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Characterization of Soft Semi Separation Axioms in Soft Quad Topological Spaces

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Abstract
Our main interest in this study is to look for soft semi separations axioms in soft quad topological spaces. We talk over and focus our attention on soft semi separation axioms in soft quad topological spaces with respect to ordinary points and soft points. Moreover study the inherited characteristics at different angles with respect to ordinary points and soft points. Some of their central properties in soft quad topological spaces are also brought under examination.

Keywords: soft sets, soft topology, soft semi open set, soft semi closed set, soft quad topological space, soft qT_0 structure, soft qT_1 structure, soft qT_2 structure, soft qT_3 structure and soft qT_4 structure.

1. Introduction.

In real life condition the complications in economics, engineering, social sciences, medical science etc. we cannot handsomely use the old-fashioned classical methods because of different types of uncertainties existing in these problems. To finish out these complications, some types of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, in which we can safely use a mathematical methods for dealing with uncertainties. But, all these theories have their inherent worries. To overcome these difficulties in the year 1999, Russian scholar Molodtsov [1] introduced the idea of soft set as a new mathematical methods to deal with uncertainties. Which is free from the above difficulties. J.C Kelly [2] studied Bi topological spaces and discussed different results.

Recently, in 2011, M. Shabir and M. Naz [3] initiated the idea of soft topological space and discussed different results with respect to ordinary points. , they beautifully defined soft topology as a collection of τ of soft sets over X. they also defined the basic concept of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axioms, soft regular and soft normal spaces and published their several performances. Soft separation axioms are also discussed at detail. Aktas and N. In the recent years, many interesting applications of soft sets theory and soft topology have been discussed at great depth [4 -26] explained Soft results included connectedness via soft ideal developed soft set theory. A. Kandil et al [27] launched Soft regularity and normality based on semi open soft sets and soft ideals.
In[28-33] discussion is launched soft semi hausdorf spaces via soft ideals, semi open and semi closed sets, separation axioms ,decomposition of some type supra soft sets and soft continuity are discussed.

In this present paper, concept of soft semi separation axioms in Soft quad topological spaces is announced with respect to ordinary and soft points.

Many mathematicians made discussion over soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open, soft α-open set and soft β-open

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set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present article hand is tried and work is encouraged over the gap that exists in soft quad-topology related to soft semi qT_0 , soft semi qT_1 soft semi qT_2 , soft semi qT_3 and soft semi qT_4structures. Some propositions in soft quid topological spaces are discussed with respect to ordinary points and soft points. When we talk about distances between the points in soft topology then the concept of soft separation axioms will automatically come in force. That is why these structures are catching our attentions. We hope that these results will be valuable for the future study on soft quad topological spaces to accomplish general framework for the practical applications and to solve the most complicated problems containing doubts in economics, engineering, medical, environment and in general mechanic systems of various varieties. In future these beautiful soft topological structures may be extended in to soft n-topological spaces provided n is even.

2. Preliminaries

The following Definition s which are pre-requisites for present study

Definition 1: **[**4].

Let X be an initial universe of discourse and E be a set of parameters. Let P(X) denotes the power set of X and A be a non-empty sub-set of E. a pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \to P(X)$

In other words, a set over X is a parameterized family of sub set of universe of discourse X . For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set $(F, A)$and if$e \notin A$ then $F(e) = \phi$ that is$F_{A=}\{F(e): e \in A \subseteq E, F: A \to P(X)\}$ the family of all these soft sets over X denoted by $SS(X)_A$

Definition 2: [4].

Let$F_{A,} G_B \in SS(X)_E$ then F_A , is a soft subset of G_B denoted by$F_A \sqsubseteq G_B$, if

1. $,A \subseteq B$ and
2. $F(e) \subseteq G(e), \forall \in A$

In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super set $F_A, G_B \sqsupseteq F_A$

Definition 3: [5].

Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A

Definition4: [6].

The complement of soft subset(F, A)ddenoted by $(F, A)^C$ is defined by $(F, A)^C = (F^C, A)$ $F^C \to P(X)$is a mapping given by $F^C(e) = U - F(e) \forall e \in A$ and $F^C$is called the soft complement function of F . Clearly $(F^C)^C$ is the same as F and $((F, A)^C)^C = (F, A)$

Definition5: [7].

The difference between two soft subset (G, E) and (G, E) over common of universe discourse X denoted by $(F, E) - (G, E)$ is the soft set(H, E) where for all $e \in E. \overline{\emptyset} or \ \emptyset_A$ if $\forall e \in A, F(e) = \emptyset$

Definition6: [7].

Let(G, E) be a soft set over X and $x \in X$ We say that $x \in (F, E)$ and read as x belong to the soft set(F, E) whenever $x \in F(e) \forall e \in E$The soft set$(F, E)$ over X such that $F(e) = \{x\} \forall e \in E$ is called singleton soft point and denoted by $x, or \ (x, E)$

Definition7: [7].

A soft set (F, A) over X is said to be Null soft set denoted by $\overline{\emptyset} \ or \ \emptyset_A$ if $\forall e \in A, F(e) = \emptyset$

Definition8: [7].

A soft set(F, A) over X is said to be an absolute soft denoted by $\bar{A} \ or \ X_A$ if $\forall e \in A, F(e) = X$

Clearly, we have, $X_A^C = \emptyset_A \ and \ \emptyset_A^C = X_A$

Definition9: [8].

The soft set $(F, A) \in SS(X)A$ is called a soft point in X_A , denoted by e_F , if for the element $e \in A, F(e) \neq \emptyset$ and $F(e^/) = \phi$ if for all $e^/ \in A - \{e\}$

Definition10:[8].

