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On New Types of Weakly Neutrosophic Crisp Open Mappings

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Abstract

This work employs the conceptions of neutrosophic crisp α-open and semi-α-open sets to distinguish some novel forms of weakly neutrosophic crisp open mappings; for instance, neutrosophic crisp α-open mappings, neutrosophic crisp α*-open mappings, neutrosophic crisp α**-open mappings, neutrosophic crisp semi-α-open mappings, neutrosophic crisp semi-α*-open mappings, and neutrosophic crisp semi-α**-open mappings. Moreover, the close connections between these forms of weakly neutrosophic crisp open mappings and the viewpoints of neutrosophic crisp open mappings are explained. Additionally, various theorems and related features and notes are submitted.

Keywords: NCα-open sets, NCSα-open sets, NCα-open mappings, NCα*-open mappings, NCα**-open mappings, NCSα-open mappings, NCSα*-open mappings and NCSα**-open mappings.

حول أنواع جديدة من التطبيقات المفتوحة النتروسوفكية الهشه الضعيفة

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الخلاصة

هذا العمل يوظف أفكار المجموعات المفتوحة α النتروسوفكية الهشه و المفتوحة شبه α النتروسوفكية الهشه لتميز أنواع جديدة وغير مألوفة للتطبيقات المفتوحة النتروسوفكية الهشه الضعيفة وعلى سبيل المثال التطبيقات المفتوحة α النتروسوفكية الهشه، التطبيقات المفتوحة *α النتروسوفكية الهشه، التطبيقات المفتوحة **α النتروسوفكية الهشه، التطبيقات شبه α المفتوحة النتروسوفكية الهشه، التطبيقات المفتوحة شبه *α النتروسوفكية الهشه، التطبيقات المفتوحة شبه **α النتروسوفكية الهشه. علاوة على ذلك، فسرت الارتباطات الوثيقة بين هذه الصيغ للتطبيقات المفتوحة النتروسوفكية الهشه الضعيفة و وجهات النظر للتطبيقات المفتوحة النتروسوفكية الهشه. بالإضافة الى ذلك، قدمت نظريات و خصائص و ملاحظات متنوعة.

الكلمات المفتاحية: المجموعات المفتوحة α النتروسوفكية الهشه، المجموعات المفتوحة شبه α النتروسوفكية الهشه، التطبيقات المفتوحة α النتروسوفكية الهشه، التطبيقات المفتوحة *α النتروسوفكية الهشه، التطبيقات المفتوحة **αالنتروسوفكية الهشه، التطبيقات المفتوحة شبه α النتروسوفكية الهشه، التطبيقات المفتوحة شبه *α النتروسوفكية الهشه، التطبيقات المفتوحة شبه **α النتروسوفكية الهشه.

1. Introduction

Salama *et al*. [1] pioneered the abstraction of neutrosophic crisp topological space. Abdel-Basset *et al*. [2-7] gave a novel neutrosophic approach. Al-Hamido *et al*. [8] submitted the intellect of

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neutrosophic crisp semi-α-closed sets. Banupriya *et al*. [9] investigated the notion of αgs continuity and αgs irresolute maps. Dhavaseelan *et al*. [10] exhibited the theme of neutrosophic α^m-continuity. Maheswari *et al*. [11] introduced gb-closed sets and gb-continuity. This study aims to establish unprecedented classes of weakly neutrosophic crisp open mappings by way of examples neutrosophic crisp α-open mappings, neutrosophic crisp α*-open mappings, neutrosophic crisp α**-open mappings, neutrosophic crisp semi-α-open mappings, neutrosophic crisp semi-α*-open mappings, and neutrosophic crisp semi-α**-open mappings. Furthermore, we shall explain the close relastionships between these categories of weakly neutrosophic crisp open mappings and the conceptions of neutrosophic crisp open mappings. Furthermore, we shall show some related theorems and their features and notes.

2. Preliminaries

For all of this work, $(\mathcal{U},\tau),(\mathcal{V},\sigma)$, and $(\mathcal{W},\rho)$ (or frugally $\mathcal{U},\mathcal{V}$, and $\mathcal{W}$) always mean neutrosophic crisp topological spaces (for brevity NCTSs). For a neutrosophic crisp set $\mathcal{M}$ in a NCTS $\mathcal{U}$, $\mathcal{M}^c=\mathcal{U}-\mathcal{M}$, $NCint(\mathcal{M})$, and $NCcl(\mathcal{M})$ signify the neutrosophic crisp complement, the neutrosophic crisp interior, and the neutrosophic crisp closure of $\mathcal{M}$, correspondingly.

