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Weak Forms of Soft (1, 2)*-Omega Open Sets in Soft Bitopological Spaces

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Abstract
In this paper, we introduce and study new classes of soft open sets in soft bitopological spaces called soft (1,2)*-omega open sets and weak forms of soft (1,2)*-omega open sets such as soft (1,2)*-α-ω-open sets, soft (1,2)*-pre-ω-opensets, soft (1,2)*-b-ω-open sets, and soft (1,2)*-β-ω-open sets. Moreover; some basic properties and the relation among these concepts and other concepts also have been studied.

Keywords: Soft (1,2)*-ω-open set, soft (1,2)*-α-ω-open set, soft (1,2)*-pre-ω-open set, soft (1,2)*-b-ω-open set and soft (1,2)*-β-ω-open set.

الصيغ الضعيفة للمجموعات المفتوحة اوميكا-*(1,2) الميسرة في الفضاءات التبولوجية الثنائية الميسرة

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الخلاصة
في هذة البحث نحن قدمنا ودرسنا اصناف جديد من المجموعات المفتوحة الميسرة في الفضاءات التبولوجية الثنائية الميسرة أسميناها بالمجموعات المفتوحة اوميكا-*(1،2) الميسرة والصيغ الضعيفة للمجموعات المفتوحة اوميكا-*(1,2) الميسرة مثل المجموعات المفتوحة - α-ω-*(1,2) الميسرة والمجموعات المفتوحة- pre-ω-*(1,2) الميسرة و المجموعات المفتوحة - b-ω-*(1,2) الميسرة و المجموعات المفتوحة - β-ω-*(1,2) الميسرة. بالاضافة الى ذلك بعض الخصائص الاساسية والعلاقة بين هذة المفاهيم ومفاهيم اخرى ايضا قد درست.

Introduction

The concept of soft set theory was firstly introduced by Molodtsov [1] in 1999. He successfully applied the soft set theory into several directions such as Operations research, Game theory, Theory of Probability, Theory of measurement, Smoothness of functions and Riemann integration, etc., Shabir, and Naz [2] in 2011 introduced and investigated the notion of soft topological spaces. Senel and Çagman [3] in 2014 introduced and study soft bitopological spaces over an initial universe set with a fixed set of parameters. Senel and Çagman [4] in 2014 and Revathi and Bageerathi [5] in 2015 introduced and study soft (1,2)*-α-open sets, soft (1,2)*-pre-open sets, soft (1,2)*-semi-open sets, soft (1,2)*-b-open sets and soft (1,2)*-β-open sets in soft bitopological spaces respectively. In this paper we introduce and study new classes of soft open sets in soft bitopological spaces called soft (1,2)*-omega open sets and weak forms of soft (1,2)*-omega open sets such as soft (1,2)*-α-ω-open sets, soft (1,2)*-pre-ω-open sets, soft (1,2)*-b-ω-open sets, and soft (1,2)*-β-ω-open sets. The fundamental

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properties as well as the relationships between these concepts and other concepts also have been studied.

1. Preliminaries:

Throughout this paper U is an initial universe set, $P(U)$ is the power set of U, P is the set of parameters and $C \subseteq P$.

Definition (1.1) [1]:

A soft set over U is a pair (H, C), where H is a function defined by $H: C \rightarrow P(U)$ and C is a non-empty subset of P.

Definition (1.2) [6]:

A soft set (H, C) over U is called a soft point if there is exactly one $e \in C$ such that $H(e) = \{u\}$ for some $u \in U$ and $H(e') = \varphi \ \forall \ e' \in C \setminus \{e\}$ and is denoted by $\tilde{u} = (e, \{u\})$

Definition (1.3) [6]:

A soft point $\tilde{u} = (e, \{u\})$ is called belongs to a soft set (H, C) if $e \in C$ and $u \in H(e)$, and is denoted by $\tilde{u} \tilde{\in} (H, C)$.

Definition (1.4) [6]:

A soft set (H, C) over U is called countable if the set $H(e)$ is countable $\forall \ e \in C$. A soft set (H, C) is called uncountable if it is not countable.

Definition (1.5)[2]:

A soft topology on U is a collection $\tilde{\tau}$ of soft subsets of $\tilde{U}$ having the following properties:

(i) $\tilde{\varphi} \tilde{\in} \tilde{\tau}$ and $\tilde{U} \tilde{\in} \tilde{\tau}$.

(ii) If $(H_1, P), (H_2, P) \tilde{\in} \tilde{\tau}$, then $(H_1, P) \tilde{\bigcap} (H_2, P) \tilde{\in} \tilde{\tau}$.

(iii) If $(H_j, P) \tilde{\in} \tilde{\tau}, \forall \ j \in \boldsymbol{\Omega}$, then $\bigcup_{j \in \boldsymbol{\Omega}} (H_j, P) \tilde{\in} \tilde{\tau}$.

The triple $(U, \tilde{\tau}, P)$ is called a soft topological space over U. The members of $\tilde{\tau}$ are called soft open sets over U. The complement of a soft open set is called soft closed.

Definition (1.6)[3]:

Let U be a non-empty set and let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two different soft topologies over U.Then $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is called a soft bitopological space over U.

