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Investigation of transition symmetry shapes of $^{160\text{-}168}$Yb nuclei using IBM

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Abstract

The interacting boson models, $IBM-1$ and $IBM-2$, were used to perform a complete study of even –even $^{160\text{-}168}$Yb isotopes .The low –lying positive parity states, dynamic symmetries, reduced electric quadrupole transition probability $B(E2)$, quadruple momentum $Q^+_{2_1}$, and potential energy surface PES for $^{160\text{-}168}$Yb were investigated. Energy level sequences and energy ratios showed the gradual transition of the properties of these nuclei from the γ-unstable features $O(6)$ to the rotational features $SU(3)$. Adding the pairing parameter a_0 to $IBM-1$ Hamiltonian had a very slight effect on this feature, but it raised the β band, since it represents symmetry breaking such as in γ-unstable features $O(6)$. This applies to the experimental decay scheme of $^{160\text{-}168}$Yb isotopes. In $IBM-2$, proton and neutron quadruple deformation parameters χ_π and χ_ν showed values equal to -1.24 and approximately 0.7, respectively, which supports the same idea in the interacting boson model $IBM-1$. A contour plot of the potential energy surface $V(\beta,\gamma)$ for $^{160\text{-}168}$Yb isotopes showed that the minimum potential occurs at approximately $\beta = 1$ and $\gamma = 60^o$.

Keywords: $IBM-1$, $IBM-2$, $^{160\text{-}168}$Yb , $O(6)-SU(3)$ limits.

شكل الانتقال التماثلي في نوى $^{160-168}$Yb باستعمال IBM

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الخلاصة

تم استخدام أنموذج البوزونات المتفاعلة الاول والثاني لأجراء دراسة كاملة عن نظائر $^{160-168}$Yb الزوجية –زوجية. تمت دراسة مستويات الطاقة الواطئة الموجبة والتناظرات الديناميكية واحتمالية الانتقال الكهربائي المختزلة $B(E2)$ وعزم رباعي القطب الكهربائي $Q^+_{2_1}$ وسطوح تساوي الجهد لنظائر $^{160-168}$Yb . يشير تتابع مستويات الطاقة ونسب الطاقة الدراسة الى الانتقال التدريجي لصفات هذه النوى من صفات نوى كاما غير المستقرة $O(6)$ الى الصفات الدورانية $SU(3)$. ان اضافة حد الازدواج a_0 الى المؤثر الهاملتوني لـ1 $IBM-$له تأثير صغير على هذه الصفات لكنة رفع حزمة β مشيرا الى كسر التناظر الديناميكي كما في نوى كاما غير المستقرة $O(6)$ والذي ينطبق مع مخطط الانحلال العملي لنظائر $^{160-168}$Yb . في نموذج $IBM-2$ كانت قيم مؤثرات التشوه رباعية القطب للبروتون والنيترون χ_π و χ_ν مساوية الى (-1.24 وتقريبا 0.7) والذي يدعم نفس الفكرة في نموذج البوزونات المتفاعلة الاول $IBM-1$. ان الرسم الكنتوري

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لسطح تساوي الجهد $V(\beta,\gamma)$ لنظائر $^{160-168}$Yb يوضح وقوع النقطة الصغرى للجهد عند قيمة $\beta =$
$.1, \gamma = 60^o$

1. Introduction

The interacting boson model (IBM) is suitable for describing the low-lying collective states in even-even nuclei by a system of interacting s and d bosons carrying angular momentums 0 and 2, respectively [1, 2]. The IBM is built on a closed shell, i.e., the total number of bosons $[N]$ depends on the number of active nucleon particle (or hole) pairs outside a closed shell. Each type of bosons, i.e. the s-and d-bosons, has its own binding energy with regard to the closed shell [3-6]. The $IBM1$ dose not distinguish between proton and neutron bosons; the total bosons number ($N = n_\pi + n_\nu$) is finite and conserved in a given nucleus and is simply given by half the total number of valence nucleons. The s ($L = 0$) and d ($L = 2$) bosons of the $IBM1$ have six sub states; therefore, they define a six-dimensional space, so that one can describe it in terms of the unitary group in six dimensions, $U(6)$. This leads to drive many of the properties of the $IBM1$ by group theoretical methods to express it analytically. The present study investigated the medium and heavy mass isotopes of Ytterbium which are located in the rear- earth mass region and are well deformed nuclei that can be populated to very high spin. There are many studies that attempted to explain the behavior of Ytterbium nuclei [7-12]. Some of them were described as having vibrattional bound to the rotational properties, while others were described as possessing unstable γ characteristics and being on their way to the rotating region by increasing the neutron number. The $^{160-168}$Yb isotopes have Z=70 and 6 hole bosons. The number of protons and neutrons are lying between 50 , 82 and 82, 126 magic shells, respectively. $^{160-168}$Yb isotopes have 90-98 neutrons, which indicates 4-8 particle neutron bosons with a total boson number of 10-14, respectively. The nucleons distributions of protons and neutrons shells are

$$\underbrace{3s_{1/2}^2}_{Z=70}\ \underbrace{1h_{9/2}^{10}}_{N=90-92}\ \underbrace{2f_{7/2}^{6}}_{N=94-98}$$

After examining decay schemes [13-22], energy level sequences and energy ratios show that the even-even $^{160-168}$Yb medium heavy nuclei are explained the moving from $\gamma -$ unstable to the $SU(3)$ leg of the symmetry triangle.

