented in section 4. In section 5, the solving model by the proposed methods is described. The proof of the convergence analysis for the proposed methods is presented in section 6. The numerical simulations and the error analysis of the approximate solutions are shown in section 7. Finally, the conclusions are presented in section 8.

**2- SIR mathematical model for epidemic diseases**

The SIR model contains three categories and cases [19], where  $t$  is the independent variable. The model can be presented as follows:

$$\frac{dS}{dt} = A - \beta SI + \gamma R - \mu S; \frac{dI}{dt} = \beta SI - \nu I - \mu I; \frac{dR}{dt} = \nu I - \gamma R - \mu R. \tag{1}$$

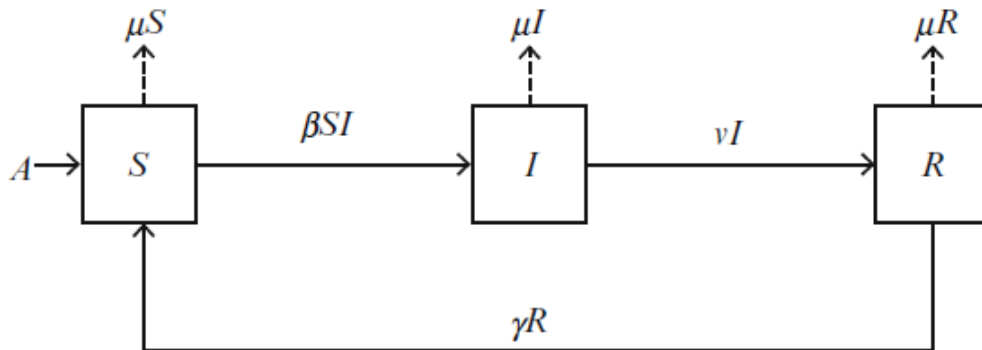
with the initial conditions

$$S(0) = r_1, I(0) = r_2 \text{ and } R(0) = r_3, \tag{2}$$

where  $A$  is vulnerable to growth,  $\beta$  is the infection rate,  $\mu$  is the death rate,  $\nu$  is the recovery rate, and  $\gamma$  is the rate of individuals who lost their immunity and became susceptible to infection after recovery. In this study, consider the following values, based on [19], that guarantee the convergence of the proposed methods:

$$r_1 = 1.8, r_2 = 0.2, r_3 = 0, A = 3, \nu = \gamma = 0.5, \mu = 0.1, \beta = 0.2. \tag{3}$$

Consequently, the following diagram shows the SIR model [20]:



**Figure 1-**The diagram of SIR model.

**3. The basics of the three iterative methods**

**3.1. The basic concept of the DJM**

Consider the following general functional equation, [9, 10]:

$$S = h + N(S), \tag{4}$$

where  $h$  is a known function and  $N$  is the nonlinear term The above equation is solved by the following series form

$$S = \sum_{j=0}^{\infty} S_j. \tag{5}$$

All terms of this series are calculated as:

Step 1:  $S_0 = h,$

Step 2:  $S_1 = N(S_0),$

$$S_2 = N(S_0 + S_1) - N(S_0),$$

$$S_3 = N(S_0 + S_1 + S_2) - N(S_0 + S_1), \tag{6}$$

⋮

$$S_n = N(S_0 + S_1 + \dots + S_{n-1}) - N(S_0 + S_1 + \dots + S_{n-2}), \tag{7}$$

$n = 2, 3 \dots$

Step 3: By putting it back (7) in (4), we shall obtain  $S_0 + S_1 + \dots + S_n = S_0 + N(S_0 + S_1 + \dots + S_{n-2}),$   $n = 1, 2, \dots$  (8)

$$\sum_{j=0}^{\infty} S_j = S_0 + N\left(\sum_{j=0}^{\infty} S_j\right). \tag{9}$$

We put Eqs. (9) and (10) next to each other and call them (9). Then we have

$$S \approx \sum_{j=0}^{n-1} S_j. \tag{10}$$

**3.2. The basic concept of TAM**

The general differential equation is expressed as follows [21]:

$$L(S(t)) = N(S(t)) + h(t) = 0, \tag{11}$$

$$D\left(S, \frac{dS}{dt}\right) = 0, \tag{12}$$

where  $t$  is the independent variable,  $S(t)$  is unknown function,  $h(t)$  is known function,  $L$  and  $N$  are linear and nonlinear terms, respectively.