Definition 2.1 [1]: For any nonempty under-consideration set $\mathcal{U}$, a neutrosophic crisp set (curtly NCS) $\mathcal{M}$ is an object holding $the\ establishment \mathcal{M}=\langle\mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3\rangle$ where $\mathcal{M}_1,\mathcal{M}_2$ and $\mathcal{M}_3$ are mutually disjoint sets included in $\mathcal{U}$.

Definition 2.2: Suppose that NCTS $\mathcal{U}$ contains a NCS $\mathcal{M}$, then we have the following:

1) If $\mathcal{M}\subseteq NCint(NCcl(NCint(\mathcal{M})))$, then $\mathcal{M}$ is named a neutrosophic crisp α-open set (concisely NCα-OS) [12]. The collection of all NCα-OSs of $\mathcal{U}$ is symbolized by $\mathrm{NC\alpha O}(\mathcal{U})$.

2) If $\mathcal{M}\subseteq NCcl(NCint(NCcl(NCint(\mathcal{M}))))$ or, in other words, for some NCα-OS $\mathcal{E}$ in $\mathcal{U}$ so that $\mathcal{E}\subseteq\mathcal{M}\subseteq NCcl(\mathcal{E})$, then $\mathcal{M}$ is a neutrosophic crisp semi-α-open set (concisely NCSα-OS) [8]. The collection of all NCSα-OSs of $\mathcal{U}$ is represented by $\mathrm{NCS\alpha O}(\mathcal{U})$.

Remark 2.3 [8,13]: For any NCTS $\mathcal{U}$, the following claims stay valid, but not vice versa:

1) For all NC-OSs are NCα-OSs and NCSα-OSs.

2) For all NCα-OSs are NCSα-OSs.

Example 2.4: Let $\mathcal{U}=\{a,b,c,d\}$.

Then, $\tau_{\mathcal{U}}=\{\phi_N,\langle\{a\},\phi,\phi\rangle,\langle\{b\},\phi,\phi\rangle,\langle\{a,b\},\phi,\phi\rangle,\langle\{a,b,c\},\phi,\phi\rangle,\mathcal{U}_N\}$ is a NCTS. The class of all NCα-OSs of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U})=\tau_{\mathcal{U}}\cup\{\langle\{a,b,d\},\phi,\phi\rangle\}$. The class of all NCSα-OSs of $\mathcal{U}$ is: $\mathrm{NCS\alpha O}(\mathcal{U})=\mathrm{NC\alpha O}(\mathcal{U})\cup\{\langle\{a,c\},\phi,\phi\rangle,\langle\{a,d\},\phi,\phi\rangle,\langle\{b,c\},\phi,\phi\rangle,\langle\{b,d\},\phi,\phi\rangle,\langle\{a,c,d\},\phi,\phi\rangle,$ $\langle\{b,c,d\},\phi,\phi\rangle\}$. The NCS $\langle\{a,b,d\},\phi,\phi\rangle$ is a NCα-OS (NCSα-OS) but not NC-OS. The NCS $\langle\{a,c,d\},\phi,\phi\rangle$ is a NCSα-OS but not NCα-OS.

Proposition 2.5 [12]: For any neutrosophic crisp subset $\mathcal{M}$ of a NCTS $(\mathcal{U},\tau)$, $\mathcal{M}\in\mathrm{NC\alpha O}(\mathcal{U})$ iff for some NC-OS $\mathcal{P}$, $\mathcal{P}\subseteq\mathcal{M}\subseteq NCint(NCcl(\mathcal{P}))$.

Definition 2.6: Let $\ell:(\mathcal{U},\tau)\longrightarrow(\mathcal{V},\sigma)$ be a mapping, then ℓ is termed:

1) Neutrosophic crisp open (briefly NC-open) [1] iff for each $\mathcal{M}$ NC-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NC-OS in $\mathcal{V}$.

2) Neutrosophic crisp α-open (briefly NCα-open) [14] iff for each $\mathcal{M}$ NC-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NCα-OS in $\mathcal{V}$.

Proposition 2.7 [1]: A mapping $\ell:(\mathcal{U},\tau)\longrightarrow(\mathcal{V},\sigma)$ is NC-open iff $\ell(NCint(\mathcal{M}))\subseteq NCint(\ell(\mathcal{M}))$, for every $\mathcal{M}\subseteq\mathcal{U}$.