2 . The interacting boson model

In the IBM1, the Hamiltonian operator contains only one body and two body terms and, thus, introducing creation ($s^\dagger, d_m^\dagger$) and annihilation (s, d_m) operators where the index $m = 0, \pm 1, \pm 2$. The most general Hamiltonian, which includes on-boson terms in boson –boson interaction, is [6].

$$H = \varepsilon_s(s^\dagger s) + \varepsilon_d \sum_m d_m^\dagger d_m + V \quad (1)$$

where $\varepsilon_s, \varepsilon_d$ are the s and d boson energies and V is the boson-boson interacting energy, which can be written as [23]:

$$H = \varepsilon_s(s^\dagger s) + \varepsilon_d \sum_m d_m^\dagger d_m + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L \left[(d^\dagger d^\dagger)^{(L)}.(dd)^L\right]^{(0)} + \frac{1}{\sqrt{2}}\upsilon_2\left[(d^\dagger d^\dagger)^{(2)}.(ds)^{(2)} + (d^\dagger s^\dagger)^{(2)}.(dd)^{(2)}\right]^{(0)} + \frac{1}{2}\upsilon_0\left[(d^\dagger d^\dagger)^{(0)}.(ss)^{(0)} + (s^\dagger s^\dagger)^{(0)}.(dd)^{(0)}\right]^{(0)} + u_2\left[(d^\dagger s^\dagger)^{(2)}.(ds)^2\right]^{(0)} + \frac{1}{2}u_0\left[(s^\dagger s^\dagger)^{(0)}.(ss)^{(2)}\right]^{(0)} \quad (2)$$

where $C_L(L = 0,2,4), \upsilon_L(L = 0,2), u_L(L = 0,2)$ describe the boson interaction. The most commonly used form of $IBM1$ Hamiltonian is [24]:

$$H = \varepsilon n_d + a_0 P^\dagger P + a_1 L.L + a_2 Q.Q + a_3 T_3 T_3 + a_4 T_4 T_4 \quad (3)$$

where $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy (for simplicity ε_s is set equal to zero and only $\varepsilon = \varepsilon_d$ appears), while a_0, a_1, a_2, a_3, a_4 designate the strengths of the quadrupole, angular momentum, pairing, octupole, and hexadecapole interacting bosons, respectively. The five components of d boson and the single component of s boson are extended across a six dimensional space. For a fixed number of bosons N, the group structure of the problem is $U(6)$. Considering the different reductions of $U(6)$, three dynamical symmetries emerge, namely $U(5)$, $SU(3)$, and $U(6)$. These symmetries are related to the geometrical idea of the spherical vibrator, deformed rotor, and symmetric ($\gamma -$soft) deformed rotor, respectively [3-6]. The dynamical symmetries in transitional Hamiltonians are related to the selection rules in electromagnetic transitions. The simplest form of $IBM-1$ transition operator is given as [25]:

$$T_m^l=\alpha_2\delta_{l2}[d^\dagger s+s^\dagger d]_m^{(2)}+\beta_l[d^\dagger d]_m^{(l)}+\gamma_0\delta_{l0}\delta_{m0}[s^\dagger s]_0^{(0)} \tag{4}$$

where $\alpha_2,\beta_l,\gamma_0$ are the coefficients of the various terms in the operators.

The general formula for the potential energy surface as a function of geometrical variables β and γ is given by [26] :

$$V(\beta,\gamma)=\frac{N(\varepsilon_s+\varepsilon_d\beta^2)}{1+\beta^2}+\frac{N(N+1)}{(1+\beta^2)^2}(\alpha_1\beta^4+\alpha_2\beta^3Cos3\gamma+\alpha_3\beta^2+\alpha_4) \tag{5}$$

$$\text{with } \alpha_1=\frac{C_0}{10}+\frac{C_2}{7}+\frac{9C_4}{35},\alpha_2=-\sqrt{\frac{8}{35}}\upsilon_2,\ \alpha_3=\frac{(\upsilon_0+u_2)}{\sqrt{5}},\alpha_4=u_0 \tag{6}$$

where N is the total boson number and β is the quadruple deformation parameter operator from $\beta=0-2.4$. γ is the distortion parameter operator or (asymmetry angle) for $0^\circ\le\gamma\le 60^\circ$. The variables $\alpha_1,\alpha_2,\ \alpha_3,\ \alpha_4$ are related to the parameters C_L,υ_L,u_L which are given in equation (2). The relationships between the variables ($\alpha's$) and these parameters was expressed by Iachello [25] as one must take into account the asymmetry angle that occurs only in the term $cos3\gamma$. Thus, the energy surfaces have minima only at $\gamma=0^\circ$ and 60°. These expressions give, at large N, $\beta_{min}=0,\sqrt{2},1$ for $U(5),SU(3)$, and $O(6)$, respectively. The Hamiltonian operator in $IBM-2$ will have three parts, one part for each of proton and neutron bosons and a third part for describing the proton-neutron interaction [26]:

