*Al-subahawi and Al-seraji* *Iraqi Journal of Science, 2020,Special Issue, pp: 204-214*
*DOI: 10.24996/ijs.2020.SI.1.28*

ISSN: 0067-2904

# Partitions on the Projective Plane Over Galois Field of Order $11^m, m = 1, 2, 3$

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Received: 1/1/ 2020 Accepted: 15/ 3/2020

**Abstract**

This research is concerned with the study of the projective plane over a finite field $F_q$. The main purpose is finding partitions of the projective line $\mathrm{PG}(1, q^3)$ and the projective plane $\mathrm{PG}(2, q^2), q = 11$, in addition to embedding $\mathrm{PG}(1,q)$ into $\mathrm{PG}(1, q^3)$ and $\mathrm{PG}(2, q)$ into $\mathrm{PG}(2, q^2)$. Clearly, the orbits of $\mathrm{PG}(1, q^3), q = 11$ are found, along with the cross-ratio for each orbit. As for $\mathrm{PG}(2, q^2)$, 13 partitions were found on $\mathrm{PG}(2, 11^2)$, each partition being classified in terms of the degree of its arc, length, its own code, as well as its error correcting. The last main aim is to classify the group actions on $\mathrm{PG}(2,11)$.

**Keywords:** Stabilizer Group, Partitions, Arcs, Cross-Ratio

# تجزئات على المستوي الاسقاطي حول حقل كالوس من الرتبة $11^m, m = 1, 2, 3$

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**الخلاصه**

هذا البحث معني بدراسة المستوي الاسقاطي حول الحقل المنتهي $F_q$, واهدافه الرئيسية في ايجاد تجزئات للخط الاسقاطي $\mathrm{PG}(1, q^3)$, والمستوي الاسقاطي $\mathrm{PG}(2, q^2), q = 11$, بالاضافة الى غمر $\mathrm{PG}(1,q)$ الى $\mathrm{PG}(1, q^3)$ و غمر $\mathrm{PG}(2, q)$ الى PG $(2, q^2)$. بشكل اوضح, المدارات ل $(1, q^3)$ تم ايجادها , وكذلك وجدت نسبة تقاطع لأجل كل مدار. كذلك لأجل $\mathrm{PG}(2, q^2)$, 13 تجزئة وجدت حول $\mathrm{PG}(2, 11^2)$, كل تجزئة صنفت في رموز درجة القوس , وطوله و كذلك تصحيح اخطاء الرموز. اخر هدف هو تصنيف فعل الزمر على $\mathrm{PG}(2,11)$.

## 1. Introduction

Today, we see projective geometry as a numerical hypothesis in its own privilege, a section of geometry is “exceptionally non-Euclidean” with no thought of separation and with quite certain topological properties. In any case, the source of projective geometry is to be found inside Euclidean geometry. For a few hundreds of years, projective “techniques” were viewed as similarly as an effective method to deal with issues in Euclidean geometry. The main topic of this paper is a partition on the projective line and the projective plane by subgeometry, depending on the projective geometry, group theory and vector space over a finite field $F_q$. Therefore, we found a non-singular matrix $T_{2\times2}$ to construct a projective line $\mathrm{PG}(1, q)$ and a matrix $T_{3\times3}$ to construct a projective plane $\mathrm{PG}(2, q)$, where $q = 11$. In this work, we partition the projective line $\mathrm{PG}(1, q^3), q = 11$ and study its properties and results. In addition, we partition the projective plane $\mathrm{PG}(2, q^2)$. Many methods and algorithms were

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used on this subject. For example, in 1998, Hirshfeld [1] partitioned PG(2,16) into three disjoint PG(2,4), while in 2010, Al-Seraji [2] classified the projective line and the plane of order 17. In 2011, Al-Zangana [3] showed the group effect on the conic in $\mathrm{PG}(2,q), q=19,$ whereas in 2014, Al-Seraji [4] studied the classification of the projective line PG(1,16). For more details about other cases, see [5-12].

**2. The group action on the projective line $\mathbf{PG(1,11)}$**

To construct PG(1,11), let $G(X)=X^2+\omega^2X+\omega^7$ , $\omega=2\in F_{11}$ be a polynomial of the degree two, where $\omega$ is a primative element and $G$ is the primitive polynomial over $F_{11}$, since:
$G(0)=2$ , $G(1)=4$ , $G(2)=8$ , $G(3)=5$, $G(4)=10$ , $G(6)=9$ , $G(7)=7$ , $G(8)=3$ , $G(9)=6$ , $G(10)=1$
Such that $G$ is irreducible polynomial over field $F_{11}$.
The non-singular matrix of size $2\times 2$ of $G(X)$,

$$T=\begin{pmatrix}0 & 1\\ \omega^2 & \omega^7\end{pmatrix} \qquad \dots (1)$$

generated 12 points on PG(1,11), such that $S_i=[1,0]T^i, i=0,1,2,\dots,11$.

**3. The group action on $\mathbf{PG(1,11^3)}$**

In regard to $\mathrm{PG}(1,11^3)$, the order becomes higher and the number of points are increased, so we found a matrix to find those points. This matrix is obtained from the polynomial of degree two $H(X)=X^2-X-\alpha^{18}$ , $\alpha\in F_{11^3}$, primitive over $F_{11^3}$. We can define this field by the form $F_{11^3}=\{0,1,\alpha,\alpha^2,\alpha^3,\dots,\alpha^{1330}|\alpha^{1331}=1\}$. To clarify more, we have 1332 points in $\mathrm{PG}(1,11^3)$ generated by the non-singular matrix of $H$ of size $2\times 2$,

$$A=\begin{pmatrix}0 & 1\\ \alpha^{18} & 1\end{pmatrix} \qquad \dots (2)$$

such that $N_i=[1,0]A^i, i=0,1,2,\dots,1331$.
The group actions of $A^{111}$ on $\mathrm{PG}(1,11^3)$ are:

