Some Results on $m_X$-N-connected Space

Ahmed A. Salih$^1$, Haider J. Ali$^2$

$^2$ Department of Mathematics, College of Science, Mustansiriyah University, Baghdad, Iraq

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Abstract

In this essay, we utilize $m$ - space to specify $m_X$-N-connected, $m_X$-N-hyper connected and $m_X$-N-locally connected spaces and some functions by exploiting the intelligible $m_X$-N-open set. Some instances and outcomes have been granted to boost our tasks.

Keywords: minimal structure, $m_X$-N-open set, $m_X$-N-connected space, $m_X$-N-hyper connected space, $m_X$-N-locally connected space, $m$-N-continuous.

المقدمة

أحمد عاشور صالح$^1$, حيدر جبر علي$^2$

$^1$ المديرية العامة ل التربية بغداد / الكرخ /3, وزارة التربية، العراق
$^2$ قسم الرياضيات، كلية العلوم، جامعة المستنصرية، العراق

في هذا البحث استعملنا الفضاء $m$ للتعريف الفضاءات $m_X$ المغلقة والمتصلة محلياً، وبعض الفروقات والمجموعات المغلقة $-N$-$m_X$ المفتوحة. بعض الحقائق والنتائج قد أُعطت معززة لعملنا.

Introduction

A. AL-Omari, and M.S.Md. Noorani [1] presented the idea of N – open sets which can be described as follows. A subcategory U of a space X is nominated to be N – open if for every $x \in U$ an open set $U_X$ is found and comprising $x$, with the end goal that $U_X / U$ is a finite. In 2000, Popa and Noiri [2,3] presented the idea of minimal structure space. They additionally characterized m-compactness and m-connectedness and analyzed their essential attributes. Hussain and Nasser [4] characterized the N – disconnected space in an association of two N – separated sets. They provided several depictions and relate them to some other recently known classes of space, for instance, N – locally connected and N – hyper connected spaces. In this paper, we first presented and studied the idea of m-connected, m-N-hyper connected, m-locally connected and m-N-locally connected spaces, by utilizing $m_X$-N-open set, and demonstrated some outcomes on this idea.

1: PRELIMINARIES

Definition (1.1) [5, 3]

Let X be a non-empty set and $\lambda(X)$ the power set of X. A subfamily $m_X$ of $\lambda(X)$ is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_X$ and $X \in m_X$. By $(X, m_X)$, we indicate a non-empty set X with an m-structure $m_X$ on X, and it is called m-space. Every individual from $m_X$ is nominated to be $m_X$-open and the complement of an $m_X$-open set is nominated to be $m_X$-closed set.

Definition (1.2) [6]

Let X be a non-empty set and $m_X$ be an m-structure on X. For a subcategory U of X, the $m_X$-closure of U and the $m_X$-interior of U are characterized as:

*Email: ahmadhmw67@gmail.com
Definition (1.3) [7]: Let $X$ be a non-empty set and $m_X$ a minimal structure on $X$. For a subcategory $U$ and $V$ of $X$, the accompanying properties hold:

i. $m_X\text{-int}(X / U) = X / m_X\text{-cl}(U)$.

ii. If $(X / U) \in m_X$, then $m_X\text{-cl}(U) = U$ and if $U \subseteq m_X$, then $m_X\text{-int}(U) = U$.

iii. If $U \subseteq V$, then $m_X\text{-cl}(U) \subseteq m_X\text{-cl}(V)$ and $m_X\text{-int}(U) \subseteq m_X\text{-int}(V)$.

iv. $m_X\text{-cl}(m_X\text{-cl}(U)) = m_X\text{-cl}(U)$ and $m_X\text{-int}(m_X\text{-int}(U)) = m_X\text{-int}(U)$.

Definition (1.4) [8]: Let $X$ be a non-empty set with a minimal structure $m_X$, and let $U$ be a subcategory of $X$. Then $x \in m_X\text{-cl}(U)$ if and only if $K \cap U \neq \emptyset$ for every $K \in m_X$ containing $x$.

Definition (1.5) [6]: An $m$-structure $m_X$ on a non-empty set $X$ is said to have property $\mathcal{B}$ if the union of any family of subsets belong to $m_X$ belongs to $m_X$.

Definition (1.6) [9]: A subcategory $U$ of an $m$-space $(X, m_X)$ is nominated to be

i. $m_X$-dense if $m_X\text{-cl}(U) = X$.

ii. $m_X$-nowhere dense if $m_X\text{-int}(m_X\text{-cl}(U)) = \emptyset$.

Definition (1.7) [10]: The subsets $U$ and $V$ of $m$-space $X$ are designated to be $m_X$-separated in $X$ if and only if $(U \cap m_X\text{-cl}(V)) \cup (m_X\text{-cl}(U) \cap V) = \emptyset$.

Definition (1.8) [10]: A subset $U$ of $X$ in $(X, m_X)$ is nominated to be $m_X$-connected in $X$ (or simply $m$-connected) if $U$ cannot be composed as the association of two non-empty $m_X$-separated subcategories of $X$. If $U$ is not $m$-connected in $X$, then we state that $U$ is $m$-disconnected in $X$. A space $(X, m_X)$ is designated to be $m$-connected if the underlying set $X$ is $m$-connected.