$$H=H_\pi+H_\nu+V_{\pi\nu} \tag{7}$$

A simple schematic Hamiltonian guided by microscopic consideration is given by [26]:

$$H=\varepsilon(n_{d\pi}+n_{d\nu})+\kappa Q_\pi.Q_\nu+V_{\pi\pi}+V_{\nu\nu}+M_{\pi\nu} \tag{8}$$

$$\text{where } Q_\rho=(d_\rho^\dagger s_\rho+s_\rho^\dagger d_\rho)_\rho^2+\chi_\rho(d_\rho^\dagger d_\rho)_\rho^2\ \ \rho=\pi,\nu \tag{9}$$

$$V_{\rho\rho}=\sum_{L=0,2,4}\frac{1}{2}(2L+1)^{\frac{1}{2}}C_L^\rho\left[(d_\rho^\dagger d_\rho^\dagger)^{(L)}.(d_\rho d_\rho)^{(L)}\right]^{(0)} \tag{10}$$

$\varepsilon_\pi,\varepsilon_\nu$ represent proton and neutron energy, respectively, and assumed as equal ($\varepsilon_\pi=\varepsilon_\nu=\varepsilon$). The last term in Eq. (8) contains the Majorana operator $M_{\pi\nu}$ and it is usually added in order to remove states of mixed proton neutron symmetry. This term can be written as [25,26]:

$$M_{\pi\nu}=\zeta_2(s_\nu^\dagger d_\pi^\dagger-d_\nu^\dagger s_\pi^\dagger)^{(2)}.(s_\nu d_\pi-d_\nu s_\pi)^{(2)}+\sum_{k=1,3}\zeta_k(d_\nu^\dagger d_\pi^\dagger)^{(k)}-(d_\nu d_\pi)^{(k)} \tag{11}$$

If there is an experimental evidence for so called “mixed symmetry state”, then the Majorana parameter is varied to fix the location of these states in the spectrum. The levels of energy are achieved by diagonalizing the Hamiltonian Eq. (8) then allowing the parameters $\varepsilon,\kappa,\chi_\pi,\chi_\nu$ and C_L to vary until one obtains the best fit to the experimental spectrum using Eq. (8). The $U(5)$ limit is when $\varepsilon\gg\kappa$, the $SU(3)$ limit is when $\varepsilon\ll\kappa$ and $\chi_\pi=\chi_\nu=-\sqrt{7}/2$, and the $O(6)$ limit is when $\varepsilon\ll\kappa$ and $\chi_\nu=-\chi_\pi$. Most nuclei do not strictly belong to any of these three limiting cases, but are somewhere between two of them. In the IBM, it is possible to make a smooth transition between the limiting cases for a series of isotopes. The general single boson transition operator of angular momentum ℓ has the same form as in eq.(4) in $IBM-1$, except the fact that in each term one has to consider π,ν degree of freedom, and this can be written as [23]:

$$T^{(l)}=\alpha_{2\rho}\delta_{\ell 2}[d^\dagger s+s^\dagger d]_\rho^{(2)}+\beta_{\ell\rho}[d^\dagger d]_\rho^{(\ell)}+\gamma_{0\rho}\delta_{\ell 0}[s^\dagger s]_\rho^{(0)}\ \ldots\rho=\pi\ or\ \nu \tag{12}$$

This equation yields transition operators for $E0,M1,E2,M3$, and $E4$.

3. Results and Discussion

The interacting boson model $IBM-1$ and the proton - neutron interacting boson $IBM-2$ were used to perform an overall investigation of $^{160-168}$Yb isotopes . The software package of , $IBMT$, and $IBMP$ computer code was used for $IBM-1$, whereas the software package of Neutron Proton Boson $NPBOS$ and Neutron Proton Boson Electromagnetic $NPBEM$ were used for $IBM-2$. The low –lying positive party states, dynamic symmetries, reduced electric quadrupole transition probabilities ($Q_{2_1^+}$) and the potential energy surface for $^{160-168}$Yb were investigated. The $IBM-1$ and $IBM-2$ Hamiltonian were used to estimate a set of parameters described in the Hamiltonian operator, as shown in equations (2) and (8). The estimated parameters for the calculations of the low- lying excited energy levels for Ytterbium isotopes are given in Table-1 and Figure-1.

Table 1-The parameters used in the $IBM-1$ and $IBM-2$ Hamiltonian for even-even $^{160\text{-}168}$Yb isotopes (in MeV), except χ , χ_ν and χ_π which were unitless.