Definition (1.7) [3]:

A soft subset (H, P) of a soft bitopological space $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$ -open if $(H, P) = (H_1, P) \tilde{\bigcup} (H_2, P)$ such that $(H_1, P) \tilde{\in} \tilde{\tau}_1$ and $(H_2, P) \tilde{\in} \tilde{\tau}_2$. The complement of a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

The collection of all soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$ need not form a soft topology over U we can see by the following example:

Example (1.8): Let $U = \{1,2,3,4,5\}$ and $P = \{p_1, p_2\}$, and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{\mathbf{U}}, \tilde{\varphi}, (H_2, P)\}$ be soft topologies over U, where $(H_1, P) = \{(p_1, \{1,2,3\}), (p_2, \{U\})\}$ and $(H_2, P) = \{(p_1, \{3,4,5\}), (p_2, \{U\})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$.Since $(H_1, P) \tilde{\bigcap} (H_2, P) = \{(p_1, \{3\}), (p_2, \{U\})\} = (H, P)$, but (H, P) is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$. Thus $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is not soft topology over U.

Definition (1.9)[3]:

Let $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ be a soft bitopological space and $(H, P) \tilde{\subseteq} \tilde{U}$. Then:

(i) $\tilde{\tau}_1\tilde{\tau}_2 cl(H, P) = \tilde{\bigcap}\{(K, P) : (H, P) \tilde{\subseteq} (K, P)$ and (K, P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set in $\tilde{U}\}$ is called the soft $\tilde{\tau}_1\tilde{\tau}_2$-closure of (H, P).

(ii) $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(H, P) = \tilde{\bigcup}\{(O, P) : (O, P) \tilde{\subseteq} (H, P)$ and (O, P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}\}$ is called the soft $\tilde{\tau}_1\tilde{\tau}_2$-interior of (H, P).

Proposition (1.10)[7]: Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space, and (H,P), $(K,P)\tilde{\subseteq}\tilde{U}$. Then:
(i) $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)\tilde{\subseteq}(H,P)$ and $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)$.
(ii) If $\{(H_j,P)\}_{j\in\Omega}$ is a collection of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$, then so is $\bigcup_{j\in\Omega}(H_j,P)$.
(iii) If $\{(H_j,P)\}_{j\in\Omega}$ is a collection of soft $\tilde{\tau}_1\tilde{\tau}_2$-closed sets in $\tilde{U}$, then so is $\bigcap_{j\in\Omega}(H_j,P)$.
(iv) $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{U}$ and $\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set in $\tilde{U}$.
(v) (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-open iff $(H,P)=\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)$.
(vi) (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed iff $(H,P)=\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)$.
(vii) $\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))=\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)$ and $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P))=\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)$.
(viii) $\tilde{U}-\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)=\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\tilde{U}-(H,P))$ and $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(\tilde{U}-(H,P))=\tilde{U}-\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)$.
(ix) If $(H,P)\tilde{\subseteq}(K,P)$, then $\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(K,P)$.
(x) If $(H,P)\tilde{\subseteq}(K,P)$, then $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(K,P)$.

Definition (1.11): A soft subset (H,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called:
(i) Soft (1, 2)*-α-open [4] if $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)))$.
(ii) Soft (1, 2)*-pre-open [4] if $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))$.
(iii) Soft (1, 2)*-semi-open [4] if $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P))$.
(iv) Soft (1, 2)*-b-open [5] if $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))\tilde{\cup}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P))$.
(v) Soft (1, 2)*-β-open [5] if $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)))$.

2. Soft (1,2)*-Omega Open Sets

Definition (2.1):
A soft subset (H,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called soft (1,2)*- omega open (briefly soft (1,2)*-ω-open) if for each $\tilde{x}\tilde{\in}(H,P)$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (O,P) in $\tilde{U}$ such that $\tilde{x}\tilde{\in}(O,P)$ and $(O,P)-(H,P)$ is a countable soft set. The complement of a soft (1,2)*-ω-open set is called soft (1,2)*-omega closed (briefly soft (1, 2)*-ω-closed).

Clearly, every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1, 2)*-ω-open, but the converse in general is not true we can see in the following example:

Example (2.2): Let $U=\{1,2,3\}$ and $P=\{p_1,p_2\}$, and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{U\}),(p_2,\{1,2\})\}$ and $(H_2,P)=\{(p_1,\{U\}),(p_2,\{1,3\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{U}$. Thus $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space and $(H,P)=\{(p_1,\{U\}),(p_2,\{1\})\}$ is a soft (1,2)*-ω-open set in $\tilde{U}$, but is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open.

Definition (2.3):
Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space and $(H,P)\tilde{\subseteq}\tilde{U}$. Then:
(i) The soft (1,2)*-omega closure (briefly soft (1,2)*-ω-closure) of (H,P), denoted by (1,2)*-$\omega\mathrm{cl}(H,P)$ is the intersection of all soft (1,2)*-ω-closed sets in $\tilde{U}$ which contains (H,P).
(ii) The soft (1,2)*-omega interior (briefly soft (1,2)*-ω-interior) of (H,P), denoted by (1,2)*-$\omega\mathrm{int}(H,P)$ is
the union of all soft (1,2)*-ω-open sets in $\tilde{U}$ which are contained in (H,P).

Theorem (2.4): If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space, and $(H,P),(K,P)\tilde{\subseteq}\tilde{U}$. Then:
(i) $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega\mathrm{int}(H,P)\tilde{\subseteq}(H,P)$.
(ii) If $\{(H_j,P)\}_{j\in\Omega}$ is a collection of soft (1,2)*-ω-open sets in $\tilde{U}$, then so is $\bigcup_{j\in\Omega}(H_j,P)$.