Definition (1.9) [3]: A non-empty set $X$ with a minimal structure $m_X$ that is fulfilling $\mathcal{B}$ property is nominated to be $m_X$-connected if $X$ cannot be composed as the association of two non-empty disjoint $m_X$-open sets.

Definition (1.10) [11]: A function $f: (X, m_X) \to (Y, m_Y)$ is nominated to be $m$-open if $f(U)$ is an $m_Y$-open set of $(Y, m_Y)$ for every $m_X$-open set $U$ of $(X, m_X)$.

2: $m_X$-N-open set

Definition (2.1) [12]: A subcategory $U$ of an $m$-space $X$ is nominated to be $m_X$-N-open set if every single $x \in U$, there exists an $m_X$-open set $V$ containing $x$ such that $V/U$ is a finite set and the complement of an $m_X$-N-open set is called $m_X$-N-closed set.

Remark (2.2): each $m_X$-open set is an $m_X$-N-open set.

Proof: Let $U$ be $m_X$-open set and $x \in U$, then $x \in U \subseteq U$ and $U/U = \emptyset$. In this manner, $U$ is an $m_X$-N-open set.

Example (2.3): Let $R$ be a set of all real numbers and $m_X = \{\emptyset, R\}$. We realize that $R/\{1\} \subseteq R$ is an $m_X$-N-open any way and it is definitely not an $m_X$-open.

Definition (2.4): Let $X$ be an $m$-space and $U$ be a subcategory of it, then $x \in X$ is designated to be $m_X$-N-interior point to $U$ if an $m_X$-N-open set $V$ is found such that $x \in V \subseteq U$. Then, the set of all $m_X$-N-interior points for $U$ is indicated by $m_X$-N-int($U$).

Definition (2.5): Let $X$ be an $m$-space and $U \subseteq X$, then $x \in X$ is nominated to be $m_X$-N-limit point to $U$ if all $m_X$-N-open sets $V$ containing $x$ We have $V \setminus \{x\} \cap U \neq \emptyset$, and the arrangement of all $m_X$-N-limit points for $U$ is indicated by $m_X$-N-d($U$).

Definition (2.6): Let $X$ be an $m$-space and $U \subseteq X$, then $x \in X$ is nominated to be $m_X$-N-adherent point to $U$ if every single $m_X$-N-open set $V$ that containing $x$ is intersected with $U$. i.e. $V \cap U \neq \emptyset$. The arrangement of all $m_X$-N-adherent points for $U$ is indicated by $m_X$-N-adh($U$) or $m_X$-N-cl($U$).
Definition (2.7)
Let X be a non-empty set and m_X an m-structure on X. For a subcategory U of X, the m_X-N-closure of U and the m_X-N-interior of U are described as:

i. m_X-N-int(U) = \bigcup \{M : M \subseteq U, M \text{ is an m_X-N-open}\}.

ii. m_X-N-cl(U) = \bigcap \{K : U \subseteq K, K \text{ is an m_X-N-closed}\}.

Proposition (2.8): Let (X, m_X) be an m-space, then the following accompanying attributes are verified:

i. The union of any family of m_X-N-open sets is an m_X-N-open set.

ii. The intersection of any family of m_X-N-closed sets is an m_X-N-closed set.

Proof:

i. Let U\alpha be an m_X-N-open set for each \alpha \in \wedge. To prove that \bigcup \{U\alpha, \alpha \in \wedge\} is m_X-N-open, let x \in \bigcup \{U\alpha, \alpha \in \wedge\}, then x \in U\alphai for some \alpha \in \wedge. Since U\alphai is an m_X-N-open, then there can be found V as an m_X-open set, such that x \in V and V / U\alpha is a finite set. Since U\alphai \subseteq \bigcup \{U\alpha, \alpha \in \wedge\}, then (\bigcup \{U\alpha, \alpha \in \wedge\})^C \subseteq (U\alphai)^C. So, V \cap (\bigcup \{U\alpha, \alpha \in \wedge\})^C \subseteq V \cap (U\alphai)^C. Hence, V \cap (\bigcup \{U\alpha, \alpha \in \wedge\}) \subseteq V / U\alpha. Since V / U\alpha is a finite set, then V / \bigcup \{U\alpha, \alpha \in \wedge\} is a finite set, too. Hence \bigcup \{U\alpha, \alpha \in \wedge\} is an m_X-N-open set.

ii. Clear by (i).

Proposition (2.9): Let U be a subcategory of m-space X, then:

i. U is m_X-N-open set if and only if m_X-N-int(U) = U.

ii. U is m_X-N-closed set if and only if m_X-N-cl(U) = U.

Proof:

i. As the union of each m_X-N-open set is m_X-N-open set, then m_X-N-int(U) is the largest m_X-N-open set contained in U. Since U is m_X-N-open set, then m_X-N-int(U) = U. Conversely, whenever m_X-N-int(U) = U, then U is m_X-N-open set, since m_X-N-int(U) is an m_X-N-open set.

ii. As the intersection of each m_X-N-closed set is m_X-N-closed set, then m_X-N-cl(U) is the smallest m_X-N-closed set that containing U. Since U is an m_X-N-closed set, then m_X-N-cl(U) = U. Conversely, whenever m_X-N-cl(U) = U, then U is an m_X-N-closed set, since m_X-N-cl(U) is an m_X-N-closed set.