Isotopes	IBM1 parameters in MeV unless χ							
	N	ε	a_0	a_1	a_2	a_3	a_4	χ
^{160}Yb	10	0.0	0.017	0.012	-0.045	0.0	0.0	-0.18
^{162}Yb	11	0.0	0.016	0.01	-0.027	0.0	0.0	-0.4
^{164}Yb	12	0.0	0.002	0.011	-0.022	0.0	0.0	-0.54
^{166}Yb	13	0.0	0.001	0.011	-0.016	0.0	0.0	-0.86
^{168}Yb	14	0.0	0.001	0.0093	-0.0143	0.0	0.0	-1

Isotopes	$IBM2$ parameters in MeV unless χ , $\chi_\pi = -1.24$, $N = 6$							
	N$_\nu$	ε_d	κ	χ_ν	ζ_2	$\zeta_{1,3}$	C_ν^L	C_π^L
^{160}Y	4	0.6	-0.21	0.72	0.02	0.01	0.9,-0.2,-0.01	-0.9,-0.16,-0.07
^{162}Y	5	0.66	-0.2	0.7	-0.04	0.02	0.04,0.0,-0.022	0.0,0.0,-0.03
^{164}Y	6	0.48	-0.15	0.7	0.008	0.01	-0.04,0.01,0.0	0.0,0.0,-0.02
^{166}Y	7	0.3	-0.13	0.6	0.03	0.01	-0.7, 0.1,0.0	0.0,0.2,-0.06
^{168}Y	8	0.25	-0.09	0.6	0.03	0.02	-0.7, 0.55,0.04	0.0,0.0 ,-0.04

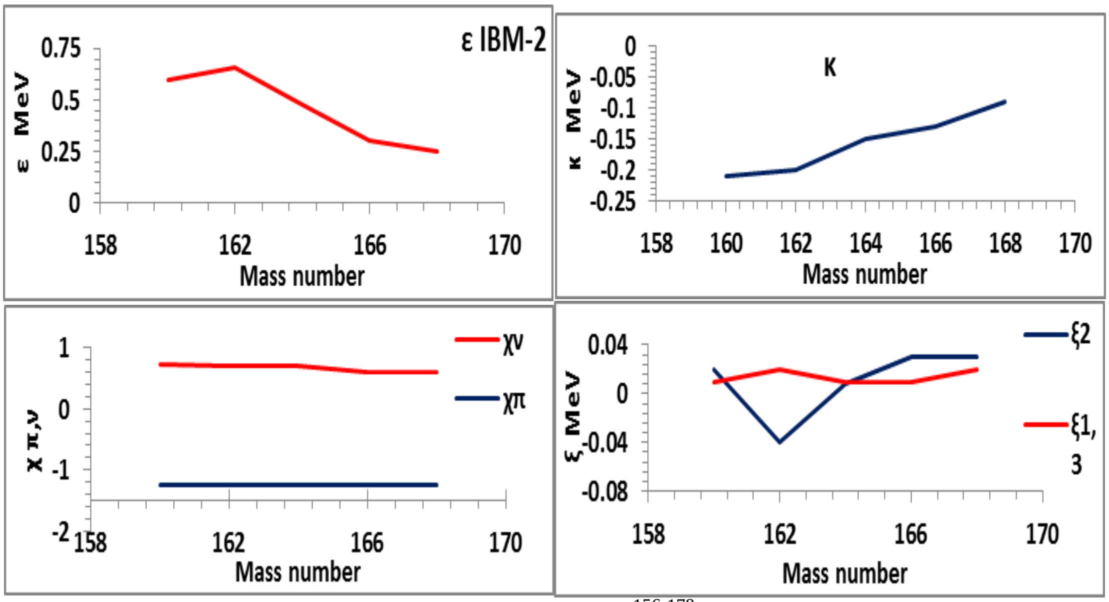


Figure 1- $IBM-2$ parameters $\varepsilon, \kappa, \chi_\pi, \chi_\nu, \zeta_2, \zeta_{1,3}$ for $^{156\text{-}178}$Yb isotopes as a function of mass numbers.

The first test to the dynamic symmetries was shown through theoretical and experimental energy levels and after a comparison with the standard values for the energy ratios [25]. Calculation of energy ratios of ($E4_1^+/E2_1^+$), ($E6_1^+/E2_1^+$), and ($E8_1^+/E2_1^+$) for all studied $^{160\text{-}168}$Yb isotopes is indicated in Figure-2. This leads to predict the nearest dynamic symmetries corresponding to the characteristics of one of the dynamic symmetries [26] or may possess transitional features between two or more symmetries. Figures-2 shows the energy ratios of ($E4_1^+/E2_1^+$), ($E6_1^+/E2_1^+$), and ($E8_1^+/E2_1^+$), respectively, as a function of mass numbers for Ytterbium isotopes. The levels of the calculated energy compared with the experimental data [13-17] for the $^{160\text{-}168}$Yb isotopes are shown in Figure-3.