The soft point e_F is said to be in the soft set (G,A) , denoted by $e_F \in (G,A)$ if for the element$e \in A, F(e) \subseteq G(e)$.

Definition11: [8].

Two soft sets $(G,A),(H,A)$ in $SS(X)_A$ are said to be soft disjoint, written $(G,A) \cap (H,A) = \emptyset_A$ If $G(e) \cap H(e) = \emptyset$ for all$e \in A$.

Definition12: [8].

The soft point e_G, e_H in X_A are disjoint, written $e_G \neq e_H$ if their corresponding soft sets (G,A) and (H,A) are disjoint.

Definition13: [8].

The union of two soft sets(F,A) and (G,B) over the common universe of discourse X is the soft set(H,C), where, $C = AUB$ For all $e \in C$

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in (B - A) \\ F(e)UG(e), & if\ e \in A \cap B \end{cases}$$

Written as $(F,A) \cup (G,B) = (H,C)$

Definition14:[8].

The intersection $(H.C)$ of two soft sets$(F,A)$and (G,B)over common universe X, denoted $(F,A) \,\overline{\cap}\, (\mathrm{G,B})$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e), \forall e \in C$

Definition15: [8].

Let (F,E) be a soft set over X and Y be a non-empty sub set of X. Then the sub soft set of (F,E) over Y denoted by (Y_F,E), is defined as follow $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \in E$ in other words $(Y_F,E) = Y \cap (F,E)$.

Definition16: [9].

Let τ be the collection of soft sets over$X$, then τ is said to be a soft topology on X, if

1. $\emptyset, X$ belong to τ
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belong to τ

The triplet (X,F,E) is called a soft topological space.

Definition17: [9].

Let (X,F,E) be a soft topological space over X, then the member of τ are said to be soft open sets in$X$.

Definition18: [9].

Let (X,F,E) be a soft topological space over$X$. A soft set (F,A) over X is said to be a soft closed set in X if its relative complement $(F,E)^C$ belong toτ.

Definition19: [16].

A soft set (A,E) in a soft topological space (X,τ,E) will be termed soft semi open (written S.S.O) if and only if there exists a soft open set (O,E) such that $(O,E) \sqsubseteq (A,E) \subseteq Cl(O,E)$.

Proposition 1. Let (X,τ,E) be a soft topological space over X.If (X,τ,E) is soft semi T_3-space, then for all $x \in X, x_E = (x,E)$ is semi-closed soft set.

Proposition 2. Let (Y,τ_Y,E) be a soft sub space of a soft topological space (X,τ,E) and $(F,E) \in SS(X)$ then

1. If (F,E) is soft semi *open* set in Y and $Y \in \tau$, then $(F,E) \in \tau$
2. (F,E) is soft semi *open soft* set in Y if and only if $(F,E) = Y \cap (G,E)$ for some $(G,E) \in \tau$.
3. (F,E) is soft semi *closed soft* set in Y if and only if $(F,E) = Y \cap (H,E)$ for some (H,E) *is* τ *soft semi closed.*

3. SOFT SEMI SEPARATION AXIOMS OF SOFT QUAD TOPOLGICAL SPACES

In this section we introduced soft Semi separation axioms in soft quad topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 20:

Let(X,τ_1,E), (X,τ_2,E) **,** (X,τ_3,E) and (X,τ_4,E)be four different soft topologies on X. Then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called a *soft quad topological space.* The soft four topologies (X,τ_1,E),

$(X,\tau_2,E),(X,\tau_3,E)$and (X,τ_4,E) are independently satisfying the axioms of soft topology. The members ofτ_1 are called τ_1 *soft open* set. And complement ofτ_1 Soft open set is called τ_1 *soft closed* set. Similarly, the member of τ_2 are called τ_2 *soft open* sets and the complement of τ_2 soft open sets are called τ_2 *soft closed* set. The members ofτ_3 are called τ_3 *soft open* set. And complement ofτ_3 Soft open set is called τ_3 *soft closed* set and the members ofτ_4 are called τ_4 *soft open* set. And complement ofτ_4 Soft open set is called τ_4 *soft closed* set.

Definition 21:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$be a soft quad topological space over X and Y be a non-empty subset of X. Then$\tau_{1Y}=\{(Y_F,E):(F,E)\in\tau_1)\}$,$\tau_{2Y}=\{(Y_G,E):(G,E)\in\tau_2)\}$, $\tau_{3Y}=\{(Y_H,E):(H,E)\in\tau_1)\}and\tau_{4Y}=\{(I_E,E):(I,E)\in\tau_2)\}$are said to be the relative topological on Y. Then $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is called relative soft quad-topological space of $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$.

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$be a soft quad topological space over X, where (X,τ_1,E), (X,τ_2,E) , (X,τ_3,E) and (X,τ_4,E)be four different soft topologies on X. Then a sub set (F,E) is said to be quad-open (in short hand q-open) if $(F,E)\subseteq\tau_1\cup\tau_2\cup\tau_3\cup\tau_4$ and its complement is said to be soft q-closed.

3.1 SOFT SEMI SEPARATION AXIOMS OF SOFT QUAD TOPOLGICAL SPACES WITH RESPECT TO ORDINARY POINTS.

In this section we introduced soft semi separation axioms in soft quad topological space with respect to ordinary points and discussed some attractive results with respect to these points in detail.

Definition 22:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft quad topological space over X and$x,y\in X$ such that $x\neq y$ If we can find soft q-open sets (F,E) and (G,E) such that $x\in(F,E)$ and $y\notin(F,E)$ or $y\in(G,E)$ and $x\notin(G,E)$ *then* $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called soft qT_0 space.

Definition 23:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft quad topological space over X and$x,y\in X$ such that $x\neq y$ If we can find two soft q-open sets (F,E) and (G,E) such that $x\in(F,E)$ and $y\notin(F,E)$ and $y\in(G,E)$ and $x\notin(G,E)$ then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called soft qT_1 space.