Definition 2.8 [1]: Let $\ell:(\mathcal{U},\tau)\longrightarrow(\mathcal{V},\sigma)$ be a mapping, then ℓ is called neutrosophic crisp continuous (briefly NC-continuous) iff for each $\mathcal{M}$ NC-OS in $\mathcal{V}$, $\ell^{-1}(\mathcal{M})$ is a NC-OS in $\mathcal{U}$.

Proposition 2.9 [1]: A mapping $\ell:(\mathcal{U},\tau)\longrightarrow(\mathcal{V},\sigma)$ is NC-continuous iff $\ell(NCcl(\mathcal{M}))\subseteq NCcl(\ell(\mathcal{M}))$, for every $\mathcal{M}\subseteq\mathcal{U}$.

3. Weakly Neutrosophic Crisp Open Mappings

Definition 3.1: Let $\ell:(\mathcal{U},\tau)\longrightarrow(\mathcal{V},\sigma)$ be a mapping, then ℓ is named:

1) Neutrosophic crisp α*-open (briefly NCα*-open) iff for each NCα-OS $\mathcal{M}$ in $\mathcal{U}$, $\ell(\mathcal{M})$ is considered a NCα-OS in $\mathcal{V}$.

2) Neutrosophic crisp α**-open (briefly NCα**-open) iff for each NCα-OS $\mathcal{M}$ in $\mathcal{U}$, $\ell(\mathcal{M})$ is considered a NC-OS in $\mathcal{V}$.

Definition 3.2: Let $\ell:(\mathcal{U},\tau)\longrightarrow(\mathcal{V},\sigma)$ be a mapping, then ℓ is termed:

1) Neutrosophic crisp semi-α-open (briefly NCSα-open) iff for each $\mathcal{M}$ as NC-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NCSα-OS in $\mathcal{V}$.
2) Neutrosophic crisp semi-α*-open (briefly NCSα*-open) iff for each $\mathcal{M}$ as NCSα-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NCSα-OS in $\mathcal{V}$.
3) Neutrosophic crisp semi-α**-open (briefly NCSα**-open) iff for each $\mathcal{M}$ as NCSα-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NC-OS in $\mathcal{V}$.

Proposition 3.3

1) Every NC-open mapping is a NCα-open, so it is NCSα-open, but the reverse is not valid in general.
2) Every NCα-open mapping is a NCSα-open, but the reverse is not valid in general.

Proof

1) Let $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ be a NC-open mapping and a NC-OS $\mathcal{M}$ be in $\mathcal{U}$. Then $\ell(\mathcal{M})$ is considered a NC-OS in $\mathcal{V}$. Because any NC-OS is NCα-OS (NCSα-OS), $\ell(\mathcal{M})$ is considered a NCα-OS (NCSα-OS) set in $\mathcal{V}$. Hence, ℓ is a NCα-open (NCSα-open) mapping.
2) Let $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ be a NCα-open mapping and $\mathcal{M}$ be a NC-OS in $\mathcal{U}$. Then $\ell(\mathcal{M})$ is a NCα-OS in $\mathcal{V}$. Because any NCα-OS is NCSα-OS, $\ell(\mathcal{M})$ considers NCSα-OS in $\mathcal{V}$. Hence ℓ is a NCSα-open mapping.

Example 3.4: Let $\mathcal{U} = \{a, b, c, d\}$.
Then, $\tau_{\mathcal{U}} = \{\phi_N, \langle\{a\}, \phi, \phi\rangle, \langle\{b\}, \phi, \phi\rangle, \langle\{a, b\}, \phi, \phi\rangle, \langle\{a, b, c\}, \phi, \phi\rangle, \mathcal{U}_N\}$ is a NCTS. The class of all NCα-OSs of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U}) = \tau_{\mathcal{U}} \cup \{\langle\{a, b, d\}, \phi, \phi\rangle\}$. The class of all NCSα-OSs of $\mathcal{U}$ is: $\mathrm{NCS\alpha O}(\mathcal{U}) = \mathrm{NC\alpha O}(\mathcal{U}) \cup \{\langle\{a, c\}, \phi, \phi\rangle, \langle\{a, d\}, \phi, \phi\rangle, \langle\{b, c\}, \phi, \phi\rangle, \langle\{b, d\}, \phi, \phi\rangle, \langle\{a, c, d\}, \phi, \phi\rangle, \langle\{b, c, d\}, \phi, \phi\rangle\}$.
We define a mapping $\ell: \mathcal{U} \longrightarrow \mathcal{U}$ by $\ell(\langle\{a\}, \phi, \phi\rangle) = \langle\{a\}, \phi, \phi\rangle, \ell(\langle\{b\}, \phi, \phi\rangle) = \langle\{b\}, \phi, \phi\rangle$, $\ell(\langle\{c\}, \phi, \phi\rangle) = \ell(\langle\{d\}, \phi, \phi\rangle) = \langle\{d\}, \phi, \phi\rangle$. We observe that ℓ is a NCα-open mapping, which is not NC-open mapping since $\langle\{a, b, c\}, \phi, \phi\rangle$ is NC-OS in $\mathcal{U}$, but $\ell(\langle\{a, b, c\}, \phi, \phi\rangle) = \langle\{a, b, d\}, \phi, \phi\rangle$ is not a NC-OS in $\mathcal{U}$.

