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# Effect of the Altitudes and Eccentricity of the Initial Orbit on Satellite Transition Efficiency

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**Abstract**

This research dealt with choosing the best satellite parking orbit and then the transition of the satellite from the low Earth orbit to the geosynchronous orbit (GEO). The aim of this research is to achieve this transition with the highest possible efficiency (lowest possible energy, time, and fuel consumption with highest accuracy) in the case of two different inclination orbits. This requires choosing a suitable primary parking orbit. All of the methods discussed in previous studies are based on two orbits at the same plane, mostly applying the circular orbit as an initial orbit. This transition required the use of the advanced technique of the Hohmann transfer method for the elliptical orbits, as we did in an earlier research, namely the transition from the perigee of the initial orbit to the final orbit and then conducting the rotation of the orbit plane to match the plane for the desired final orbit.

The effect of the perigee altitude of the initial orbit on the transition efficiency calculated for the values between 300 to 3000 km. It was found that increasing the altitude reduces the energy and fuel needed for transportation, but the time required for transportation increases, into account that the increased height of the initial or parking orbit also implies the requirement of higher energy to reach it.

The effects of eccentricity (e) values of the initial orbit between 0.01 to 0.2 on the transition efficiency were calculated. It was found that the increase in (e) reduces the energy and fuel, but does not affect the time, required for transportation.

**Keyword**: transfer orbit, Hohman methods, orbital elements, parking orbit, and perturbations.

# تأثير الارتفاعات والشذوذ المركزي للمدار الأولي على كفائة النقل للأقمار الصناعية

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**الخلاصة**

تناول هذا البحث اختيار مدار الاستراحة الأفضل ثم نقل القمر الاصطناعي من حضيض مدار قطع ناقص واطئ الى المدار المتزامن الارضي (GEO) . والهدف من البحث تحقيق هذا الانتقال بأفضل كفاءة ممكنة ( أقل طاقة ممكنة وأقل زمن وأقل وقود وأعلى دقة ) في حالة مدارين مختلفين بالميل. وهذا يتطلب اختيار مدار وقوف ابتدائي مناسب. كل الطرائق المتناولة سابقا تعد المدارين لهما نفس الميل, وأغلب البحوث تعد المدار الابتدائي هو دائري. تطلب هذا الانتقال استخدام تقنية مطورة من طريقة انتقال هوهمان الخاصة بالمدارات البيضوية قمنا بها في بحث اخر, وهي الانتقال من حضيض المدار الابتدائي الى المدار النهائي ثم

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اجراء تدوير مستوى المدار ليتطابق مع المستوى للمدار النهائي المطلوب. تم حساب تأثير ارتفاع حضيض المدار الابتدائي للقيم بين 300 الى 3000 كم على كفاءة النقل وقد تبين ان زيادة الارتفاع يقلل من الطاقة والوقود اللازمين للنقل كما يزيد الزمن اللازم للانتقال, ولا ننسى ان ارتفاع المدار الابتدائي يعني أيضا طاقة أعلى للوصول اليه. تم حساب تغير الشذوذ المركزي (e) للمدار الابتدائي بين 0.01 الى 0.2 على كفاءة النقل وقد تبين ان زيادة (e) يقلل من الطاقة والوقود اللازمين للنقل ولا يؤثر على الزمن اللازم للنقل .

## 1. Introduction

Satellites undergo several maneuvers to correct their location size, and shape, as well as the inclination of the orbit [1]. One of the classification methods of orbits is that classifying them according to altitude into the following: Low Earth Orbits (LEO), in which the altitude is lower than 2000 km from earth surface, Mid Earth Orbits (MEO), with an altitude range of 2000-35000 km, and High Earth Orbits (HEO), with an altitude of higher than 35000 km [2, 3]. The shape of the orbit is classified by (e) into:- circular ($e = 0$), elliptical ($e <1$), parabolic ($e =1$) and hyperbolic ($e >1$) [4]. There are many types of transition using the Hohmann transfer method; the first is when an elliptical orbit is used to transfer between circular orbits of different radii in the same plane. In general, a transition orbit uses the least amount of energy possible to travel. It transfers from initial to final orbits during less than one period [5]. The second type of Hohmann transfer method is that between coaxial elliptical orbits. This method contains two-impulsive maneuvers that occur between elliptical orbits, called coaxial elliptical orbits. The transition occurs from the original orbit to the target orbit in two cases, either from perigee or from apogee [6]. The third type is the Bi-Elliptic Transfer method, when the transfer occurs from elliptical initial orbit to elliptical final orbit by using Bi-Elliptic Transfer, as in the following cases:

a-The first pulse is in the perigee of the initial orbit, the second pulse is in the apogee of the transfer orbit, and the third pulse is in the final orbit [7].

b-The first pulse is in the apogee of the initial orbit, the second pulse is in the apogee of the transfer orbit, and the third pulse is in the final orbit [7].