$$\mathrm{N}_0\xrightarrow{A^{111}}\mathrm{N}_{111}\xrightarrow{A^{111}}\mathrm{N}_{222}\xrightarrow{A^{111}}\mathrm{N}_{333}\xrightarrow{A^{111}}\dots\xrightarrow{A^{111}}\mathrm{N}_{1221}\xrightarrow{A^{111}}\mathrm{N}_0$$
$$\mathrm{N}_1\xrightarrow{A^{111}}\mathrm{N}_{112}\xrightarrow{A^{111}}\mathrm{N}_{223}\xrightarrow{A^{111}}\mathrm{N}_{334}\xrightarrow{A^{111}}\dots\xrightarrow{A^{111}}\mathrm{N}_{1222}\xrightarrow{A^{111}}\mathrm{N}_1$$
$$\mathrm{N}_2\xrightarrow{A^{111}}\mathrm{N}_{113}\xrightarrow{A^{111}}\mathrm{N}_{224}\xrightarrow{A^{111}}\mathrm{N}_{335}\xrightarrow{A^{111}}\dots\xrightarrow{A^{111}}\mathrm{N}_{1223}\xrightarrow{A^{111}}\mathrm{N}_2$$
$$\vdots$$
$$\mathrm{N}_{110}\xrightarrow{A^{111}}\mathrm{N}_{221}\xrightarrow{A^{111}}\mathrm{N}_{332}\xrightarrow{A^{111}}\mathrm{N}_{443}\xrightarrow{A^{111}}\dots\xrightarrow{A^{111}}\mathrm{N}_{1233}\xrightarrow{A^{111}}\mathrm{N}_{110}$$

**Theorem 3.1:** On $\mathrm{PG}(1,11^3)$, we have the following properties:
1. The projective line $\mathrm{PG}(1,11^3)$ is partitioned into 111 orbits;
2. Any one of the pervious orbit represents PG(1,11) and all of them are equivalent under the effect of the matrix $A$;
3. All the orbits are disjoint on the effect of $A$ as 12-transtive;
4. The action of $A^{111}$ cycles 12 points.

**4. Partitions on $\mathbf{PG(1,11^{\mathit{i}}), \mathit{i}=1,3}$**

Let

$$\Upsilon=\frac{(A_1-A_3)}{(A_1-A_4)}\frac{(A_2-A_4)}{(A_2-A_3)} \qquad \dots\dots\dots\dots\dots\dots\dots\dots.. \qquad (3)$$

be a cross-ratio of any four points $\{A_1,A_2,A_3,A_4\}$, such that these points will be an elements of $F_q$. On the PG(1,11$^i$), $i$ in {1,3} , the number of orbits were formed and founded cross ratio for each orbit. The formed orbit gave a permutation of 4-set (tetrad), therefore this 4-set is described as:

**1.** Harmonic (H) if $\Upsilon=\frac{1}{\Upsilon}$ , $\Upsilon=1-\Upsilon$ or $\Upsilon=\frac{\Upsilon}{\Upsilon-1}$;

**2.** Equianharmonic (E) if $\Upsilon=\frac{1}{1-\Upsilon}$ or $\Upsilon=\frac{\Upsilon-1}{\Upsilon}$;

**3.** Superharmonic if it is both (1&2);

**4.** Neither harmonic nor equianharmonic (N) if the cross-ratio is of another value.

Evaluate the tetrads $\{\infty,0,1,t\}$ with $t\in F_{11^i}, i\in\{0,1\}$, hence there are four classes of tetrads:
$Y_1=$ {class of H tetrads}
$Y_2=$ {class of E tetrads}
$Y_3=$ {class of S tetrads}

$\Upsilon_4 =$ {class of N tetrads}
The three orbits formed by (1) on PG(1,11) are:

$$P_0 \xrightarrow{T^3} P_3 \xrightarrow{T^3} P_6 \xrightarrow{T^3} P_9 \xrightarrow{T^3} P_0$$
$$P_1 \xrightarrow{T^3} P_4 \xrightarrow{T^3} P_7 \xrightarrow{T^3} P_{10} \xrightarrow{T^3} P_1$$
$$P_2 \xrightarrow{T^3} P_5 \xrightarrow{T^3} P_8 \xrightarrow{T^3} P_{11} \xrightarrow{T^3} P_2$$

The next table shows the properties for cross-ratio on PG(1,11).
**Table 1-**Some 4-set on PG(1,11).

| No. | The 4- set (tetrads) | The cross-ratio | Class of tetrads | Stabilizer |
|---|---|---|---|---|
| 1 | $\{\omega^2, \omega^5, \omega^8, \omega^9\}$ | $\Upsilon=\omega^9$ | S | $Z_2 \times Z_2$ |
| 2 | $\{\omega^2, \omega^5, \omega^9, \omega^8\}$ | $\Upsilon = \omega$ | N | $Z_2 \times Z_2$ |
| 3 | $\{\omega^2, \omega^8, \omega^5, \omega^9\}$ | $\Upsilon = \omega^9$ | S | $Z_2 \times Z_2$ |
| 4 | $\{\omega^2, \omega^8, \omega^9, \omega^5\}$ | $\Upsilon = \omega$ | N | $Z_2 \times Z_2$ |
| 5 | $\{\omega^2, \omega^9, \omega^5, \omega^8\}$ | $\Upsilon = \omega^5$ | H | $Z_2 \times Z_2$ |

From Table-1, the following theorem is established.
**Theorem 4.1:** Partitions on PG(1,11) satisfy the following properties :
1. We divide the projective line PG(1,11) into three orbits, each one of them containing four points called tetrads;
2. For each orbit, the values of a cross-ratio are ($\Upsilon_1 = \omega$ , $\Upsilon_2 = \omega^5$, $\Upsilon_3 = \omega^9$);
3. The cross-ratio of $\{\infty, 0,1, \omega\}$ belongs to the class N , $\{\infty, 0,1, \omega^5\}$ belongs to the class H, and$\{\infty, 0,1, \omega^9\}$ belongs to the class S;
4. Each of $\Upsilon_1, \Upsilon_2$ and $\Upsilon_3$ gave eight permutations from the same class to which they belong.