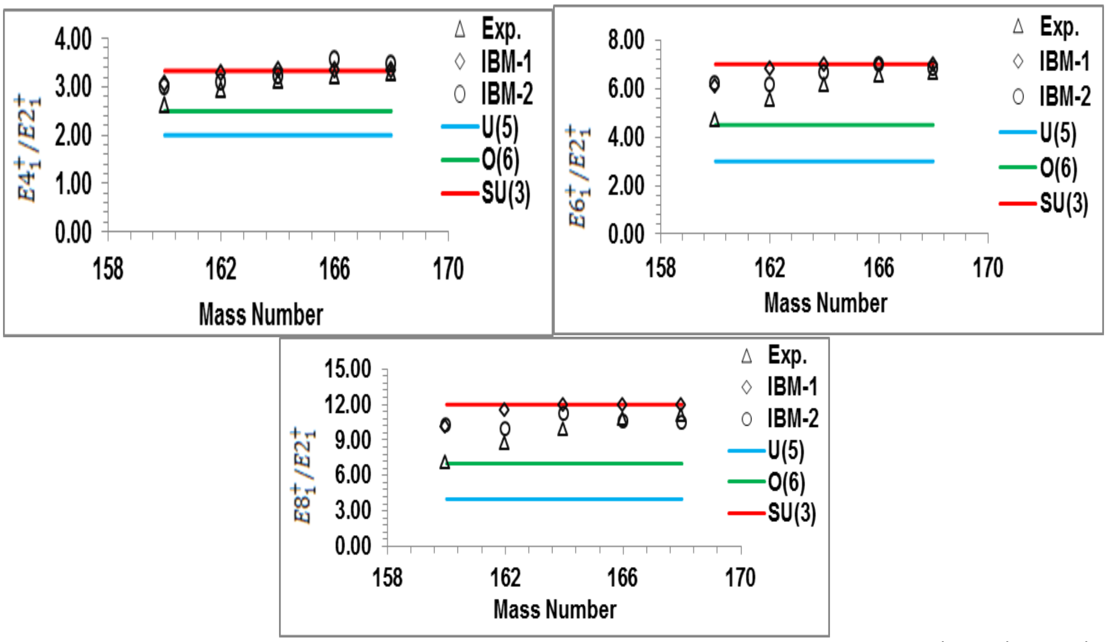
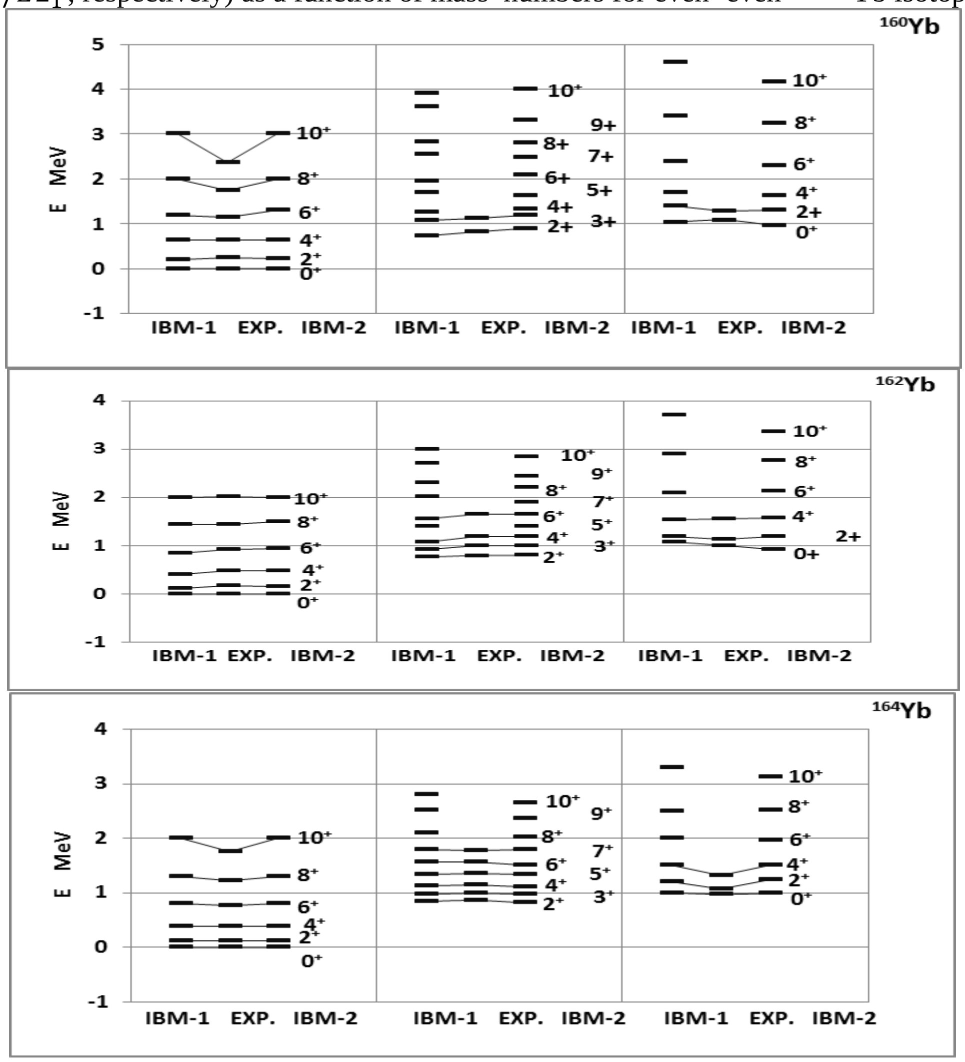


Figure 2-The experimental [13-17], theoretical, and standard [25] energy ratios ($E4_1^+/E2_1^+$, $E6_1^+/E2_1^+$, and $E8_1^+/E2_1^+$, respectively) as a function of mass numbers for even -even $^{160-168}$Yb isotopes.