In this work, a technique of modified Hohmann transfer method between coaxial elliptical orbits was used for the transmission of the satellite [8]. This research applied the process of transiting the satellite by an elliptical orbit with ($e$ =0.01, to 0.2 step 0.01) to geostationary orbit (42164 km) through an elliptical transition orbit, from initial orbit with altitude ( 300, 400, 500, … 3000) . In this research, we transfer the satellite from the perigee of the initial orbit and then rotate it at the apogee of the transfer orbit or at the final orbit. There are some perturbations that affect the satellite orbit, such as the atmospheric drag perturbation, solar radiation pressure, tidal friction effect, lunisolar gravitational attraction, and acceleration due to the non-spherical shape of the Earth. Our transfer orbit was not significantly affected by the perturbation effects because the transition occurs in less than one orbit . Therefor the perturbations are very small and can be neglected [9].

**2.** Orbital elements [10,11,12]:

The movement of any two objects under the influence of the force of gravity shared between them (such as the movement of satellites around the Earth) can be described mathematically through three differential equations of second order. From the integration of these equations, we obtain

Six parameters that are called state vectors are used to obtain the orbital elements (Keplerian elements), which include two groups:

a- Dimension and shape elements: that determines the orbit's dimensions.

1- Semi-major axis (a): which determines the size of the orbit.

2- Eccentricity (e): Which determines the shape of the orbit.

3- the time after the perigee passage (t): It represents the relationship between the positions of the satellite within its orbit and the time.

b- Orientation element: Which determines the orbit’s position in space.

1- Inclination angle (i): It is the angle between the orbit plane and the celestial equatorial plane. The value of this angle ranges between (0-180 degree) [10].

2- Right ascension of the ascending node ($\Omega$): It is the angle measured from the direction of the vernal equinox (which represents the intersection of the equator with ecliptic circle) to the ascending node (which is the intersection point of the satellite's orbit heading from south to north of the equator). The value of this angle ranges between (0-360 degree) [13].

3- Argument of perigee (w): It represents the angular displacement from the ascending node to the line between the center of the Earth and the perigee, with a value of (0-360) degree.

3. The theory of transfer technique:

The issue of the movement of two bodies (the Keplarian movement) is an approximate case in calculating the location and velocity of satellites and their orbital elements because they relate to movement in isolation from any external influence.

Satellite motion without perturbation on the elliptical orbit is calculated using the following equation [13].

$$\ddot{r} = -\frac{\mu}{r^3}\vec{r} \tag{1}$$

where µ is a constant resulting from G Me= 398602.4415 at r in km unit where G is the gravitational constant = 6.67408 × $10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$ and Me is the mass of the Earth= 5.972E24 kg $\vec{r}$ is the positon vector for the satellite, while r is the distance between the earth center and satellite at time (t).

The satellite motion with perturbation on the elliptical orbit is calculated using the equation (2) [3,14].

$$\ddot{r} = -\frac{\mu}{r^3}\vec{r} + \ddot{r_p} \tag{2}$$

Where $\ddot{r_p}$ is the perturbation acceleration, which can be written as follows [15].

$\ddot{r_p} = \ddot{r}_E + \ddot{r}_s + \ddot{r}_M + \ddot{r}_{sp} + \ddot{r}_A + \ddot{r}_{drag}$

where $\ddot{r}_E$ is the non-spherical earth, $\ddot{r}_s$ and $\ddot{r}_M$ are the sun's and the moon's acceleration attraction on the satellite, respectively, $\ddot{r}_{sp}$ and $\ddot{r}_A$ are the accelerations to direct and Earth-reflected solar radiation pressures, respectively, and $\ddot{r}_{drag}$ is the Earth atmospheric drag. The latter perturbation is accumulated and important for the low Earth orbit because of the high density of air [3].