Definition 24:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft quad topological space over X and $x,y\in X$ such that $x\neq y$. If we can find two q- open soft sets such that $x\in(F,E)$and $y\in(G,E)$ moreover $(F,E)\cap(G,E)=\phi$ Then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called a soft qT_2 space.

Definition 25:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft topological space (G,E) be q-closed soft set in X and $x\in X_A$ such that $x\notin(G,E)$. If there occurs soft q-open sets (F_1,E) and (F_2,E) such that $x\in(F_1,E),(G,E)\subseteq(F_2,E)$ and $(F_1,E)\cap(F_2,E)=\varphi$. Then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called soft q-regular spaces. A soft q-regular qT_1 *Space* is called soft qT_3space.

Then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$is called a soft q-regular spaces. A soft q-regular T_1 *Space*.is called soft qT_3space.

Definition 26:

$(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a *soft quad topological* space $(F_1,E),(G,E)$ be closed soft sets in X such that $(F,E)\cap(G,E)=\varphi$ If there exists q-open soft sets (F_1,E) and(F_2,E) such that $(F,E)\subseteq(F_1,E),(G,E)\subseteq(F_2,E)$ and$(F_1,E)\cap(F_2,E)=\varphi$. Then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$is called a q-soft normal space. A soft q-normal qT_1 *Space* is called soft qT_4 *Space*.

Definition 27:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft Topological space over X and $e_G,e_H\in X_A$ such that $e_G\neq e_H$ if there can happen at least one soft q-open set (F_1,A) or (F_2,A) such that $e_G\in(F_1,A)$, $e_H\notin(F_1,A)$ or $e_H\in(F_2,A)$, $e_G\notin((F_2,A)$ then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called a *soft* qT_0 *space.*

Definition 28:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$be a soft Topological spaces over X and $e_G,e_H\in X_A$ such that $e_G\neq e_H$ if there can happen *soft q-open sets* $(F_1,A)$and (F_2,A) such that $e_G\in(F_1,A)$, $e_H\notin(F_1,A)$ and $e_H\in(F_2,A)$, $e_G\notin((F_2,A)$ then $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is called soft qT_1 *space*.

Definition 29:

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a *soft Topological* space over X and $e_G,e_H\in X_A$ such that $e_G\neq e_H$ if there can happen *soft q-open sets* $(F_1,A)$and (F_2,A) such that $e_G\in(F_1,A)$, and $e_H\in(F_2,A)$

$(F_1, A) \cap (F_2, A) = \phi_A$ .Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_2 *space*

Definition 30:

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space (G, E) be q-closed soft set in X and $e_G \in X_A$ such that $e_G \notin (G, E)$. If there occurs soft q-open sets (F_1, E) and (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft q-regular spaces. A soft q- regular qT_1 *Space* is called soft qT_3space.

Definition 31:

In a *soft quad topological* space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1) $\tau_1 \cup \tau_2$ is said to be *soft semi* T_0 space with respect to $\tau_3 \cup \tau_4$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ soft semi open set (F, E) and a to $\tau_3 \cup \tau_4$ soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (G, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ similarly, to $\tau_3 \cup \tau_4$ is said to be *soft semi* T_0 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_3 \cup \tau_4$ *soft semi open set* (F, E) and $\tau_1 \cup \tau_2$ soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. *Soft quad topological spaces*$(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be *pair wise soft semi* T_0 space if $\tau_1 \cup \tau_2$ is *soft semi* T_0 *space* with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$ and is *soft semi* T_0 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft semi T_1 space with respect to $\tau_3 \cup \tau_4$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_1 \cup \tau_2$ *soft semi open set* (F, E) and to $\tau_3 \cup \tau_4$ soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (G, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi T_1 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $x, y \in X$ such that $x \neq y$ there exists $\tau_3 \cup \tau_4$ soft semi open set (F, E) and a to $\tau_1 \cup \tau_2$ soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. *Soft quad topological spaces* $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be *pair wise soft semi* T_1 space if $\tau_1 \cup \tau_2$ is *soft semi* T_1 *space* with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$is *soft semi* T_1 space with respect to $\tau_1 \cup \tau_2$.

3) $\tau_1 \cup \tau_2$ is said to be soft semi T_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_1 \cup \tau_2$ soft semi open set (F, E) and a $\tau_3 \cup \tau_4$ soft semi open set (G, E) such that $x \in (F, E)$ and $y \in (G, E)$, $(F, E) \cap (G, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$is said to be *soft semi* T_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ *soft semi open* set (F, E) and a $\tau_1 \cup \tau_2$ *soft semi open* set (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft semi T_2 space if $\tau_1 \cup \tau_2$ is soft semi T_2 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_2 space with respect to$\tau_1 \cup \tau_2$.

