On New Types of Weakly Neutrosophic Crisp Open Mappings

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Abstract

This work employs the conceptions of neutrosophic crisp α-open and semi-α-open sets to distinguish some novel forms of weakly neutrosophic crisp open mappings; for instance, neutrosophic crisp α-open mappings, neutrosophic crisp α*-open mappings, neutrosophic crisp semi-α-open mappings, neutrosophic crisp semi-α*-open mappings, and neutrosophic crisp semi-α**-open mappings. Moreover, the close connections between these forms of weakly neutrosophic crisp open mappings and the viewpoints of neutrosophic crisp open mappings are explained. Additionally, various theorems and related features and notes are submitted.

Keywords: NCα-open sets, NCSα-open sets, NCα*-open mappings, NCα**-open mappings, NCSα-open mappings, NCSα*-open mappings and NCSα**-open mappings.

1. Introduction


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neutrosophic crisp semi-α-closed sets. Banupriya et al. [9] investigated the notion of αgs continuity and αgs irresolute maps. Dhavaseelan et al. [10] exhibited the theme of neutrosophic αα*-continuity. Maheswari et al. [11] introduced gb-closed sets and gb-continuity. This study aims to establish unprecedented classes of weakly neutrosophic crisp open mappings by way of examples neutrosophic crisp α-open mappings, neutrosophic crisp α*-open mappings, neutrosophic crisp semi-α-open mappings, neutrosophic crisp semi-α*-open mappings, and neutrosophic crisp semi-α**-open mappings. Furthermore, we shall explain the close relationships between these categories of weakly neutrosophic crisp open mappings and the conceptions of neutrosophic crisp open mappings. Furthermore, we shall show some related theorems and their features and notes.

2. Preliminaries

For all of this work, (U, τ), (V, σ), and (W, ρ) (or frugally U, V, and W) always mean neutrosophic crisp topological spaces (for brevity NCTSs). For a neutrosophic crisp set M in a NCTS U, MC = U − M, NCint (M), and NCcl (M) signify the neutrosophic crisp complement, the neutrosophic crisp interior, and the neutrosophic crisp closure of M, correspondingly.

Definition 2.1 [1]: For any nonempty under consideration set U, a neutrosophic crisp set (curtly NCS) M is an object holding the establishment M = (M_1, M_2, M_3) where M_1, M_2, and M_3 are mutually disjoint sets included in U.

Definition 2.2: Suppose that NCTS U contains a NCS M, then we have the following:
1) If M ⊆ NCint (NCcl (NCint (M))), then M is named a neutrosophic crisp α-open set (concisely NCα-OS) [12]. The collection of all NCα-OSs of U is symbolized by NCαO(U).
2) If M ⊆ NCcl (NCint (NCcl (NCint (M)))) or, in other words, for some NCα-OS E in U so that E ⊆ M ⊆ NCcl (E), then M is a neutrosophic crisp semi-α-open set (concisely NCα-OS) [8]. The collection of all NCα-OSs of U is represented by NCαO(U).

Remark 2.3 [8,13]: For any NCTS U, the following claims stay valid, but not vice versa:
1) For all NCα-OSs are NCα-OSs and NCα-OSs.
2) For all NCα-OSs are NCα-OSs.

Example 2.4: Let U = {a, b, c, d}. Then, τ_U = {Φ, N, {{a}, φ, φ}, {{b}, φ, φ}, {{a, b}, φ, φ}, {{a, b, c}, φ, φ}}. U_N is a NCTS. The class of all NCα-OSs of U is: NCαO(U) = τ_U ∪ {{a, b, d}, φ, φ}. The class of all NCα-OSs of U is: NCαO(U) = NCαO(U) ∪ {{a, c}, φ, φ}, {{a, d}, φ, φ}, {{b, c}, φ, φ}, {{b, d}, φ, φ}, {{a, c, d}, φ, φ}, {{b, c, d}, φ, φ}}.

Proposition 2.5 [12]: For any neutrosophic crisp subset M of a NCTS (U, τ), M ∈ NCαO(U) iff for some NC-OS P, P ⊆ M ⊆ NCint (NCcl (P)).

Definition 2.6: Let ℓ: (U, τ) → (V, σ) be a mapping, then ℓ is termed:
1) Neutrosophic crisp open (briefly NC-open) [1] iff for each M NCα-OS in U, ℓ(M) is a NC-OS in V.
2) Neutrosophic crisp α-open (briefly NCα-open) [14] iff for each M NCα-OS in U, ℓ(M) is a NCα-OS in V.