The solution of the Satellite motion without perturbation on the elliptical orbit are [16]:

$$r = \frac{h^2}{\mu}\frac{1}{1+e\cos(f)} \tag{3}$$

$f$: True anomaly angle (0,360°), h: angular momentum per unit mass.

From equation (3) the perigee and apogee distance $r_p$ , $r_a$ are calculated at $f$ = 0,180 degree [16].

$$r_a = a\,(1+e) \quad , \quad r_p = a\,(1-e) \tag{4}$$

Where a is the semi-major axis of the elliptical orbit.

The Eccentricity of the orbit can be calculated as the following [16, 17]:

$$e = \frac{ra-rp}{ra+rp} \tag{5}$$

The angular momentum can be calculated from following [16]:

$$h = \sqrt{2\mu}\sqrt{\frac{ra\,rp}{ra+rp}} \qquad or \qquad h = \sqrt{\mu * a(1-e^2)} \tag{6}$$

The velocity for elliptical orbit can be calculated from following [16]:

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \tag{7}$$

The semi major axis (a) calculated from equation (4)

For circular orbit (r=a) the velocity at equation (7) become:

$$v^2 = \frac{\mu}{r} \tag{8}$$

The transition time at µ=398602.4415 and the semi-major axis (a) in km. for half period in second can be calculated from the following equation [9]:

$$Ttran = \frac{1}{2}\left(\frac{2\pi}{\sqrt{\mu}}a^{\frac{3}{2}}\right) \qquad \text{(in sec )} \tag{9}$$

The mass of satellite is not constant through transition orbit because mass burn. This can be calculated as the following [6,9]:

$$\frac{\Delta m}{m} = 1 - e^{-\frac{\Delta v}{Isp\,go}} \tag{10}$$

Where:

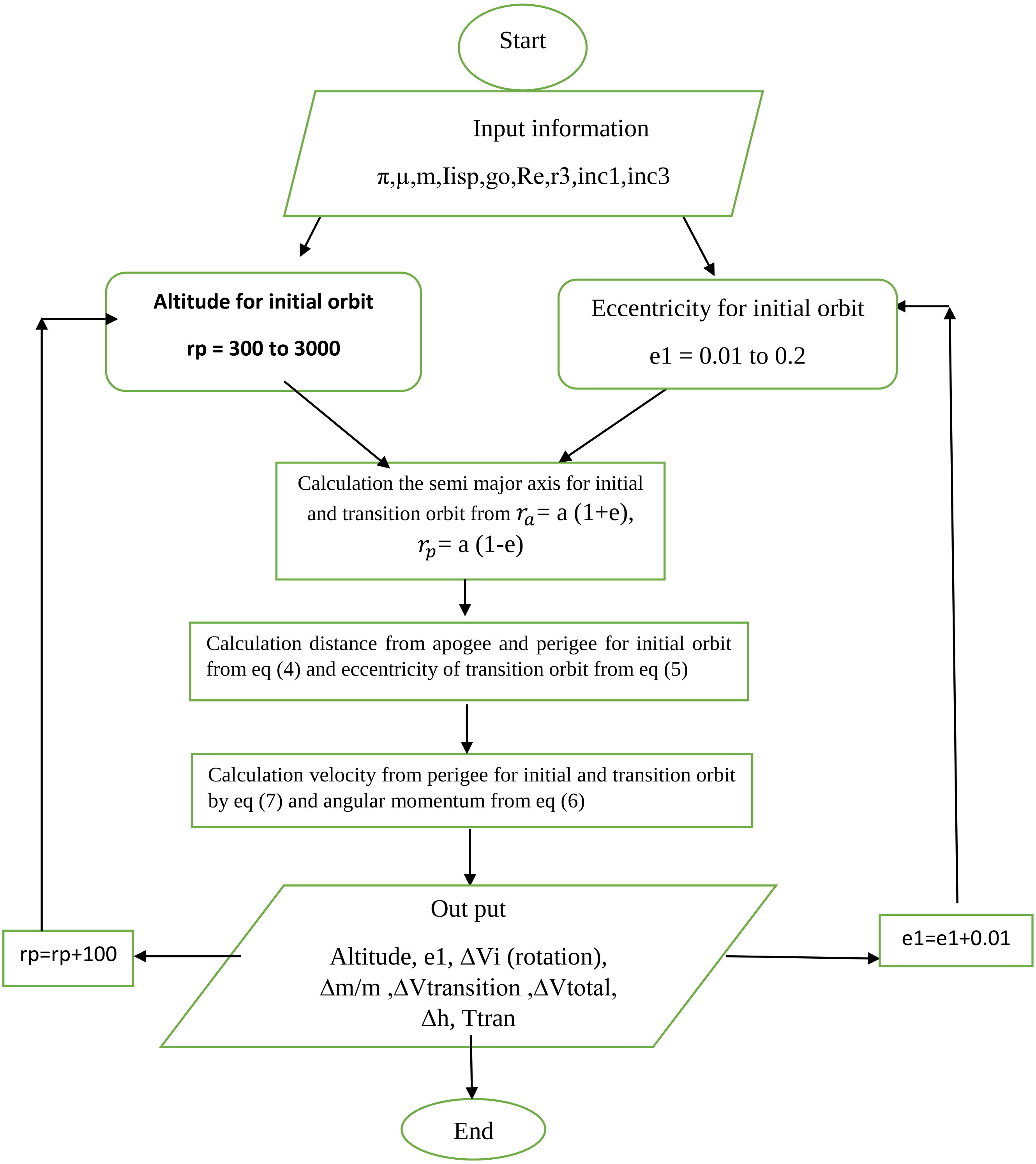
Δ m is the consume mass for propellant.

