DOI: 10.24996/ijs.2021.62.2.26

ISSN: 0067-2904

Some Results on m_X-N-connected Space

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Received: 30/12/2019 Accepted: 15/3/2020

Abstract

In this essay, we utilize m - space to specify m_X-N-connected, m_X-N-hyper connected and m_X-N-locally connected spaces and some functions by exploiting the intelligible m_X-N-open set. Some instances and outcomes have been granted to boost our tasks.

Keywords: minimal structure, m_X-N-open set, m_X-N-connected space, m_X-N-hyper connected space, m_X-N-locally connected space, m-N-continuous.

بعض النتائج حول الفضاء المتصل –m_X–N

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الخلاصة

في هذا البحث استعملنا الفضاء – m لتعريف الفضاءات –N–m المتصلة و–N–m المتصلة محليا وبعض الدوال باستخدام مفهوم المجموعات m_X–N– المفتوحة. بعض الحقائق والنتائج قد أعطيت معززة لعملنا.

Introduction

A. AL-Omari, and M.S.Md. Noorani [1] presented the idea of N – open sets which can be described as follows. A subcategory U of a space X is nominated to be N – open if for every $x \in U$ an open set U_X is found and comprising x, with the end goal that U_X / U is a finite. In 2000, Popa and Noiri [2,3] presented the idea of minimal structure space. They additionally characterized m-compactness and m-connectedness and analyzed their essential attributes. Hussain and Nasser [4] characterized the N – disconnected space in an association of two N – separated sets. They provided several depictions and relate them to some other recently known classes of space, for instance, N – locally connected and N – hyper connected spaces. In this paper, we first presented and studied the idea of m-N-connected, m-N-hyper connected, m-locally connected and m-N-locally connected spaces, by utilizing m_X-N-open set, and demonstrated some outcomes on this idea.

1: PRELIMINARIES

Definition (1.1) [5, 3]

Let X be a non-empty set and ƛ(X) the power set of X. A subfamily m_X of ƛ(X) is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_X$ and $X \in m_X$. By (X, m_X), we indicate a non-empty set X with an m-structure m_X on X, and it is called m-space. Every individual from m_X is nominated to be m_X-open and the complement of an m_X-open set is nominated to be m_X-closed set.

Definition (1.2) [6]

Let X be a non-empty set and m_X be an m-structure on X. For a subcategory U of X, the m_X-closure of U and the m_X-interior of U are characterized as:

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i. m_X-cl(U) = ∩{M: U⊆ M, X / M ∈ m_X}.
ii. m_X-int(U) = ⋃ {K: K ⊆ U, K ∈ m_X}.

Lemma (1.3) [7]: Let X be a non-empty set and m_X a minimal structure on X. For a subcategory U and V of X, the accompanying properties hold:
i. m_X-int(X / U) = X / m_X-cl(U).
ii. If (X / U) ∈ m_X, then m_X-cl(U) = U and if U∈ m_X, then m_X-int(U) = U.
iii. If U⊆ V, then m_X-cl(U) ⊆ m_X-cl(V) and m_X-int(U) ⊆ m_X-int(V).
iv. m_X-cl(m_X-cl(U)) = m_X-cl(U) and m_X-int(m_X-int(U)) = m_X-int(U).

Lemma (1.4) [8]: Let X be a non-empty set with a minimal structure m_X, and let U be a subcategory of X. Then x∈ m_X-cl(U) if and only if K ∩ U ≠ Ø for every K ∈ m_X containing x.

Definition (1.5) [6]
An m-structure m_X on a non-empty set X is said to have property $\mathfrak{B}$ if the union of any family of subsets belong to m_X belongs to m_X.

Definition (1.6) [9]
A subcategory U of a an m-space (X, m_X) is nominated to be
i. m_X-dense if m_X-cl(U) = X.
ii. m_X-nowhere dense if m_X-int(m_X-cl(U)) = Ø.

Definition (1.7) [10]
The subsets U and V of m-space X are designated to be m_X-separated in X if and only if (U∩ m_X-cl(V)) ⋃ (m_X-cl(U) ∩ V) = Ø.

Definition (1.8) [10]
A subset U of X in (X, m_X) is nominated to be m-connected in X (or simply m-connected) if U cannot be composed as the association of two non-empty m_X-separated subcategories of X. If U is not m-connected in X, then we state that U is m-disconnected in X. A space (X, m_X) is designated to be m-connected if the underlying set X is m-connected.

Definition (1.9) [3]
A non-empty set X with a minimal structure m_X that is fulfilling $\mathfrak{B}$ property is nominated to be m_X-connected if X cannot be composed as the association of two non-empty disjoint m_X-open sets.

Definition (1.10) [11]
A function f: (X, m_X) → (Y, m_Y) is nominated to be m-open if f(U) is an m_Y-open set of (Y, m_Y) for every m_X-open set U of (X, m_X).

2: m_X-N-open set

Definition (2.1) [12]
A subcategory U of an m-space X is nominated to be m_X-N-open set if every single x ∈ U, there exists an m_X-open set V containing x such that V/U is a finite set and the complement of an m_X-N-open set is called m_X-N-closed set.

Remark (2.2): each m_X-open set is an m_X-N-open set.

Proof: Let U be m_X-open set and x ∈ U, then x ∈ U⊆U and U/U = Ø. In this manner, U is an m_X-N-open set.