Proof 4 : The eight permutations for the cross-ratio $\Upsilon_1$ are:

$$\{\omega^2, \omega^5, \omega^9, \omega^8\}$$
$$\{\omega^2, \omega^8, \omega^9, \omega^5\}$$
$$\{\omega^5, \omega^2, \omega^8, \omega^9\}$$
$$\{\omega^5, \omega^9, \omega^8, \omega^2\}$$
$$\{\omega^8, \omega^2, \omega^5, \omega^9\}$$
$$\{\omega^8, \omega^9, \omega^5, \omega^2\}$$
$$\{\omega^9, \omega^5, \omega^2, \omega^8\}$$
$$\{\omega^9, \omega^8, \omega^2, \omega^5\}$$

Also, the eight permutations for $\Upsilon_2$ are:

$$\{\omega^2, \omega^9, \omega^5, \omega^8\}$$
$$\{\omega^2, \omega^9, \omega^8, \omega^5\}$$
$$\{\omega^5, \omega^8, \omega^2, \omega^9\}$$
$$\{\omega^5, \omega^8, \omega^9, \omega^2\}$$
$$\{\omega^8, \omega^5, \omega^2, \omega^9\}$$
$$\{\omega^8, \omega^5, \omega^9, \omega^2\}$$
$$\{\omega^9, \omega^2, \omega^5, \omega^8\}$$
$$\{\omega^9, \omega^2, \omega^8, \omega^5\}$$

The eight permutations for $\Upsilon_3$ are:

$$\{\omega^2, \omega^5, \omega^8, \omega^9\}$$
$$\{\omega^2, \omega^8, \omega^5, \omega^9\}$$
$$\{\omega^5, \omega^2, \omega^9, \omega^8\}$$
$$\{\omega^5, \omega^9, \omega^2, \omega^8\}$$
$$\{\omega^8, \omega^2, \omega^9, \omega^5\}$$
$$\{\omega^8, \omega^9, \omega^2, \omega^5\}$$
$$\{\omega^9, \omega^5, \omega^8, \omega^2\}$$
$$\{\omega^9, \omega^8, \omega^5, \omega^2\}.$$

In regard to the partition on $PG(1, 11^3)$, there are 333 orbits formed by (2), some of these orbits are:

$$P_0 \xrightarrow{A^{333}} P_{333} \xrightarrow{A^{333}} P_{666} \xrightarrow{A^{333}} P_{999} \xrightarrow{A^{333}} P_0$$
$$P_1 \xrightarrow{A^{333}} P_{334} \xrightarrow{A^{333}} P_{667} \xrightarrow{A^{333}} P_{1000} \xrightarrow{A^{333}} P_1$$
$$P_2 \xrightarrow{A^{333}} P_{335} \xrightarrow{A^{333}} P_{668} \xrightarrow{A^{333}} P_{1001} \xrightarrow{A^{333}} P_2$$
$$\vdots$$
$$P_{332} \xrightarrow{A^{333}} P_{665} \xrightarrow{A^{333}} P_{998} \xrightarrow{A^{333}} P_{1331} \xrightarrow{A^{333}} P_{332}$$

The second table shows the properties of these partitions.

**Table 2-**Some 4-set on $PG(1, 11^3)$.

| No. | The 4-set(tetrads) | The cross-ratio | Class of tetrad | Stabilizer |
|---|---|---|---|---|
| 1 | $\{\alpha^{18}, \alpha^{136}, \alpha^{561}, \alpha^{952}\}$ | $\Upsilon = \alpha^9$ | E | $Z_2 \times Z_2$ |
| 2 | $\{\alpha^{18}, \alpha^{561}, \alpha^{136}, \alpha^{952}\}$ | $\Upsilon = \alpha^9$ | E | $Z_2 \times Z_2$ |
| 3 | $\{\alpha^{18}, \alpha^{952}, \alpha^{136}, \alpha^{561}\}$ | $\Upsilon = \alpha^5$ | H | $Z_2 \times Z_2$ |
| 4 | $\{\alpha^{18}, \alpha^{952}, \alpha^{561}, \alpha^{136}\}$ | $\Upsilon = \alpha^5$ | H | $Z_2 \times Z_2$ |

**Theorem 4.2:** On $PG(1, 11^3)$ the partition satisfies the following properties:

1. We divide $PG(1, 11^3)$ into 333 orbits, each one of them containing four points called tetrads;
2. From the 333 orbits, we obtain two cross-ratio ($\Upsilon_1 = \alpha^5$ and $\Upsilon_2 = \alpha^9$)
3. The cross-ratio of $\{\infty, 0, 1, \alpha^5\}$ belongs to the class H and that of $\{\infty, 0, 1, \alpha^9\}$ belongs to the class E;
4. Each of $\Upsilon_1$ and $\Upsilon_2$ gave 16 permutations from the same class to which they belong.

## 5. Partitions on $PG(2, 11^i), i = 1, 2$

After the projective line has been partitioned, we now partition the projective plane on the two fields $F_{11}$ & $F_{11^2}$. The used symbols are:

- $n$: The size of orbit.
- $k$: The degree of arc.
- $d$ : The minimum distance of projective code.
- $e = [d - 1/2]$ error correcting .

### 5.1_ Partition on the projective plane $PG(2, 11)$

At PG(2,11) there are 133 points and 133 lines, with 12 points on lines, and 12 lines passing through points. To construct PG(2,11), let $W(X) = X^3 + \tau^8 X^2 + X + \tau, \tau \in F_{11}$, where $\tau = 2$ is a primitive element and $W$ is a primitive polynomial over $F_{11}$. The non- singular matrix

$$M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \tau^6 & \tau^5 & \tau^3 \end{pmatrix} \quad \ldots (4)$$

generated 133 points and lines on PG(2,11), such that $P_i = [1,0,0]\, M^i, i = 0,1,2, \cdots, 132$.

The actions of $M^7$ and $M^{19}$ are given in Tables-(3 and 4).

**Table 3**-The first partition on PG(2,11)

| NO. | The orbits | *n* | *k* | *d* | *e* | Description |
|---|---|---|---|---|---|---|
| $M^7$ | $P_0 \xrightarrow{M^7} P_7 \xrightarrow{M^7} P_{14} \xrightarrow{M^7} P_{21} \cdots \xrightarrow{M^7} P_{126} \xrightarrow{M^7} P_0$<br>$P_1 \xrightarrow{M^7} P_8 \xrightarrow{M^7} P_{15} \xrightarrow{M^7} P_{22} \cdots \xrightarrow{M^7} P_{127} \xrightarrow{M^7} P_1$<br>$P_2 \xrightarrow{M^7} P_9 \xrightarrow{M^7} P_{16} \xrightarrow{M^7} P_{23} \cdots \xrightarrow{M^7} P_{128} \xrightarrow{M^7} P_2$<br>$P_3 \xrightarrow{M^7} P_{10} \xrightarrow{M^7} P_{17} \xrightarrow{M^7} P_{24} \cdots \xrightarrow{M^7} P_{129} \xrightarrow{M^7} P_3$<br>$P_4 \xrightarrow{M^7} P_{11} \xrightarrow{M^7} P_{18} \xrightarrow{M^7} P_{25} \cdots \xrightarrow{M^7} P_{130} \xrightarrow{M^7} P_4$<br>$P_5 \xrightarrow{M^7} P_{12} \xrightarrow{M^7} P_{19} \xrightarrow{M^7} P_{26} \cdots \xrightarrow{M^7} P_{131} \xrightarrow{M^7} P_5$<br>$P_6 \xrightarrow{M^7} P_{13} \xrightarrow{M^7} P_{20} \xrightarrow{M^7} P_{27} \cdots \xrightarrow{M^7} P_{131} \xrightarrow{M^7} P_6$ | **19** | **3** | **16** | **7** | **$C_0 \neq 0$ incomplete** |