In the first step, we assuming that  $S_0(t)$  is a first evaluation for (11) and (12),

$$L(S_0(t)) + h(t) = 0; D\left(S_0, \frac{dS_0}{dt}\right) = 0, \tag{13}$$

and the next approximate  $S_1(t)$ ,

$$L(S_1(t)) + N(S_0(t)) + h(t) = 0; D\left(S_1, \frac{dS_1}{dt}\right) = 0. \tag{14}$$

We continue to the  $n + 1$  iteration approximation, as follows

$$L(S_{n+1}(t)) + N(S_n(t)) + h(t) = 0; D\left(S_{n+1}, \frac{dS_{n+1}}{dt}\right) = 0, n = 0, 1, 2.. \tag{15}$$

Each term represents a solution to Eq. (11).

Finally,  $S(t) = \lim_{n \rightarrow \infty} S_{n+1}(t).$

**3.3. The basic concept of the BCM [18]**

The following successive approximations were considered to solve Eq. (4):

$$S_0(t) = h(t),$$

$$S_1(t) = S_0(t) + N(S_0(t)),$$

$$S_2(t) = S_0(t) + N(S_1(t)),$$

⋮

$$S_n(t) = S_0(t) + N(S_{n-1}(t)), n = 1, 2, .. \tag{16}$$

In the final step, the solution is  $S(t) = \lim_{n \rightarrow \infty} S_n(t).$

**4. Convergence of the proposed methods**

In this section, the convergence for the proposed techniques will be discussed. The convergence for the DJM can be applied directly by selecting the obtained component terms, but the convergence between TAM and BCM is shown below.

First, for the TAM:

$$L(B_k(t)) + h(t) + N\left(\sum_{i=0}^{k-1} B_i(t)\right) = 0, \quad k = 1, 2, \dots \tag{17}$$

Second, for the BCM:

$$B_k(t) = B_0(t) + N\left(\sum_{i=0}^{k-1} B_i(t)\right), \quad k = 1, 2, \dots \tag{18}$$

where,

$$B_0 = S_0(t),$$

$$B_1 = V(B_0),$$

$$B_2 = V(B_0 + B_1),$$

:

$$B_{n+1} = V(B_0 + B_1 + \dots + B_n). \tag{19}$$

where  $V$  represents the following operator. In general, the term  $W_k$  is the solution for the problem in the form

$$\text{where } V(B_k) = W_k(t) - \sum_{i=0}^{k-1} B_i(t), \quad k = 1, 2, \dots \tag{20}$$

By using (19) and (20), one can obtain the solution by

$$S(t); I(t); R(t) = \lim_{k \rightarrow \infty} S_k; I_k; R_k = \sum_{i=0}^{\infty} B_i. \tag{21}$$

The following theorems [22] are carried out for the convergences of DJM, TAM, and BCM methods:

*Theorem 1* [22]. "Let  $V$ , which is introduced in Eq. (20), be an operator from Hilbert space

$H$  to  $H$ .  $S_n(t); I_n(t); R_n(t) = \sum_{i=0}^n B_i$  converges; and  $\exists 0 < \alpha < 1$  if  $\|V[B_0 + B_1 + \dots + B_{i+1}]\| \leq \alpha \|V[B_0 + B_1 + \dots + B_i]\|$  (so that  $\|B_{i+1}\| \leq \alpha \|B_i\|$ )  $\forall i = 0, 1, 2, \dots$ "

The condition for studying the convergence of our proposed iterative techniques is based on the Banach fixed-point theorem.