Example (2.3*)*: Let R be a set of all real numbers and m_X = {Ø, R}. We realize that R/{1}⊆R is an m_X-N-open anyway and it is definitely not an m_X-open.

Definition (2.4)
Let X be an m-space and U be a subcategory of it, then x ∈ X is designated to be m_X-N-interior point to U if an m_X-N-open set V is found such that x∈V⊆U. Then, the set of all m_X-N-interior points for U is indicated by m_X-N-int(U).

Definition (2.5)
Let X be an m-space and U⊆X, then x ∈X is nominated to be m_X-N-limit point to U if all m_X-N-open sets V containing x We have V /{x}) ∩U ≠ Ø, and the arrangement of all m_X-N-limit points for U is indicated by m_X-N-d(U).

Definition (2.6)
Let X be an m-space and U⊆X, then x∈X is nominated to be m_X-N- adherent point to U if every single m_X-N-open set V that containing x is intersected with U. i.e. V∩U ≠ Ø. The arrangement of all m_X-N-adherent points for U is indicated by m_X-N-adh(U) or m_X-N-cl(U).

Definition (2.7)

Let X be a non-empty set and m_X an m-structure on X. For a subcategory U of X, the m_X-N-closure of U and the m_X-N-interior of U are described as:

i. m_X-N-int(U) = $\bigcup${M: M ⊆ U, M is an m_X-N-open}.

ii. m_X-N-cl(U) = ∩{K: U ⊆ K, K is an m_X-N-closed}.

Proposition (2.8): Let (X, m_X) be an m-space, then the following accompanying attributes are verified: i. The union of any family of m_X-N-open sets is an m_X-N-open set.

ii.The intersection of any family of m_X-N-closed sets is an m_X-N-closed set.

Proof:

i. Let U_α be an m_X-N-open set for each $\alpha\in\wedge$. To prove that $\bigcup\{U_\alpha, \alpha\in\wedge\}$ is m_X-N-open, let $x\in\bigcup\{U_\alpha, \alpha\in\wedge\}$, then $x\in U_{\alpha i}$ for some $\alpha i\in\wedge$. Since $U_{\alpha i}$ is an m_X-N-open, then there can be found V as an m_X-open set, such that $x\in V$ and $V / U_{\alpha i}$ is a finite set. Since $U_{\alpha I}\subseteq\bigcup\{U_\alpha, \alpha\in\wedge\}$, then $(\bigcup\{U_\alpha, \alpha\in\wedge\})^C\subseteq(U_{\alpha i})^C$. So, $V\cap(\bigcup\{U_\alpha, \alpha\in\wedge\})^C\subseteq V\cap(U_{\alpha i})^C$. Hence, $V / \bigcup\{U_\alpha, \alpha\in\wedge\}\subseteq V/ U_{\alpha i}$. Since $V / U_{\alpha i}$ is a finite set, then $V / \bigcup\{U_\alpha, \alpha\in\wedge\}$ is a finite set, too. Hence $\bigcup\{U_{\alpha i}, \alpha\in\wedge\}$ is an m_X-N-open set.

ii. Clear by (i).

Proposition (2.9): Let U be a subcategory of m-space X, then:

i.U is m_X-N-open set if and only if m_X-N-int(U) = U.

ii.U is m_X-N-closed set if and only if m_X-N-cl(U) = U.

proof:

i. As the union of each m_X-N-open set is m_X-N-open set, then m_X-N-int(U) is the largest m_X-N-open set contained in U. Since U is m_X-N-open set, then m_X-N-int(U) = U. Conversely, whenever m_X-N-int(U) = U, then U is m_X-N-open set, since m_X-N-int(U) is an m_X-N-open set.

ii. As the intersection of each m_X-N-close set is m_X-N-close set, then m_X-N-cl(U) is the smallest m_X-N-close set that containing U. Since U is an m_X-N-closed set, then m_X-N-cl(U) = U. Conversely, whenever m_X-N-cl(U) = U, then U is an m_X-N-closed set, since m_X-N-cl(U) is an m_X-N-closed set.

Proposition (2.10): Let U, V be a subcategory of m-space X and $U\subseteq V$, then :

i.m_X -int(U) $\subseteq$ m_X-N-int(U). ii. m_X -N-cl(U) $\subseteq$ m_X-cl(U).

iii.m_X -N-cl(U) $\subseteq$ m_X-N-cl(V). iv.m_X -N-int(U) $\subseteq$ m_X-N-int(V)

v.m_X-N-int(X) = X and m_X-N-int(Ø) = Ø. vi.m_X-N-cl(X) = X and m_X-N-cl(Ø) = Ø.

vii.m_X-N-int(U) $\subseteq$ U and U $\subseteq$ m_X-N-cl(U) viii.m_X-N-int(m-N-int(U))= m_X-N-int(U)

x.m_X-N-cl(m-N-cl(U))= m_X-N-cl(U) ix.m_X-N-cl(U^C) = (m_X-N-int(U))^C .

x.m_X-N-int(U^C) = (m_X-N-cl(U))^C.