Proposition (2.10): Let U, V be a subcategory of m-space X and U \subseteq V, then:

i. m_X-N-int(U) \subseteq m_X-N-int(U).

ii. m_X-N-cl(U) \subseteq m_X-N-cl(U).

iii. m_X-N-cl(U) \subseteq m_X-N-cl(V).

iv. m_X-N-int(U) \subseteq m_X-N-int(V).

v. m_X-N-int(X) = X and m_X-N-int(\emptyset) = \emptyset.

vi. m_X-N-cl(X) = X and m_X-N-cl(\emptyset) = \emptyset.

vii. m_X-N-int(m_X-N-int(U)) \subseteq m_X-N-int(U).

viii. m_X-N-cl(m_X-N-cl(U)) \subseteq m_X-N-cl(U).

ix. m_X-N-int(m_X-N-int(U)^C) = (m_X-N-int(U)^C)^C.

x. m_X-N-cl(m_X-N-cl(U)^C) = (m_X-N-cl(U)^C)^C.

Proof:

i. Let x \in m_X-N-int(U), then there can be found m_X-open set U_X such that x \in U_X \subseteq U. For the reason that every m_X-open set is an m_X-N-open set, therefore x \in m_X-N-int(U).

ii. Let x \notin m_X-N-cl(U), then there can be found M as an m_X-open set, such that x \in M and M \cap U = \emptyset. For the reason that every m_X-open set is an m_X-N-open set, then x \notin m_X-N-cl(U) and consequently m_X-N-cl(U) \subseteq m_X-N-cl(U).

iii. Postulate that x \in m_X-N-cl(U), then each m_X-N-open set K containing x intersect U. Since U \subseteq V, then the set K intersect V. Consequently, x \in m_X-N-cl(V) and, in this way, m_X-N-cl(U) \subseteq m_X-N-cl(V).

iv. Let x \in m_X-N-int(U), then there can be found an m_X-N-open set U_X such that x \in U_X \subseteq U. For the reason that U \subseteq V, then x \in U_X \subseteq V. Consequently, x \in m_X-N-int(V). Therefore, m_X-N-int(U) \subseteq m_X-N-int(V).

v. For the reason that X and \emptyset are m_X-N-open sets, then by definition 2.7, m_X-N-int(X) = \bigcup \{U : U \text{ is an m_X-N-open, } U \subseteq X\} = X \bigcup \text{all m_X-N-open sets} = X. In this manner, m_X-N-int(X) = X. Since \emptyset is the only m_X-N-open set contained in \emptyset, henceforth, m_X-N-int(\emptyset) = \emptyset.

vi. By definition 2.7, then m_X-N-cl(X) = \bigcap \{V : \emptyset \subseteq V, V \text{ is m_X-N-closed}\} is m_X-N-closed set. But X is the only m_X-N-closed set comprising X. In this way m_X-N-cl(X) = X. Thus, m_X-N-cl(X) = X. By the definition of m_X-N-cl(\emptyset), m_X-N-cl(\emptyset) = \bigcap \{V : \emptyset \subseteq V, V \text{ is m_X-N-closed}\} = \emptyset \bigcap \text{any m_X-N-closed sets comprising } \emptyset = \emptyset. In this way m_X-N-cl(\emptyset) = \emptyset.
viii. Clear.
ix. By definition 2.7 and proposition 2.8, we note that \( m_X \)-N-int(U) is an \( m_X \)-N-open set. Furthermore, by proposition 2.9, we conclude that \( m_X \)-N-int(m-N-int(U)) = \( m_X \)-N-int(U).
x. By definition 2.7 and proposition 2.8, we note that \( m_X \)-N-cl(U) is an \( m_X \)-N-closed set. Furthermore, by proposition 2.9, we conclude that \( m_X \)-N-cl(m-N-cl(U)) = \( m_X \)-N-cl(U).
xii. Let \( x \not\in (m_X \text{-N-int}(U))^C \), then \( x \in m_X \text{-N-int}(U) \). Thus, there is an \( m_X \)-N-open set \( U_X \) such that \( x \in U_X \subseteq U \). In this way, \( x \in U_X \cup U_X^C = \emptyset \). So, \( x \not\in m_X \text{-N-int}(U^C) \). Thus, we get \( m_X \text{-N-cl}(U^C) \subseteq (m_X \text{-N-int}(U))^C \). Now, let \( x \not\in m_X \text{-N-cl}(U^C) \), then there is an \( m_X \)-N-open set \( U_X \) such that \( x \in U_X \cup U_X^C = \emptyset \). Hence, \( x \in U_X \subseteq U \) and, in this manner, \( x \in m_X \text{-N-int}(U) \). Consequently, \( x \not\in (m_X \text{-N-int}(U))^C \).

Thus, we get \( (m_X \text{-N-int}(U))^C \subseteq m_X \text{-N-cl}(U^C) \).

\textbf{Definition (2.11)}

Let \( (X, m_X) \) be an \( m \)-space, then two non-empty subcategories \( U \) and \( V \) of \( X \) are nominated to be \( m_X \)-N-separated if \( U \cap m_X \text{-N-cl}(V) = \emptyset \) and \( V \cap m_X \text{-N-cl}(U) = \emptyset \).

\textbf{Proposition (2.12):} Two \( m_X \)-N-closed (open) subcategories \( U \) and \( V \) of \( X \) are \( m_X \)-N-separated iff they are disjoint.