Definition 32:

In a soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1) $\tau_1 \cup \tau_2$ is said to be soft semi qT_3 space with respect to a $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi T_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ Soft Semi closed set (G, E) such that $x \notin (G, E)$, a $\tau_1 \cup \tau_2$soft semi open set (F_1, E) and $\tau_3 \cup \tau_4$ soft semi open set (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and$(F_1, E) \cap (F_2, E) = \emptyset$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi T_3 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft semi T_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ soft semi closed set (G, E) such that $x \notin (G, E)$, $\tau_3 \cup \tau_4$soft semi open set (F_1, E) and $\tau_1 \cup \tau_2$ soft semi open set (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft semi T_3 space if $\tau_1 \cup \tau_2$ is soft semi T_3 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_3 space with respect to $\tau_1 \cup \tau_2$

2) $\tau_1 \cup \tau_2$ is said to be soft semi T_4 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi T_1 space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft semi closed set (F_1, E) and $\tau_3 \cup \tau_4$ soft semi closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \emptyset$. Also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_1 \cup \tau_2$ semi open set, (G_1, E) is soft $\tau_3 \cup \tau_4$ semi open set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi T_4 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft semi T_1 space with respect to$\tau_1 \cup \tau_2$, there exists $\tau_3 \cup \tau_4$ soft semi closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft semi closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. Also there exist (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ semi open set, (G_1, E) is soft $\tau_1 \cup \tau_2$ semi open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Thus, (X, τ_1, τ_2, E) is

said to be pair wise soft semi T_4 space if $\tau_1 \cup \tau_2$ is soft semi T_4 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_4 space with respect to$\tau_1 \cup \tau_2$.

Proposition 3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. Then, if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft semi T_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi T_2 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft semi T_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $x, y \in X$, which distinct, by using Proposition 1, (Y, E) is soft semi closed set in $\tau_3 \cup \tau_4$ and $x \notin (Y, E)$ there exists a $\tau_1 \cup \tau_2$ soft semi open set (F, E) and a $\tau_3 \cup \tau_4$ soft semi open set (G, E) such that $x \in (F, E)$, $y \in (Y, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $\tau_1 \cup \tau_2$ is soft semi T_2 space with respect to$\tau_3 \cup \tau_4$. Similarly, if (X, τ_3, τ_4, E) is a soft semi T_3 space with respect to (X, τ_1, τ_2, E) then according to definition for $x, y \in X, x \neq y$, by using Theorem 2, (x, E) is semi closed soft set in $\tau_1 \cup \tau_2$ and $y \notin (x, E)$ there exists a $\tau_3 \cup \tau_4$ soft semi open set (F, E) and a $\tau_1 \cup \tau_2$ soft semi open set (G, E) such that $y \in (F, E)$, $x \in (x, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$ .Hence $\tau_3 \cup \tau_4$ is soft semi T_2 space. This implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi T_2 space.

Proposition 4. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. if $(X, \tau_1, \tau_2, E)$and (X, τ_3, τ_4, E) are soft semi T_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi T_3 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft semi T_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $x, y \in X, x \neq y$ there exists a(X, τ_1, τ_2, E) soft semi open set (F, E) and a (X, τ_3, τ_4, E) soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each (X, τ_1, τ_2, E) semi closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists a (X, τ_1, τ_2, E) soft semi open set (F_1, E) and (X, τ_3, τ_4, E) soft semi open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, to (X, τ_3, τ_4, E) is a soft semi T_3 space with respect to(X, τ_1, τ_2, E) So according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_3, τ_4, E) soft semi open set (F, E) and a (X, τ_1, τ_2, E) soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each (X, τ_3, τ_4, E)semi closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists (X, τ_3, τ_4, E) soft semi open set (F_1, E) and a (X, τ_1, τ_2, E) soft semi open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi T_3 space.

Proposition 5.If $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft semi T_4 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi T_4 space.

Proof: Suppose (X, τ_1, τ_2, E) is soft semi T_4 space with respect to (X, τ_3, τ_4, E).So according to definition for $x, y \in X, x \neq y$ there exist a (X, τ_1, τ_2, E) soft semi open set (F, E) and a(X, τ_3, τ_4, E) soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ each (X, τ_1, τ_2, E) soft semi closed set (F_1, E) and a (X, τ_3, τ_4, E) soft semi closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. There exist (F_3, E) and (G_1, E) such that (F_3, E) is soft (X, τ_3, τ_4, E) and soft semi open set (G_1, E) is soft (X, τ_1, τ_2, E) semi open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and$(F_3, E) \cap (G_1, E) = \phi$. Similarly, (X, τ_3, τ_4, E) is soft semi T_4 space with respect to τ_1 so according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_3, τ_4, E) soft semi open set (F, E) and a (X, τ_1, τ_2, E) soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each (X, τ_3, τ_4, E) soft semi closed set (F_1, E) and (X, τ_1, τ_2, E) soft semi closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$.there exists soft semi open sets (F_3, E) and(G_1, E) such that (F_3, E) is soft (X, τ_3, τ_4, E)semi open set (G_1, E) is soft (X, τ_1, τ_2, E) semi open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi T_4 space.

Proposition 6. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a non-empty subset of X. if $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft semi T_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft semi T_3 space.

Proof: First we prove that $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft semi T_1 space.

Let $x, y \in X, x \neq y$ if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise space then this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft space. So there exists (X, τ_1, τ_2, E) soft semi open (F, E) and (X, τ_3, τ_4, E) soft semi open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ now $x \in Y \, and \, x \notin (G, E)$. Hence $x \in Y \cap (F, E) = (Y_F, E)$ then $y \notin Y \cap (\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$.