Proof:

i. Let $x\in m_X$-int(U), then there can be found m_X-open set U_X such that $x\in U_X\subseteq U$. For the reason that every m_X-open set is an m_X-N-open set, therefore $x\in m_X$-N-int(U).

ii. Let $x\notin m_X$-cl(U), then there can be found M as an m_X-open set, such that $x\in$ M and M ∩ U = Ø. For the reason that every m_X-open set is an m_X-N-open set, then $x\notin m_X$-N-cl(U) and consequently m_X-N-cl(U) $\subseteq$ m_X -cl(U)**.**

iii. Postulate that $x\in m_X$-N-cl(U), then each m_X-N-open set K containing x intersect U, Since $U\subseteq V$, then the set K intersect V. Consequently, $x\in m_X$-N-cl(V) and, in this way, m_X-N-cl(U) $\subseteq$ m_X-N-cl(V).

iv. Let $x\in m_X$-N-int(U), then there can be found an m_X-N-open set U_X such thatx$\in U_X\subseteq U$. For the reason that $U\subseteq V$, then $x\in U_X\subseteq V$. Consequently, $x\in m_X$-N-int(V). Therefore, m_X-N-int(U) $\subseteq$ m_X-N-int(V).

v. For the reason that X and Ø are m_X-N-open sets, then by definition 2.7, m_X-N-int (X) = $\bigcup$ {U: U is an m_X-N-open, $U\subseteq X$} = X $\bigcup$ all m_X-N-open sets = X. In this manner, m_X-N-int (X) = X. Since Ø is the only m_X-N-open set contained in Ø, henceforth, m_X-N-int (Ø) = Ø.

vi. By definition 2.7, then m_X-N-cl(X)= ∩{V: X⊆ V, V} is m_X-N-closed set. But X is the only m_X-

vii. N-closed set comprising X. In this way m_X-N-cl(X) = X. Thus, m_X-N-cl(X) = X. By the definition of m_X-N-cl(Ø), m_X-N-cl(Ø) = ∩{V: Ø ⊆ V, V is an m_X-N-closed} = Ø ∩ any m_X-N-closed sets comprising Ø = Ø. In this way m_X-N-cl(Ø) = Ø.

viii. Clear.

ix. By definition 2.7 and proposition 2.8, we note that m_X-N-int(U) is an m_X-N-open set. Furthermore, by proposition 2.9, we conclude that m_X-N-int(m-N-int(U)) = m_X-N-int(U).

x. By definition 2.7 and proposition 2.8, we note that m_X-N-cl(U) is an m_X-N-closed set. Furthermore, by proposition 2.9, we conclude that m_X-N-cl(m-N-cl(U)) = m_X-N-cl(U).

xi. Let $x \notin (m_X\text{-N-int}(U))^C$, then $x \in m_X$-N-int(U). Thus, there is an m_X-N-open set U_X such that$x \in U_X \subseteq$ U. In this way, $x \in U_X$ and $U_X \cap U^C = \emptyset$. So, $x \notin m_X$-N-cl(U^C). Thus, we get m_X-N-cl(U^C) $\subseteq$ (m_X-N-int(U))^C. Now, let $x \notin m_X$-N-cl(U^C), thus there is an m_X-N-open set U_X such that $x \in U_X$ and $U_X \cap U^C = \emptyset$. Hence, $x \in U_X \subseteq$ U and, in this manner, $x \in m_X$-N-int(U). Consequently, $x \notin (m_X\text{-N-int}(U))^C$. Thus, we get $(m_X\text{-N-int}(U))^C \subseteq m_X$-N-cl($U^C$).

xii. Let $x \in m_X$-N-int(U^C). Accordingly, there is an m_X-N-open set U_X such that$x \in U_X \subseteq U^C$. In this manner, $x \in U_X$ and $U_X \cap U = \emptyset$. Consequently, $x \notin m_X$-N-cl(U). Thus, we get $x \in$ (m-N-cl(U))^C and lastly m_X-N-int(U^C) $\subseteq$ (m_X-N-cl(U))^C. Then again, let $x \in (m_X\text{-N-cl}(U))^C$, then $x \notin m_X$-N-cl(U) and, in this way, there is an m_X-N-open set U_X such that$x \in U_X$ and $U_X \cap U = \emptyset$. Hence, $x \in U_X$ and $U_X \subseteq U^C$. Therefore, $x \in m_X$-N-int(U^C) and hence $(m_X\text{-N-cl}(U))^C \subseteq m_X$-N-int($U^C$). Finally, $(m_X\text{-N-cl}(U))^C$ = m_X-N-int(U^C).

Definition (2.11)

Let (X, m_X) be an m-space, then two non-empty subcategories U and V of X are nominated to be m_X-N-separated if U∩ m_X-N-cl(V) = Ø and V ∩ m_X-N-cl(U) = Ø.

Proposition (2.12): Two m_X-N-closed (open) subcategories U and V of X are m_X-N-separated iff they are disjoint.

Proof: Let U and V are both disjoint and m_X-N-open sets, then U^C and V^C are m_X-N-closed. As $U \subseteq V^C$, then m_X-N-cl(U) $\subseteq V^C$, thus m_X-N-cl(U) ∩ V = Ø. Likewise, we demonstrated that m_X-N-cl(V)∩U = Ø. Hence, both U and V are m_X-N-separated. Convrsely, if U and V are m_X-N-separated, then both U and V are disjoint in light of the fact that U ∩ V $\subseteq$ m_X-N-cl(U)∩ V.

Definition (2.13)

Let X be an m-space, then U⊆X, U is nominated to be m_X-N-dense in X if m_X-N-cl(U) = X.

Example (2.14): Let R be the arrangement of all genuine numbers and m_X ={Ø, R}, then we realize that all R/finite sets are m_X-N-dense.

Definition (2.15)

A subcategory U of an m-space (X, m_X) is nominated to be m_X-N-nowhere dense if m_X-N-int(m_X-cl(U)) = Ø.