Proposition 2.7 [1]: A mapping ℓ: (U, τ) → (V, σ) is NC-open iff ℓ(NCint (M)) ⊆ NCint (ℓ(M)), for every M ⊆ U.

Definition 2.8 [1]: Let ℓ: (U, τ) → (V, σ) be a mapping, then ℓ is called neutrosophic crisp continuous (briefly NC-continuous) iff for each M NC-OS in U, ℓ^{-1} (M) is a NC-OS in U.

Proposition 2.9 [1]: A mapping ℓ: (U, τ) → (V, σ) is NC-continuous iff ℓ(NCcl (M)) ⊆ NCcl (ℓ(M)), for every M ⊆ U.

3. Weakly Neutrosophic Crisp Open Mappings

Definition 3.1: Let ℓ: (U, τ) → (V, σ) be a mapping, then ℓ is named:
1) Neutrosophic crisp α*-open (briefly NCα*-open) iff for each NCα-OS M in U, ℓ(M) is considered a NCα-OS in V.
2) Neutrosophic crisp α**-open (briefly NCα**-open) iff for each NCα-OS M in U, ℓ(M) is considered a NC-OS in V.

Definition 3.2: Let ℓ: (U, τ) → (V, σ) be a mapping, then ℓ is termed:
1) Neutrosophic crisp semi-$\alpha$-open (briefly NCS$\alpha$-open) iff for each $\mathcal{M}$ as NC-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NCS$\alpha$-OS in $\mathcal{V}$.
2) Neutrosophic crisp semi-$\alpha^*$-open (briefly NCS$\alpha^*$-open) iff for each $\mathcal{M}$ as NCS$\alpha$-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NCS$\alpha$-OS in $\mathcal{V}$.
3) Neutrosophic crisp semi-$\alpha^{**}$-open (briefly NCS$\alpha^{**}$-open) iff for each $\mathcal{M}$ as NCS$\alpha$-OS in $\mathcal{U}$, $\ell(\mathcal{M})$ is a NC-OS in $\mathcal{V}$.

**Proposition 3.3**

1) Every NC-open mapping is a NCS$\alpha$-open, so it is NCS$\alpha$-open, but the reverse is not valid in general.
2) Every NCS$\alpha$-open mapping is a NCS$\alpha$-open, but the reverse is not valid in general.

**Proof**

1) Let $\ell: (\mathcal{U}, \tau) \rightarrow (\mathcal{V}, \sigma)$ be a NC-open mapping and a NC-OS $\mathcal{M}$ be in $\mathcal{U}$. Then $\ell(\mathcal{M})$ is considered a NC-OS in $\mathcal{V}$. Because any NC-OS is NCS$\alpha$-OS (NCS$\alpha$-OS), $\ell(\mathcal{M})$ is considered a NCS$\alpha$-OS (NCS$\alpha$-OS) set in $\mathcal{V}$. Hence, $\ell$ is a NC-open (NCS$\alpha$-open) mapping.

2) Let $\ell: (\mathcal{U}, \tau) \rightarrow (\mathcal{V}, \sigma)$ be a NCS$\alpha$-open mapping and $\mathcal{M}$ be a NC-OS in $\mathcal{U}$. Then $\ell(\mathcal{M})$ is a NCS$\alpha$-OS in $\mathcal{V}$. Because any NC-OS is NCS$\alpha$-OS, $\ell(\mathcal{M})$ considers NCS$\alpha$-OS in $\mathcal{V}$. Hence $\ell$ is a NCS$\alpha$-open mapping.

**Example 3.4:** Let $\mathcal{U} = \{a, b, c, d\}$.

Then, $\tau_{\mathcal{U}} = \{\phi_{\mathcal{N}}, \{\alpha\}, \phi, \phi, \phi\}, \{(a, b), \phi, \phi\}, \{\{a, b, c\}, \phi, \phi\}, \{\{a, b, d\}, \phi, \phi\}, \{a, \phi, \phi\}, \{b, \phi, \phi\}, \{c, \phi, \phi\}, \{d, \phi, \phi\}, \{(a, b, c, d), \phi, \phi\}\}$ is a NCTS. The class of all NCS$\alpha$-OSs of $\mathcal{U}$ is: $\text{NCS} \alpha \text{O} (\mathcal{U}) = \tau_{\mathcal{U}} \cup \{(a, b, d), \phi, \phi\}$. The class of all NCS$\alpha$-OSs of $\mathcal{U}$ is: $\text{NCS} \alpha \text{O} (\mathcal{U}) = \tau_{\mathcal{U}} \cup \{(a, b, d), \phi, \phi\}$. Then, $\ell(\mathcal{M})$ is a NC-OS in $\mathcal{V}$.