*Theorem 2* [22]. "If the series  $S(t); I(t); R(t) = \sum_{i=0}^{\infty} B_i$  converges, then this series represents the exact solution  $S; I; R$ ."

**Remark**

For Eq. (4), theorems 1 and 2 state that the solution obtained by the DJM given in (9), by the TAM given in (15), by the BCM given in (16), or obtained by (19), is a convergent solution to the exact solution under the given condition:  $\exists 0 < \alpha < 1$  such that  $\|V[B_0 + B_1 + \dots + B_{i+1}]\| \leq \alpha \|V[B_0 + B_1 + \dots + B_i]\|$  (that is  $\|B_{i+1}\| \leq \alpha \|B_i\|$ )  $\forall i = 0, 1, 2, \dots$ . In a different way, if the parameters for each iteration  $i$  takes the form:

$$\rho_i^1 = \begin{cases} \frac{\|B_{i+1}\|}{\|B_i\|}, & \|B_i\| \neq 0 \\ 0, & \|B_i\| = 0 \end{cases} \text{ for } S(t); \quad \rho_i^2 = \begin{cases} \frac{\|B_{i+1}\|}{\|B_i\|}, & \|B_i\| \neq 0 \\ 0, & \|B_i\| = 0 \end{cases} \text{ for } I(t);$$

$$\rho_i^3 = \begin{cases} \frac{\|B_{i+1}\|}{\|B_i\|}, & \|B_i\| \neq 0 \\ 0, & \|B_i\| = 0 \end{cases} \text{ for } R(t). \tag{22}$$

then  $\sum_{i=0}^{\infty} B_i$  of Eqs. (1) and (2) converges to the exact solution  $S(t); I(t); R(t)$  when  $0 \leq \rho_i^j < 1, \forall i = 0, 1, 2, \dots, j = 1, 2, 3$ .

**5- Solving the SIR model by the proposed methods**

In this section, the three iterative methods are used to solve the nonlinear differential systems (1) with the initial condition in (2).

**5.1. Solving SIR model by the DJM**

To solve the SIR model (1) and (2) by the DJM:

First, rewrite Eq. (1) as:

$$S'(t) = N_1(S(t)); I'(t) = N_2(I(t)); R'(t) = N_3(R(t)). \tag{23}$$

where,

$$N_1(S(t)) = A - \beta S(t)I(t) + \gamma R(t) - \mu S(t); N_2(I(t)) = \beta S(t)I(t) - \nu I(t) - \mu I(t); N_3(R(t)) = \nu I(t) - \gamma R(t) - \mu R(t). \tag{24}$$

By integrating Eq. (23) from 0 to  $t$ , and using Eq. (3), we have

$$S(t) = 1.8 + 3t + \int_0^t ((-\beta S(\omega)I(\omega) + \gamma R(\omega) - \mu S(\omega))d\omega, I(t) = 0.2 + \int_0^t ((\beta S(\omega)I(\omega) - \nu I(\omega) - \mu I(\omega))d\omega, R(t) = 0 + \int_0^t ((\nu I(\omega) - \gamma R(\omega) + \mu R(\omega))d\omega, \tag{25}$$

Then, as in the first step in section 3.1 (i.e  $S_0 = h$ )

$$\widehat{S}_0(t) = 1.8 + 3t; \widehat{I}_0(t) = 0.2; \widehat{R}_0(t) = 0.$$

Sequentially, we applied step 2 in section 3.1

$$\begin{aligned} \widehat{S}_{n+1}(t) &= N_1(\widehat{S}_0(t) + \widehat{S}_1(t) + \dots + \widehat{S}_n(t)) - N_1(\widehat{S}_0(t) + \widehat{S}_1(t) + \dots + \widehat{S}_{n-1}(t)), \\ \widehat{I}_{n+1}(t) &= N_2(\widehat{I}_0(t) + \widehat{I}_1(t) + \dots + \widehat{I}_n(t)) - N_2(\widehat{I}_0(t) + \widehat{I}_1(t) + \dots + \widehat{I}_{n-1}(t)), \\ \widehat{R}_{n+1}(t) &= N_3(\widehat{R}_0(t) + \widehat{R}_1(t) + \dots + \widehat{R}_n(t)) - N_3(\widehat{R}_0(t) + \widehat{R}_1(t) + \dots + \widehat{R}_{n-1}(t)), \end{aligned} \tag{26}$$