(iii) $(1,2)^*\text{-}\omega\mathrm{int}(H,P)$ is the largest soft (1,2)*-ω-open set in $\tilde{U}$ which contained in (H,P).
(iv) (H,P) is soft (1,2)*-ω-open iff $(1,2)^*\text{-}\omega\mathrm{int}(H,P)=(H,P)$.
(v) $(1,2)^*\text{-}\omega\mathrm{int}\,((1,2)^*\text{-}\omega\mathrm{int}(H,P))=(1,2)^*\text{-}\omega\mathrm{int}(H,P)$.
(vi) $\tilde{U}\text{-}(1,2)^*\text{-}\omega\mathrm{int}(H,P)=(1,2)^*\text{-}\omega\mathrm{cl}\,(\tilde{U}\text{-}(H,P))$.
(vii) $\tilde{x}\,\tilde{\in}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)$ iff there is a soft (1,2)*-ω-open set (O,P) in $\tilde{U}$ s.t $\tilde{x}\,\tilde{\in}\,(O,P)\,\tilde{\subseteq}\,(H,P)$.
(viii) If $(H,P)\,\tilde{\subseteq}\,(K,P)$, then $(1,2)^*\text{-}\omega\mathrm{int}(H,P)\,\tilde{\subseteq}\,(1,2)^*\text{-}\omega\mathrm{int}(K,P)$.
(ix) $(1,2)^*\text{-}\omega\mathrm{int}((H,P)\,\tilde{\cap}\,(K,P))\,\tilde{\subseteq}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)\,\tilde{\cap}\,(1,2)^*\text{-}\omega\mathrm{int}(K,P)$.
(x) $\bigcup_{j\in\Omega}(1,2)^*\text{-}\omega\mathrm{int}(H_j,P)\,\tilde{\subseteq}\,(1,2)^*\text{-}\omega\mathrm{int}(\bigcup_{j\in\Omega}(H_j,P))$.

Proof: (ii) Let $\{(H_j,P)\}_{j\in\Omega}$ be a collection of soft (1,2)*-ω-open sets in $\tilde{U}$. To prove that $\bigcup_{j\in\Omega}(H_j,P)$ is soft (1,2)*-ω-open. Let $\tilde{x}\,\tilde{\in}\,\bigcup_{j\in\Omega}(H_j,P)$ $\Rightarrow$ $\tilde{x}\,\tilde{\in}\,(H_{j_0},P)$ for some $j_0\in\Omega$. Since (H_{j_0},P) is a soft (1,2)*-ω-open set in $\tilde{U}$, then there is a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (O,P) in $\tilde{U}$ s.t $\tilde{x}\,\tilde{\in}\,(O,P)$ and $(O,P)-(H_{j_0},P)$ is a countable soft set. Since $(H_{j_0},P)\,\tilde{\subseteq}\,\bigcup_{j\in\Omega}(H_j,P)$ $\Rightarrow$ $\tilde{U}$-$(\bigcup_{j\in\Omega}(H_j,P))\,\tilde{\subseteq}\,\tilde{U}\text{-}(H_{j_0},P)$ $\Rightarrow$ $(O,P)\,\tilde{\cap}\,(\tilde{U}\text{-}(\bigcup_{j\in\Omega}(H_j,P)))\,\tilde{\subseteq}\,(O,P)\,\tilde{\cap}\,(\tilde{U}\text{-}(H_{j_0},P))$ $\Rightarrow$ $(O,P)-(\bigcup_{j\in\Omega}(H_j,P))\,\tilde{\subseteq}\,(O,P)-(H_{j_0},P)$. Since $(O,P)-(H_{J_0},P)$ is a countable soft set, then so is $(O,P)-(\bigcup_{j\in\Omega}(H_j,P))$. Therefore $\bigcup_{j\in\Omega}(H_j,P)$ is soft (1,2)*-ω-open.

(iii) It is obvious.
(iv) $(\Rightarrow)$ Suppose that (H,P) is a soft (1,2)*-ω-open set in $\tilde{U}$. To prove that $(1,2)^*\text{-}\omega\mathrm{int}(H,P)=(H,P)$.
By (i), we have $(1,2)^*\text{-}\omega\mathrm{int}(H,P)\,\tilde{\subseteq}\,(H,P)$, to prove that $(H,P)\,\tilde{\subseteq}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)$. Since (H,P) is soft (1,2)*-ω-open and $(H,P)\,\tilde{\subseteq}\,(H,P)$, then by (iii), we get $(H,P)\,\tilde{\subseteq}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)$. Thus $(1,2)^*\text{-}\omega\mathrm{int}(H,P)=(H,P)$ $(\Leftarrow)$ Suppose that $(1,2)^*\text{-}\omega\mathrm{int}(H,P)=(H,P)$. Since $(1,2)^*\text{-}\omega\mathrm{int}(H,P)$ is a soft (1,2)*-ω-open set in $\tilde{U}$, then so is (H,P).
(v) Since $(1,2)^*\text{-}\omega\mathrm{int}(H,P)$ is soft (1,2)*-ω-open, then by (iv), we get $(1,2)^*\text{-}\omega\mathrm{int}\,((1,2)^*\text{-}\omega\mathrm{int}(H,P))=(1,2)^*\text{-}\omega\mathrm{int}(H,P)$.
(vi) Since $(1,2)^*\text{-}\omega\mathrm{int}(H,P)=\tilde{\bigcup}\{(O,P):(O,P)$ is soft (1,2)*-ω-open in $\tilde{U}$ and $(O,P)\,\tilde{\subseteq}\,(H,P)\}$.
Hence $\tilde{U}\text{-}(1,2)^*\text{-}\omega\mathrm{int}(H,P)=\tilde{U}\text{-}\tilde{\bigcup}\{(O,P):(O,P)$ is soft (1,2)*-ω-open in $\tilde{U}$ and $(O,P)\,\tilde{\subseteq}\,(H,P)\}$.
$=\tilde{\bigcap}\{\tilde{U}\text{-}(O,P):\tilde{U}\text{-}(O,P)$ is soft (1,2)*-ω-closed in $\tilde{U}$ and $\tilde{U}\text{-}(H,P)\,\tilde{\subseteq}\,\tilde{U}\text{-}(O,P)\}=(1,2)^*\text{-}\omega\mathrm{cl}(\tilde{U}\text{-}(H,P))$.
(vii) If $\tilde{x}\,\tilde{\in}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)=\tilde{\bigcup}\{(O,P):(O,P)$ is soft (1,2)*-ω-open in $\tilde{U}$ and $(O,P)\,\tilde{\subseteq}\,(H,P)\}$, then $\exists$ a soft (1,2)*-ω-open set (O,P) in $\tilde{U}$ s.t $\tilde{x}\,\tilde{\in}\,(O,P)\,\tilde{\subseteq}\,(H,P)$. Conversely, suppose that there is a soft (1,2)*-ω-open set (O,P) in $\tilde{U}$ such that $\tilde{x}\,\tilde{\in}\,(O,P)\,\tilde{\subseteq}\,(H,P)$. But $(1,2)^*\text{-}\omega\mathrm{int}(H,P)$ is the largest soft (1,2)*-ω-open set which contained in (H,P), thus $(O,P)\,\tilde{\subseteq}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)$. Therefore $\tilde{x}\,\tilde{\in}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)$.
(viii) Let $\tilde{x}\,\tilde{\in}\,(1,2)^*\text{-}\omega\mathrm{int}(H,P)$**,** then by (vii), there is a soft (1,2)*-ω-open set (O,P) in $\tilde{U}$ such that $\tilde{x}\,\tilde{\in}\,(O,P)\,\tilde{\subseteq}\,(H,P)$. Since $(H,P)\,\tilde{\subseteq}\,(K,P)$, then there is a soft (1,2)*-ω-open set (O,P) in $\tilde{U}$ such