From the third table we obtained the following results:

- PG(2,11) is partitioned into 7 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $M$;
- The values of the parameters of the projective code are $n = 19, k = 3, d = 16, e = 7$;
- The matrix $M^7$ cycles 19 points.
- Any orbit in Table- 3 represents an incomplete arc since $c_0 \neq 0$;

**Table 4-**The second partition on PG(2,11).

| NO. | The orbits | $n$ | $k$ | $d$ | $e$ | Description |
|---|---|---|---|---|---|---|
| $M^{19}$ | $P_0 \xrightarrow{M^{19}} P_{19} \xrightarrow{M^{19}} P_{38} \xrightarrow{M^{19}} P_{57} \xrightarrow{M^{19}} \dots \xrightarrow{M^{19}} P_{114} \xrightarrow{M^{19}} P_0$<br>$P \xrightarrow{M^{19}} P_{20} \xrightarrow{M^{19}} P_{39} \xrightarrow{M^{19}} P_{58} \xrightarrow{M^{19}} \dots \xrightarrow{M^{19}} P_{115} \xrightarrow{M^{19}} P_1$<br>$P_2 \xrightarrow{M^{19}} P_{21} \xrightarrow{M^{19}} P_{40} \xrightarrow{M^{19}} P_{59} \xrightarrow{M^{19}} \dots \xrightarrow{M^{19}} P_{116} \xrightarrow{M^{19}} P_2$<br>$P_3 \xrightarrow{M^{19}} P_{22} \xrightarrow{M^{19}} P_{41} \xrightarrow{M^{19}} P_{60} \xrightarrow{M^{19}} \dots \xrightarrow{M^{19}} P_{117} \xrightarrow{M^{19}} P_3$<br>$P_4 \xrightarrow{M^{19}} P_{23} \xrightarrow{M^{19}} P_{42} \xrightarrow{M^{19}} P_{61} \xrightarrow{M^{19}} \dots \xrightarrow{M^{19}} P_{118} \xrightarrow{M^{19}} P_4$<br>$\vdots$<br>$P_{18} \xrightarrow{M^{19}} P_{37} \xrightarrow{M^{19}} P_{56} \xrightarrow{M^{19}} P_{75} \xrightarrow{M^{19}} \dots \xrightarrow{M^{19}} P_{132} \xrightarrow{M^{19}} P_{18}$ | 7 | 2 | 5 | 2 | $C_0 = 0$ complete |

From the forth table, we obtain the following:

- PG(2,11) is partitioned into 19 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $M$;
- The values of the parameters of the projective code are $n = 7, k = 2, d = 5, e = 2$;
- The matrix $M^{19}$ cycles 7 points.
- Any orbit in Table- 4 represents a complete arc since $c_0 = 0$;

**5.2_The group action on the projective plane PG$(2, 11^2)$**

Let $Y(X) = X^3 - \rho^{22}X^2 - \rho^{61}, \rho \in F_{11^2}$ be a polynomial, to construct PG$(2, 11^2)$, since $Y$ is primitive over a field $F_{11^2}$. We have 14763 points and lines on PG$(2, 11^2)$, The non-singular matrix that generated these points is

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \rho^{61} & 0 & \rho^{22} \end{pmatrix} \quad \dots (4)$$

Such that $V_i = [1,0,0]B^i, i = 0, \dots, 14762$.

We have 13 partitions on PG$(2, 11^2)$ as illustrated by the table below.

**Table 5-**The first partition on PG$(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^3$ | $P_0 \xrightarrow{B^3} P_3 \xrightarrow{B^3} P_6 \xrightarrow{B^3} P_9 \xrightarrow{B^3} P_{12} \xrightarrow{B^3} \dots \xrightarrow{B^3} P_{14760} \xrightarrow{B^3} P_0$<br>$P_1 \xrightarrow{B^3} P_4 \xrightarrow{B^3} P_7 \xrightarrow{B^3} P_{10} \xrightarrow{B^3} P_{13} \xrightarrow{B^3} \dots \xrightarrow{B^3} P_{14761} \xrightarrow{B^3} P_1$<br>$P_2 \xrightarrow{B^3} P_5 \xrightarrow{B^3} P_8 \xrightarrow{B^3} P_{11} \xrightarrow{B^3} P_{14} \xrightarrow{B^3} \dots \xrightarrow{B^3} P_{14762} \xrightarrow{B^3} P_2$ | 4921 | 48 | 4873 | 2436 |

From Table-5, we obtain the following:

- PG$(2, 11^2)$ is partitioned into 3 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 4921, k = 48, d = 4873,$ $e = 2436$;
- The matrix $B^3$ cycles 4921 points.