such that the first approximation is

$$\widehat{S}_1(t) = -0.252t - 0.21t^2; \widehat{I}_1(t) = -0.048t + 0.06t^2; \widehat{R}_1(t) = 0.1t.$$

and the second approximation

$$\begin{aligned} \widehat{S}_2(t) &= 5.5511210^{-17}t + 0.05128t^2 + 0.0113936t^3 - 0.008748t^4 + 0.000504t^5, \\ \widehat{I}_2(t) &= 1.3877810^{-17}t + 0.00072t^2 - 0.0163936t^3 + 0.008748t^4 - 0.000504t^5, \\ \widehat{R}_2(t) &= 0.042t^2 + 0.01t^3. \end{aligned}$$

In the following steps we apply step 3 in section 3.1

$$S_0(t) = \widehat{S}_0(t) = 1.8 + 3t; I_0(t) = \widehat{I}_0(t) = 0.2; R_0(t) = \widehat{R}_0(t) = 0. \tag{27}$$

and,

$$S_1(t) = \widehat{S}_0(t) + \widehat{S}_1(t) = 1.8 + 2.748t - 0.21t^2; I_1(t) = \widehat{I}_0(t) + \widehat{I}_1(t) = 0.2 - 0.048t + 0.06t^2; R_1(t) = \widehat{R}_0(t) + \widehat{R}_1(t) = 0.1t. \tag{28}$$

$$S_2(t) = \widehat{S}_0(t) + \widehat{S}_1(t) + \widehat{S}_2(t) = 1.8 + 2.748t - 0.15872t^2 + 0.0113936t^3 - 0.008748t^4 + 0.000504t^5;$$

$$I_2(t) = \widehat{I}_0(t) + \widehat{I}_1(t) + \widehat{I}_2(t) = 0.2 - 0.048t + 0.06072t^2 - 0.0163936t^3 + 0.008748t^4 - 0.000504t^5;$$

$$R_2(t) = \widehat{R}_0(t) + \widehat{R}_1(t) + \widehat{R}_2(t) = 0. + 0.1t - 0.042t^2 + 0.01t^3. \tag{29}$$

We continue with the iterations to  $n = 6$ , for  $S_n(t)$ ,  $I_n(t)$  and  $R_n(t)$ , where these terms are not listed to ensure brevity.

### 5.2. Solving SIR model by the TAM

By applying TAM to the equations (1) and (2), we have

$$L_1(S(t)) = S'(t); L_2(I(t)) = I'(t); L_3(R(t)) = R'(t).$$

$$\text{and } N_1(S(t)) = -(-\beta S(t)I(t) + \gamma R(t) - \mu S(t)); N_2(I(t)) = -(\beta S(t)I(t) - \nu I(t) - \mu I(t)); N_3(R(t)) = -(\nu I(t) - \gamma R(t) - \mu R(t)).$$

such that,

$$h_1(t) = 3 = A; h_2(t) = 0; h_3(t) = 0. \tag{30}$$

Then, Eq. (1) has changed to the following, after using Eq. (15),

$$L_1(S_{n+1}(t)) + N_1(S_n(t)) + h_1(t) = 0, S_{n+1}(0) = 1.8,$$

$$L_2(I_{n+1}(t)) + N_2(I_n(t)) + h_2(t) = 0, I_{n+1}(0) = 0.2,$$

$$L_3(R_{n+1}(t)) + N_3(R_n(t)) + h_3(t) = 0, R_{n+1}(0) = 0, \tag{31}$$

and Eq. (3) was used for Eq. (2).

$$S_0(0) = 1.8; I_0(0) = 0.2; R_0(0) = 0. \tag{32}$$

In addition, from (13) the initial approximation is

$$S_0(t) = 1.8 + 3t; I_0(t) = 0.2; R_0(t) = 0.$$

In the second step for the first approximation, we have

$$S'_1(t) = N_1(S_0), S_1(0) = 1.8; I'_1(t) = N_2(I_0), I_1(0) = 0.2; R'_1(t) = N_3(R_0), R_1(0) = 0 \tag{33}$$

Meanwhile, the same result (28) was obtained when going through the same processes (25) to solve (33).