We define a mapping $\ell: \mathcal{U} \rightarrow \mathcal{V}$ by $\ell((\{a\}, \phi, \phi)) = \{(a, \phi, \phi), \ell((\{b\}, \phi, \phi)) = \{(b, \phi, \phi), \ell((\{c\}, \phi, \phi)) = \{(c, \phi, \phi), \ell((\{d\}, \phi, \phi)) = \{(d, \phi, \phi). We observe that $\ell$ is a NC-open mapping, which is not NC-OS mapping since $\{a, b, c, \phi, \phi\}$ is NC-OS in $\mathcal{U}$, but $\ell(\{(a, b, c, \phi, \phi)) = \{(a, b, d, \phi, \phi)$ is not a NC-OS in $\mathcal{U}$.

**Example 3.5:** Let $\mathcal{U} = \{a, b, c, d\}$.

Then, $\tau_{\mathcal{U}} = \{\phi_{\mathcal{N}}, \{\alpha\}, \phi, \phi, \phi\}, \{(a, b), \phi, \phi\}, \{\{a, b, c\}, \phi, \phi\}, \{\{a, b, d\}, \phi, \phi\}, \{a, \phi, \phi\}, \{b, \phi, \phi\}, \{c, \phi, \phi\}, \{d, \phi, \phi\}, \{(a, b, c, \phi, \phi)\}\}$ is a NCTS. The class of all NCS$\alpha$-OSs of $\mathcal{U}$ is: $\text{NCS} \alpha \text{O} (\mathcal{U}) = \tau_{\mathcal{U}} \cup \{(a, b, d), \phi, \phi\}$. Then, $\ell(\mathcal{M})$ is a NC-OS in $\mathcal{V}$.

We define a mapping $\ell: \mathcal{U} \rightarrow \mathcal{V}$ by $\ell((\{a\}, \phi, \phi)) = \{(a, \phi, \phi), \ell((\{b\}, \phi, \phi)) = \{(b, \phi, \phi), \ell((\{c\}, \phi, \phi)) = \{(c, \phi, \phi), \ell((\{d\}, \phi, \phi)) = \{(d, \phi, \phi). It is easily seen that $\ell$ is a NC-open mapping, but it is not NC-OS mapping since $\{a, b, c, \phi, \phi\}$ is NC-OS in $\mathcal{U}$, but $\ell(\{(a, b, c, \phi, \phi)) = \{(a, c, \phi, \phi)$ is not a NC-OS in $\mathcal{U}$. Then, $\ell$ is a NCS$\alpha$-open mapping, but it is not NC$\alpha$-open mapping.

**Remark 3.6:** The ideas of NC-open mapping and NCS$\alpha^*$-open mapping are self-regulating, as in the further examples below:

**Example 3.7:** Let $\mathcal{U} = \{a, b, c, d\}$.

Then, $\tau_{\mathcal{U}} = \{\phi_{\mathcal{N}}, \{\alpha\}, \phi, \phi\}, \{(a, b), \phi, \phi\}, \{\{a, b, c\}, \phi, \phi\}, \{\{a, b, d\}, \phi, \phi\}, \{a, \phi, \phi\}, \{b, \phi, \phi\}, \{c, \phi, \phi\}, \{d, \phi, \phi\}, \{(a, b, c, d), \phi, \phi\}\}$ is a NCTS. The class of all NCS$\alpha$-OSs of $\mathcal{U}$ is: $\text{NCS} \alpha \text{O} (\mathcal{U}) = \tau_{\mathcal{U}} \cup \{(a, b, d), \phi, \phi\}$. Then, $\ell(\mathcal{M})$ is a NC-OS in $\mathcal{V}$.

We define a mapping $\ell: \mathcal{U} \rightarrow \mathcal{V}$ by $\ell((\{a\}, \phi, \phi)) = \{(a, \phi, \phi), \ell((\{b\}, \phi, \phi)) = \{(b, \phi, \phi), \ell((\{c\}, \phi, \phi)) = \{(c, \phi, \phi), \ell((\{d\}, \phi, \phi)) = \{(d, \phi, \phi). It is easily seen that $\ell$ is a NC-open mapping, which is not NC$\alpha^*$-open mapping since $\{a, b, d, \phi, \phi\} \in \text{NCS} \alpha \text{O} (\mathcal{U})$, but $\ell(\{(a, b, d, \phi, \phi)) = \{(a, c, \phi, \phi) \notin \text{NCS} \alpha \text{O} (\mathcal{U})$.