\textbf{Proof:} Let \( U \) and \( V \) be both disjoint and \( m_X \)-N-open sets, then \( U^C \) and \( V^C \) are \( m_X \)-N-closed. As \( U \subseteq V^C \), then \( m_X \text{-N-cl}(U) \subseteq V^C \), thus \( m_X \text{-N-cl}(U) \cap V = \emptyset \). Likewise, we demonstrated that \( m_X \text{-N-cl}(V) \cap U = \emptyset \). Hence, both \( U \) and \( V \) are \( m_X \)-N-separated.

\textbf{Definition (2.13)}

Let \( X \) be an \( m \)-space, then \( U \subseteq X \), \( U \) is nominated to be \( m_X \)-N-dense in \( X \) if \( m_X \text{-N-cl}(U) = X \).

\textbf{Example (2.14):} Let \( R \) be the arrangement of all genuine numbers and \( m_X = \{ \emptyset, R \} \), then we realize that all \( R/ \text{finite sets are } m_X \)-N-dense.

\textbf{Definition (2.15)}

A subcategory \( U \) of an \( m \)-space \( (X, m_X) \) is nominated to be \( m_X \)-N-nowhere dense if \( m_X \text{-N-int}(m_X \text{-cl}(U)) = \emptyset \).

\textbf{Proposition (2.16):} Every \( m_X \)-N-nowhere dense \( X \) is \( m_X \)-N-nowhere dense.

\textbf{Proof:} Clear.

\textbf{Remark (2.17):} The opposite of the above proposition might be not valid as a rule.

\textbf{Example (2.18):} Let \( X = \{ x_1, x_2, x_3 \} \) and \( m_X = \{ \emptyset, \{ x_1 \}, \{ x_2 \}, \{ x_3 \} \} \), then we realize that the set \( \{ x_1 \} \) is \( m_X \)-nowhere dense but it is not \( m_X \)-N-nowhere dense.

\textbf{Definition (2.19)}

A subcategory \( U \) of an \( m \)-space \( (X, m_X) \) is nominated to be \( m_X \)-N*-nowhere dense if \( m_X \text{-N-int}(m_X \text{-N-cl}(U)) = \emptyset \).

\textbf{Proposition (2.20):} Every \( m_X \)-N-nowhere dense \( X \) is \( m_X \)-N*-nowhere dense.

\textbf{Proof:} Clear.

\textbf{Remark (2.21):} The opposite of the above proposition might be not valid as a rule.

\textbf{Example (2.22):} Let \( R \) be the arrangement of all genuine numbers and \( m_X = \{ \emptyset, R \} \), then we realize that every finite set is \( m_X \)-N*-nowhere dense, yet it is not \( m_X \)-N-nowhere dense.

\textbf{Remark (2.23):} Let \( (X, m_X) \) be an \( m \)-space and \( U \) be a subcategory of \( X \). If \( U \) is an \( m_X \)-N*-nowhere dense then it is not necessary to be an \( m_X \)-nowhere dense, as well the converse.

\textbf{Example (2.24):} Let \( X = \mathbb{R} \) set all real numbers and \( m_X = \{ \emptyset, \mathbb{R} \} \), then we realize that the set \( \{ 1 \} \) is \( m_X \)-N*-nowhere dense, but not \( m_X \)-nowhere dense. On the other hand, let \( X = \{ x_1, x_2, x_3 \} \) and \( m_X = \{ \emptyset, \{ x_1, x_2 \}, \{ x_3 \} \} \), then we realize that the set \( \{ x_1 \} \) is an \( m \)-nowhere dense, but not \( m_X \)-N*-nowhere.

\textbf{Proposition (2.25):} Let \( m_X \), \( m_X' \) be an \( m \)-structure on the set \( X \) such that \( m_X \subseteq m_X' \) and \( U \) are a subcategory of \( X \). Then,
i. \( U \) is an \( m_X \)-N-open set whenever \( U \) is an \( m_X \)-N-open set.
ii. \( m_{X}' \text{-N-cl}(U) \subseteq m_X \text{-N-cl}(U) \).

**Proof:**

i. Let \( U \) be an \( m_X \)-N-open set, then a piece \( x \in U \), and there will be found an \( m_X \)-open set \( U_X \) of \( X \), such that \( x \in U_X \) and \( U_X / U \) is a finite set. Since \( m_X \subseteq m_{X}' \), then \( U_X \subseteq m_{X}' \) and therefore \( U \) is an \( m_{X}' \)-N-open set.

ii. Let \( a \notin m_X \text{-N-cl}(U) \), then there will be found an \( m_X \)-N-open set \( K \) such that \( a \in K \) and \( K \cap U = \emptyset \). By (i), \( K \) is an \( m_{X}' \)-N-open set and therefore \( a \notin m_{X}' \text{-N-cl}(U) \). Subsequently, \( m_{X}' \text{-N-cl}(U) \subseteq m_X \text{-N-cl}(U) \).

**Definition (2.26):**

Let \( f : (X, m_X) \rightarrow (Y, m_Y) \) be a function, then:

i. \( f \) is nominated to be \( m \)-continuous if \( f^{-1}(U) \) is an \( m_X \)-open subcategory in \( X \) for a piece \( m_Y \)-open subcategory \( U \) of \( Y \).

ii. \( f \) is nominated to be \( m \)-N-continuous function if \( f^{-1}(U) \) is an \( m_X \)-open subcategory in \( X \) for a piece \( m_Y \)-open subcategory \( U \) of \( Y \).

iii. \( f \) is nominated to be \( m \)-N*-continuous function if \( f^{-1}(U) \) is an \( m_X \)-open subcategory in \( X \) for a piece \( m_Y \)-open subcategory \( U \) of \( Y \).

iv. \( f \) is nominated to be \( m \)-N**-continuous function if \( f^{-1}(U) \) is an \( m_X \)-open subcategory in \( X \) for a piece \( m_Y \)-open subcategory \( U \) of \( Y \).