$g_o$ is the gravity standard of acceleration.

Isp is the impulsive specific of the propellants.

Calculation of the impulse transition velocity (ΔVhohman) and rotational velocity required (ΔVi) and total velocity required (ΔVt) from the following equations by us:

ΔVhohman= (ΔV3+ ΔV2) (11a)
From [16] the required velocity to rotation the orbit is:
ΔVi = 2*(v3 or vp2)* sin ((Δi/2)) (11b)
ΔVt = (ΔVhohman + ΔVi). (11c)
In equation (11b) use vp2 at rotation before transition and use v3 at transition before rotation.
The angular momentum needed to transition is:
Δh= h3-h1 (12)
3. The flowchart of program:
1 is the initial orbit. 2 is the transition orbit. 3 is the final orbit



**3.** Results and discussion:

The best parking orbit is the more stability elliptical orbit, have less effect of perturbations. It chosen depended on references [9 ,18 ]. The best orbit have inclination 63.5 degree and normalized orbit, which have a suitable value of eccentricity with semi major axis (a), a proportional with the perigee altitude ($r_p$). When the $r_p$ decrease the eccentricity (e) must be also decrease, the best value of e=0.001 at $r_p$< 300 km. where the suitable parking orbit have e=0.01 at $r_p$=300km. e can be increased 0.01 when $r_p$ increased 200 km.

For any value of rotation angle with neglict the perturbations of the transition orbit,these perturbations are very small through transition because the transtion happen through haif revolution only. The best techniqe to transition is transition and then rotation orbit [8], in this work the alttude of initial in km orbit from 6678 to 9678 step 100 and the Eccentricity of initial orbit from 0.01 to 0.2 step 0.01, the program was designed by us to get the output date (ΔVhohman, ΔVi, ΔVtotal, Δm/m, Ttran, Δh).

4.1 At constant rp=300 km and different e:

The variation of the required velocity to make a transition of the orbit is decreases with the Eccentricity increases because the vp1 is greater, the total velocity also decreases with the Eccentricity increases due as the increases (e) for initial orbit The velocity increases at perigee while the final orbital velocity is constant so the difference between them is constant as show in Figure-1. Since the velocity of transmission decreases with increasing of Eccentricity this mean the energy needed to transition the satellite will be reduced by an increase (e), in addition to the amount of fuel required for transition will decrease with increase (e) as show in Figure-2. The process of transition of the satellite will happen at perigee, so the transition time will be constant as show in Figure-3. The variation of the angular momentum for transition from the selected initial orbits to the final orbit is linear decreases with eccentricity of the initial orbit as show in Figure-4, it also has the same behavior with the perigee high of the initial orbit as shown in Figure-5. This behavior of the needed angular momentum like the behavior of the velocity needed to transition because the relationship between them is directly proportional, or h= r x v=constant, h1 is depend on e and a as in equation (6).