that $\tilde{x} \tilde{\in} (O,P) \tilde{\subseteq} (K,P)$. Thus $\tilde{x} \tilde{\in} (1,2)^*\text{-}\omega\mathrm{int}(K,P)$.

(ix) Since $(H,P) \tilde{\cap} (K,P) \tilde{\subseteq} (H,P)$ and $(H,P) \tilde{\cap} (K,P) \tilde{\subseteq} (K,P)$, then by (viii), we get $(1,2)^*\text{-}\omega\mathrm{int}((H,P) \tilde{\cap} (K,P)) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(H,P)$ and $(1,2)^*\text{-}\omega\mathrm{int}((H,P) \tilde{\cap} (K,P)) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(K,P)$.

Thus $(1,2)^*\text{-}\omega\mathrm{int}((H,P) \tilde{\cap} (K,P)) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(H,P) \tilde{\cap} (1,2)^*\text{-}\omega\mathrm{int}(K,P)$.

(x) Since $(H_j,P) \tilde{\subseteq} (\bigcup_{j\in\Omega}(H_j,P)), \forall\ j\in\Omega \Rightarrow (1,2)^*\text{-}\omega\mathrm{int}(H_j,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(\bigcup_{j\in\Omega}(H_j,P))$, $\forall\ j\in\Omega$.

Thus $\bigcup_{j\in\Omega}(1,2)^*\text{-}\omega\mathrm{int}(H_j,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(\bigcup_{j\in\Omega}(H_j,P))$.

Theorem (2.5): If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space, and $(H,P),(K,P) \tilde{\subseteq} \tilde{U}$. Then:

(i) $(H,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{cl}(H,P) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)$.

(ii) If $\{(H_j,P)\}_{j\in\Omega}$ is a collection of soft (1,2)*-ω-closed sets in $\tilde{U}$, then so is $\bigcap_{j\in\Omega}(H_j,P)$.

(iii) $(1,2)^*\text{-}\omega\mathrm{cl}(H,P)$ is the smallest soft (1,2)*-ω-closed set in $\tilde{U}$ which contains (H,P).

(iv) (H,P) is soft (1,2)*-ω-closed iff $(1,2)^*\text{-}\omega\mathrm{cl}(H,P) = (H,P)$.

(v) $(1,2)^*\text{-}\omega\mathrm{cl}((1,2)^*\text{-}\omega\mathrm{cl}(H,P)) = (1,2)^*\text{-}\omega\mathrm{cl}(H,P)$.

(vi) $\tilde{U}\text{-}(1,2)^*\text{-}\omega\mathrm{cl}(H,P) = (1,2)^*\text{-}\omega\mathrm{int}(\tilde{U}\text{-}(H,P))$.

(vii) $\tilde{x} \tilde{\in} (1,2)^*\text{-}\omega\mathrm{cl}(H,P)$ iff for every soft (1,2)*-ω-open set (O,P) containing $\tilde{x}$, $(O,P) \tilde{\cap} (H,P) \neq \tilde{\varphi}$.

(viii) If $(H,P) \tilde{\subseteq} (K,P)$, then $(1,2)^*\text{-}\omega\mathrm{cl}(H,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{cl}(K,P)$.

(ix) $(1,2)^*\text{-}\omega\mathrm{cl}((H,P) \tilde{\cap} (K,P)) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{cl}(H,P) \tilde{\cap} (1,2)^*\text{-}\omega\mathrm{cl}(K,P)$.

(x) $\bigcup_{j\in\Omega}(1,2)^*\text{-}\omega\mathrm{cl}(H_j,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{cl}(\bigcup_{j\in\Omega}(H_j,P))$.