**Theorem (2.27):** Let \( f : (X, m_X) \rightarrow (Y, m_Y) \) be a function, then the accompanying proclamations are identical:

i. \( f \) is an \( m \)-N**-continuous.

ii. The inverse image of every \( m_Y \)-N-closed set is an \( m_X \)-N-closed.

iii. For a piece subcategory \( U \) of \( Y \), \( m_X \text{-N-cl}(f^{-1}(U)) \subseteq f^{-1}(m_Y \text{-N-cl}(U)) \).

**Proof:**

(i)\(\rightarrow\)(ii). Let \( V \) be a subset of \( Y \) be an \( m_Y \)-N-closed. Then \( V^c \) is an \( m_X \)-N-open and, by (i), \( f^{-1}(V^c) = (f^{-1}(V))^c \) is an \( m_X \)-N-open set. Consequently, \( f^{-1}(V) \) is an \( m_X \)-N-closed set in \( X \).

(ii)\(\rightarrow\)(iii). Let \( U \) be a subcategory in \( Y \). As \( U \subseteq m_Y \text{-N-cl}(U) \), then \( f^{-1}(U) \subseteq f^{-1}(m_Y \text{-N-cl}(U)) \), \( m_X \text{-N-cl}(f^{-1}(U)) \subseteq m_X \text{-N-cl}(f^{-1}(m_Y \text{-N-cl}(U))) \). Since \( f^{-1}(m_Y \text{-N-cl}(U)) \) is an \( m_X \)-N-closed set in \( X \), then \( m_X \text{-N-cl}(f^{-1}(m_Y \text{-N-cl}(U))) = f^{-1}(m_Y \text{-N-cl}(U)) \). Hence, \( m_X \text{-N-cl}(f^{-1}(U)) \subseteq f^{-1}(m_Y \text{-N-cl}(U)) \).

(iii)\(\rightarrow\)(i). Let \( U \) be an \( m_Y \)-N-open set in \( Y \), then \( U^c \) is an \( m_Y \)-N-closed set and therefore \( U^c = m_Y \text{-N-cl}(U^c) \). As \( m_X \text{-N-cl}(f^{-1}(U)) \subseteq f^{-1}(m_Y \text{-N-cl}(U^c)) \), then \( m_X \text{-N-cl}(f^{-1}(U^c)) \subseteq f^{-1}(U^c) \). Hence, \( m_X \text{-N-cl}(f^{-1}(U^c)) = f^{-1}(U^c) \) and therefore \( f^{-1}(U^c) = f^{-1}(U)^c \) is an \( m_X \)-N-closed set. So, \( f^{-1}(U) \) is an \( m_X \)-N-open set in \( X \).

**Theorem (2.28):** Let \( f : (X, m_X) \rightarrow (Y, m_Y) \) be a function, then \( f \) is an \( m \)-N**-continuous iff \( f \) is an \( m_X \)-N-cl(f(U)) \subseteq m_Y \text{-N-cl}(f(U)) \) for a piece subcategory \( U \) of \( X \).

**Proof:** Let \( f \) be an \( m \)-N**-continuous. As \( m_X \text{-N-cl}(f(U)) \) is an \( m_X \)-N-closed, then \( f^{-1}(m_X \text{-N-cl}(f(U))) = m_X \text{-N-cl}(f^{-1}(U)) \) is an \( m_X \)-N-closed. Consequently, \( m_X \text{-N-cl}(f^{-1}(U)) = m_X \text{-N-cl}(f^{-1}(U)) \). As \( U \subseteq m_X \text{-N-cl}(f(U)) \), then \( f^{-1}(m_X \text{-N-cl}(f(U))) \) and therefore \( m_X \text{-N-cl}(f^{-1}(U)) \subseteq m_X \text{-N-cl}(f^{-1}(U)) \). So, \( f^{-1}(U) \) is an \( m_X \)-N-open set in \( X \).

Conversely, let \( f \) be an \( m \)-N**-continuous. As \( m_X \text{-N-cl}(f(U)) \) is an \( m_X \)-N-closed, then \( f^{-1}(m_X \text{-N-cl}(f^{-1}(U))) \subseteq m_X \text{-N-cl}(f^{-1}(U)) \subseteq m_X \text{-N-cl}(f^{-1}(U)) \). Hence, \( m_X \text{-N-cl}(f^{-1}(U)) \subseteq f^{-1}(U) \) and, in this manner, \( m_X \text{-N-cl}(f^{-1}(V)) = f^{-1}(V) \).

So, \( f^{-1}(V) \) is an \( m_X \)-N-closed set in \( X \). In this way, \( f \) is an \( m \)-N**-continuous.