The same step for finding  $S_2(t)$ ,  $I_2(t)$ , and  $R_2(t)$  are use, which means solving the following problem

$$S'_2(t) = N_1(S_1), S_2(0) = 1.8, I'_2(t) = N_2(I_1), I_2(0) = 0.2, R'_2(t) = N_3(R_1), R_2(0) = 0 \quad (34)$$

The steps in section 5.1 were used to get  $S_2(t)$ ,  $I_2(t)$  and  $R_2(t)$ , where the result is quite similar to (29).

Furthermore, we continue to achieve the approximations to  $n = 6$  for  $S_n(t)$ ,  $I_n(t)$  and  $R_n(t)$ .

### 5.3. Solving SIR model by the BCM

Let us start with the same steps of integration processes given in section (5.1), which implies that we obtain an integral (25). After following the BCM steps, we re-rewrite (1) as (23), (24) and (25) with (27)

$$S'(t) = N_1(S(t)); I'(t) = N_2(I(t)); R'(t) = N_3(R(t)). \quad (35)$$

where,

$$N_1(S(t)) = A - \beta S(t)I(t) + \gamma R(t) - \mu S(t); N_2(I(t)) = \beta S(t)I(t) - \nu I(t) - \mu I(t); N_3(R(t)) = \nu I(t) - \gamma R(t) - \mu R(t). \quad (36)$$

In general, we have

$$S_{n+1}(t) = S_0(t) + \int_0^t N_1(S_n(\omega))d\omega; I_{n+1}(t) = I_0(t) + \int_0^t N_2(I_n(\omega))d\omega; R_{n+1}(t) = R_0(t) + \int_0^t N_3(R_n(\omega))d\omega. \quad (37)$$

where,  $S_0(t) = 1.8 + 3t$ ;  $I_0(t) = 0.2$ ;  $R_0(t) = 0$ .

Therefore,

$S_1(t)$ ,  $I_1(t)$  and  $R_1(t)$  are similar to Eq. (28),

and

$S_2(t)$ ,  $I_2(t)$  and  $R_2(t)$  are similar to Eq. (29)

## 6. Proof of the convergence analysis for the proposed methods

In this section, we prove the convergence analysis for the DJM, TAM and BCM and calculate the values of  $\rho_i^j$ .

First, for  $S(t)$

$$\rho_0^1 = \frac{\|S_1\|}{\|S_0\|} = 0.06925 < 1; \rho_1^1 = \frac{\|S_2\|}{\|S_1\|} = 0.117813 < 1; \rho_2^1 = \frac{\|S_3\|}{\|S_2\|} = 0.121651 < 1; \rho_3^1 = \frac{\|S_4\|}{\|S_3\|} = 0.140792 < 1; \rho_4^1 = \frac{\|S_5\|}{\|S_4\|} = 0.116682 < 1; \rho_5^1 = \frac{\|S_6\|}{\|S_5\|} = 0.0972425 < 1.$$

Second, for  $I(t)$

$$\rho_0^2 = \frac{\|I_1\|}{\|I_0\|} = 0.06 < 1; \rho_1^2 = \frac{\|I_2\|}{\|I_1\|} = 0.619133 < 1; \rho_2^2 = \frac{\|I_3\|}{\|I_2\|} = 0.0483844 < 1; \rho_3^2 = \frac{\|I_4\|}{\|I_3\|} = 0.139044 < 1; \rho_4^2 = \frac{\|I_5\|}{\|I_4\|} = 0.111496 < 1; \rho_5^2 = \frac{\|I_6\|}{\|I_5\|} = 0.0957768 < 1.$$