Proof: (vii) ($\Rightarrow$) Assume that $\tilde{x} \tilde{\in} (1,2)^*\text{-}\omega\mathrm{cl}(H,P)$ and $(O,P) \tilde{\cap} (H,P) = \tilde{\varphi}$ for some soft (1,2)*-ω-open set (O,P) containing $\tilde{x}$. Hence $(H,P) \tilde{\subseteq} \tilde{U}\text{-}(O,P)$, since $\tilde{U}\text{-}(O,P)$ is soft (1,2)*-ω-closed, then by (iv), we get $(1,2)^*\text{-}\omega\mathrm{cl}(H,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{cl}(\tilde{U}-(O,P)) = \tilde{U}-(O,P)$. Since $\tilde{x} \tilde{\notin} \tilde{U}-(O,P)$, then $\tilde{x} \tilde{\notin} (1,2)^*\text{-}\omega\mathrm{cl}(H,P)$. This contradicts with the hypothesis, therefore $(O,P) \tilde{\cap} (H,P) \neq \tilde{\varphi}$ for every soft (1,2)*-ω-open set (O,P) containing $\tilde{x}$.

($\Leftarrow$) To prove that $\tilde{x} \tilde{\in} (1,2)^*\text{-}\omega\mathrm{cl}(H,P)$. If not, then $\tilde{x} \tilde{\notin} (1,2)^*\text{-}\omega\mathrm{cl}(H,P) \Rightarrow \tilde{x} \tilde{\in} \tilde{U}\text{-}(1,2)^*\text{-}\omega\mathrm{cl}(H,P)$ and $\tilde{U}\text{-}(1,2)^*\text{-}\omega\mathrm{cl}(H,P)$ is soft (1,2)*-ω-open set. Since $(H,P) \tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{cl}(H,P) \Rightarrow (H,P) \tilde{\cap} (\tilde{U}\text{-}(1,2)^*\text{-}\omega\mathrm{cl}(H,P)) = \tilde{\varphi}$. This is a contradiction. Therefore $\tilde{x} \tilde{\in} (1,2)^*\text{-}\omega\mathrm{cl}(H,P)$.

By definition (2.3), we can prove other casses.

Remark (2.6): The intersection of two soft (1,2)*-ω-open sets in general is not soft (1,2)*-ω-open we can see in the following example:

Example (2.7): Let $\mathbf{U} = \mathfrak{R}$, $P = \{p_1,p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U},\tilde{\varphi},(H_1,P)\}$ and $\tilde{\tau}_2 = \{\tilde{U},\tilde{\varphi},(H_2,P)\}$ be soft topologies over U, where $(H_1,P) = \{(p_1,(0,1]),(p_2,(0,1])\}$, $(H_2,P) = \{(p_1,[1,2)),(p_2,[1,2))\}$, and $(H_3,P) = \{(p_1,(0,2)),(p_2,(0,2))\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P),(H_3,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Hence (H_1,P) and (H_2,P) are soft (1, 2)*-ω-open sets in $\tilde{U}$, but $(H_1,P) \tilde{\cap} (H_2,P) = \{(p_1,\{1\}),(p_2,\{1\})\}$ is not soft (1,2)*-ω-open.

3. Weak Forms of Soft (1,2)*-Omega Open Sets

Now, we introduce and study new kinds of soft (1,2)*-omega open sets in soft bitopological spaces called soft (1,2)*-α-ω-open sets, soft (1,2)*-pre-ω-open sets, soft (1,2)*-b-ω-open sets and soft (1,2)*-β-ω-open sets which are weaker than soft (1,2)*-omega open sets. We discussed the fundamental properties of these soft open sets and the relationships between these sets and other soft open sets.

Definitions (3.1): A soft subset (H, P) of a soft bitopological space $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is called:

(i) Soft (1, 2)*-α-ω-open if $(H, P) \tilde{\subseteq} (1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P)))$.

(ii) Soft (1, 2)*-pre-ω-open if $(H, P) \tilde{\subseteq} (1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P))$.

(iii) Soft (1, 2)*-b-ω-open if $(H, P) \tilde{\subseteq} (1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)) \tilde{\cup} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P))$.

(iv) Soft (1, 2)*-β-ω-open if $(H, P) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)))$.

Proposition (3.2): If $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft bitopological space, then the following hold:

(i) Every soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1, 2)*-α-open, soft (1, 2)*-pre-open, soft (1,2)*-b-open, soft (1,2)*-β-open) set is soft (1, 2)*-ω-open (resp. soft (1, 2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b- ω-open, soft (1,2)*-β-ω-open).

(ii) Every soft (1, 2)*-ω-open set is soft (1, 2)*-α-ω-open.

(iii) Every soft (1, 2)*-α-ω-open set is soft (1, 2)*-pre-ω-open.

(iv) Every soft (1, 2)*-pre-ω-open set is soft (1, 2)*-b-ω-open.

(v) Every soft (1, 2)*-b-ω-open set is soft (1, 2)*-β-ω-open.

Proof: (i) It is obvious. *(ii)* ***If*** (H, P) ***is a soft (1,2)*-ω-open set, then by theorem (2.4), no. (iv), we get (1,2)*-*** $\omega\mathrm{int}(H, P) = (H, P)$.

Since $(H, P) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)$, then $(H, P) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P))$. Hence $(H, P) \tilde{\subseteq}$ (1,2)*-$\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P)))$. Therefore (H, P) is a soft (1, 2)*-α-ω-open set in $\tilde{U}$.

(iii) If (H, P) is a soft (1, 2)*-α-ω-open set, then $(H, P) \tilde{\subseteq} (1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P))) \tilde{\subseteq}$ $(1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P))$. Therefore (H, P) is a soft (1,2)*-pre-ω-open set in $\tilde{U}$.