3: \( m_X \)-N-connected spaces

**Definition (3.1):**

A subset \( U \) of \( X \) (\( m_X \)) is nominated to be \( m_X \)-N-connected in \( X \) if \( U \) cannot be composed as the association of two non-empty \( m_X \)-N-separated subsets of \( X \). If \( U \) is not \( m_X \)-N-connected in \( X \), then we state that \( U \) is an \( m_X \)-N-disconnected in \( X \). A space \( (X, m_X) \) is designated to be \( m_X \)-N-connected if the fundamental set \( X \) is an \( m_X \)-N-connected.

**Definition (3.2):**

A subset \( U \) of \( m \)-space \( X \) is nominated to be \( m_X \)-N-clopen set if \( U \) is both \( m_X \)-N-open set and \( m_X \)-N-closed set.

**Proposition (3.3):** Let \( X \) be an \( m \)-space, then the following is identical:

i. \( X \) is an \( m_X \)-N-connected space.
ii. The only $m_X$-N-clopen sets in the space are $X$ and $\emptyset$.

iii. $X$ is not able to compose as the association of two non-empty disjoint $m_X$-N-open sets.

**Proof:**

1. Let $X$ be $m_X$-N-connected space, to demonstrate that the only $m_X$-N-clopen sets in the space are $X$ and $\emptyset$. Let $U$ be an $m_X$-N-clopen set such that $U \neq \emptyset$ and $U \neq X$ and let $U \subset V$. Consequently, $X = U \cup V$ and $V$ is additionally $m_X$-N-clopen set. As $V$ is an $m_X$-N-closed, then $m_X$-N-cl($V$) = $V$, $U \cap m_X$-N-cl($V$) = $U \cap V = \emptyset$, and $V \cap m_X$-N-cl($U$) = $V \cap U = \emptyset$. Subsequently, $X$ is not an $m_X$-N-connected space, which is a logical inconsistency. Consequently, the only $m_X$-N-clopen sets in the space are $X$ and $\emptyset$.

2. Let the only $m_X$-N-clopen sets in the space be $X$ and $\emptyset$ and assume that $X = U \cup V$ such that $U$ and $V$ are non-empty disjoint $m_X$-N-open sets. Then $U \neq V$ and, in this way, $U$ is an $m_X$-N-closed set. Consequently, $U$ is $m_X$-N-clopen set, which is a logical inconsistency. So, $X$ is not able to compose as the association of two non-empty disjoint $m_X$-N-open sets.

3. Let $X$ be not able to compose as the association of two non-empty disjoint $m_X$-N-open sets, and suppose that $X$ is an $m_X$-N-disconnected space. Then there will be found non-empty subcategories $U$, $V$ of $X$ such that $U \cap m_X$-N-cl($V$) = $\emptyset$, $V \cap m_X$-N-cl($U$) = $\emptyset$, and $U \cup V = X$. Since $V \subseteq m_X$-N-cl($V$), then $U \cap V = \emptyset$. Since $V \cap m_X$-N-cl($U$) = $\emptyset$, then $m_X$-N-cl($U$) $\subseteq V = U$. Consequently, $m_X$-N-cl($U$) $\subseteq U$. Hence, $U$ is an $m_X$-N-closed set. As $U \subseteq V$, then $V$ is an $m_X$-N-open set. It is similarly proved that $U$ is an $m_X$-N-open set, which is an inconsistency. So, $X$ is an $m_X$-N-connected space.

**Proposition (3.4):** Every $m_X$-N-connected space is an $m_X$-N-connected space.

**Proof:** Let $X$ be an $m_X$-N-connected space and assume that $X$ is not $m_X$-connected, then there will be found $U$, $V$ as non-empty subsets of $X$ such that $X = U \cup V$, $U \cap m_X$-cl($V$) = $\emptyset$, and $V \cap m_X$-cl($U$) = $\emptyset$. By proposition 2.10-ii, we deduced that $U \cap m_X$-N-cl($V$) = $\emptyset$ and $V \cap m_X$-N-cl($U$) = $\emptyset$. Accordingly, $X$ is an $m_X$-N-disconnected space, which is a logical inconsistency. Consequently, $X$ is an $m_X$-N-connected space.

**Remark (3.5):** The opposite of the above proposition might not be valid as a rule.

**Example (3.6):** Let $X = \{a_1, a_2, a_3\}$ and $m_X = \{\emptyset, X\}$, we realize that $X$ is an $m_X$-connected, nevertheless it is not $m_X$-N-connected.

**Theorem (3.7):** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be surjective $m$-N-continuous. If $(X, m_X)$ is an $m_X$-N-connected space and $(Y, m_Y)$ possess attributes $\mathcal{B}$, then $(Y, m_Y)$ is $m_Y$-connected.

**Proof:** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be surjective $m$-N-continuous, $X$ is an $m_X$-N-connected space, and $(Y, m_Y)$ possess attributes $\mathcal{B}$. To demonstrate that $Y$ is $m_Y$-connected, assume that $Y$ is an $m$-N-disconnected space, then $Y = U \cup V$ such that $U$, $V$ are non-empty disjoint $m_Y$-open sets. Subsequently $X = f^{-1}(Y) = f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$. Since $f$ is an $m$-N-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are $m_X$-N-open sets in $X$, and since $U \neq \emptyset$, $V \neq \emptyset$ and $f$ are surjective functions, then $f^{-1}(U) \neq \emptyset$, $f^{-1}(V) \neq \emptyset$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence, $X$ is an $m_X$-N-disconnected space, which is an inconsistency. In this way, $Y$ is an $m_Y$-connected space.