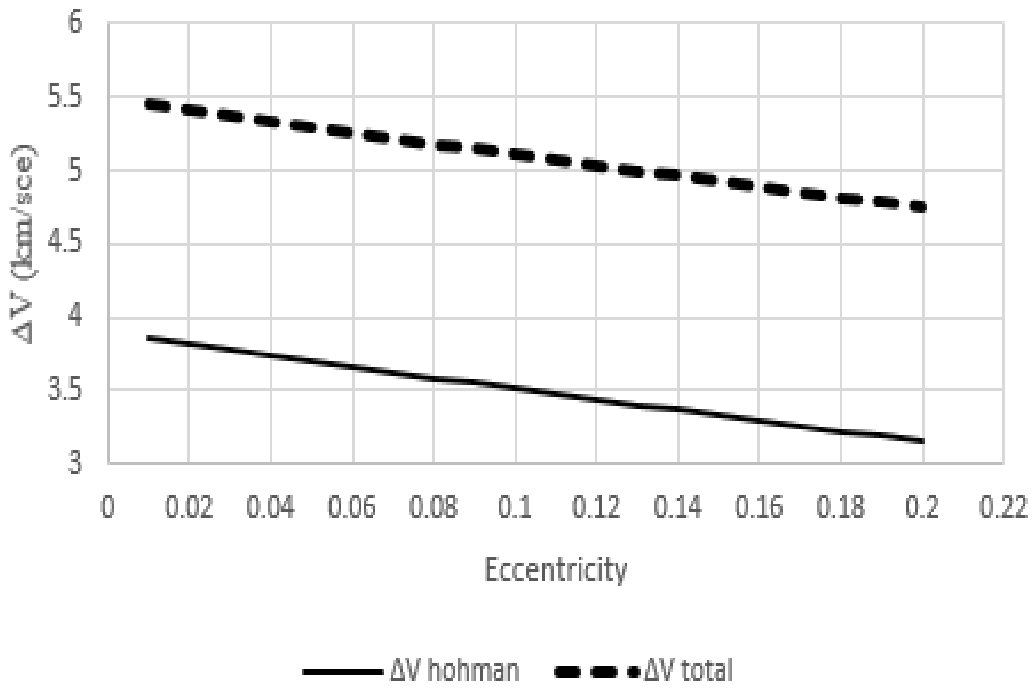



**Figure 1-** ΔVtotal and ΔVhohman with Eccentricity.

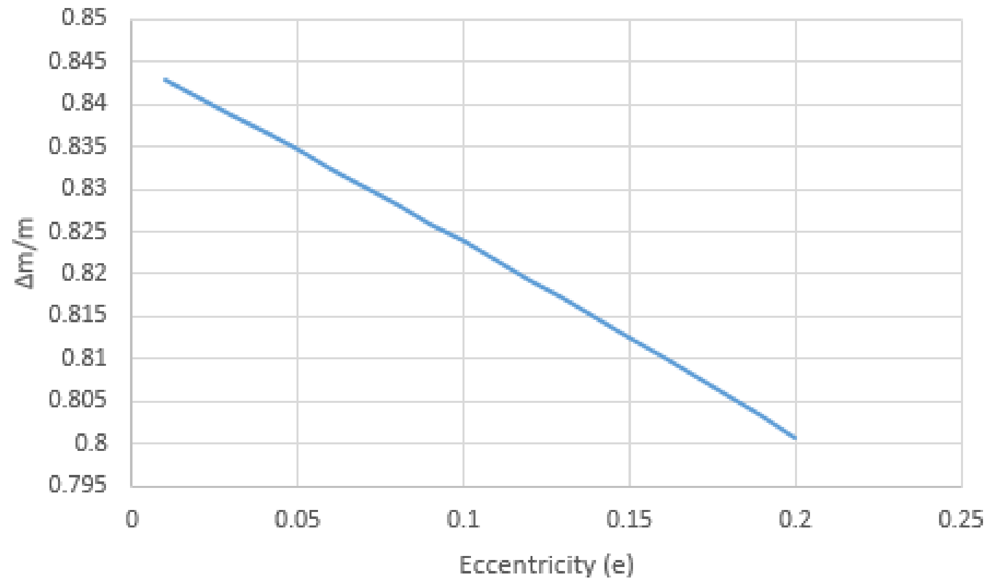



**Figure 2-**Δm/m with Eccentricity.

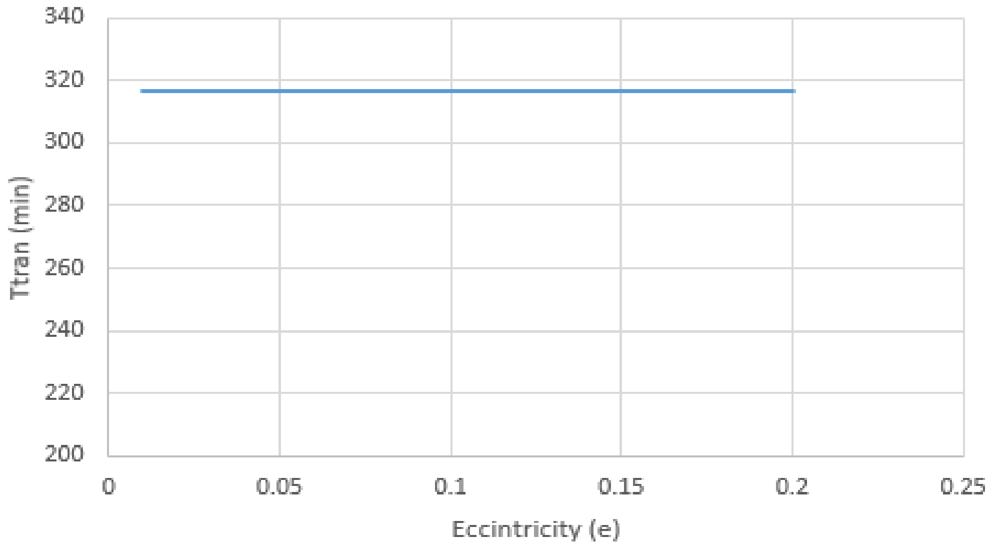



**Figure 3-** the transition time with Eccentricity.

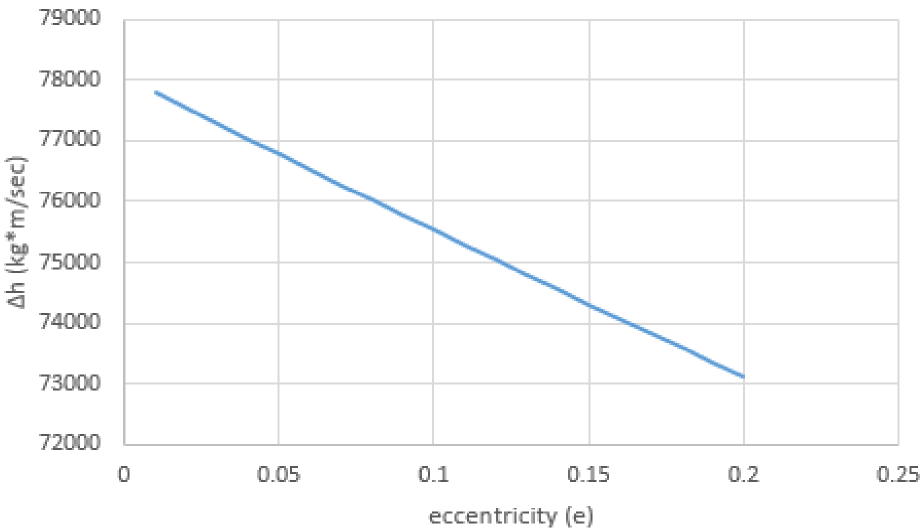



**Figure 4-** the Δh with Eccentricity.

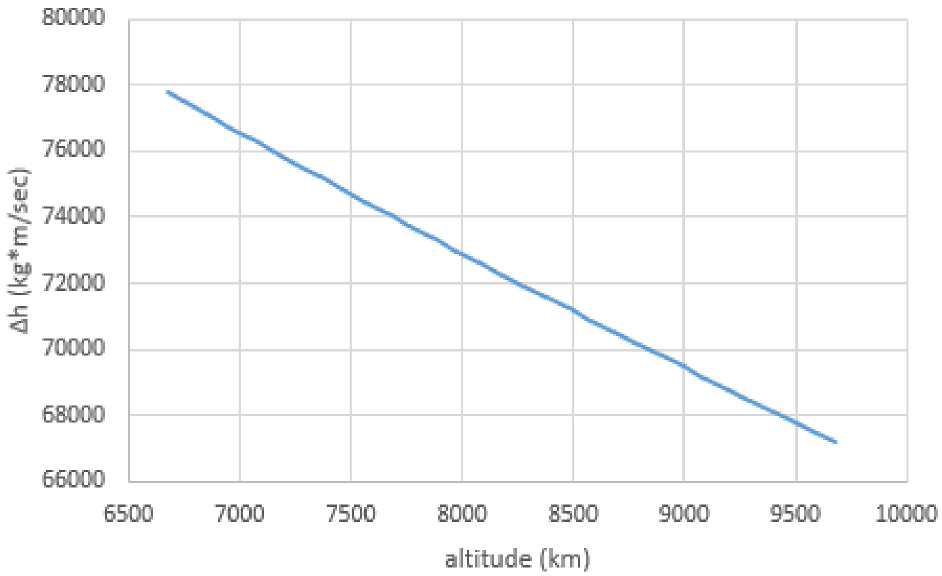



**Figure 5-the** Δh with altitude of initial orbit.

The velocity required to transmit the satellite is decrease when the altitude of initial orbit is increasing, also the total velocity is decrease with altitude of initial orbit the reason is that the higher initial orbit, the lower velocity required to reach the final orbits as show in Figure-6. Since the velocity decrease by increasing the height of the initial orbit, this means that the amount of energy required will also decrease, and therefore the rate of fuel required for the satellite transfer process to the final orbit will be decrease as show in Figure-7. We notice that the transmit time proportional with rp or altitude. The reason for this is that the velocity of the satellite at launch (perigee orbital transition) is less and less more after launch, but it increases when it reaches the final orbit because the satellite will receive another boost there as show in Figure-8.