Therefore, $y \notin Y \cap (F,E) = (Y_F,E)$. Now $y \in Y$ and $y \in (G,E)$. Hence $y \in Y \cap (G,E) = (G_Y,E)$ where $(G,E) \in (X,\tau_3,\tau_4,E)$. Consider $x \notin (G,E)$ this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. Therefore $x \notin Y \cap (G,E) = (G_Y,E)$ thus $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_1 space.
Now we prove that $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_3 space then $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi regular space.
Let $y \in Y$ and (G,E) be a soft semi closed set in Y such that $y \notin (G,E)$ where $(G,E) \in (X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ then $(G,E) = (Y,E) \cap (F,E)$ for some soft semi closed set in $(X,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$. Hence $y \notin (Y,E) \cap (F,E)$ but $y \in (Y,E)$, so $y \notin (F,E)$ since $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is soft semi T_3 space
$(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is soft semi regular space so there exists (X,τ_1,τ_2,E) soft semi open set (F_1,E) and (X,τ_3,τ_4,E) soft semi open set (F_2,E) such that
$y \in (F_1,E), (G,E) \subseteq (F_2,E)$
$$(F_1,E)(F_2,E) = \phi$$
Take $(G_1,E) = (Y,E) \cap (F_2,E)$ then $(G_1,E),(G_2,E)$ are soft semi open set in Y such that
$y \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$
$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$
$(G_1,E) \cap (G_2,E) = \phi$
There fore $\tau_{1Y} \cup \tau_{2Y}$ is soft semi regular space with respect to $\tau_{3Y} \cup \tau_{4Y}$. Similarly, Let $y \in Y$ and (G,E) be a soft semi closed sub set in Y such that $y \notin (G,E)$, where $(G,E) \in (X,\tau_3,\tau_4,E)$ then $(G,E) = (Y,E) \cap (F,E)$ where (F,E) is some soft semi closed set in (X,τ_3,τ_4,E). $y \notin (Y,E) \cap (F,E)$ But $y \in (Y,E)$ so $y \notin (F,E)$ since (X,τ_1,τ_2,E) is soft semi regular space so there exists (X,τ_3,τ_4,E) soft semi open set (F_1,E) and (X,τ_1,τ_2,E) soft semi open set (F_2,E). Such that
$y \in (F_1,E), (G,E) \subseteq (F_2,E)$
$(F_1,E) \cap (F_2,E) = \phi$
Take $(G_1,E) = (Y,E) \cap (F_1,E)$
$$(G_1,E) = (Y,E) \cap (F_1,E)$$
Then (G_1,E) and (G_2,E) are soft semi open set in Y such that
$$y \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$
There fore $\tau_{3Y} \cup \tau_{4Y}$ is soft semi regular space with respect $\tau_{1Y} \cup \tau_{2Y}$
$\Rightarrow (Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_3 space.

Proposition 7. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft quad topological space over X and Y be a soft semi closed sub space of X. if $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is pair wise soft semi T_4 space then $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_4 space.

Proof: Since $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is pair wise soft semi T_4 space so this implies that $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is pair wise soft semi T_1 space as proved above.
We prove $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is pair wise soft semi normal space.
Let $(G_1,E),(G_2,E)$ be soft semi closed sets in Y such that
$$(G_1,E) \cap (G_2,E) = \phi$$
Then $(G_1,E) = (Y,E) \cap (F_1,E)$
And $(G_2,E) = (Y,E) \cap (F_2,E)$
For some soft semi closed sets such that (F_1,E) is soft semi closed set in $\tau_1 \cup \tau_2$ soft semi closed set (F_2,E) in $\tau_3 \cup \tau_4$.
And $(F_1,E) \cap (F_2,E) = \phi$
From Proposition 2**.** Since, Y is soft semi closed sub set of X then $(G_1,E),(G_2,E)$ are soft semi closed sets in X such that
$$(G_1,E) \cap (G_2,E) = \phi$$
Since $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is pair wise soft semi normal space. So there exists soft semi open sets (H_1,E) and (H_2,E) such that
(H_1,E) is soft semi open set in $\tau_1 \cup \tau_2$ and (H_2,E) is soft semi open set in $\tau_3 \cup \tau_4$ such that
$$(G_1,E) \subseteq (H_1,E)$$
$$(G_2,E) \subseteq (H_2,E)$$
$$(H_1,E) \cap (H_2,E) = \phi$$
Since $(G_1,E),(G_2,E) \subseteq (Y,E)$

Then $(G_1,E) \subseteq (Y,E) \cap (H_1,E)$
$(G_2,E) \subseteq (Y,E) \cap (H_2,E)$
And $[(Y,E) \cap (H_1,E)] \cap [(Y,E) \cap (H_2,E)] = \phi$
Where $(Y,E) \cap (H_1,E)$ and $(Y,E) \cap (H_2,E)$ are soft semi open sets in Y there fore $\tau_{1Y} \cup \tau_{2Y}$ is soft semi normal space with respect to$\tau_{3Y} \cup \tau_{4Y}$. Similarly, let $(G_1,E),(G_2,E)$ be soft semi closed sub set in Y such that

$(G_1,E) \cap (G_2,E) = \phi$
Then $(G_1,E) = (Y,E) \cap (F_1,E)$
And $(G_2,E) = (Y,E) \cap (F_2,E)$
For some soft semi closed sets such that (F_1,E) is soft semi closed set in $\tau_3 \cup \tau_4 and (F_2,E)$ soft semi closed set in $\tau_1 \cup \tau_2$and

$(F_1,E)(F_2,E) = \phi$
From Proposition 2. Since, Y is soft semi closed sub set in X then $(G_1,E),(G_2,E)$ are soft semi closed sets in X such that

$(G_1,E) \cap (G_2,E) = \phi$
Since $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$ is pair wise soft semi normal space so there exists soft semi open sets (H_1,E) and (H_2,E)
Such that (H_1,E) is soft semi open set is $\tau_3 \cup \tau_4$ and (H_2,E) is soft semi open set in $\tau_1 \cup \tau_2$ such that
$(G_1,E) \subseteq (H_1,E)$
$(G_2,E) \subseteq (H_2,E)$
$(H_1,E) \cap (H_2,E) = \phi$
Since $(G_1,E),(G_2,E) \subseteq (Y,E)$
Then $(G_1,E) \subseteq (Y,E) \cap (H_1,E)$
$(G_2,E) \subseteq (Y,E) \cap (H_2,E)$
And $[(Y,E) \cap (H_1,E)] \cap [(Y,E) \cap (H_2,E)] = \phi$
Where $(Y,E) \cap (H_1,E)$ and $(Y,E) \cap (H_2,E)$ are soft semi open sets in Y there fore $\tau_{3Y} \cup \tau_{4Y}$ is soft semi normal space with respect to $\tau_{1Y} \cup \tau_{2Y}$
$\Rightarrow (Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y}E)$ is pair wise soft semi T_4 space.