**Table 6-**The second partition on $PG(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^7$ | $P_0 \xrightarrow{B^7} P_7 \xrightarrow{B^7} P_{14} \xrightarrow{B^7} P_{21} \xrightarrow{B^7} P_{28} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14756} \xrightarrow{B^7} P_0$<br>$P_1 \xrightarrow{B^7} P_8 \xrightarrow{B^7} P_{15} \xrightarrow{B^7} P_{22} \xrightarrow{B^7} P_{29} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14757} \xrightarrow{B^7} P_1$<br>$P_2 \xrightarrow{B^7} P_9 \xrightarrow{B^7} P_{16} \xrightarrow{B^7} P_{23} \xrightarrow{B^7} P_{30} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14758} \xrightarrow{B^7} P_2$<br>$P_3 \xrightarrow{B^7} P_{10} \xrightarrow{B^7} P_{17} \xrightarrow{B^7} P_{24} \xrightarrow{B^7} P_{31} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14759} \xrightarrow{B^7} P_3$<br>$P_4 \xrightarrow{B^7} P_{11} \xrightarrow{B^7} P_{18} \xrightarrow{B^7} P_{25} \xrightarrow{B^7} P_{32} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14760} \xrightarrow{B^7} P_4$<br>$P_5 \xrightarrow{B^7} P_{12} \xrightarrow{B^7} P_{19} \xrightarrow{B^7} P_{26} \xrightarrow{B^7} P_{33} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14761} \xrightarrow{B^7} P_5$<br>$P_6 \xrightarrow{B^7} P_{13} \xrightarrow{B^7} P_{20} \xrightarrow{B^7} P_{27} \xrightarrow{B^7} P_{34} \xrightarrow{B^7} \dots \xrightarrow{B^7} P_{14762} \xrightarrow{B^7} P_6$ | 2109 | 20 | 2089 | 1004 |

From Table-6, we obtain the following:

- $PG(2, 11^2)$ is partitioned into 7 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 2109, k = 20, d = 2009$, $e = 1004$;
- The matrix $B^7$ cycles 2109 points.

**Table 7-**The third partition on $PG(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{19}$ | $P_0 \xrightarrow{B^{19}} P_{19} \xrightarrow{B^{19}} P_{38} \xrightarrow{B^{19}} P_{57} \xrightarrow{B^{19}} \dots \xrightarrow{B^{19}} P_{14717} \xrightarrow{B^{19}} P_0$<br>$P \xrightarrow{B^{19}} P_{20} \xrightarrow{B^{19}} P_{39} \xrightarrow{B^{19}} P_{58} \xrightarrow{B^{19}} \dots \xrightarrow{B^{19}} P_{14718} \xrightarrow{B^{19}} P_1$<br>$P_2 \xrightarrow{B^{19}} P_{21} \xrightarrow{B^{19}} P_{40} \xrightarrow{B^{19}} P_{59} \xrightarrow{B^{19}} \dots \xrightarrow{B^{19}} P_{14719} \xrightarrow{B^{19}} P_2$<br>$P_3 \xrightarrow{B^{19}} P_{22} \xrightarrow{B^{19}} P_{41} \xrightarrow{B^{19}} P_{60} \xrightarrow{B^{19}} \dots \xrightarrow{B^{19}} P_{14720} \xrightarrow{B^{19}} P_3$<br>$P_4 \xrightarrow{B^{19}} P_{23} \xrightarrow{B^{19}} P_{42} \xrightarrow{B^{19}} P_{61} \xrightarrow{B^{19}} \dots \xrightarrow{B^{19}} P_{14721} \xrightarrow{B^{19}} P_4$<br>$\vdots$<br>$P_{18} \xrightarrow{B^{19}} P_{37} \xrightarrow{B^{19}} P_{56} \xrightarrow{B^{19}} P_{75} \xrightarrow{B^{19}} \dots \xrightarrow{B^{19}} P_{14745} \xrightarrow{B^{19}} P_{18}$ | 777 | 9 | 768 | 383 |

From Table-7, we obtain the following:

- $PG(2, 11^2)$ is partitioned into 19 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 777, k = 9, d = 768$, $e = 383$;
- The matrix $B^{19}$ cycles 777 points.

**Table 8-**The forth partition on $PG(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{37}$ | $P_0 \xrightarrow{B^{37}} P_{37} \xrightarrow{B^{37}} P_{74} \xrightarrow{B^{37}} P_{111} \xrightarrow{B^{37}} \dots \xrightarrow{B^{37}} P_{14726} \xrightarrow{B^{37}} P_0$<br>$P_1 \xrightarrow{B^{37}} P_{38} \xrightarrow{B^{37}} P_{75} \xrightarrow{B^{37}} P_{112} \xrightarrow{B^{37}} \dots \xrightarrow{B^{37}} P_{14727} \xrightarrow{B^{37}} P_1$<br>$P_2 \xrightarrow{B^{37}} P_{39} \xrightarrow{B^{37}} P_{76} \xrightarrow{B^{37}} P_{113} \xrightarrow{B^{37}} \dots \xrightarrow{B^{37}} P_{14728} \xrightarrow{B^{37}} P_2$<br>$P_3 \xrightarrow{B^{37}} P_{40} \xrightarrow{B^{37}} P_{77} \xrightarrow{B^{37}} P_{114} \xrightarrow{B^{37}} \dots \xrightarrow{B^{37}} P_{14729} \xrightarrow{B^{37}} P_3$<br>$P_4 \xrightarrow{B^{37}} P_{41} \xrightarrow{B^{37}} P_{78} \xrightarrow{B^{37}} P_{115} \xrightarrow{B^{37}} \dots \xrightarrow{B^{37}} P_{14730} \xrightarrow{B^{19}} P_4$<br>$\vdots$<br>$P_{36} \xrightarrow{B^{37}} P_{73} \xrightarrow{B^{37}} P_{110} \xrightarrow{B^{37}} P_{147} \xrightarrow{B^{37}} \dots \xrightarrow{B^{37}} P_{14761} \xrightarrow{B^{37}} P_{36}$ | 399 | 14 | 385 | 192 |

From Table-8, we obtain the following:

- $PG(2,11^2)$ is partitioned into 37 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 399, k = 14, d = 385$, $e = 192$;
- The matrix $B^{37}$ cycles 399 points