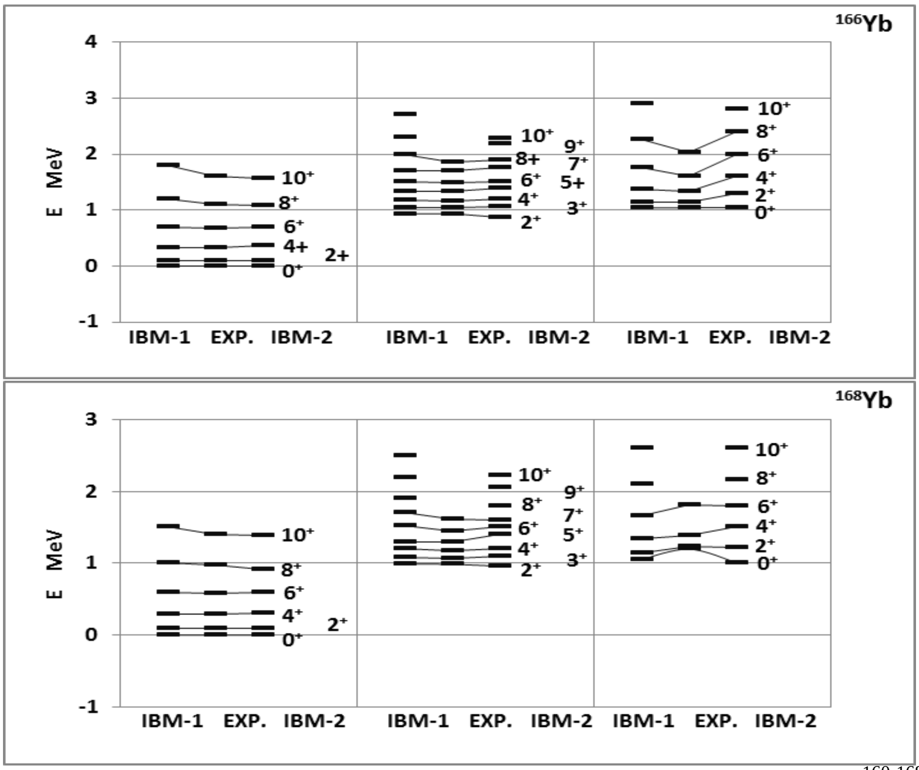


Figure 3-Comparison between experimental [13-17] and calculated energy levels for $^{160\text{-}168}$Yb isotope.

The effective boson charges estimated from equations (4) and (12) were used in $IBM-1$ and $IBM-2$ to calculate the reduced electric quadrupole transition probability $B(E2)$ that was compared with the experimental values [13-22] for $^{160\text{-}168}$Yb isotopes, as listed in Tables-(2, 3).

Table 2-The used effective boson charges in $IBM-1$ and $IBM-2$ to calculate $B(E2)$ transition for $^{160\text{-}168}$Yb isotopes.

Isotopes	The effective boson charges (eb)			
	IBM-1		IBM-2	
	E2SD*	E2DD	e_ν	e_π
^{160}Yb	0.113	-0.132	0.012	0.19
^{162}Yb	0.131	-0.111	0.033	0.21
^{164}Yb	0.134	-0.112	0.03	0.23
^{166}Yb	0.121	-0.1464	0.04	0.24
^{168}Yb	0.116	-0.152	0.031	0.25

*$E2SD$ and $E2DD$ are $IBMT$ parameters, where α_2 and β_2 are the boson effective charges for $IBM-1$, $E2SD=\alpha_2, E2DD=\sqrt{5}\beta_2$, where $\beta_2=\frac{-0.7}{\sqrt{5}}\alpha_2, \frac{-\sqrt{7}}{2}\alpha_2,$ and $\beta_2=0$ in $U(5), SU(3)$, and $O(6)$, respectively.

Table 3-Calculated reduced electric quadruple transitions probability $B(E2)$ in unit (e^{2}b^2) and electric quadruple moment of 2_1^+ state in unit (eb) in comparison with the experimental values [13-22] for $^{160\text{-}168}$Yb isotopes.

Isotopes	B(E2) (e^{2}b^2) ^{160}Yb			B(E2) (e^{2}b^2) ^{162}Yb			B(E2) (e^{2}b^2) ^{166}Yb		
$J_i^+ \to J_f^+$	$Exp.$	IBM	IBM	$Exp.$	IBM	IBM	$Exp.$	IBM	IBM
$2_1 \to 0_1$	0.48	0.48	0.482	0.72	0.728	0.72	0.906	0.906	0.906
$4_1 \to 2_1$	0.67	0.681	0.677	1.1	1.028	1.027	1.36	1.27	1.36
$6_1 \to 4_1$	0.728	0.728	0.73	1.001	1.103	0.097	1.447	1.37	1.442
$8_1 \to 6_1$	0.761	0.719	0.68	1.3	1.107	1.073	1.44	1.38	1.45
10_1	0.464	0.674	0.584	0.944	1.06	0.98	1.35	1.34	1.34
$0_2 \to 2_1$	--	0.0022	0.0035	--	0.0004	0.0006	--	0.0002	0.0027