Proposition (2.16): Every m_X-N-nowhere dense is an m_X-nowhere dense.

Proof: Clear.

Remark (2.17): The opposite of the above proposition might be not valid as a rule.

Example (2.18): Let X = {x_1, x_2, x_3} and m_X = {Ø, {x_1}, {x_2}, X}, then we realize that the set {x_3}is m_X-nowhere dense but it is not m_X-N-nowhere dense**.**

Definition (2.19)

A subcategory U of an m-space (X, m_X) is nominated to be m_X-N*-nowhere dense if m_X-N-int(m_X-N-cl(U)) = Ø.

Proposition (2.20): Every m_X-N-nowhere dense is an m_X-N*nowhere dense.

Proof: Clear.

Remark (2.21): The opposite of the above proposition might be not valid as a rule.

Example (2.22): Let R be the arrangement of all genuine numbers and m_X = {Ø, R}, then we realize that every finite set is m_X-N*-nowhere dense, yet it is not m_X-N-nowhere dense**.**

Remark (2.23): Let (X, m_X) be an m-space and U be a subcategory of X. If U is an m_X-N*-nowhere dense then it is not necessary to be an m_X-nowhere dense, as well the converse.

Example (2.24): Let X=R set all real numbers and m_X = {Ø, R}, then we realize that the set {1} is m_X-N*-nowhere dense, but not m_X-nowhere dense. On the other hand, let X = {x_1, x_2, x_3} and m_X ={Ø, {x_1, x_2}, X}, then we realize that the set {x_3} is an m-nowhere dense, but not m_X-N*-nowhere.

Proposition (2.25): Let m_X, m_X' be an m-structure on the set X such that $m_X \subseteq m_X'$ and U are a subcategory of X. Then ,

i. U is an m_X'-N-open set whenever U is an m_X-N-open set.

ii. $m_X^{/}$-N-cl(U) $\subseteq$ m_X-N-cl(U).

Proof:

i. Let U be an m_X-N-open set, then apiece $x \in U$, and there will be found an m_X-open set U_X of X, such that $x \in U_X$ and U_X / U is a finite set. Since $m_X \subseteq m_X^{/}$, then $U_X \in m_X^{/}$ and therefore U is an $m_X^{/}$-N-open set.

ii. Let $a \notin m_X$-N-cl(U), then there will be found m_X-N-open set K such that $a \in K$ and K∩U = Ø. By (i), K is an $m_X^{/}$-N-open set and therefore $a \notin m_X^{/}$-N-cl(U). Subsequently, $m_X^{/}$-N-cl(U) $\subseteq$ m_X-N-cl(U).

Definition (2.26)

Let f: (X, m_X)→(Y, m_Y) be a function, then :

i. **[7]** f is nominated to be m-continuous if f^{-1} (U) is m_X-open subcategory in X for a piece m_Y-open subcategory U of Y.

ii. f is nominated to be m-N-continuous function if f^{-1} (U) is an m_X-N-open subcategory in X for a piece m_Y-open subcategory U of Y.

iii. f is nominated to be m-N*-continuous function if f^{-1} (U) is an an m_X-open subcategory in X for a piece m_Y-N-open subcategory U of Y.

iv. f is nominated to be m-N**- continuous function if f^{-1} (U) is an m_X-N-open subcategory in X for a piece m_Y-N-open subcategory U of Y.

Theorem (2.27): Let f: (X, m_X) → (Y, m_Y) be a function, then the accompanying proclamations are identical:

i. f is an m-N**-continuous.

ii. The inverse image of every m_Y-N-closed set is an m_X-N-closed.

iii. For a piece subcategory U of Y, m_X-N-cl (f^{-1}(U)) $\subseteq$ f^{-1} (m_Y-N-cl(U)).

Proof:

(i)→(ii). let V be a subset of Y be an m_Y-N-closed. Then V^C is m_Y-N-open and, by (i),
$f^{-1}(V^C) = (f^{-1}(V))^C$ is an m_X-N-open set. Consequently, f^{-1} (V) is an m_X-N-closed set in X.

(ii)→(iii). Let U be a subcategory in Y. As U$\subseteq$$m_Y$-N-cl(U), then f^{-1}(U) $\subseteq$ f^{-1} (m_Y-N-cl(U)), m_X-N-cl(f^{-1} (U)) $\subseteq$ m_X-N-cl(f^{-1} (m_Y-N-cl(U))). Since f^{-1} (m_Y-N-cl(U)) is an m_X-N-closed set in X, then m_X-N-cl(f^{-1} (m_Y-N-cl(U)) = f^{-1} (m_Y-N-cl(U)). Hence, m_X-N-cl(f^{-1}(U)) $\subseteq$ f^{-1} (m_Y-N-cl(U)).

(iii)→(i). Let U be an m_Y-N-open set in Y, then U^C is m_Y-N-closed set and therefore U^C = m_Y-N-cl(U^C). As m_X-N-cl($f^{-1}(U^C)$) $\subseteq$ f^{-1} (m_Y-N-cl(U^C)), then m_X-N-cl($f^{-1}(U^C)$) $\subseteq$ $f^{-1}(U^C)$. Hence, m_X-N-cl($f^{-1}(U^C)$) = $f^{-1}(U^C)$ and therefore $f^{-1}(U^C) = (f^{-1}(U))^C$ is an m_X-N-closed set. So,
f^{-1} (U) is an m_X-N-open set in X.