Example 3.5: Let $\mathcal{U} = \{a, b, c, d\}$.
Then, $\tau_{\mathcal{U}} = \{\phi_N, \langle\{a\}, \phi, \phi\rangle, \langle\{b\}, \phi, \phi\rangle, \langle\{a, b\}, \phi, \phi\rangle, \langle\{a, b, c\}, \phi, \phi\rangle, \mathcal{U}_N\}$ is a NCTS. The class of all NCα-OSs of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U}) = \tau_{\mathcal{U}} \cup \{\langle\{a, b, d\}, \phi, \phi\rangle\}$. The class of all NCSα-OSs of $\mathcal{U}$ is: $\mathrm{NCS\alpha O}(\mathcal{U}) = \mathrm{NC\alpha O}(\mathcal{U}) \cup \{\langle\{a, c\}, \phi, \phi\rangle, \langle\{a, d\}, \phi, \phi\rangle, \langle\{b, c\}, \phi, \phi\rangle, \langle\{b, d\}, \phi, \phi\rangle, \langle\{a, c, d\}, \phi, \phi\rangle, \langle\{b, c, d\}, \phi, \phi\rangle\}$.
We define a mapping $\ell: \mathcal{U} \longrightarrow \mathcal{U}$ by $\ell(\langle\{a\}, \phi, \phi\rangle) = \ell(\langle\{b\}, \phi, \phi\rangle) = \langle\{a\}, \phi, \phi\rangle, \ell(\langle\{c\}, \phi, \phi\rangle) = \ell(\langle\{d\}, \phi, \phi\rangle) = \langle\{c\}, \phi, \phi\rangle$. It is easily seen that ℓ is a NCSα-open mapping, but it is not NC-open mapping since $\langle\{a, b, c\}, \phi, \phi\rangle$ is NC-OS in $\mathcal{U}$, but $\ell(\langle\{a, b, c\}, \phi, \phi\rangle) = \langle\{a, c\}, \phi, \phi\rangle$ is not a NC-OS in $\mathcal{U}$. Then, ℓ is a NCSα-open mapping, but it is not NCα-open mapping.

Remark 3.6: The ideas of NC-open mapping and NCα*-open mapping are self-regulating, as in the further examples below:

Example 3.7: Let $\mathcal{U} = \{a, b, c, d\}$.
Then, $\tau_{\mathcal{U}} = \{\phi_N, \langle\{a\}, \phi, \phi\rangle, \langle\{b\}, \phi, \phi\rangle, \langle\{a, b\}, \phi, \phi\rangle, \langle\{a, b, c\}, \phi, \phi\rangle, \mathcal{U}_N\}$ is a NCTS. The class of all NCα-OSs of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U}) = \tau_{\mathcal{U}} \cup \{\langle\{a, b, d\}, \phi, \phi\rangle\}$. The class of all NCSα-OSs of $\mathcal{U}$ is: $\mathrm{NCS\alpha O}(\mathcal{U}) = \mathrm{NC\alpha O}(\mathcal{U}) \cup \{\langle\{a, c\}, \phi, \phi\rangle, \langle\{a, d\}, \phi, \phi\rangle, \langle\{b, c\}, \phi, \phi\rangle, \langle\{b, d\}, \phi, \phi\rangle, \langle\{a, c, d\}, \phi, \phi\rangle, \langle\{b, c, d\}, \phi, \phi\rangle\}$.
We define a mapping $\ell: \mathcal{U} \longrightarrow \mathcal{U}$ by $\ell(\langle\{a\}, \phi, \phi\rangle) = \ell(\langle\{b\}, \phi, \phi\rangle) = \langle\{a\}, \phi, \phi\rangle, \ell(\langle\{c\}, \phi, \phi\rangle) = \langle\{b\}, \phi, \phi\rangle$, $\ell(\langle\{d\}, \phi, \phi\rangle) = \langle\{c\}, \phi, \phi\rangle$. It is easily seen that ℓ is a NC-open mapping, which is not NCα*-open mapping since $\langle\{a, b, d\}, \phi, \phi\rangle \in \mathrm{NC\alpha O}(\mathcal{U})$, but $\ell(\langle\{a, b, d\}, \phi, \phi\rangle) = \langle\{a, c\}, \phi, \phi\rangle \notin \mathrm{NC\alpha O}(\mathcal{U})$.