Finally, for  $R(t)$

$$\rho_0^3 = \frac{\|R_1\|}{\|R_0\|} = 0 < 1; \rho_1^3 = \frac{\|R_2\|}{\|R_1\|} = 0.32 < 1; \rho_2^3 = \frac{\|R_3\|}{\|R_2\|} = 0.181362 < 1; \rho_3^3 = \frac{\|R_4\|}{\|R_3\|} = 0.150153 < 1; \rho_4^3 = \frac{\|R_5\|}{\|R_4\|} = 0.118191 < 1; \rho_5^3 = \frac{\|R_6\|}{\|R_5\|} = 0.0974857 < 1.$$

All values of  $\rho_i^j$  are less than one, for  $i=1, 2 \dots 6, 0 < t < 1$ . From theorems (1) and (2), the DJM, TAM and BCM methods provided convergence.

## 7. The numerical results

In this section, we carry out more numerical calculations and test some of the error indicators to verify the accuracy of the proposed methods. The residual error function is calculated by [23]

$$\begin{aligned} ER_{1,n}(t) &= S'_n - (A - \beta S_n I_n + \gamma R_n - \mu S_n), \\ ER_{2,n}(t) &= I'_n - (\beta S_n I_n - \nu I_n - \mu I_n), \\ ER_{3,n}(t) &= R'_n - (\nu I_n - \gamma R_n - \mu R_n), \end{aligned} \quad (38)$$

and the maximal error remainders are

$$MER_{k,n} = \max_{0 \leq t \leq 1} |ER_{k,n}(t)|, k = 1,2,3$$

Tables-1-3 show the maximal error remainder  $MER_{k,n}$  values at  $0 \leq t \leq 1$  for the numerical solutions with the comparison with other iterative methods, such as the ADM [24- 25] and the VIM [26, 27, 28]. It can be seen that the error decreases with increasing the iterations.

**Table 1-**  $MER_{1,n}$  comparison of the proposed methods solution with those of ADM and VIM methods for  $S(t)$ .

$n$	$MER_{1,n}$ by the proposed methods	$MER_{1,n}$ by ADM	$MER_{1,n}$ by VIM
1	0.104269	0.104269	0.291059
2	0.017224	0.016078	0.0471248
3	0.00351953	0.002823	0.00972674
4	0.000523294	0.00051242	0.00162459
5	0.0000619428	0.0000778422	0.000197424
6	$6.0431710^{-6}$	$9.0244810^{-6}$	0.0000196617

**Table 2-**  $MER_{2,n}$  comparison of the proposed methods solution with those of ADM and VIM methods for  $I(t)$ .

$n$	$MER_{2,n}$ by proposed methods	$MER_{2,n}$ by ADM	$MER_{2,n}$ by VIM
1	0.0152688	0.0152688	0.142291
2	0.000636481	0.000915055	0.00996162
3	0.000171276	0.000549563	0.000323757
4	0.0000243462	0.0000487929	0.0000781984
5	$2.9116810^{-6}$	$4.4301610^{-6}$	$9.2168710^{-6}$
6	$2.8541510^{-7}$	$1.5256910^{-6}$	$9.2834610^{-7}$

**Table 3-**  $MER_{3,n}$  comparison of the proposed methods solution with those of ADM and VIM methods for  $R(t)$ .

$n$	$MER_{3,n}$ by proposed methods	$MER_{3,n}$ by ADM	$MER_{3,n}$ by VIM
1	0.054	0.054	0.084
2	0.0154852	0.01546	0.0511632
3	0.00330242	0.00332673	0.00992314
4	0.000497864	0.00056013	0.00155806
5	0.0000590103	0.0000733912	0.00018844
6	$5.7574210^{-6}$	$7.4984510^{-6}$	0.0000187372

We note that the error value of the proposed methods is lower than those of the ADM and the VIM, which indicates that the proposed methods converged faster. Further investigations were carried out by comparing the numerical results with the classical Runge-Kutta 4 (RK4) method. In Figures- 2-4, a good agreement between the proposed methods and RK4 can be observed for  $S(t)$ ,  $I(t)$  and  $R(t)$ .