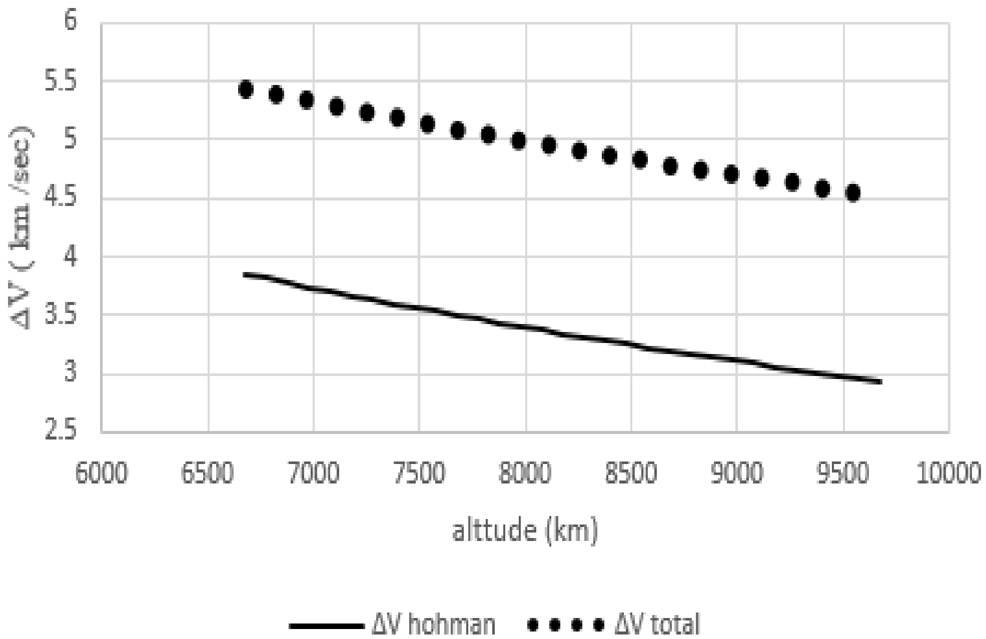



**Figure 6-** ΔV hohman and ΔV total with altitude of initial orbit.

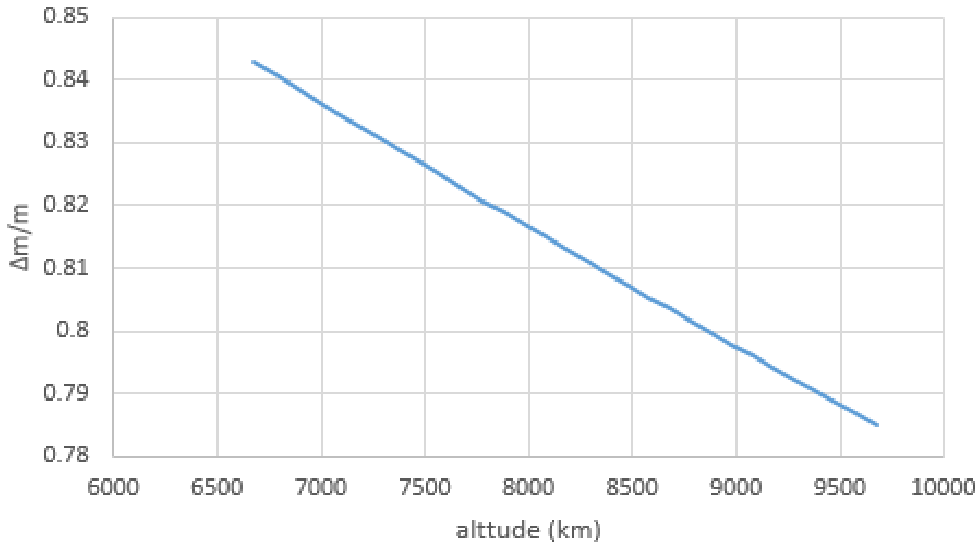



**Figure 7-**the Δm/m with altitude of initial orbit.

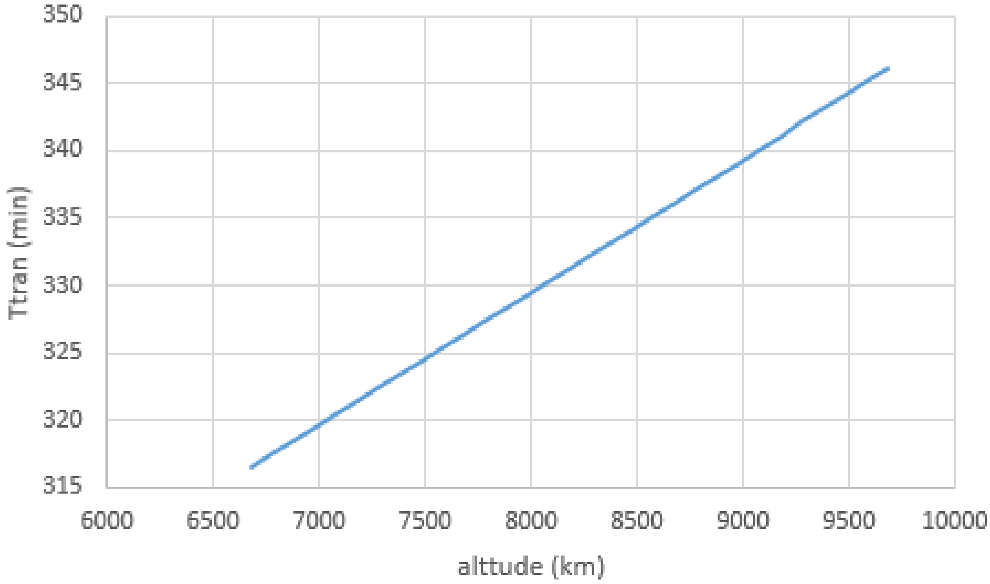



**Figure 8-**the time transition with altitude of initial orbit.

**Conclusions:**

1- The correct selection of the parking orbit makes for an easier and accurate transition to the desired orbit.

2- The increase e1 for initial orbit causes a decrease in the energy needed to transfer to a higher orbit, a decrease in the fuel needed for transportation, and no change in transfer time

3- The increase in the perigee's height for the initial orbit causes a decrease in both the energy needed to transfer to a higher orbit and the fuel needed for that transfer with an increase in the time required.

4- The momentum needed to move the satellite Δh= h3-h1 Since h3 is constant then Δh decrease when h1 increase, h1 increase when altitude and e1 is increase.

5- The best and easier transition happen at high initial orbit with suitable eccentricity.

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