Example 3.8: In Example 3.4, it is easily seen that ℓ is a NCα*-open mapping, but it is not NC-open since $\langle\{a, b, c\}, \phi, \phi\rangle \in \tau_{\mathcal{U}}$, but $\ell(\langle\{a, b, c\}, \phi, \phi\rangle) = \langle\{a, b, d\}, \phi, \phi\rangle \notin \tau_{\mathcal{U}}$.

Proposition 3.9

1) If $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ is a NC-open, NC-continuous mapping, then ℓ is a NCα*-open mapping.
2) $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ is a NCα*-open mapping iff $\ell: \left(\mathcal{U}, \mathrm{NC\alpha O}(\mathcal{U})\right) \longrightarrow \left(\mathcal{V}, \mathrm{NC\alpha O}(\mathcal{V})\right)$ is a NC-open.

Proof

1) Let $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ be a NC-open, NC-continuous mapping. To prove that ℓ is a NCα*-open mapping, let $\mathcal{M} \in$ NCαO($\mathcal{U}$), then for some NC-OS $\mathcal{F}$, such that $\mathcal{F} \subseteq \mathcal{M} \subseteq NCint(NCcl(\mathcal{F}))$ (by Proposition 2.5). Hence, $\ell(\mathcal{F}) \subseteq \ell(\mathcal{M}) \subseteq \ell\left(NCint\left(NCcl(\mathcal{F})\right)\right)$, but $\ell(NCint(NCcl(\mathcal{F}))) \subseteq NCint(\ell(NCcl(\mathcal{F})))$ (since ℓ is a NC-open mapping).

Then, $\ell(\mathcal{F}) \subseteq \ell(\mathcal{M}) \subseteq \ell(NCint(NCcl(\mathcal{F})))) \subseteq NCint(\ell(NCcl(\mathcal{F})))$.

But, $NCint(\ell(NCcl(\mathcal{F}))) \subseteq NCint(NCcl(\ell(\mathcal{F})))$ (since ℓ is a NC-continuous mapping). Therefore, we get $\ell(\mathcal{F}) \subseteq \ell(\mathcal{M}) \subseteq NCint(NCcl(\ell(\mathcal{F})))$. However, $\ell(\mathcal{F})$ is a NC-OS in $\mathcal{V}$ (since ℓ is a NC-open mapping). Hence, $\ell(\mathcal{M}) \in$ NCαO($\mathcal{V}$) (by Proposition 2.5). Thus, it is a NCα*-open mapping.

2) The proof of a part (2) is easily reached.

Proposition 3.10: Every NCα*-open mapping is a NCα-open and NCSα-open, but the reverse is not valid in general.

Proof: Let $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ be a NCα*-open mapping and $\mathcal{M}$ be NC-OS in $\mathcal{U}$. Then, we have that $\mathcal{M}$ is considered a NCα-OS in $\mathcal{U}$ [from Proposition 2.5]. Because ℓ is a NCα*-open, then $\ell(\mathcal{M})$ is considered a NCα-OS in $\mathcal{V}$. Therefore, ℓ is a NCα-open. Also, ℓ is a NCSα-open.

Example 3.11: In Example 3.7, it is easily seen that ℓ is a NCα-open mapping and NCSα-open mapping, but not NCα*-open.

Remark 3.12: The ideas of NC-open mapping and NCSα*-open mapping are independent, as explained in the examples below.