Theorem (2.28): Let f: (X, m_X)→(Y, m_Y) be a function, then f is an m-N**-continuous iff f (m_X-N-cl(U)) $\subseteq$ m_Y-N-cl(f(U)) for a piece subcategory U of X.

Proof: Let f be an m-N**-continuous. As m_Y-N-cl(f (U)) is an m_Y-N-closed, then f^{-1} (m_Y-N-cl(f (U))) is an m_X-N-closed. Consequently, m_X-N-cl(f^{-1} (m_Y-N-cl(f (U)))) = f^{-1} (m_Y-N-cl(f (U))). As f (U) $\subseteq$ m_Y-N-cl(f (U)), then U $\subseteq$ f^{-1} (m_Y-N-cl(f (U)) and therefore m_X-N-cl(U) $\subseteq$ m_X-N-cl(f^{-1}(m_Y-N-cl(f (U))) = f^{-1}(m_Y-N-cl(f (U)). So, f (m_X-N-cl(U)) $\subseteq$ m_Y-N-cl(f (U)).

Conversely, let f (m_X-N-cl(U)) $\subseteq$ m_Y-N-cl(f (U)) for a piece subcategory U of X. Let V be an m_Y-N-closed, then f (m_X-N-cl(f^{-1} (V))) $\subseteq$ m_Y-N-cl(f (f^{-1} (V))) $\subseteq$ m_Y-N-cl(V) = V. Hence, m_X-N-cl(f^{-1} (V)) $\subseteq$ f^{-1} (V) and, in this manner, m_X-N-cl(f^{-1} (V)) = f^{-1} (V).

So, f^{-1}(V) is an m_X-N-closed set in X. In this way, f is an m-N**-continuous.

3: m_X-N-connected spaces

Definition (3.1)

A subset U of X in (X, m_X) is nominated to be m_X-N-connected in X if U cannot be composed as the association of two non-empty m_X-N-separated subsets of X. If U is not m_X-N-connected in X, then we state that U is an m_X-N-disconnected in X. A space (X, m_X) is designated to be m_X-N-connected i the fundamental set X is an m_X-N-connected**.**

Definition (3.2)

A subset U of m-space X is nominated to be m_X-N-clopen set if U is both m_X-N-open set and m_X-N-closed set.

Proposition (3.3): Let X be an m-space, then the following is identical:

i. X is an m_X-N-connected space.

ii. The only m_X-N-clopen sets in the space are X and Ø.
iii. X is not able to compose as the association of two non-empty disjoint m_X-N-open sets.

Proof:

i⇒ ii. Let X be m_X-N-connected space, to demonstrate that the only m_X-N-clopen sets in the space are X and Ø. Let U be an m_X-N-clopen set such that U≠Ø and U≠X and let U^C = V. Consequently, X= U ⋃ V and V is additionally m_X-N-clopen set. As V is an m_X-N-closed, then m_X-N-cl(V) = V, U∩ m_X-N-cl(V) = U∩V = Ø, and V ∩ m_X-N-cl(U) = V∩U= Ø. Subsequently, X is not an m_X-N-connected space, which is a logical inconsistency. Consequently, the only m_X-N-clopen sets in the space are X and Ø.

ii⇒iii. Let the only m_X-N-clopen sets in the space be X and Ø and assume that X=U⋃ V such that U and V are non-empty disjoint m_X-N-open sets. Then $U=V^C$ and, in this way, U is an m_X-N-closed set. Consequently, U is m_X-N-clopen set, which is a logical inconsistency. So, X is not able to compose as the association of two non-empty disjoint m_X-N-open sets.

iii⇒i. Let X be not able to compose as the association of two non-empty disjoint m_X-N-open sets, and suppose that X is an m_X-N-disconnected space. Then there will be found non-empty subcategories U, V of X such that U∩ m_X-N-cl(V) = Ø, V∩ m_X-N-cl(U) = Ø, and U⋃ V = X. Since V ⊆ m_X-N-cl(V), then U∩V= Ø. Since V∩ m_X-N-cl(U) = Ø, then m_X-N-cl(U) ⊆ V^C = U. Consequently, m_X-N-cl(U) = U. Hence, U is an m_X-N-closed set. As U^C = V, then V is an m_X-N-open set. It is similarly proved that U is an m_X-N-open set, which is an inconsistency. So, X is an m_X-N-connected space.

Proposition (3.4): Every m_X-N-connected space is an m_X-connected space.

Proof: Let X be an m_X-N-connected space and assume that X is not m_X-connected, then there will be found U, V as non-empty subsets of X such that X =U⋃ V, U ∩ m_X-cl(V) = Ø, and V ∩ m_X-cl(U) = Ø. By proposition 2.10 - ii, we deduced that U ∩ m_X-N-cl(V) = Ø and V ∩ m_X-N-cl(U) = Ø. Accordingly, X is an m_X-N-disconnected space, which is a logical inconsistency. Consequently, X is an m_X-connected space.

Remark (3.5): The opposite of the above proposition might be not valid as a rule.

Example (3.6): Let X = {a_1, a_2, a_3} and m_X = {Ø, X}, we realize that X is an m_X-connected, nevertheless it is not m_X-N-connected.

Theorem (3.7): Let f: (X, m_X) → (Y, m_Y) be surjective m-N-continuous. If (X, m_X) is an m_X-N-connected space and (Y, m_Y) possess attributes 𝔅, then (Y, m_Y) is m_Y-connected.