3.2 SOFT SEMI SEPARATION AXIOMS IN SOFT QUAD TOPOLGICAL SPACES WITH RESPECT TO SOFT POINTS.

In this section, we introduced soft topological structures known as semi separation axioms in soft quad topology with respect to soft points. With the applications of these soft semi separation axioms different result are brought under examination.

Definition 33:

In a *soft quad topological* space $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$
1) $\tau_1 \cup \tau_2$ said to be *soft semi* T_0 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\tau_1 \cup \tau_2$ soft semi open set (F,E) and a $\tau_3 \cup \tau_4$ soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (G,E)$, Similarly, $\tau_3 \cup \tau_4$ is said to be *soft semi* T_0 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\tau_3 \cup \tau_4$ *soft semi open set* (F,E) and a $\tau_1 \cup \tau_2$ semi soft open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$. *Soft quad topological spaces* $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$ is said to be *pair wise soft semi* T_0 space if $\tau_1 \cup \tau_2$ is *soft semi* T_0 *space* with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_0 spaces with respect to $\tau_1 \cup \tau_2$
2) $\tau_1 \cup \tau_2$ is said to be soft semi T_1 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$there happens a $\tau_1 \cup \tau_2$ *soft semi open set* $(F.E)$ and $\tau_3 \cup \tau_4$ soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (G,E)$ and $e_H \in (G,E)$ and $e_G \notin (G,E)$.Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi T_1 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$there exist a $\tau_3 \cup \tau_4$ soft semi open set (F,E) and a $\tau_1 \cup \tau_2$ Soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (G,E)$ and $e_H \in (G,E)$ and $e_G \notin (G,E)$. *Soft quad topological space* $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$ is said to be *pair wise soft semi* T_1 space if $\tau_1 \cup \tau_2$ is *soft semi* T_1 *space* with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_1 spaces with respect to $\tau_1 \cup \tau_2$.

3) $\tau_1 \cup \tau_2$ is said to be soft semi T_2 space with respect to $\tau_3 \cup \tau_4$, if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\tau_1 \cup \tau_2$ soft semi open set (F,E) and a $\tau_3 \cup \tau_4$ soft semi open set (G,E)

such that $e_G \in (F,E)$ and $e_H \notin (G,E)$ and $e_H \in (G,E)$ and $e_G \notin (G,E)$ and $(F,E) \cap (G,E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is aid to be *soft semi* T_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_G \in X_A$ there happens a $\tau_3 \cup \tau_4$ *soft semi open* set (F,E) and a $\tau_1 \cup \tau_2$ *soft semi open* set (G,E) such that $e_G \in (F,E)$ and $e_G \in (G,E)$ and $(F,E) \cap (G,E) = \phi$. The soft quad topological space $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$ is said to be pair wise soft semi T_2 space if $\tau_1 \cup \tau_2$ is soft semi T_2 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_2 space with respect to$\tau_1 \cup \tau_2$.

Definition 34:

In a soft quad topological space $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$

1) $\tau_1 \cup \tau_2$is said to be soft semi T_3 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi T_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of distinct points $e_G, e_H \in X_A$,there exists a $\tau_1 \cup \tau_2$ Semi closed soft set (G,E) such that$e_G \notin (G,E)$, $\tau_1 \cup \tau_2$ soft semi open set (F_1,E) and $\tau_3 \cup \tau_4$ soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G,E) \subseteq (F_2,E)$ and$(F_1,E) \cap (F_2,E) = \emptyset$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi T_3 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft semi T_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of distinct points $e_G, e_H \in X_A$ there exists a $\tau_3 \cup \tau_4$ soft semi closed set (G,E) such that $e_G \notin (G,E)$, $\tau_3 \cup \tau_4$ soft semi open set (F_1,E) amd $\tau_1 \cup \tau_2$ soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$.
$(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$ is said to be pair wise soft semi T_3 space if $\tau_1 \cup \tau_2$ is soft semi T_3 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi T_3 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft semi T_4 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi T_1 space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft semi closed set (F_1,E) and $\tau_3 \cup \tau_4$ soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \emptyset$,also, there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft $\tau_1 \cup \tau_2$ semi open set, (G_1,E) is soft $\tau_3 \cup \tau_4$ semi open set such that $(F_1,E) \subseteq (F_3,E)$, $(F_2,E) \subseteq (G_1,E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi T_4 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft semi T_1 space with respect to $\tau_1 \cup \tau_2$ there exists $\tau_3 \cup \tau_4$ soft semi closed set (F_1,E) and $\tau_1 \cup \tau_2$ soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$. Also there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft $\tau_3 \cup \tau_4$ semi open set, (G_1,E) is soft $\tau_1 \cup \tau_2$ semi soft set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$. Thus, $(X,\tau_1,\tau_2,\tau_1,\tau_2,E)$ is said to be pair wise soft semi T_4 space if $\tau_1 \cup \tau_2$ is soft semi T_4 space with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$ is soft semi T_4 space with respect to $\tau_1 \cup \tau_2$.

Proposition 8. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ be a soft topological space over$X$. $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is soft semi T_3space, then for all $e_G \in X_E$ $e_G = (e_G,E)$ is soft semi-closed set.