**Example 3.8:** In Example 3.4, it is easily seen that $\ell$ is a NC$\alpha^*$-open mapping, but it is not NC-open since $\{(a, b, c), \phi, \phi\} \in \tau_{\mathcal{U}}$, but $\ell(\{(a, b, c), \phi, \phi\}) = \{(a, b, d), \phi, \phi\} \notin \tau_{\mathcal{U}}$.

**Proposition 3.9**

1) If $\ell: (\mathcal{U}, \tau) \rightarrow (\mathcal{V}, \sigma)$ is a NC-open, NC-continuous mapping, then $\ell$ is a NC$\alpha^*$-open mapping.

2) $\ell: (\mathcal{U}, \tau) \rightarrow (\mathcal{V}, \sigma)$ is a NC$\alpha^*$-open mapping iff $\ell: (\mathcal{U}, \text{NCS} \alpha \text{O} (\mathcal{U})) \rightarrow (\mathcal{V}, \text{NCS} \alpha \text{O} (\mathcal{V}))$ is a NC-open.
Proof

1) Let \( \ell : (U, \tau) \rightarrow (V, \sigma) \) be a NC-open, NC-continuous mapping. To prove that \( \ell \) is a NC\( \alpha^* \)-open mapping, let \( M \in \text{NC}_0(U) \), then for some NC-OS \( F \), such that \( F \subseteq M \subseteq \text{NCint}(\text{NCcl}(F)) \) (by Proposition 2.5). Hence, \( \ell(F) \subseteq \ell(M) \subseteq \ell \left( \text{NCint}(\text{NCcl}(F)) \right) \), but \( \ell(\text{NCint}(\text{NCcl}(F))) \subseteq \text{NCint}(\ell(\text{NCcl}(F))) \) (since \( \ell \) is a NC-open mapping).

Then, \( \ell(F) \subseteq \ell(M) \subseteq \ell(\text{NCint}(\text{NCcl}(F)))) \subseteq \text{NCint}(\ell(\text{NCcl}(F))). \) Hence, \( \ell(M) \subseteq \ell(\text{NCint}(\text{NCcl}(F)))) \) (by Proposition 2.5). Thus, it is a NC\( \alpha^* \)-open mapping.

2) The proof of a part (2) is easily reached.

Proposition 3.10: Every NC\( \alpha^* \)-open mapping is a NC\( \alpha \)-open and NC\( \alpha \)-open, but the reverse is not valid in general.

Proof: Let \( \ell : (U, \tau) \rightarrow (V, \sigma) \) be a NC\( \alpha^* \)-open mapping and \( M \) be NC-OS in \( U \). Then, we have that \( M \) is considered a NC\( \alpha \)-OS in \( U \) [from Proposition 2.5]. Because \( \ell \) is a NC\( \alpha^* \)-open, then \( \ell(M) \) is considered a NC\( \alpha \)-OS in \( V \). Therefore, \( \ell \) is a NC\( \alpha \)-open. Also, \( \ell \) is a NC\( \alpha \)-open.

Example 3.11: In Example 3.7, it is easily seen that \( \ell \) is a NC\( \alpha \)-open mapping and NC\( \alpha \)-open mapping, but not NC\( \alpha^* \)-open.

Remark 3.12: The ideas of NC-open mapping and NC\( \alpha^* \)-open mapping are independent, as explained in the examples below.

Example 3.13: Let \( U = \{a, b, c\} \). Then, \( \tau = \{\phi_N, \{(a), \phi, \phi\}, \cup_N\} \) is a NCTS. The class of all NC\( \alpha \)-OSs (NC\( \alpha \)-OSs) of \( U \) is: \( \text{NC}_0(U) = \text{NC}_0(U) = \tau \cup \{(a, b, \phi, \phi), (a, c, \phi, \phi)\} \). Let \( V = \{p, q, r, s\} \). Then, \( \sigma = \{\phi_N, \{(p), \phi, \phi\}, \{(q, r), \phi, \phi\}, \{(p, q, r), \phi, \phi\}, \{(p, q, r), \phi, \phi\} \). Let \( \nu = \{p, q, r, s\} \).