Proof: Let f: (X, m_X)→(Y, m_Y) be surjective m-N-continuous, X is an m_X-N-connected space, and (Y, m_Y) possess attributes 𝔅. To demonstrate that Y is m_Y-connected, assume that Y is an m-disconnected space, then Y =U⋃ V such thatU, V are non-empty disjoint m_Y-open sets. Subsequently X= f^{-1}(Y) = f^{-1}(U⋃ V) = f^{-1}(U) ⋃ f^{-1}(V). Since f is an m-N-continuous, then f^{-1}(U) and f^{-1}(V) are m_X-N-open sets in X, and since U≠∅, V≠∅ and f are surjective functions, then
f^{-1}(U) ≠∅, f^{-1} (V) ≠ ∅ and f^{-1}(U) ∩ f^{-1}(V) = ∅. Hence, X is an m_X-N-disconnected space, which is an inconsistency. In this way, Y is an m_Y-connected space.

Proposition (3.8): Let f: (X, m_X) → (Y, m_Y) be surjective m-N**-continuous and (X, m_X) is an m_X-N-connected space, then (Y, m_Y) is m_Y-N-connected.

Proof: Let f: (X, m_X) → (Y, m_Y) be surjective m-N**- continuous function such that X is an m_X-N-connected space. To demonstrate that Y is m_Y-N-connected, suppose that Y is an m_Y-N-disconnected space, then Y = U⋃ V such thatU, V are non-empty disjoint m_Y-N-open sets, therefore X = f^{-1} (Y) = f^{-1} (U⋃ V) = f^{-1} (U) ⋃ f^{-1} (V). Since$f$ is m-N**-continuous, thus f^{-1} (U) and f^{-1} (V) are m_X-N-open in X and since U≠∅, V≠∅, and f are surjective functions, then f^{-1}(U) ≠∅, f^{-1} (V) ≠ ∅ and f^{-1}(U) ∩ f^{-1} (V) = ∅. Subsequently, X is an m_X-N-disconnected space, which is an inconsistency. Subsequently, Y is an m_Y-N-connected space .

Remark (3.9): The above Proposition is also true if f is m-N*-continuous.

4: m-N-hyper connected space

Definition (4.1) [9]

A space X is nominated to be m-hyper connected space if every non-empty m_X-open subcategory of X is m_X-dense.

Definition (4.2)
A space X is nominated to be m_X-N-hyper connected space if every non-empty m_X-N-open subcategory of X is an m_X-N-dense.

Proposition (4.3): Every m_X-N-hyper connected space is m_X-hyper connected.

Proof: Let X be an m_X-N-hyper connected space. To demonstrate that X is an m_X-hyper connected space, let U be an m_X-open set in X. Consequently, it is an m_X-N-open set. As X is an m_X-N-hyper connected space, then m_X-N-cl(U) = X. By proposition 2.10-ii, we conclude that m_X-cl(U) = X and, subsequently, X is an m_X-hyper connected space.

Remark (4.4): The opposite of the above proposition might be not valid as a rule.

Example (4.5): Let X= $\{x_1, x_2, x_3\}$, $m_X = \{\emptyset, X\}$. Obviously, X is m_X-hyper connected, however, it is not m-N-hyper connected, since $\{x_1\}$ is an m_X-N-open set and m_X-N-cl$\{x_1\} = \{x_1\} \neq$ X.

Proposition (4.6): Every m_X-N-hyper connected space is an m_X-N-connected space.

Proof: Let X be an m_X-N-hyper connected space and assume that X is not m_X-N-connected. Then, it can be found that a subset U of X is an m_X-N-clopen set such thatU$\neq\emptyset$ and U $\neq$ X. Consequently, U= m_X-N-cl(U), which is a logical inconsistency, since X is m-N-hype connected. Therefore, X is an m_X-N-connected space.

Remark (4.7): The opposite of the above proposition might be not valid as a rule.

Example (4.8): Let X= R be the arrangement of all genuine numbers and m_X={Ø, (-1,1], [1,3], R}, then m_X-N-open sets are {Ø, R, (-1,1], [1,3], (-1,1), (1,3), (1,3], [1,3), (-1,3], R/finite set,….}. Therefore, m_X-N-closed sets are {Ø, R, R/(-1,1], R/[1,3], R/(-1,1), R/(1,3), R/(1,3], R/[1,3), R/(-1,3], finite set,….}. We realize that R is an m_X-N-connected space and m_X-N-cl(-1,1)=R\[1,3]≠R, therefore R is not m_X-N-hyper connected space.

Remark (4.9): The essential attribute of m-space X being m_X-N-hyper connected is not a hereditary property.

Example (4.10): Let (R, m_X) be an m-space and m_X = {Ø, R}, then we realize that R is an m_X-N-hyper connected but a subcategory A= {1, 2, 3} with a relative m-structure is not m_X-N-hyper connected, since m_X-N-cl{1} = {1}.

Proposition (4.11): Let m_X, $m_X^{/}$ be m-structure on the set X such that $m_X \subseteq m_X^{/}$, If (X, $m_X^{/}$) is $m_X^{/}$-N-hyper connected space, then (X, m_X) is m_X-N-hyper connected space.

Proof: Let U be an m_X-N-open set, then by proposition 2.25, U is an $m_X^{/}$-N-open set, but (X, $m_X^{/}$) is an $m_X^{/}$-N-hyper connected space, so $m_X^{/}$-N-cl(U) = X. Then, by the same (Proposition 2.25) we get that m_X-N-cl(U) = X. Hence, (X, m_X) is an m_X-N-hyper connected space.