(iv) If (H, P) is a soft (1, 2)*-pre-ω-open set, then $(H, P) \tilde{\subseteq} (1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)) \tilde{\subseteq}$ $(1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)) \tilde{\cup} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P))$. Therefore (H, P) is a soft (1, 2)*-b-ω-open set in $\tilde{U}$.

(v) If (H, P) is a soft (1, 2)*-b-ω-open set in $\tilde{U}$, then $(H, P) \tilde{\subseteq} (1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)) \tilde{\cup}$ $\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(H, P))) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P))) \tilde{\cup} \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*$ $\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P))) = \tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\,\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H, P)))$. Therefore (H, P) is a soft (1,2)*-β-ω-open set in $\tilde{U}$.

The converse of proposition (3.2) in general is not true we can see in the following examples.

Example (3.3): Let $\mathbf{U} = \{1,2,3,4\}$ and $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P)\}$ be soft topologies over U, where $(H_1, P) = \{(p_1, \{1\}), (p_2, \{1\})\}$, $(H_2, P) = \{(p_1, \{2\}), (p_2, \{2\})\}$ and $(H_3, P) = \{(p_1, \{1,2\}), (p_2, \{1,2\})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P), (H_3, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(H, P) = \{(p_1, \{1,2\}), (p_2, \{1\})\}$ is a soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open) set, but is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-α-open).

Example (3.4): Let $U = N$, $P = \{p_1, p_2, p_3\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P), (H_3, P)\}$ be soft topologies over U, where $(H_1, P) = \{(p_1, \{1\}), (p_2, \{1\}), (p_3, \{1\})\}$, $(H_2, P) = \{(p_1, \{1,2\}), (p_2, \{1,2\}), (p_3, \{1,2\})\}$ and $(H_3, P) = \{(p_1, \{2\}), (p_2, \{2\}), (p_3, \{2\})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P), (H_3, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(H, P) = \{(p_1, N - \{1\}), (p_2, N - \{1\}), (p_3, N - \{1\})\}$ is a soft (1,2)*-pre-ω-open set, but is not soft (1,2)*-pre-open.

Example (3.5): Let $U = N$, $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P)\}$ be soft topologies over U, where $(H_1, P) = \{(p_1, \{1\}), (p_2, \{1\})\}$ and $(H_2, P) = \{(p_1, \{1,2\}), (p_2, \{1,2\})\}$.

The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Then $(H, P) = \{(p_1, N - \{1\}), (p_2, N - \{1\})\}$ is a soft (1,2)*-b-ω-open set, but is not soft (1,2)*-b-open.

Example (3.6): Let $U = \mathfrak{R}$, $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P)\}$ be soft topologies over U, where $(H_1, P) = \{(p_1, \{1,2,3\}), (p_2, \{4,5\})\}$, $(H_2, P) = \{(p_1, \mathfrak{R} - \{10\}), (p_2, \mathfrak{R} - \{4,5\})\}$ and $(H_3, P) = \{(p_1, \mathfrak{R} - \{10\}), (p_2, \mathfrak{R})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P), (H_3, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Then $(H, P) = \{(p_1, \mathfrak{R} - \{1,2,3\}), (p_2, \mathfrak{R} - \{4,5\})\}$ is a soft (1,2)*-β-ω-open set, but is not soft (1,2)*-β-open.

Example(3.7): Let $U = \mathfrak{R}$, $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}\}$ be soft topologies over U, where $(H, P) = \{(p_1, \{1\}), (p_2, \{1\})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Then $(K, P) = \{(p_1, \{1,2\}), (p_2, \{1,2\})\}$ is a soft (1,2)*-α-ω-open set, but is not soft (1,2)*-ω-open.

Example (3.8): Let $U = \mathfrak{R}$, $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P)\}$ be soft topologies over X, where $(H_1, P) = \{(p_1, \mathfrak{R} - \{1\}), (p_2, \mathfrak{R} - \{1\})\}$ and $(H_2, P) = \{(p_1, \mathfrak{R} - \{1,2\}), (p_2, \mathfrak{R} - \{1,2\})\}$. The soft sets in $\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Then $(H, P) = \{(p_1, \{3\}), (p_2, \{3,4\})\}$ is a soft (1,2)*-pre-ω-open set (since (H, P) is soft (1,2)*-pre-open), but is not soft (1,2)*-α-ω-open set.

Example (3.9): Let $U = \mathfrak{R}$, $P = \{p\}$ and let $(\mathfrak{R}, \tilde{\mu}, P)$ be the soft usual topological space over U and $(\mathfrak{R}, \tilde{I}, P)$ be the soft indiscrete topological space over U, then $(U, \tilde{\mu}, \hat{I}, P)$ is a soft bitopological space over U and $(H, P) = \{(p, (0,1])\}$ is a soft (1,2)*-b-ω-open set (since (H, P) is soft (1,2)*-b-open), but is not soft (1,2)*-pre-ω-open.

Example (3.10): Let $U = \mathfrak{R}$, $P = \{p\}$ and let $(\mathfrak{R}, \tilde{\mu}, P)$ be the soft usual topological space over U and $(\mathfrak{R}, \tilde{I}, P)$ be the soft indiscrete topological space over U, then $(U, \tilde{\mu}, \hat{I}, P)$ is a soft bitopological space over U and $(H, P) = \{(p, Q \cap [0,1])\}$ is a soft (1,2)*-β-ω-open set (since (H, P) is soft (1,2)*-β-open), but is not soft (1,2)*-b-ω-open.