**Proposition (3.8):** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be surjective $m$-$N^*$-continuous and $(X, m_X)$ is an $m_X$-N-connected space, then $(Y, m_Y)$ is $m_Y$-N-connected.

**Proof:** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be surjective $m$-$N^*$-continuous function such that $X$ is an $m_X$-N-connected space. To demonstrate that $Y$ is $m_Y$-N-connected, suppose that $Y$ is an $m_Y$-N-disconnected space, then $Y = U \cup V$ such that $U$, $V$ are non-empty disjoint $m_Y$-N-open sets, therefore $X = f^{-1}(Y) = f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$. Since $f$ is $m$-$N^*$-continuous, thus $f^{-1}(U)$ and $f^{-1}(V)$ are $m_X$-N-open in $X$ and since $U \neq \emptyset$, $V \neq \emptyset$, and $f$ are surjective functions, then $f^{-1}(U) \neq \emptyset$, $f^{-1}(V) \neq \emptyset$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Subsequently, $X$ is an $m_X$-N-disconnected space, which is an inconsistency. Subsequently, $Y$ is an $m_Y$-N-connected space.

**Remark (3.9):** The above Proposition is also true if $f$ is $m$-$N^*$-continuous.

4: $m$-N-hyper connected space

**Definition (4.1)** [9]
A space $X$ is nominated to be $m$-hyper connected space if every non-empty $m_X$-open subcategory of $X$ is $m_X$-dense.
**Definition (4.2)**

A space $X$ is nominated to be $m_X$-$N$-hyper connected space if every non-empty $m_X$-$N$-open subcategory of $X$ is an $m_X$-$N$-dense.

**Proposition (4.3):** Every $m_X$-$N$-hyper connected space is $m_X$-hyper connected.

**Proof:** Let $X$ be an $m_X$-$N$-hyper connected space. To demonstrate that $X$ is an $m_X$-hyper connected space, let $U$ be an $m_X$-open set in $X$. Consequently, it is an $m_X$-$N$-open set. As $X$ is an $m_X$-$N$-hyper connected space, then $m_X$-$N$-$\text{cl}(U) = X$. By proposition 2.10-ii, we conclude that $m_X$-$\text{cl}(U) = X$ and, subsequently, $X$ is an $m_X$-hyper connected space.

**Remark (4.4):** The opposite of the above proposition might not be valid as a rule.

**Example (4.6):** Let $X= \{x_1, x_2, x_3\}$, $m_X = \{\emptyset, X\}$. Obviously, $X$ is $m_X$-hyper connected, however, it is not $m$-$N$-hyper connected, since $\{x_1\}$ is an $m_X$-$N$-open set and $m_X$-$N$-$\text{cl}(\{x_1\}) = \{x_1\} \neq X$.

**Proposition (4.6):** Every $m_X$-$N$-hyper connected space is an $m_X$-$N$-connected space.

**Proof:** Let $X$ be an $m_X$-$N$-hyper connected space and assume that $X$ is not $m_X$-$N$-connected. Then, it can be found that a subset $U$ of $X$ is an $m_X$-$N$-clopen set such that $U \neq \emptyset$ and $U \neq X$. Consequently, $U = m_X$-$N$-$\text{cl}(U)$, which is a logical inconsistency, since $X$ is $m$-$N$-hype connected. Therefore, $X$ is an $m_X$-$N$-connected space.

**Remark (4.7):** The opposite of the above proposition might not be valid as a rule.

**Example (4.8):** Let $X = R$ be the arrangement of all genuine numbers and $m_X = \{\emptyset, (1,1], [1,3], R\}$, then $m_X$-$N$-open sets are $\{\emptyset, R, (-1,1], [1,3], (-1,1), (1,3), [1,3], (-1,3), R/\text{finite set}, \ldots\}$. Therefore, $m_X$-$N$-closed sets are $\{\emptyset, R, R/(-1,1], R/[1,3], R/(-1,1), R/(1,3], R/[1,3], R/(1,3], R/(-1,3), R/((-1,3], \text{finite set}, \ldots\}$. We realize that $R$ is an $m_X$-$N$-connected space and $m_X$-$N$-$\text{cl}(-1,1) = R/[1,3] \neq R$, therefore $R$ is not $m_X$-$N$-hyper connected space.

**Remark (4.9):** The essential attribute of $m$-space $X$ being $m_X$-$N$-hyper connected is not a hereditary property.

**Example (4.10):** Let $(R, m_X)$ be an $m$-space and $m_X = \{\emptyset, R\}$, then we realize that $R$ is an $m_X$-$N$-hyper connected, but a subcategory $A= \{1, 2, 3\}$ with a relative $m$-structure is not $m_X$-$N$-hyper connected, since $m_X$-$N$-$\text{cl}\{1\} = \{1\}$.

**Proposition (4.11):** Let $m_X, m^{'X}$- $m$-structure on the set $X$ such that $m_X \subseteq m^{'X}$. If $(X, m^{'X})$ is an $m^{'X}$-$N$-hyper connected space, then $(X, m_X)$ is $m_X$-$N$-hyper connected space.

**Proof:** Let $U$ be an $m_X$-$N$-open set, then by proposition 2.25, $U$ is an $m^{'X}$-$N$-open set, but $(X, m^{'X})$ is an $m^{'X}$-$N$-hyper connected space, so $m_X^{'X}$-$N$-$\text{cl}(U) = X$. Then, by the same (Proposition 2.25) we get that $m_X^{'X}$-$N$-$\text{cl}(U) = X$. Hence, $(X, m_X)$ is an $m_X$-$N$-hyper connected space.