Example 3.13: Let $\mathcal{U} = \{a, b, c\}$. Then, $\tau = \{\phi_N, \langle\{a\}, \phi, \phi\rangle, \mathcal{U}_N\}$ is a NCTS. The class of all NCα-OSs (NCSα-OSs) of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U}) = \mathrm{NCS\alpha O}(\mathcal{U}) = \tau \cup \{\langle\{a,b\}, \phi, \phi\rangle, \langle\{a,c\}, \phi, \phi\rangle\}$. Let $\mathcal{V} = \{p, q, r, s\}$. Then, $\sigma = \{\phi_N, \langle\{p\}, \phi, \phi\rangle, \langle\{q,r\}, \phi, \phi\rangle, \langle\{p,q,r\}, \phi, \phi\rangle, \mathcal{V}_N\}$ is a NCTS. The class of all NCα-OSs of $\mathcal{V}$ is: $\mathrm{NC\alpha O}(\mathcal{V}) = \sigma.$ The class of all NCSα-OSs of $\mathcal{V}$ is: $\mathrm{NCS\alpha O}(\mathcal{V}) = \mathrm{NC\alpha O}(\mathcal{V}) \cup \{\langle\{q,r,s\}, \phi, \phi\rangle\}$.

We define a mapping $\ell: \mathcal{U} \longrightarrow \mathcal{V}$ by $\ell(\langle\{a\}, \phi, \phi\rangle) = \langle\{p\}, \phi, \phi\rangle, \ell(\langle\{b\}, \phi, \phi\rangle) = \langle\{q\}, \phi, \phi\rangle$, $\ell(\langle\{c\}, \phi, \phi\rangle) = \langle\{r\}, \phi, \phi\rangle$. It is easily seen that ℓ is a NC-open mapping, but it is not NCSα*-open mapping, since $\langle\{a,b\}, \phi, \phi\rangle \in \mathrm{NCS\alpha O}(\mathcal{U})$, but $\ell(\langle\{a,b\}, \phi, \phi\rangle) = \langle\{p,q\}, \phi, \phi\rangle \notin \mathrm{NCS\alpha O}(\mathcal{V})$.

Example 3.14: Let $\mathcal{U} = \{a, b, c\}$. Then, $\tau_{\mathcal{U}} = \{\phi_N, \langle\{a\}, \phi, \phi\rangle, \langle\{b\}, \phi, \phi\rangle, \langle\{a,b\}, \phi, \phi\rangle, \mathcal{U}_N\}$ is a NCTS. The family of all NCα-OSs of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U}) = \tau_{\mathcal{U}}$. The family of all NCSα-OSs of $\mathcal{U}$ is: $\mathrm{NC\alpha O}(\mathcal{U}) = \mathrm{NC\alpha O}(\mathcal{U}) \cup \{\langle\{a,c\}, \phi, \phi\rangle, \langle\{b,c\}, \phi, \phi\rangle\}$.

We define a mapping $\ell: \mathcal{U} \longrightarrow \mathcal{U}$ by $\ell(\langle\{a\}, \phi, \phi\rangle) = \ell(\langle\{b\}, \phi, \phi\rangle) = \langle\{a\}, \phi, \phi\rangle, \ell(\langle\{c\}, \phi, \phi\rangle) = \langle\{c\}, \phi, \phi\rangle$. It is easily observed that ℓ is a NCSα*-open mapping, but it is not NC-open mapping since $\mathcal{U}_N \in \tau_{\mathcal{U}}$, but $\ell(\mathcal{U}_N) = \langle\{a,c\}, \phi, \phi\rangle \notin \tau_{\mathcal{U}}$.

Proposition 3.15: A mapping $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ is a NCSα*-open iff $\ell: \left(\mathcal{U}, \mathrm{NCS\alpha O}(\mathcal{U})\right) \longrightarrow \left(\mathcal{V}, \mathrm{NCS\alpha O}(\mathcal{V})\right)$ is a NC-open mapping.

Proof: Obvious.

Remark 3.16: The ideas of NCα*-open mapping and NCSα*-open mapping are independent as the further examples demonstrate:

Example 3.17: In Example 3.14, it is easily seen that ℓ is a NCSα*-open mapping but it is not NCα*-open since $\mathcal{U}_N \in \mathrm{NC\alpha O}(\mathcal{U})$, but $\ell(\mathcal{U}_N) = \langle\{a,c\}, \phi, \phi\rangle \notin \mathrm{NC\alpha O}(\mathcal{U})$.

Example 3.18: Let $\mathcal{U} = \{a, b, c, d\}$. Then $\tau = \{\phi_N, \langle\{a\}, \phi, \phi\rangle, \langle\{b,d\}, \phi, \phi\rangle, \langle\{a,b,d\}, \phi, \phi\rangle, \mathcal{U}_N\}$ is a NCTS. Let $\mathcal{V} = \{p, q, r, s\}$. Then $\sigma = \{\phi_N, \langle\{p\}, \phi, \phi\rangle, \langle\{q,s\}, \phi, \phi\rangle, \langle\{p,q,s\}, \phi, \phi\rangle, \mathcal{V}_N\}$ is a NCTS.