**Table 9-**The fifth partition on $PG(2,11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{21}$ | $P_0 \xrightarrow{B^{21}} P_{21} \xrightarrow{B^{21}} P_{42} \xrightarrow{B^{21}} P_{63} \xrightarrow{B^{21}} \dots \xrightarrow{B^{21}} P_{14742} \xrightarrow{B^{21}} P_0$<br>$P_1 \xrightarrow{B^{21}} P_{22} \xrightarrow{B^{21}} P_{43} \xrightarrow{B^{21}} P_{64} \xrightarrow{B^{21}} \dots \xrightarrow{B^{21}} P_{14743} \xrightarrow{B^{21}} P_1$<br>$P_2 \xrightarrow{B^{21}} P_{23} \xrightarrow{B^{21}} P_{44} \xrightarrow{B^{21}} P_{65} \xrightarrow{B^{21}} \dots \xrightarrow{B^{21}} P_{14744} \xrightarrow{B^{21}} P_2$<br>$P_3 \xrightarrow{B^{21}} P_{24} \xrightarrow{B^{21}} P_{45} \xrightarrow{B^{21}} P_{66} \xrightarrow{B^{21}} \dots \xrightarrow{B^{21}} P_{14745} \xrightarrow{B^{21}} P_3$<br>$P_4 \xrightarrow{B^{21}} P_{25} \xrightarrow{B^{21}} P_{46} \xrightarrow{B^{21}} P_{67} \xrightarrow{B^{21}} \dots \xrightarrow{B^{21}} P_{14746} \xrightarrow{B^{21}} P_4$<br>$\vdots$<br>$P_{20} \xrightarrow{B^{21}} P_{41} \xrightarrow{B^{21}} P_{62} \xrightarrow{B^{21}} P_{83} \xrightarrow{B^{21}} \dots \xrightarrow{B^{21}} P_{14761} \xrightarrow{B^{21}} P_{20}$ | 703 | 11 | 692 | 345 |

From Table-9, we obtain the following:

- $PG(2,11^2)$ is partitioned into 21 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 703, k = 11, d = 692$, $e = 345$;
- The matrix $B^{21}$ cycles 703 points.

**Table 10**-The 6th partition on $PG(2,11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{57}$ | $P_0 \xrightarrow{B^{57}} P_{57} \xrightarrow{B^{57}} P_{114} \xrightarrow{B^{57}} P_{171} \xrightarrow{B^{57}} \dots \xrightarrow{B^{57}} P_{14706} \xrightarrow{B^{57}} P_0$<br>$P_1 \xrightarrow{B^{57}} P_{58} \xrightarrow{B^{57}} P_{115} \xrightarrow{B^{57}} P_{172} \xrightarrow{B^{57}} \dots \xrightarrow{B^{57}} P_{14707} \xrightarrow{B^{57}} P_1$<br>$P_2 \xrightarrow{B^{57}} P_{59} \xrightarrow{B^{57}} P_{116} \xrightarrow{B^{57}} P_{173} \xrightarrow{B^{57}} \dots \xrightarrow{B^{57}} P_{14708} \xrightarrow{B^{57}} P_2$<br>$P_3 \xrightarrow{B^{57}} P_{60} \xrightarrow{B^{57}} P_{117} \xrightarrow{B^{57}} P_{174} \xrightarrow{B^{57}} \dots \xrightarrow{B^{57}} P_{14709} \xrightarrow{B^{57}} P_3$<br>$P_4 \xrightarrow{B^{57}} P_{61} \xrightarrow{B^{57}} P_{118} \xrightarrow{B^{57}} P_{175} \xrightarrow{B^{57}} \dots \xrightarrow{B^{57}} P_{14710} \xrightarrow{B^{57}} P_4$<br>$\vdots$<br>$P_{56} \xrightarrow{B^{57}} P_{113} \xrightarrow{B^{57}} P_{170} \xrightarrow{B^{57}} P_{227} \xrightarrow{B^{57}} \dots \xrightarrow{B^{57}} P_{14761} \xrightarrow{B^{57}} P_{56}$ | 259 | 4 | 255 | 127 |

From Table-10, we have the following:

- $PG(2,11^2)$ is partitioned into 57 orbits;
- Any one of the pervious orbits represents $PG(2,11^2)$ and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 4, k = 259, d = 255$, $e = 127$;
- All the orbits are disjoint on the effect of $B$ as 14763-transtive;
- The set $B^{57}$ is cyclic on 259 points.

**Table 11-** The 7th partition on $PG(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{111}$ | $P_0 \xrightarrow{B^{111}} P_{111} \xrightarrow{B^{111}} P_{222} \xrightarrow{B^{111}} P_{333} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14652} \xrightarrow{B^{111}} P_0$<br>$P_1 \xrightarrow{B^{111}} P_{112} \xrightarrow{B^{111}} P_{223} \xrightarrow{B^{111}} P_{334} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14653} \xrightarrow{B^{111}} P_1$<br>$P_2 \xrightarrow{B^{111}} P_{113} \xrightarrow{B^{111}} P_{224} \xrightarrow{B^{111}} P_{335} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14654} \xrightarrow{B^{111}} P_2$<br>$P_3 \xrightarrow{B^{111}} P_{114} \xrightarrow{B^{111}} P_{225} \xrightarrow{B^{111}} P_{336} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14655} \xrightarrow{B^{111}} P_3$<br>$P_4 \xrightarrow{B^{111}} P_{115} \xrightarrow{B^{111}} P_{226} \xrightarrow{B^{111}} P_{337} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14656} \xrightarrow{B^{111}} P_4$<br>$\vdots$<br>$P_{110} \xrightarrow{B^{111}} P_{221} \xrightarrow{B^{111}} P_{332} \xrightarrow{B^{111}} P_{443} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14761} \xrightarrow{B^{111}} P_{11}$ | 133 | 12 | 121 | 60 |

From Table-11, we obtain the following:

- $PG(2, 11^2)$ is partitioned into 111 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 133, k = 12, d = 121,$ $e = 60$;
- The matrix $B^{111}$ cycles 133 points.

**Table 12-**The 8th partition on $PG(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{133}$ | $P_0 \xrightarrow{B^{133}} P_{133} \xrightarrow{B^{133}} P_{266} \xrightarrow{B^{133}} P_{399} \xrightarrow{B^{133}} \dots \xrightarrow{B^{133}} P_{14630} \xrightarrow{B^{133}} P_0$<br>$P_1 \xrightarrow{B^{133}} P_{134} \xrightarrow{B^{133}} P_{267} \xrightarrow{B^{133}} P_{400} \xrightarrow{B^{133}} \dots \xrightarrow{B^{133}} P_{14631} \xrightarrow{B^{133}} P_1$<br>$P_2 \xrightarrow{B^{133}} P_{135} \xrightarrow{B^{133}} P_{268} \xrightarrow{B^{133}} P_{401} \xrightarrow{B^{133}} \dots \xrightarrow{B^{133}} P_{14632} \xrightarrow{B^{133}} P_2$<br>$P_3 \xrightarrow{B^{133}} P_{136} \xrightarrow{B^{133}} P_{269} \xrightarrow{B^{133}} P_{402} \xrightarrow{B^{133}} \dots \xrightarrow{B^{133}} P_{14633} \xrightarrow{B^{133}} P_3$<br>$P_4 \xrightarrow{B^{111}} P_{137} \xrightarrow{B^{111}} P_{270} \xrightarrow{B^{111}} P_{403} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14634} \xrightarrow{B^{111}} P_4$<br>$\vdots$<br>$P_{132} \xrightarrow{B^{111}} P_{256} \xrightarrow{B^{111}} P_{390} \xrightarrow{B^{111}} P_{531} \xrightarrow{B^{111}} \dots \xrightarrow{B^{111}} P_{14761} \xrightarrow{B^{111}} P_{13}$ | 111 | 2 | 109 | 54 |

From Table-12, we have the following:

- $PG(2, 11^2)$ is partitioned into 133 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 111, k = 2, d = 109,$ $e = 54$;
- The matrix $B^{133}$ cycles 111 points.