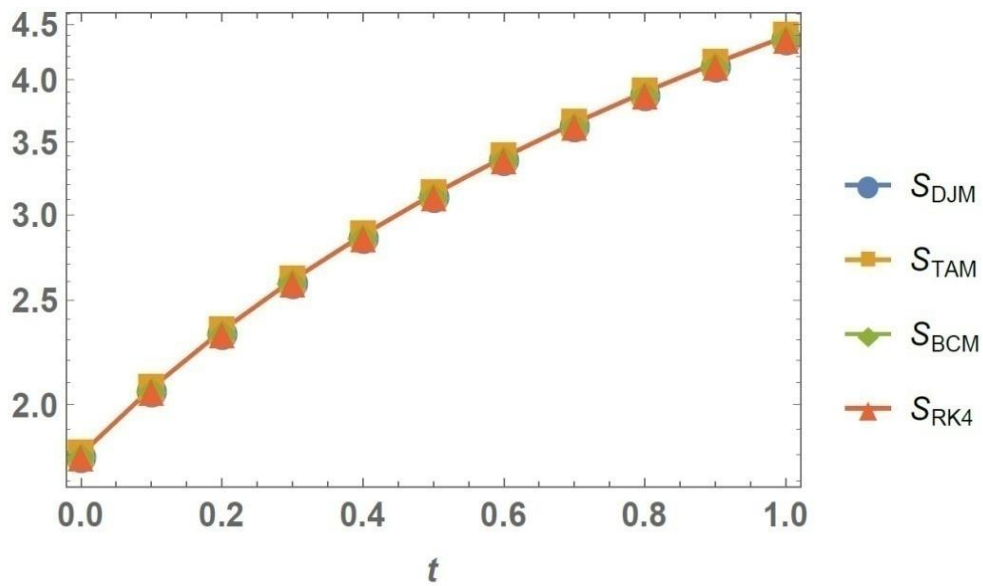


Figure 2- Comparison of the RK4 method with the proposed methods solution for  $S(t)$ .

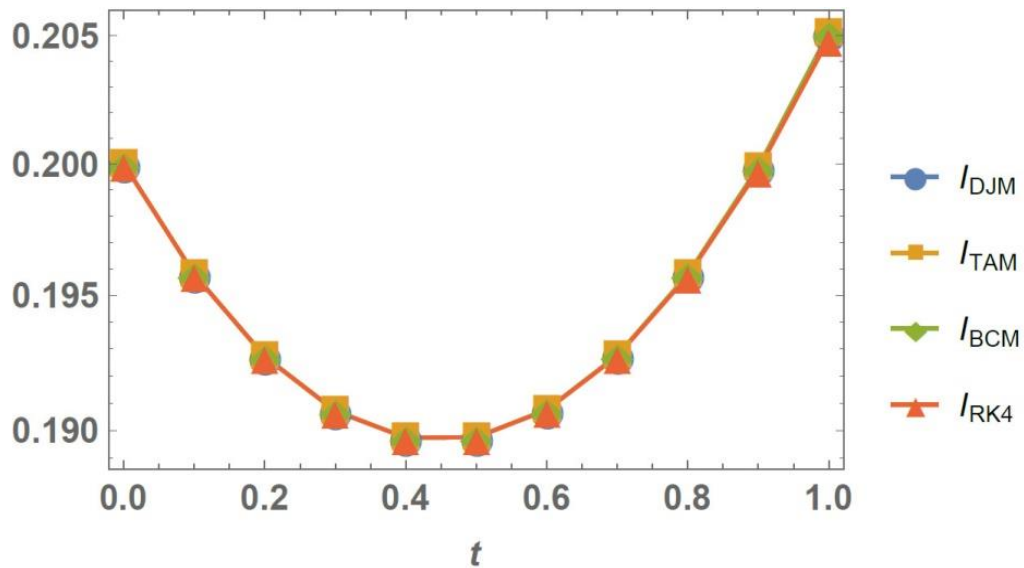


Figure 3-Comparison of the RK4 method with the proposed methods solution for  $I(t)$ .