Define a mapping $\ell: \mathcal{U} \longrightarrow \mathcal{V}$ by $\ell(\langle\{a\}, \phi, \phi\rangle) = \langle\{p\}, \phi, \phi\rangle, \ell(\langle\{b\}, \phi, \phi\rangle) = \ell(\langle\{c\}, \phi, \phi\rangle) = \langle\{q\}, \phi, \phi\rangle$, $\ell(\langle\{d\}, \phi, \phi\rangle) = \langle\{s\}, \phi, \phi\rangle$. It is easily seen that ℓ is a NCα*-open mapping, but it is not NCSα*-open mapping.

Theorem 3.19: If a mapping $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ is NCα*-open and NC-continuous, then it is NCSα*-open.

Proof: Let $\ell: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ be a NCα*-open and NC-continuous mapping. Let $\mathcal{M}$ be a NCSα-OS in $\mathcal{U}$. Then, we have for some NCα-OS, say $\mathcal{S}$, such that $\mathcal{S} \subseteq \mathcal{M} \subseteq NCcl(\mathcal{S})$. Therefore, $\ell(\mathcal{S}) \subseteq \ell(\mathcal{M}) \subseteq \ell(NCcl(\mathcal{S})) \subseteq NCcl(\ell(\mathcal{S}))$ (since ℓ is a NC-continuous), but $\ell(\mathcal{S}) \in \mathrm{NC\alpha O}(\mathcal{U})$ (since ℓ is a NCα*-open mapping). Hence, $\ell(\mathcal{S}) \subseteq \ell(\mathcal{M}) \subseteq NCcl(\ell(\mathcal{S}))$. Thus, $\ell(\mathcal{M}) \in \mathrm{NCS\alpha O}(\mathcal{U})$. Therefore, ℓ is a NCSα*-open.

Theorem 3.20: Let $\ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{V}, \sigma)$ and $\ell_2: (\mathcal{V}, \sigma) \longrightarrow (\mathcal{W}, \rho)$ be two mappings, then:
1) If ℓ_1 is NC-open mapping and ℓ_2 is NCα-open mapping, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα-open mapping.
2) If ℓ_1 is NCα-open mapping and ℓ_2 is NCα*-open mapping, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα-open mapping.
3) If ℓ_1 and ℓ_2 are NCα*-open mappings, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα*-open mapping.
4) If ℓ_1 and ℓ_2 are NCSα*-open mappings, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCSα*-open mapping.
5) If ℓ_1 and ℓ_2 are NCα**-open mappings, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα**-open mapping.
6) If ℓ_1 and ℓ_2 are NCSα**-open mappings, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCSα**-open mapping.
7) If ℓ_1 is NCα**-open mapping and ℓ_2 is NCα*-open mapping, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα*-open mapping.
8) If ℓ_1 is NCα-open mapping and ℓ_2 is NCα**-open mapping, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NC-open mapping.
9) If ℓ_1 is NCα**-open mapping and ℓ_2 is NCα-open mapping, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα*-open mapping.
10) If ℓ_1 is NCα**-open mapping and ℓ_2 is NC-open mapping, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα**-open mapping.

Proof

1) Let a NC-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NC-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NC-OS in $\mathcal{V}$. Because ℓ_2 is a NCα-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NCα-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα-open mapping.
2) Let a NC-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NCα-OS in $\mathcal{V}$. Because ℓ_2 is a NCα*-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NCα-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα-open mapping.
3) Let a NCα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα*-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NCα-OS in $\mathcal{V}$. Because ℓ_2 is a NCα*-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NCα-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα*-open mapping.
4) Let a NCSα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCSα*-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NCSα-OS in $\mathcal{V}$. Because ℓ_2 is a NCSα*-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NCSα-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCSα*-open mapping.
5) Let a NCα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα**-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NC-OS in $\mathcal{V}$. Because any NC-OS is NCα-OS, $\ell_1(\mathcal{M})$ is considered as a NCα-OS in $\mathcal{V}$. Meanwhile ℓ_2 is a NCα**-open mapping, then $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NC-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα**-open mapping.
6) Let a NCSα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCSα**-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NC-OS in $\mathcal{V}$. Because any NC-OS is NCSα-OS, $\ell_1(\mathcal{M})$ is considered as a NCSα-OS in $\mathcal{V}$. Meanwhile, ℓ_2 is a NCSα**-open mapping and $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NC-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCSα**-open mapping.
7) Let a NCα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα**-open mapping, $\ell_1(\mathcal{M})$ is considered as a NC-OS in $\mathcal{V}$. Because any NC-OS is NCα-OS, $\ell_1(\mathcal{M})$ is considered as a NCα-OS in $\mathcal{V}$. Meanwhile ℓ_2 is a NCα*-open mapping and $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NCα-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα*-open mapping.
8) Let a NC-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NCα-OS in $\mathcal{V}$. Because ℓ_2 is a NCα**-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NC-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NC-open mapping.
9) Let a NCα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα**-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NC-OS in $\mathcal{V}$. Because ℓ_2 is a NCα-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NCα-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα*-open mapping.