Remark (4.12): The opposite of the above proposition might be not valid as a rule.

Example (4.13): Let X=R be the arrangement of all genuine numbers and m_X = {Ø, R}, then R is an m_X-N-hyper connected space, but whenever $m_X^{/}$ = {Ø, 1, R}, then R is not an $m_X^{/}$-N-hyper connected space, since $m_X^{/}$-N-cl{1} = {1} ≠ R.

Theorem (4.14): Let (X, m_X) be an m-space and U be a subcategory of X. Then, the following is identical:

i. X is m_X-N-hyper connected.
ii. U is m_X-N-dense or m_X-N*-nowhere dense, for each subcategory U of X.
iii. U∩V≠ Ø, for each non-empty m_X-N-open subcategory U and V of X.

Proof: (i) →(ii). Let (X, m_X) be m_X-N-hyper connected and U be a subcategory of X. Assume that U is not m_X-N*-nowhere dense, then m_X-N-int(m_X-N-cl(U)) ≠ Ø, so by (i), m_X-N-cl(m_X-N-int(m_X-N-cl(U))) = X. Since m_X-N-int(m_X-N-cl(U) $\subseteq$ m_X-N-cl(U), then m_X-N-cl(U) = X and, therefore, U is m_X-N-dense. Also, if U is not m_X-N-dense, then m_X-N-cl(U) ≠ X. Assume that m_X-N-int(m_X-N-cl(U)) ≠ Ø, then by (i), m_X-N-cl(m_X-N-int(m_X-N-cl(U))) = X $\subseteq$ m_X-N-cl(U). Therefore, m_X-N-cl(U) = X, which is a contradiction. So, m_X-N-int(m_X-N-cl(U)) = Ø. Thus, U is an m_X-N*-nowhere dense.

(ii) → (iii). Suppose that U∩V=Ø for some non-empty m_X-N-open subcategory U and V of X. Then, m_X-N-cl(U)∩V=Ø and, therefore, U is not m_X-N-dense. Since U is an m_X-N-open set, so Ø≠U $\subseteq$ m_X-N-int(m_X-N-cl(U)). Subsequently, U is not m_X-N*-nowhere dense, which is an inconsistency. So U∩V≠Ø for each non-empty m_X-N-open subset U and V of X.

(iii) →(i). Let U∩V≠ Ø for each non-empty m_X-N-open subcategories U and V of X and assume that (X, m_X) is not m_X-N-hyper connected space, then there will be found, at any rate, m_X-N-open subset W of X that is not m_X-N-dense in X. So, m_X-N-cl(W) ≠ X. In this manner, X / m_X-N-cl(W) and W are disjoint non-empty m_X-N-open subcategories of X, which is a logical inconsistency. Subsequently, (X, m_X) is an m_X-N-hyper connected space.

Theorem 4.15: Let f: (X, m_X)→ (Y, m_Y) be surjective m-N^{**}-continuous and (X, m_X) is an m_X-N-hyper connected space, then (Y, m_Y) is an m_Y-N- hyper connected.

Proof: Let f: (X, m_X)→(Y, m_Y) be a surjective m-N**-continuous function and U is an m_Y-N-open set. To demonstrate that m_Y-N-cl(U) = Y, since f is m-N**-continuous and X is an m_X-N-hyper connected space, then f^{-1} (U) is m_X-N-open and X = m_X-N-cl(f^{-1} (U)) $\subseteq$ f^{-1}(m_Y-N-cl(U)). Consequently, m_Y-N-cl(U) = Y. Accordingly, Y is an m_Y-N-hyper connected space.

5: m_X-N- locally connected space

Definition (5.1) Let X be an m-space, then (X, m_X) is nominated to be m_X-locally connected space if, for each point $x \in X$ and each m_X-open set U such that $x \in U$, there will be found m_X-connected open set V such that$x \in V \subseteq U$.

Definition (5.2) Let X be an m-space, then (X, m_X) is nominated to be m_X-N-locally connected space if, for every point $x \in X$ and every m_X-N-open set such that $x \in U$, there will be found an m_X-N-connected open set V such that $x \in V \subseteq U$.

Proposition (5.3): Every m_X-N-locally connected space is an m_X-locally connected space.

Proof: Let X be an m_X-N-locally connected space and let $a \in X$ and U be an m_X-open set in X such that$a \in U$, as every m_X-open set is an m_X-N-open set and X is an m_X-N-locally connected space. Then , there will be found an m_X-N-connected open set V such that$a \in V \subseteq U$. By proposition 3.4, we conclude that V is an m_X-connected open set in X. Consequently, X is an m_X-locally connected space.

Remark (5.4): The opposite of the above proposition might be not valid as a rule.

Example (5.5): Let X= {a, b, c}, m_X = {Ø, {b, c}, X}. The m_X-N-open set is {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, X}.

Obviously (X, m_X) is an m_X-locally connected space, however (X, m_X) is not m_X-N-locally connected space since a ∈ {a, b} and exist no U is an m_X-N-connected m_X-open set such that a ∈ U ⊆ {a, b}.

Remark (5.6): If (X, m_X) is m_X-N-locally connected space, then it needs not to be m_X-N-connected and if (X, m_X) is an m_X-N-connected space, then it needs not to be an m_X-N-locally connected space.