The following diagram shows the relation between the different types of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets and types of weak soft (1,2)*-ω-open sets

$$\begin{tikzcd}
\textbf{Soft } \tilde{\tau}_1\tilde{\tau}_2 \textbf{-open set} \arrow[r, shift right] \arrow[d, shift right] & \textbf{soft (1,2)*-}\omega\textbf{-open set} \arrow[l, shift right, "|"'] \arrow[d, shift right] \\
\textbf{soft (1,2)*-}\alpha\textbf{-open set} \arrow[u, shift right, "-"'] \arrow[r, shift right] \arrow[d, shift right] & \textbf{soft (1,2)*-}\alpha\textbf{-}\omega\textbf{-open set} \arrow[u, shift right, "-"'] \arrow[l, shift right, "|"'] \arrow[d, shift right] \\
\textbf{soft (1,2)*-pre-open set} \arrow[u, shift right, "-"'] \arrow[r, shift right] \arrow[d, shift right] & \textbf{soft (1,2)*-pre-}\omega\textbf{-open set} \arrow[u, shift right, "-"'] \arrow[l, shift right, "|"'] \arrow[d, shift right] \\
\textbf{soft (1,2)*-b-open set} \arrow[u, shift right, "-"'] \arrow[r, shift right] \arrow[d, shift right] & \textbf{soft (1,2)*-b-}\omega\textbf{-open set} \arrow[u, shift right, "-"'] \arrow[l, shift right, "|"'] \arrow[d, shift right] \\
\textbf{soft (1,2)*-}\beta\textbf{-open set} \arrow[u, shift right, "-"'] \arrow[r, shift right] & \textbf{soft (1,2)*-}\beta\textbf{-}\omega\textbf{-open set} \arrow[u, shift right, "-"'] \arrow[l, shift right, "|"']
\end{tikzcd}$$

Theorem (3.11): If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space and (H,P) is a soft (1, 2)*-pre-ω-open set in $\tilde{U}$ such that $(O,P)\tilde{\subseteq}(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(O,P)$ for a soft subset (O,P) of $\tilde{U}$, then (O,P) is also soft (1, 2)*-pre-ω-open.

Proof: Since $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(O,P) \Rightarrow \tilde{\tau}_1\tilde{\tau}_2cl(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2cl(O,P))=\tilde{\tau}_1\tilde{\tau}_2cl(O,P) \Rightarrow$ $(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P))\tilde{\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(O,P))$. Since $(H,P)\tilde{\subseteq}(1,\ 2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P))$ and $(O,P)\tilde{\subseteq}(H,P) \Rightarrow (O,P)\tilde{\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(O,P))$. Thus (O,P) is a soft (1,2)*-pre-ω-open set in $\tilde{U}$.

Theorem (3.12): A soft subset (H,P) of a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is soft (1, 2)*-semi-open if and only if (H,P) is soft (1,2)*-β-ω-open and $(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P))\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P))$.

Proof: If (H,P) is a soft (1, 2)*-semi-open set in $\tilde{U}$, then $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P))\tilde{\subseteq}$ $\tilde{\tau}_1\tilde{\tau}_2cl((1,\ 2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P)))$ and hence (H,P) is a soft (1, 2)*-β-ω-open set. Also, since $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P)) \Rightarrow \tilde{\tau}_1\tilde{\tau}_2cl(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P)) \Rightarrow (1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P))$ $\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P))$. Conversely, let (H,P) be a soft (1,2)*-β-ω-open set in $\tilde{U}$ and $(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P))\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P))$. Then $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl((1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(H,P)))$

$\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P)))=\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\tau}_1\tilde{\tau}_2 int(H,P))$ and hence (H,P) is a soft (1,2)*-semi-open set in $\tilde{U}$.

Remark(3.13): The intersection of two soft (1,2)*-α-ω-open (resp. soft (1,2)*-pre-ω-open, soft (1,2)* -b-ω-open, soft (1,2)*-β-ω-open) sets need not be soft (1,2)*-α-ω-open (resp. soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-pen) we can see in the following examples:

Example(3.14): Let $U=\mathfrak{R}$, $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P)\}$ and $\tilde{\tau}_2=\{\tilde{U},\tilde{\varphi},(H_2,P),(H_3,P)\}$ be soft topologies over U, where $(H_1,P)=\{(p_1,\{1\}),(p_2,\{1\})\}$, $(H_2,P)=\{(p_1,\{1,2\}),(p_2,\{1,2\})\}$ and $(H_3,P)=\{(p_1,\{2\}),(p_2,\{2\})\}$. The soft sets in $\{\tilde{U},\tilde{\varphi},(H_1,P),(H_2,P),(H_3,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(K_1,P)=\{(p_1,\mathfrak{R}-\{1\}),(p_2,\mathfrak{R}-\{1\})\}$ and $(K_2,P)=\{(p_1,\{1,3\}),(p_2,\{1,3\})\}$ are soft (1,2)*-α-ω-open sets, but $(K_1,P)\tilde{\cap}(K_2,P)=(K,P)=\{(p_1,\{3\}),(p_2,\{3\})\}$ is not soft (1,2)*-α-ω-open, since $(K,P)\tilde{\not\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl((1,2)^*\text{-}\omega int(K,P)))=(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\varphi}))=\tilde{\varphi}$.