10) Let a NCα-OS $\mathcal{M}$ be in $\mathcal{U}$. Since ℓ_1 is a NCα**-open mapping, then $\ell_1(\mathcal{M})$ is considered as a NC-OS in $\mathcal{V}$. Because ℓ_2 is a NC-open mapping, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is considered as a NC-OS in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1 : (\mathcal{U}, \tau) \longrightarrow (\mathcal{W}, \rho)$ is a NCα**-open mapping.

Remark 3.21: The illustration demonstrated in Figure 1 explains the relationships between weakly NC-open mappings.

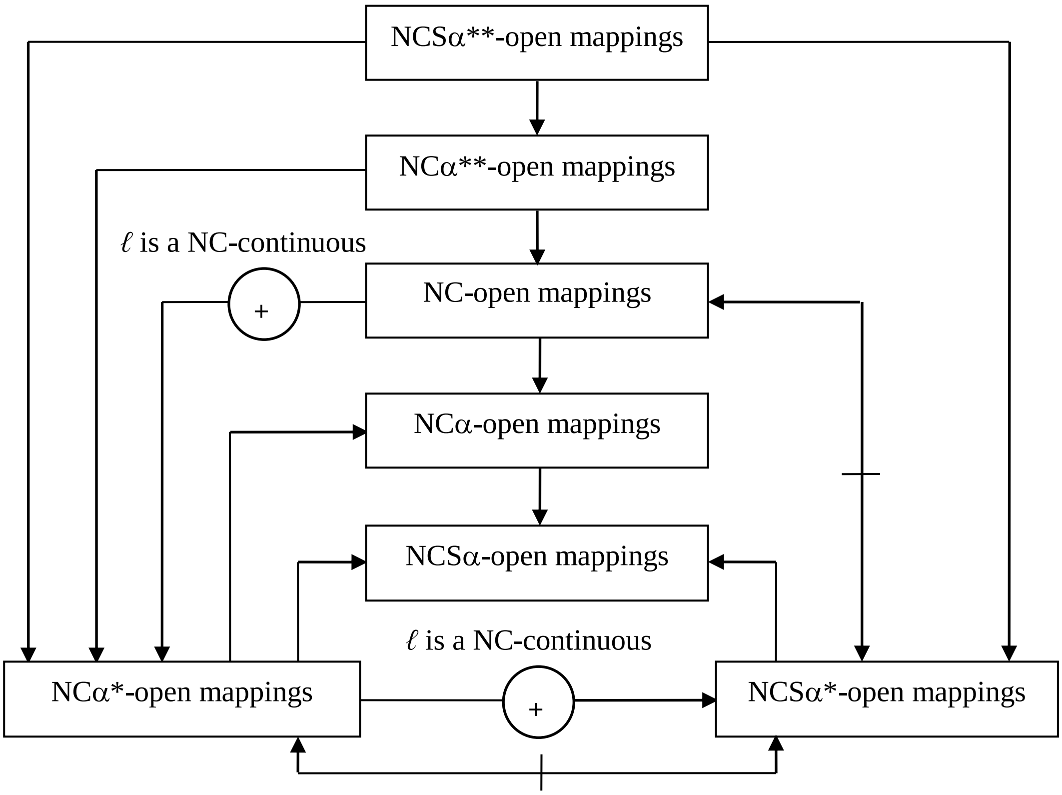

Figure 1- The relationships between weakly NC-open mappings.

4. Conclusions

We developed the thoughts of NCα-open and NCSα-open sets to describe some fresh types of weakly neutrosophic crisp open mappings, such as NCα-open, NCα*-open, NCα**-open, NCSα-open, NCSα*-open, and NCSα**-open mappings. The NCα-closed and NCSα-closed sets can be manipulated to obtain some innovative kinds of weakly neutrosophic crisp closed mappings.

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