Example (5.7): Let X= {a, b, c}, m_X be discrete m-structure. Unmistakably, (X, m_X) is m_X-N-locally connected, but (X, m_X) is not m_X-N-connected space, Since{a}, {b, c} are m_X-N-open sets in X such thatX = {a}⋃{b, c} and {a}∩{b, c} = ∅. Furthermore, let *N* be the arrangement of every single natural numbers and m_X = {∅, N}, then the arrangements of all m_X-N-open sets are {∅, N, N / finite set}. Obviously, (N, m_X) is m_X-N-connected, but it is not m_X-N- locally connected. Since if M = N / finite set, then for a piece $a \in M$, there will be found no m_X-N-connected open set U such that $a \in U \subseteq$ M .

Proposition (5.8): Let m_X, $m_X^{/}$ be two diverse m-structures defined on the set X, such that $m_X \subseteq m_X^{/}$. Then:

i. If (X, $m_X^{/}$) is $m_X^{/}$-N-locally connected space, then (X, m_X) might be not m_X-N-locally connected space.
ii. If (X, m_X) is m_X -N-locally connected space, then (X, $m_X^{/}$) might be not $m_X^{/}$-N-locally connected space.

Examples (5.9):

i. Let X={a, b, c}, m_X ={Ø,{b, c}, X}, and $m_X^{/}$={X, Ø, {a}, {b}, {c}, {b,c}}. Obviously, (X, $m_X^{/}$) is an $m_X^{/}$-N-locally connected space, however, (X, m_X) is not m_X-N-locally connected space.
ii. Let X = R be the arrangement of all genuine numbers and $m_X^{/}$ = {Ø, R/finite set, {1,2}, R} and m_X = {Ø, R/finite, R} set. Obviously, (X, m_X) is an m_X-N-locally connected space, yet (X, $m_X^{/}$) is not m_X-N-locally space, Since$1 \in R$ and {1} are an $m_X^{/}$-N-open set, $1 \in \{1\}$, but there is no U as an
iii. $m_X^{/}$-N-connected open set such that$1 \in U \subseteq \{1\}$.

Proposition (5.10): Let f: (X, m_X) → (Y, m_Y) be a surjective m-N-continuous open. If (X, m_X) is an m_X-N-locally connected space and (Y, m_Y) has the property 𝔅, then (Y, m_Y) is locally connected.

Proof: Let the data of the proposition be achieved. To demonstrate that (Y, m_Y) is m_Y -locally connected, let $b \in Y$ and V be an m_Y-open set in Y such that$b \in V$, Since$f$ is onto, then there will be found $a \in X$ such that$f$ (a) = b. Since$f$ is m-N-continuous, then f^{-1} (V) is m_X-N-open set in X such that$a \in f^{-1}(V)$. SinceX is an m_X-N-locally connected space, then there will be found U as an m_X-N-connected open set in X such that $a \in U \subseteq f^{-1}(V)$. Hence, $b = f(a) \in f(U) \subseteq V$. Since f is an m-open function, then $f(U)$ is an m_Y –open set, and by theorem 3.7, then $f(U)$ is m_Y-connected. Therefore, Y is m_Y-locally connected.

Remark (5.11): The above Proposition is also true if we change the property m-N-continuous to m -continuous.

Remark (5.12): If f: $(X, m_X) \rightarrow (Y, m_Y)$ be an m-N-continuous or m-continuous or m-N^{**}-continuous image of m_X-N-locally connected space need not be m_Y-N-locally connected space.

Example (5.13): Let X= {a, b, c}, Y= {1, 2, 3}, m_X = {∅, {a}, {b}, {c}, X}, and m_Y = {∅, {1}, Y}. Define f: $(X, m_X) \rightarrow (Y, m_Y)$ such that $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. It is obvious that f is m-N-continuous, m-continuous and m-N^{**}- continuous and (X, m_X) is an m_X-N-locally connected space. Yet (Y, m_Y) is not m_Y-N-locally connected space, Since{2,3} is an m_Y-N-open set, $3 \in \{2,3\}$, and there exists no m_Y-N-connected open set U to such an extent that $3 \in U \subseteq \{2,3\}$**.**

Proposition (5.14): Let f: $(X, m_X) \rightarrow (Y, m_Y)$ be a surjective m-N^{**}-continuous, m-open. If (X, m_X) is an m_X-N-locally connected space, then (Y, m_Y) is an m_Y-N-locally connected space.

Proof: Let f: $(X, m_X) \rightarrow (Y, m_Y)$ is an m-N**- continuous, m-open, and onto function and (X, m_X) is an m_X-N-locally connected space. To demonstrate that (Y, m_Y) is m_Y-N-locally connected, let $b \in Y$ and U is an m_X-N-open set in Y such that $b \in U$. Since f is onto, then there will be found $a \in X$ such that $f(a) = b$. Since f is an m-N**- continuous, then $f^{-1}(U)$ is an m_X-N-open set in X, such that $a \in f^{-1}(U)$. Since (X, m_X) is an m_X-N-locally connected space, then there will be found V as an m_X-N-connected open set in X such that $a \in V \subseteq f^{-1}(U)$. Hence, $b = f(a) \in f(V) \subseteq U$. Since f is m-open, then $f(V)$ is an m_Y-open set, and by Proposition 3.8, $f(V)$ is m_Y-N-connected. Thus, Y is an m_Y-N-locally connected space.

Remark (5.15): The above Proposition is also true if we change the property m-N**-continuous to m-N*-continuous.

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