**Table 13-**The 9th partition on $PG(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{259}$ | $P_0 \xrightarrow{B^{259}} P_{259} \xrightarrow{B^{259}} P_{518} \xrightarrow{B^{259}} P_{777} \xrightarrow{B^{259}} \dots \xrightarrow{B^{259}} P_{14504} \xrightarrow{B^{259}} P_0$<br>$P_1 \xrightarrow{B^{259}} P_{260} \xrightarrow{B^{259}} P_{519} \xrightarrow{B^{259}} P_{778} \xrightarrow{B^{259}} \dots \xrightarrow{B^{259}} P_{14505} \xrightarrow{B^{259}} P_1$<br>$P_2 \xrightarrow{B^{259}} P_{261} \xrightarrow{B^{259}} P_{520} \xrightarrow{B^{259}} P_{779} \xrightarrow{B^{259}} \dots \xrightarrow{B^{259}} P_{14506} \xrightarrow{B^{259}} P_2$<br>$P_3 \xrightarrow{B^{259}} P_{262} \xrightarrow{B^{259}} P_{521} \xrightarrow{B^{259}} P_{780} \xrightarrow{B^{259}} \dots \xrightarrow{B^{259}} P_{14507} \xrightarrow{B^{259}} P_3$<br>$P_4 \xrightarrow{B^{259}} P_{263} \xrightarrow{B^{259}} P_{522} \xrightarrow{B^{259}} P_{781} \xrightarrow{B^{259}} \dots \xrightarrow{B^{259}} P_{14508} \xrightarrow{B^{259}} P_4$<br>$\vdots$<br>$P_{258} \xrightarrow{B^{259}} P_{517} \xrightarrow{B^{259}} P_{776} \xrightarrow{B^{259}} P_{1035} \xrightarrow{B^{259}} \dots \xrightarrow{B^{259}} P_{14761} \xrightarrow{B^{259}} P_{2}$ | 57 | 4 | 54 | 26 |

From Table-13, we have the following:

- $\mathrm{PG}(2, 11^2)$ is partitioned into 259 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 57, k = 4, d = 54$, $e = 26$;
- The matrix $B^{259}$ cycles 57 points.

**Table 14-**The 10th partition on $\mathrm{PG}(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{703}$ | $P_0 \xrightarrow{B^{703}} P_{703} \xrightarrow{B^{703}} P_{1406} \xrightarrow{B^{703}} P_{2109} \xrightarrow{B^{703}} \dots \xrightarrow{B^{703}} P_{14060} \xrightarrow{B^{703}} P_0$<br>$P_1 \xrightarrow{B^{703}} P_{704} \xrightarrow{B^{703}} P_{1407} \xrightarrow{B^{703}} P_{2110} \xrightarrow{B^{703}} \dots \xrightarrow{B^{703}} P_{14061} \xrightarrow{B^{703}} P_1$<br>$P_2 \xrightarrow{B^{703}} P_{705} \xrightarrow{B^{703}} P_{1408} \xrightarrow{B^{703}} P_{2111} \xrightarrow{B^{703}} \dots \xrightarrow{B^{703}} P_{14062} \xrightarrow{B^{703}} P_2$<br>$P_3 \xrightarrow{B^{703}} P_{706} \xrightarrow{B^{703}} P_{1409} \xrightarrow{B^{703}} P_{2112} \xrightarrow{B^{703}} \dots \xrightarrow{B^{703}} P_{14063} \xrightarrow{B^{703}} P_3$<br>$P_4 \xrightarrow{B^{703}} P_{707} \xrightarrow{B^{703}} P_{1410} \xrightarrow{B^{703}} P_{2113} \xrightarrow{B^{703}} \dots \xrightarrow{B^{703}} P_{14064} \xrightarrow{B^{703}} P_4$<br>$\vdots$<br>$P_{702} \xrightarrow{B^{703}} P_{1405} \xrightarrow{B^{703}} P_{2108} \xrightarrow{B^{703}} \dots \xrightarrow{B^{703}} P_{14761} \xrightarrow{B^{703}} P_{702}$ | 21 | 2 | 19 | 9 |

From Table-14, we obtain the following:

- $\mathrm{PG}(2, 11^2)$ is partitioned into 703 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 21, k = 2, d = 19$, $e = 9$;
- The matrix $B^{703}$ cycles 21 points.

**Table 15-**The 11th partition on $\mathrm{PG}(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{399}$ | $P_0 \xrightarrow{B^{399}} P_{399} \xrightarrow{B^{399}} P_{798} \xrightarrow{B^{399}} P_{1197} \xrightarrow{B^{399}} \dots \xrightarrow{B^{399}} P_{14364} \xrightarrow{B^{399}} P_0$<br>$\square_1 \xrightarrow{B^{399}} P_{400} \xrightarrow{B^{399}} P_{799} \xrightarrow{B^{399}} P_{1198} \xrightarrow{B^{399}} \dots \xrightarrow{B^{399}} P_{14365} \xrightarrow{B^{399}} P_1$<br>$P_2 \xrightarrow{B^{399}} P_{401} \xrightarrow{B^{399}} P_{800} \xrightarrow{B^{399}} P_{1199} \xrightarrow{B^{399}} \dots \xrightarrow{B^{399}} P_{14366} \xrightarrow{B^{399}} P_2$<br>$P_3 \xrightarrow{B^{399}} P_{402} \xrightarrow{B^{399}} P_{801} \xrightarrow{B^{399}} P_{1200} \xrightarrow{B^{399}} \dots \xrightarrow{B^{399}} P_{14367} \xrightarrow{B^{399}} P_3$<br>$P_4 \xrightarrow{B^{399}} P_{403} \xrightarrow{B^{399}} P_{802} \xrightarrow{B^{399}} P_{1201} \xrightarrow{B^{399}} \dots \xrightarrow{B^{399}} P_{14368} \xrightarrow{B^{399}} P_4$<br>$\vdots$<br>$P_{398} \xrightarrow{B^{399}} P_{797} \xrightarrow{B^{399}} P_{1196} \xrightarrow{B^{399}} P_{1595} \xrightarrow{B^{399}} \dots \xrightarrow{B^{399}} P_{14761} \xrightarrow{B^{399}} P_{398}$ | 37 | 3 | 34 | 16 |