Example (3.15): Let $U=\mathfrak{R}$, $P=\{p\}$ and let $(\mathfrak{R},\tilde{\mu},P)$ be the soft usual topological space over U and $(\mathfrak{R},\tilde{I},P)$ be the soft indiscrete topological space over U, then $(X,\tilde{\mu},\hat{I},P)$ is a soft bitopological space over U. Observe that $(K_1,P)=\{(p,Q)\}$ and $(K_2,P)=\{(p,Q^c\cup\{1\})\}$ are soft (1,2)*-pre-ω-open sets, but $(K_1,P)\tilde{\cap}(K_2,P)=(K,P)=\{(p,\{1\})\}$ is not soft (1,2)*-β-ω-pen, since $(K,P)\tilde{\not\subseteq}\tilde{\tau}_1\tilde{\tau}_2cl((1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl(K,P)))=\tilde{\tau}_1\tilde{\tau}_2cl((1,2)^*\text{-}\omega int(K,P))=\tilde{\tau}_1\tilde{\tau}_2cl(\tilde{\varphi})=\tilde{\varphi}$.

Theorem (3.16): If $\{(H_j,P): j\in\boldsymbol{\Omega}\}$ is a collection of soft (1,2)*-α-ω-open (resp. soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-pen) sets in a soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$, then $\bigcup_{j\in\boldsymbol{\Omega}}(H_j,P)$ is soft (1,2)*-α-ω-open (resp. soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open).

Proof: Since $(H_j,P)\tilde{\subseteq}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl((1,2)^*\text{-}\omega int(H_j,P)))$ for every $j\in\boldsymbol{\Omega}$, we have:

$$\bigcup_{j\in\boldsymbol{\Omega}}(H_j,P)\tilde{\subseteq}\bigcup_{j\in\boldsymbol{\Omega}}(1,2)^*\text{-}\omega int(\tilde{\tau}_1\tilde{\tau}_2cl((1,2)^*\text{-}\omega int(H_j,P))).$$

$$\tilde{\subseteq}(1,2)^*\text{-}\omega int(\bigcup_{j\in\boldsymbol{\Omega}}\tilde{\tau}_1\tilde{\tau}_2((1,2)^*\text{-}\omega int(H_j,P))).\ \text{(By theorem (2.4), no. (x))}$$

$$\tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(\bigcup_{j\in\Omega}(1,2)^*\text{-}\omega\mathrm{int}(H_j,P)).$$ (By theorem (2.5), no. (x))

$$\tilde{\subseteq} (1,2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\omega\mathrm{int}(\bigcup_{j\in\Omega}(H_j,P)))).$$ (By theorem (2.4), no. (x))

Therefore $\bigcup_{j\in\Omega}(H_j,P)$ is soft (1,2)*-α-ω-open. Similarly, we can show other cases.

Proposition (3.17): If $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological spaces and (H,P) is a soft (1, 2)*-b-ω-open set in $\tilde{U}$ such that $(1, 2)^*\text{-}\omega\mathrm{int}(H,P)=\tilde{\varphi}$, then (H,P) is soft (1, 2)*-pre-ω-open.

Proof: If (H,P) is a soft (1, 2)*-b-ω-open set in $\tilde{U}$, then $(H,P)\tilde{\subseteq}(1, 2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))\tilde{\cup}$ $\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1, 2)^*\text{-}\omega\mathrm{int}(H,P))$. But $(1, 2)^*\text{-}\omega\mathrm{int}(H,P)=\tilde{\varphi}$, then $\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1, 2)^*\text{-}\omega\mathrm{int}(H,P))=\tilde{\varphi}$, thus $(H,P)\tilde{\subseteq}(1, 2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))$. Therefore (H,P) is a soft (1, 2)*-pre-ω-open set in $\tilde{U}$.

Definition (3.18): A soft bitopological space $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called soft (1, 2)*-door space if every soft subset of $\tilde{U}$ is either soft $\tilde{\tau}_1\tilde{\tau}_2$-open or soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Propositions (3.19): Let $(U,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft (1, 2)*-door space, then:

(i) Soft (1, 2)*-α-ω-open set is soft (1, 2)*-ω-open.

(ii) Soft (1, 2)*-pre-ω-open set is soft (1, 2)*-α-ω-open.

(iii) Soft (1, 2)*-β-ω-open set is soft (1, 2)*-b-ω-open.

Proof: (i) Let (H,P) be a soft (1, 2)*-α-ω-open set in $\tilde{U}$. If (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-open, then (H,P) is soft (1,2)*-ω-open. Otherwise, (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed. Thus $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))=$ $(1,2)^*\text{-}\omega\mathrm{int}(H,P)$, hence $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega\mathrm{int}(H,P)$. Since $(1,2)^*\text{-}\omega\mathrm{int}(H,P)\tilde{\subseteq}(H,P)$, then $(1,2)^*\text{-}\omega\mathrm{int}(H,P)=(H,P)$. Therefore (H,P) is a soft (1,2)*-ω-open set in $\tilde{U}$.

(ii) Let (H,P) be a soft (1,2)*-pre-ω-open set in $\tilde{U}$. If (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-open, then (H,P) is soft (1,2)*-α-ω-open. Otherwise, (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed. Thus $(H,P)\tilde{\subseteq}(1,2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))$ $=(1,2)^*\text{-}\omega\mathrm{int}(H,P)$. Hence (H,P) is a soft (1,2)*-ω-open set and by proposition (3.2), no. (ii), (H,P) is a soft (1,2)*-α-ω-open set in $\tilde{U}$.

(iii) Let (H,P) be a soft (1,2)*-β-ω-open set. If (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-open, then (H,P) is soft (1,2)*-b-ω-open. Otherwise, (H,P) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed. Thus $(H,P)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P)))$ $=\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\omega\mathrm{int}(H,P))\tilde{\subseteq}(1,2)^*\text{-}\omega\mathrm{int}(\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(H,P))\tilde{\cup}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}((1,2)^*\text{-}\omega\mathrm{int}(H,P))$. Hence (H,P) is a soft (1,2)*-b-ω-open set.

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