We define a mapping \( \ell : U \rightarrow V \) by \( \ell(\{(a), \phi, \phi\}) = \{(p), \phi, \phi\}, \ell(\{(b), \phi, \phi\}) = \{(q), \phi, \phi\}, \ell(\{(c), \phi, \phi\}) = \{(r), \phi, \phi\} \). It is easily seen that \( \ell \) is a NC-open mapping, but it is not NC\( \alpha^* \)-open mapping, since \( (a, b, \phi, \phi) \in \text{NC}_0(U) \), but \( \ell(\{(a, b), \phi, \phi\}) = \{(p, q), \phi, \phi\} \notin \text{NC}_0(V) \).

Example 3.14: Let \( U = \{a, b, c\} \). Then, \( \tau_U = \{\phi_N, \{(a), \phi, \phi\}, \{(b), \phi, \phi\}, \{(a, b), \phi, \phi\}, \{(a, c), \phi, \phi\}, \cup_N\} \) is a NCTS. The family of all NC\( \alpha \)-OSs of \( U \) is: \( \text{NC}_0(U) = \text{NC}_0(U) \cup \{(a, c), \phi, \phi\} \). Let \( V = \{p, q, r, s\} \). Then, \( \sigma = \{\phi_N, \{(p), \phi, \phi\}, \{(q, s), \phi, \phi\}, \{(p, q, s), \phi, \phi\}, \{(p, q, s), \phi, \phi\}, \{(p, q, s), \phi, \phi\} \).

We define a mapping \( \ell : U \rightarrow V \) by \( \ell(\{(a), \phi, \phi\}) = \ell(\{(b), \phi, \phi\}) = \{(a), \phi, \phi\}, \ell(\{(c), \phi, \phi\}) = \{(c), \phi, \phi\} \). It is easily observed that \( \ell \) is a NC\( \alpha^* \)-open mapping, but it is not NC\( \alpha \)-open mapping since \( \cup_N \notin \tau_U \), but \( \ell(\cup_N) = \{(a, c), \phi, \phi\} \notin \tau_U \).

Proposition 3.15: A mapping \( \ell : (U, \tau) \rightarrow (V, \sigma) \) is a NC\( \alpha^* \)-open iff \( \ell : (U, \text{NC}_0(U)) \rightarrow (V, \text{NC}_0(V)) \) is a NC-open mapping.

Proof: Obvious.

Remark 3.16: The ideas of NC\( \alpha \)-open mapping and NC\( \alpha^* \)-open mapping are independent as the further examples demonstrate:

Example 3.17: In Example 3.14, it is easily seen that \( \ell \) is a NC\( \alpha^* \)-open mapping but it is not NC\( \alpha \)-open since \( \cup_N \notin \text{NC}_0(U) \), but \( \ell(\cup_N) = \{(a, c), \phi, \phi\} \notin \text{NC}_0(U) \).

Example 3.18: Let \( U = \{a, b, c, d\} \). Then \( \tau = \{\phi_N, \{(a), \phi, \phi\}, \{(b), \phi, \phi\}, \{(a, b), \phi, \phi\}, \{(a, c), \phi, \phi\} \). Let \( V = \{p, q, r, s\} \) is a NCTS. Let \( \sigma = \{\phi_N, \{(p), \phi, \phi\}, \{(q, s), \phi, \phi\}, \{(p, q, s), \phi, \phi\}, \{(p, q, s), \phi, \phi\} \). Let \( \nu = \{p, q, r, s\} \).

Define a mapping \( \ell : U \rightarrow V \) by \( \ell(\{(a), \phi, \phi\}) = \{(p), \phi, \phi\}, \ell(\{(b), \phi, \phi\}) = \{(q), \phi, \phi\}, \ell(\{(d), \phi, \phi\}) = \{(s), \phi, \phi\} \). It is easily seen that \( \ell \) is a NC\( \alpha^* \)-open mapping, but it is not NC\( \alpha \)-open mapping.

Theorem 3.19: If a mapping \( \ell : (U, \tau) \rightarrow (V, \sigma) \) is NC\( \alpha^* \)-open and NC-continuous, then it is NC\( \alpha^* \)-open.

Proof: Let \( \ell : (U, \tau) \rightarrow (V, \sigma) \) be a NC\( \alpha^* \)-open and NC-continuous mapping. Let \( M \) be a NC\( \alpha \)-OS in \( U \). Then, we have for some NC\( \alpha \)-OS, say \( S \), such that \( S \subseteq M \subseteq NCcl(S) \). Therefore, \( \ell(S) \subseteq \ell(M) \subseteq \ell(\text{NC}(S)) \subseteq \text{NC}(\ell(S)) \) (since \( \ell \) is a NC-continuous), but \( \ell(S) \notin \text{NC}(U) \) (since \( \ell \) is a NC\( \alpha^* \)-open mapping).