$2_2 \rightarrow 0_2$	--	0.049	0.05	--	0.0408	0.09	--	0.026	0.06
$2_2 \rightarrow 2_1$	--	0.0732	0.07	--	0.0322	0.056	--	0.023	0.026
$4_2 \rightarrow 2_2$	--	0.233	0.177	--	0.35	0.4	--	0.43	0.48
$3_1 \rightarrow 2_2$	--	0.523	0.572	--	0.95	0.826	--	1.257	1.251
$3_1 \rightarrow 4_3$	--	0.21	0.25	--	0.0713	0.075	--	0.0164	0.0146
$5_1 \rightarrow 3_1$	--	0.326	0.29	--	0.53	0.57	--	0.672	0.7
$7_1 \rightarrow 5_1$	--	0.417	0.409	--	0.69	0.7	--	0.891	0.896
$9_1 \rightarrow 7_1$	--	0.425	0.418	--	0.72	0.8	--	0.947	0.98
$5_1 \rightarrow 4_2$	--	0.230	0.325	--	0.455	0.415	--	0.629	0.663
$7_1 \rightarrow 6_2$	--	0.116	0.102	--	0.227	0.24	--	0.321	0.37
$9_1 \rightarrow 8_2$	--	0.064	0.059	--	0.124	0.159	--	0.179	0.23
$4_2 \rightarrow 4_1$	--	.058	0.069	--	0.036	0.031	--	0.027	0.0229
$3_1 \rightarrow 1_1$	--	--	0.004	--	--	0.0158	--	--	0.006
$1_1 \rightarrow 2_1$	--	--	0.0201	--	--	0.0052	--	--	0.0035
$1_1 \rightarrow 2_2$	--	--	0.0029	--	--	0.0142	--	--	0.0222
$Q_{2_1^+}(eb)$	--	-1.77	-1.9	--	-2.2	-2.2	--	-2.5	-2.48

Isotopes	B(E2) (e^2b^2) ^{166}Yb			B(E2) (e^2b^2) ^{168}Yb		
$J_i^+ \rightarrow J_f^+$	$Exp.$	$IBM-1$	$IBM-2$	$Exp.$	$IBM-1$	$IBM-2$
$2_1 \rightarrow 0_1$	1.035	1.035	1.036	1.15	1.15	1.15
$4_1 \rightarrow 2_1$	1.47	1.46	1.47	--	1.62	1.64
$6_1 \rightarrow 4_1$	1.57	1.57	1.56	--	1.757	1.759
$8_1 \rightarrow 6_1$	1.73	1.58	1.71	--	1.78	1.774
$10_1 \rightarrow 8_1$	1.68	1.55	1.66	--	1.755	1.722
$0_2 \rightarrow 2_1$	--	0.00012	0.0025	--	0.000083	0.0016
$2_2 \rightarrow 0_2$	--	0.011	0.122	--	0.0095	0.0025
$2_2 \rightarrow 2_1$	--	0.0024	0.0027	0.042	0.0053	0.0422
$4_2 \rightarrow 2_2$	--	0.483	0.472	--	0.546	0.576
$3_1 \rightarrow 2_2$	--	1.462	1.338	--	1.655	1.511
$3_1 \rightarrow 4_3$	--	0.0015	0.0022	--	1.59	1.57
$5_1 \rightarrow 3_1$	--	0.762	0.793	--	0.863	0.823
$7_1 \rightarrow 5_1$	--	1.014	1.103	--	1.155	1.123
$9_1 \rightarrow 7_1$	--	1.085	1.083	--	1.246	
$5_1 \rightarrow 4_2$	--	0.767	0.675	--	0.874	0.729
$7_1 \rightarrow 6_2$	--	0.415	0.372	--	0.479	0.462
$9_1 \rightarrow 8_2$	--	0.246	0.25	--	0.29	0.21
$4_2 \rightarrow 4_1$	--	0.003	0.008	--	0.00066	0.00059
$3_1 \rightarrow 1_1$	--	--	0.0034	--	--	0.0169
$1_1 \rightarrow 2_1$	--	--	0.0219	--	--	0.0283
$1_1 \rightarrow 2_2$	--	--	0.0016	--	--	0.0067
$Q_{2_1^+}(eb)$	--	-2.7	-2.68	--	-2.87	-2.79

The surfaces of the potential energy as a function of β along with the contour diagrams for $^{160\text{-}168}$Yb isotopes that have been calculated from equation (5) using IBMP computer code are presented in Figure-4.