Proof: We want to prove that e_G is semi closed soft set, which is sufficient to prove that e_G^c is semi open soft set for all $e_H \in \{e_G\}^c$.Since $(X,\tau_1,\tau_2,\tau_3,\tau_4,E)$ is soft semi T_3 space, then there exists soft semi sets$(F,E)_{e_H}$ and (G,E) such that $e_{H_E} \subseteq (F,E)_{e_H}$ and $e_{G_E} \cap (F,E)_{e_H} = \phi$ and $e_{G_E} \subseteq (G,E)$ and $e_{H_E} \cap (G,E) = \phi$.It follows that, $\cup_{e_H \in (e_G)^c (F,E)_{e_H} \subseteq e_{G_E}^c}$ Now, we want to prove that $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$.Let $\cup_{e_H \in (e_G)^c} (F,E)_{e_H} = (H,E)$.Where $H(e) = \cup_{e_H \in (e_G)^c (F(e)_{e_H}}$ for all $e \in E$.Since $e_{G_E}^c(e) = (e_G)^c$for all $e \in E$ from Definition 9, so, for all $e_H \in \{e_G\}^c$and $e \in E$ $e_{G_E}^c(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c F(e)_{e_H} = H(e)}$.Thus, $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$ from Definition 2, and so, $e_{G_E}^c = \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$.
This means that, $e_{G_E}^c$ is soft semi open set for all $e_H \in \{e_G\}^c$.Therefore, e_{G_E} is semi closed soft set.

Proposition 9. Let $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ be a soft quad topological space over X. Then, if (X,τ_1,τ_2,E) and (X,τ_3,τ_4,E) are soft semi qT_3 space, then $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is a pair wise soft semi T_2 space.

Proof: Suppose if (X,τ_1,τ_2,E) is a soft semi T_3 space with respect to (X,τ_3,τ_4,E) , then according to definition for, $e_G \neq e_{H\,,} e_G, e_H \in X_A$, by using Theorem 8, ($e_H$, E) is soft semi closed set in (X,τ_3,τ_4,E) and $e_G \notin (e_H,E)$ there exist a (X,τ_1,τ_2,E) soft semi open set (F,E) and a (X,τ_3,τ_4,E)soft semi open set (G,E) such that $e_G \in (F,E)$, $e_H \in (y,E) \subseteq (G,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence, (X,τ_1,τ_2,E) is soft semi T_2 space with respect to(X,τ_3,τ_4,E) Similarly, if (X,τ_3,τ_4,E) is a soft semi T_3 space with respect to (X,τ_1,τ_2,E), then according to definition for , $e_G \neq e_{H\,,} e_G, e_H \in X_A$, by using Theorem 8, ($e_G$, E) is semi closed soft set in (X,τ_1,τ_2,E) is and $y \notin (x,E)$ there exists a (X,τ_3,τ_4,E)soft semi open set (F,E) and a (X,τ_1,τ_2,E)soft semi open set

(G,E) such that $e_H \in (F,E)$, $e_G \in (x,E) \subseteq (G,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence, $(X,\tau_3,\tau_4,E)$is a soft semi T_2 space. Thus $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is a pair wise soft semi T_2 space.

Proposition 10. Let $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ be a soft quad topological space over X. If if (X,τ_1,τ_2,E) and (X,τ_3,τ_4,E) are soft semi T_3 space then $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is a pair wise soft semi T_3 space.

Proof: Suppose $(X,\tau_1,\tau_2,E)$is a soft semi T_3 space with respect to (X,τ_3,τ_4,E)then according to definition for $e_G, e_H \in X_A$ $e_G \neq e_H$ there happens $\tau_1 \cup \tau_2$ soft semi open set (F,E) and a $\tau_3 \cup \tau_4$ soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each point $e_G \in X_A$and each $\tau_1 \cup \tau_2$semi closed soft set (G_1,E) such that $e_G \notin (G_1,E)$ there happens a $\tau_1 \cup \tau_2$ soft semi open set (F_1,E) and $\tau_3 \cup \tau_4$ soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$.Similarly$(X,\tau_3,\tau_4,E)$is a soft semi T_3 space with respect to(X,τ_1,E). So according to definition for$e_G, e_H \in X_A$, $e_G \neq e_H$ there exists a $\tau_3 \cup \tau_4$ soft semi open set (F,E) and $\tau_1 \cup \tau_2$ soft semi open set (G,E) such that $e_H \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each point $e_G \in X_A$ and each $\tau_3 \cup \tau_4$ semi closed soft set (G_1,E) such that $e_G \notin (G_1,E)$ there exists $\tau_3 \cup \tau_4$ soft semi open set (F_1,E) and $\tau_1 \cup \tau_2$ soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is pair wise soft semi T_3 space.

Proposition 11. If $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ be a soft quad topological space over X. $(X,\tau_1,\tau_2,E)$and $(X,\tau_3,\tau_4,E)$are soft semi T_4 space then $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is pair wise soft semi T_4 space.

Proof: Suppose $((X,\tau_1,\tau_2,E))$ is soft semi T_4 space with respect to (X,τ_3,τ_4,E)So according to definition for $e_G, e_H \in X_A$, $e_G \neq e_H$ there happens a $\tau_1 \cup \tau_2$ soft semi open set (F,E) and a $\tau_3 \cup \tau_4$ soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ each $\tau_1 \cup \tau_2$ soft semi closed set (F_1,E) and a $\tau_3 \cup \tau_4$ soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$.There occurs (F_3,E) and (G_1,E) such that (F_3,E) is soft $\tau_3 \cup \tau_4$ semi open set (G_1,E) is soft a $\tau_1 \cup \tau_2$ semi open set $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and$(F_3,E) \cap (G_1,E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is soft semi T_4 space with respect to $\tau_1 \cup \tau_2$ so according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a $\tau_3 \cup \tau_4$ soft semi open set (F,E) and a $\tau_1 \cup \tau_2$ soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each $\tau_3 \cup \tau_4$ soft semi closed set (F_1,E) and $\tau_1 \cup \tau_2$soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$.there occurs (F_3,E) and(G_1,E) such that (F_3,E) is soft $\tau_3 \cup \tau_4$ semi open set (G_1,E) is soft $\tau_1 \cup \tau_2$semi open set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$ hence $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is pair wise soft semi T_4 space.