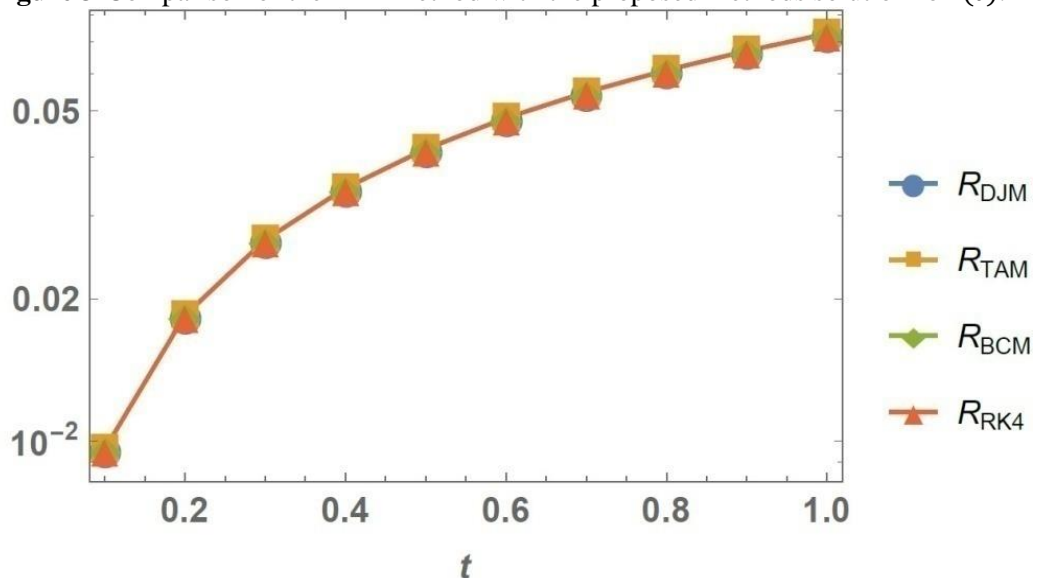


Figure 4-Comparison of the RK4 method with the proposed methods solution for  $R(t)$ .



Table-4 shows the maximum errors remainder  $MER_{k,n}$  values at  $0 \leq t \leq 3.5$ . The accuracy deteriorates and  $MER_{k,n}$  increases if the interval of  $t$  is extended. If the interval of  $t$  is extended, we actually move away from the starting point. As a result, the accuracy of the new methods is less reliable than the original value.

**Table 4**-The maximal error remainder  $MER_{k,n}$  of SIR by the proposed methods where  $0 \leq t \leq 3.5$  and  $n = 1,2,..6$

$n$	$MER_{1,n}$	$MER_{2,n}$	$MER_{3,n}$
1	0.34445	0.5247	0.0735
2	0.691357	0.429229	0.22844
3	0.108786	0.13244	0.0395636
4	0.118443	0.0571519	0.0610343
5	0.0101842	0.00455294	0.0147208
6	0.0109364	0.00369686	0.0072386

## 8-Conclusions

In this paper, three iterative methods, namely DJM TAM and BCM, were used to solve the epidemic model SIR. All the proposed methods provided an approximate solution in a number of terms. The convergence of the proposed methods is proved on the basis of the Banach theorem. In addition, the obtained numerical results were compared with the numerical results of the Runge-Kutta 4 (RK4). It is worth to mention that we modified the work previously achieved [27- 28] to solve the model given in Eq. (1), where good matches were achieved. The proposed methods were used to solve the nonlinear problem without additional assumptions, in order to work with the nonlinear term which is used in other methods, such as ADM and VIM.

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