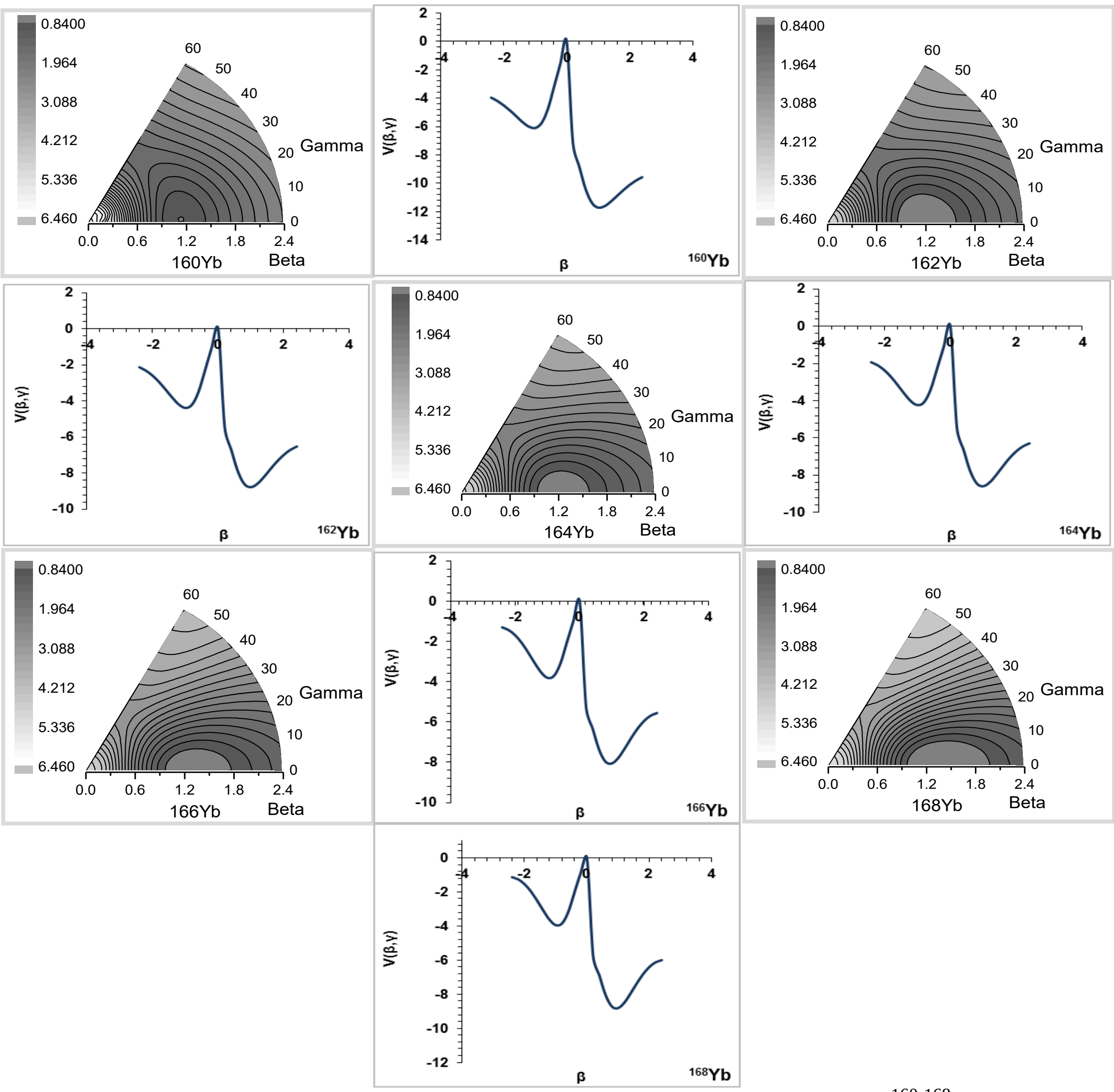


Figure 4-The potential energy surface $V(\beta,\gamma)$ as a function of β for even- even $^{160\text{-}168}$Yb isotopes with its $\gamma - \beta$ plot.

4. Conclusions

There are two approaches of theoretical nuclear models ($IBM-1$ and $IBM-2$) which are were used to predict the behavior of $^{160\text{-}168}$Yb nuclei. The most striking feature of $^{160\text{-}168}$Yb medium heavy even- even nuclei of the level structure at low excitation energy is the occurrence of collective quadruple excitation near the line of stability. These excitations may be studied in a variety of ways, only few of which are applicable to nuclei far from the stability neutron deficient nuclei in the same region. Moving from $\gamma-$unstable to the $SU(3)$ leg of the symmetry triangle clearly indicates the gradual transition of the properties of these nuclei from the $\gamma-$unstable features to the rotational features. However, adding a pairing parameter to $IBM1$ Hamiltonian has a very slight effect on this feature, but does raise the β band since it represents a symmetry breaking such as in $O(6)$. This applies to the experimental decay scheme of $^{160\text{-}168}$Yb isotopes. In $IBM-2$, the proton and neutron quadruple deformation parameters χ_π and χ_ν were equal to -1.24 and about 0.7, respectively, which supports the same idea as shown in the energy ratios that are being transitioned gradually from $\gamma-$unstable $O(6)$ towards rotational $SU(3)$ features. The values of the calculated reduced electric quadrupole transition probability and quadruple electrical transitions in $^{160\text{-}168}$Yb clearly show the transitional characteristics of these nuclei between $O(6)$ and $SU(3)$. A contour plot of $V(\beta,\gamma)$ for $^{160\text{-}168}$Yb isotopes showed the minimum potential that occurs at approximately $\beta = 1, \gamma = 60^o$ for all nuclei potential, which implies that the $^{160-168}$Yb isotopes have prolate shapes; they also indicate a good agreement with the typical axial symmetry of $O(6) - SU(3)$ limits.

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