Proposition 12. Let $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ be a soft quad topological space over X and Y be a non-empty subset of X. if $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_3 space. Then $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_3 space.

Proof. First we prove that $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$ is pair wise soft semi T_1 space.

Let $e_G, e_H \in X_A, e_G \neq e_H$ if $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is pair wise soft space then this implies that $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is pair wise soft $\tau_1 \cup \tau_2$ space. So there exists $\tau_1 \cup \tau_2$soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ now $e_G \in Y \; and \; e_G \notin (G,E)$. Hence $e_G \in Y \cap (F,E) = (Y_F,E)$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$.

There fore, $e_H \notin Y \cap (F,E) = (Y_F,E)$. Now $e_H \in Y$ and $e_H \in (G,E)$. Hence, $e_H \in Y \cap (G,E) = (G_Y,E)$ where $(G,E) \in \tau_3 \cup \tau_4$. Consider $x \notin (G,E)$ .this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $e_G \notin Y \cap (G,E) = (G_Y,E)$ thus $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E)$is pair wise soft semi T_1 space.

Now, we prove that $(Y,\tau_{1Y},\tau_{2Y},\tau_{3Y},\tau_{4Y},E\;)$ is pair wise soft semi T_3 space.

Let $e_H \in Y$ and (G,E) be soft semi closed set in Y such that $e_H \notin (G,E)$ where $(G,E) \in \tau_1 \cup \tau_2$ then $(G,E) = (Y,E) \cap (F,E)$ for some soft semi closed set in $\tau_1 \cup \tau_2$hence $e_H \notin (Y,E) \cap (F,E)$ but $e_H \in (Y,E)$, so $e_H \notin (F,E)$ since $(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$is soft semi T_3 space

$(X,\tau_1,\tau_{2,}\tau_3,\tau_4,E)$ is soft semi regular space so there happens $\tau_1 \cup \tau_2$ soft semi open set (F_1,E) and $\tau_3 \cup \tau_4$ soft semi open set (F_2,E) such that

$e_H \in (F_1,E), (G,E) \subseteq (F_2,E)$

$$(F_1,E)(F_2,E) = \phi$$

Take $(G_1,E) = (Y,E) \cap (F_2,E)$ then $(G_1,E), (G_2,E)$ are soft semi open sets in Y such that

$e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$
$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$
$(G_1, E) \cap (G_2, E) = \phi$
Therefore, (τ_{1Y}, τ_{2Y}) is soft semi regular space with respect to (τ_{3Y}, τ_{4Y}) Similarly, Let $e_H \in Y$ and (G, E) be a soft semi closed sub set in Y such that $e_H \notin (G, E)$, Where $(G, E) \in \tau_3 \cup \tau_4$ then $(G, E) = (Y, E) \cap (F, E)$ where (F, E) is some soft semi closed set in$\tau_3 \cup \tau_4$. $e_H \notin (Y, E) \cap (F, E)$ but $e_H \in (Y, E)$ so $e_H \notin (F, E)$ since $(X, \tau_1, \tau_{2,} \tau_3, \tau_4, E)$is soft semi regular space so there happens $\tau_3 \cup \tau_4$ soft semi open set (F_1, E) and $\tau_1 \cup \tau_2$ soft semi open set (F_2, E). Such that
$e_H \in (F_1, E), (G, E) \subseteq (F_2, E)$
$(F_1, E) \cap (F_2, E) = \phi$
Take $(G_1, E) = (Y, E) \cap (F_1, E)$
$(G_1, E) = (Y, E) \cap (F_1, E)$
Then (G_1, E) and (G_2, E) are soft semi open set in Y such that
$$e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$$
$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi.$$
Therefore (τ_{3Y}, τ_{4Y}) is soft semi regular space.

4. Conclusion

A soft set with single specific topological structure is unable to shoulder up the responsibility to construct the whole theory. So to make the theory healthy, some additional structures on soft set has to be introduced. It makes, it more springy to develop the soft topological spaces with its infinite applications. In this regard we introduce strong topological structure known as soft quad topological structure in this paper.

Topology is the supreme branch of pure mathematics which deals with mathematical structures. Freshly, many scholars have studied the soft set theory which is coined by Molodtsov [1] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz [3] familiarized and deeply studied the origin of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we constantly study the behavior of soft semi separation axioms in soft quad topological spaces with respect to soft points as well as ordinary points of a soft topological space. We introduce soft semi qT_0 structure, soft semi qT_1 structure, soft semi qT_2 structure, Soft semi qT_3 and soft semi qT_4 structure with respect to soft and ordinary points. In future we will plant these structures in different results. More over defined soft semi T_0 structure w.r.t. soft semi T_1 structure and vice versa, soft semi T_1 structure w.r.t soft semi T_2 structure and vice versa and soft semi T_3 space w.r.t soft semi T_4 and vice versa with respect to ordinary and soft points in soft quad topological spaces and studied their activities in different results with respect to ordinary and soft points . We also planted these axioms to different results. These soft semi separation axioms in quad structure would be valuable for the development of the theory of soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. We expect that these results in this article will do help the researchers for strengthening the toolbox of soft topological structures. In the forthcoming spread the idea of soft α- open, and soft b^{**}open sets in soft quad topological structure with respect to ordinary and soft points.

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