Hence, \( \ell(S) \subseteq \ell(M) \subseteq \text{NC}(\ell(S)) \). Thus, \( \ell(M) \notin \text{NC}(U) \). Therefore, \( \ell \) is a NC\( \alpha \)-open.
Theorem 3.20: Let \( \ell_1: (U, \tau) \rightarrow (V, \sigma) \) and \( \ell_2: (V, \sigma) \rightarrow (W, \rho) \) be two mappings, then:
1) If \( \ell_1 \) is NC-open mapping and \( \ell_2 \) is NC\(\sigma\)-open mapping, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
2) If \( \ell_1 \) is NC\(\sigma\)-open mapping and \( \ell_2 \) is NC\(\sigma\)-open mapping, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
3) If \( \ell_1 \) and \( \ell_2 \) are NC\(\sigma\)-open mappings, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
4) If \( \ell_1 \) and \( \ell_2 \) are NC\(\sigma\)-open mappings, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
5) If \( \ell_1 \) and \( \ell_2 \) are NC\(\sigma\)-open mappings, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
6) If \( \ell_1 \) and \( \ell_2 \) are NC\(\sigma\)-open mappings, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
7) If \( \ell_1 \) is NC\(\sigma\*-\)open mapping and \( \ell_2 \) is NC\(\sigma\)-open mapping, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
8) If \( \ell_1 \) is NC\(\sigma\)-open mapping and \( \ell_2 \) is NC\(\sigma\)-open mapping, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
9) If \( \ell_1 \) is NC\(\sigma\)-open mapping and \( \ell_2 \) is NC\(\sigma\)-open mapping, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.
10) If \( \ell_1 \) is NC\(\sigma\)-open mapping and \( \ell_2 \) is NC\(\sigma\)-open mapping, then \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC\(\sigma\)-open mapping.

Proof
1) Let a NC-OS \( M \) be in \( U \). Since \( \ell_1 \) is a NC-open mapping, then \( \ell_1(M) \) is considered as a NC-OS in \( V \). Because \( \ell_2 \) is a NC-open mapping, \( \ell_2 \circ \ell_1(M) = \ell_2(\ell_1(M)) \) is considered as a NC-OS in \( W \). Thus, \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC-OS-open mapping.
2) Let a NC-OS \( M \) be in \( U \). Since \( \ell_1 \) is a NC-open mapping, then \( \ell_1(M) \) is considered as a NC-OS in \( V \). Because \( \ell_2 \) is a NC-OS-open mapping, \( \ell_2 \circ \ell_1(M) = \ell_2(\ell_1(M)) \) is considered as a NC-OS in \( W \). Thus, \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC-OS-open mapping.
3) Let a NC-OS \( M \) be in \( U \). Since \( \ell_1 \) is a NC-OS-open mapping, then \( \ell_1(M) \) is considered as a NC-OS in \( V \). Because \( \ell_2 \) is a NC-OS-open mapping, \( \ell_2 \circ \ell_1(M) = \ell_2(\ell_1(M)) \) is considered as a NC-OS in \( W \). Thus, \( \ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho) \) is a NC-OS-open mapping.
10) Let a NCα-OS $M$ be in $U$. Since $\ell_1$ is a NCα**-open mapping, then $\ell_1(M)$ is considered as a NC-OS in $V$. Because $\ell_2$ is a NC-open mapping, $\ell_2 \circ \ell_1(M) = \ell_2(\ell_1(M))$ is considered as a NC-OS in $W$. Thus, $\ell_2 \circ \ell_1: (U, \tau) \rightarrow (W, \rho)$ is a NCα**-open mapping.

**Remark 3.21:** The illustration demonstrated in Figure 1 explains the relationships between weakly NC-open mappings.

**Figure 1-** The relationships between weakly NC-open mappings.

4. Conclusions

We developed the thoughts of NCα-open and NCSα-open sets to describe some fresh types of weakly neutrosophic crisp open mappings, such as NCα-open, NCSα*-open, NCα**-open, NCSα-open, NCSα*-open, and NCSα**-open mappings. The NCα-closed and NCSα-closed sets can be manipulated to obtain some innovative kinds of weakly neutrosophic crisp closed mappings.

**References**