From Table-15, we have the following:

- $\mathrm{PG}(2, 11^2)$ is partitioned into 399 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 37, k = 3, d = 34$, $e = 16$;
- The matrix $B^{399}$ cycles 27 points.

**Table 16-**The 12th partition on $\mathrm{PG}(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{777}$ | $P_0 \xrightarrow{B^{777}} P_{777} \xrightarrow{B^{777}} P_{1554} \xrightarrow{B^{777}} P_{2331} \xrightarrow{B^{777}} \ldots \xrightarrow{B^{777}} P_{13986} \xrightarrow{B^{777}} P_0$<br>$\square_1 \xrightarrow{B^{777}} P_{778} \xrightarrow{B^{777}} P_{1555} \xrightarrow{B^{777}} P_{2332} \xrightarrow{B^{777}} \ldots \xrightarrow{B^{777}} P_{13987} \xrightarrow{B^{777}} P_1$<br>$P_2 \xrightarrow{B^{777}} P_{779} \xrightarrow{B^{777}} P_{1556} \xrightarrow{B^{777}} P_{2333} \xrightarrow{B^{777}} \ldots \xrightarrow{B^{777}} P_{13988} \xrightarrow{B^{777}} P_2$<br>$P_3 \xrightarrow{B^{777}} P_{780} \xrightarrow{B^{777}} P_{1557} \xrightarrow{B^{777}} P_{2334} \xrightarrow{B^{777}} \ldots \xrightarrow{B^{777}} P_{13989} \xrightarrow{B^{777}} P_3$<br>$P_4 \xrightarrow{B^{777}} P_{781} \xrightarrow{B^{777}} P_{1558} \xrightarrow{B^{777}} P_{2335} \xrightarrow{B^{777}} \ldots \xrightarrow{B^{777}} P_{13990} \xrightarrow{B^{777}} P_4$<br>$\vdots$<br>$P_{776} \xrightarrow{B^{777}} P_{1553} \xrightarrow{B^{777}} P_{2330} \xrightarrow{B^{777}} \ldots \xrightarrow{B^{777}} P_{13985} \xrightarrow{B^{777}} P_{776}$ | *19* | *3* | *16* | *7* |

From Table-16, we obtain the following:

- $\mathrm{PG}(2, 11^2)$ is partitioned into 777 orbits;
- Any one of the pervious orbits represents an arc and all of them are equivalent under the effect of the matrix $B$;
- The values of the parameters of the projective code are $n = 19, k = 3, d = 16, e = 7$;
- The matrix $B^{777}$ cycles 19 points.

**Table 17**-The 13th partition on $\mathrm{PG}(2, 11^2)$.

| No. | Orbits | $n$ | $k$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $B^{4921}$ | $P_0 \xrightarrow{B^{4921}} P_{4921} \xrightarrow{B^{4921}} P_{9842} \xrightarrow{B^{4921}} P_0$<br>$P_1 \xrightarrow{B^{4921}} P_{4922} \xrightarrow{B^{4921}} P_{9843} \xrightarrow{B^{4921}} P_1$<br>$P_2 \xrightarrow{B^{4921}} P_{4923} \xrightarrow{B^{4921}} P_{9844} \xrightarrow{B^{4921}} P_2$<br>$P_3 \xrightarrow{B^{4921}} P_{4924} \xrightarrow{B^{4921}} P_{9845} \xrightarrow{B^{4921}} P_3$<br>$P_4 \xrightarrow{B^{4921}} P_{4925} \xrightarrow{B^{4921}} P_{9846} \xrightarrow{B^{4921}} P_4$<br>$\vdots$<br>$P_{4920} \xrightarrow{B^{4921}} P_{9841} \xrightarrow{B^{4921}} P_{14762} \xrightarrow{B^{4921}} P_0$ | *3* | *2* | *1* | *0* |

From Table-17, we obtain the following:

- $\mathrm{PG}(2, 11^2)$ is partitioned into 4921 orbits;
- Any one of the pervious orbits a triangle and all of them are equivalent under the effect of the matrix $B^{4921}$;
- The values of the parameters of the projective code are $n = 3, k = 2, d = 1, e = 0$;
- The matrix $B^{4921}$ cycles 3 points.

**Conclusions**

The summary of this research is given in the following table.

| Matrix | No. of orbits | $n$ | $k$ | $d$ |
|---|---|---|---|---|
| $M^7$ | *7* | *19* | *3* | *16* |
| $M^{19}$ | *19* | *7* | *2* | *5* |
| $B^3$ | *3* | *4921* | *48* | *4873* |
| $B^7$ | *7* | *2109* | *20* | *2089* |
| $B^{19}$ | *19* | *777* | *9* | *768* |

| | | | | |
|---|---|---|---|---|
| $B^{37}$ | *37* | *399* | *14* | *385* |
| $B^{21}$ | *21* | *703* | *11* | *692* |
| $B^{57}$ | *57* | *259* | *4* | *255* |
| $B^{111}$ | *111* | *133* | *12* | *121* |
| $B^{133}$ | *133* | *111* | *2* | *109* |
| $B^{259}$ | *259* | *57* | *4* | *54* |
| $B^{703}$ | *703* | *21* | *2* | *19* |
| $B^{399}$ | *399* | *37* | *3* | *34* |
| $B^{777}$ | *777* | *19* | *3* | *16* |
| $B^{4921}$ | *4921* | *3* | *2* | *1* |

**Acknowledgements**

We are thankful to all professors in the department of Mathematics, Collage of Science, Mustansiriyah University for their positive support for us.

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