**Remark (4.12):** The opposite of the above proposition might not be valid as a rule.

**Example (4.13):** Let $X=R$ be the arrangement of all genuine numbers and $m_X = \{\emptyset, R\}$, then $R$ is an $m_X$-$N$-hyper connected space, but whenever $m_X^{'X} = \{\emptyset, 1, R\}$, then $R$ is not an $m_X$-$N$-hyper connected space, since $m_X^{'X}$-$N$-$\text{cl}\{1\} = \{1\} \neq R$.

**Theorem (4.14):** Let $(X, m_X)$ be an $m$-space and $U$ be a subcategory of $X$. Then, the following is identical:

i. $X$ is $m_X$-$N$-hyper connected.

ii. $U$ is $m_X$-$N$-dense or $m_X$-$N^*$-nowhere dense, for each subcategory $U$ of $X$.

iii. $U \cap \emptyset \neq \emptyset$, for each non-empty $m_X$-$N$-open subcategory $U$ and $V$ of $X$.

**Proof:** (i) $\rightarrow$ (ii). (Let $(X, m_X)$ be $m_X$-$N$-hyper connected and $U$ be a subcategory of $X$. Assume that $U$ is not $m_X$-$N^*$-nowhere dense, then $m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U)) \neq \emptyset$, so by (i), $m_X$-$N$-$\text{cl}(m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U))) = X$. Since $m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U)) \subseteq m_X$-$N$-$\text{cl}(U)$, then $m_X$-$N$-$\text{cl}(U) = X$ and, therefore, $U$ is $m_X$-$N$-dense. Also, if $U$ is not $m_X$-$N$-dense, then $m_X$-$N$-$\text{cl}(U) \neq X$. Assume that $m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U)) \neq \emptyset$, then by (i), $m_X$-$N$-$\text{cl}(m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U))) = X \subseteq m_X$-$N$-$\text{cl}(U)$. Therefore, $m_X$-$N$-$\text{cl}(U) = X$, which is a contradiction. So, $m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U)) = \emptyset$. Thus, $U$ is an $m_X$-$N^*$-nowhere dense. (ii) $\rightarrow$ (iii). Suppose that $U \cap V \neq \emptyset$ for some non-empty $m_X$-$N$-open subcategory $U$ and $V$ of $X$. Then, $m_X$-$N$-$\text{cl}(U) \cap V = \emptyset$ and, therefore, $U$ is not $m_X$-$N$-dense. Since $U$ is an $m_X$-$N$-open set, so $\emptyset \neq U \subseteq m_X$-$N$-$\text{int}(m_X$-$N$-$\text{cl}(U))$. Subsequently, $U$ is not $m_X$-$N^*$-nowhere dense, which is an inconsistency. So $U \cap V \neq \emptyset$ for each non-empty $m_X$-$N$-open subset $U$ and $V$ of $X$. (iii) $\rightarrow$ (i). Let $U \cap V \neq \emptyset$ for each non-empty $m_X$-$N$-open subcategories $U$ and $V$ of $X$ and assume that $(X, m_X)$ is not $m_X$-$N$-hyper connected space, then there will be found, at any rate, $m_X$-$N$-open subset $W$ of $X$ that is not $m_X$-$N$-dense in $X$. So, $m_X$-$N$-$\text{cl}(W) \neq X$. In this manner, $X / m_X$-$N$-$\text{cl}(W)$ and $W$ are
disjoint non-empty $m_X$-N-open subcategories of $X$, which is a logical inconsistency. Subsequently, $(X, m_X)$ is an $m_X$-N-hyper connected space.

**Theorem 4.15:** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be surjective $m$-N$^{**}$-continuous and $(X, m_X)$ is an $m_X$-N-hyper connected space, then $(Y, m_Y)$ is an $m_Y$-N-hyper connected space.

**Proof:** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be a surjective $m$-N$^{**}$-continuous function and $U$ is an $m_Y$-N-open set. To demonstrate that $m_Y$-N-cl$(U) = Y$, since $f$ is $m$-N$^{**}$-continuous and $X$ is an $m_X$-N-hyper connected space, then $f^{-1}(U)$ is $m_X$-N-open and $X = m_X$-N-cl$(f^{-1}(U)) \subseteq f^{-1}(m_Y$-N-cl$(U))$. Consequently, $m_Y$-N-cl$(U) = Y$. Accordingly, $Y$ is an $m_Y$-N-hyper connected space.

5: $m_X$-N-locally connected space

**Definition (5.1):** Let $X$ be an $m$-space, then $(X, m_X)$ is nominated to be $m_X$-locally connected space if, for each point $x \in X$ and each $m_X$-open set $U$ such that $x \in U$, there will be found $m_X$-connected open set $V$ such that $x \in V \subseteq U$.

**Definition (5.2):** Let $X$ be an $m$-space, then $(X, m_X)$ is nominated to be $m_X$-N-locally connected space if, for every point $x \in X$ and every $m_X$-N-open set such that $x \in U$, there will be found an $m_X$-N-connected open set $V$ such that $x \in V \subseteq U$.

**Proposition (5.3):** Every $m_X$-N-locally connected space is an $m_X$-locally connected space.

**Proof:** Let $X$ be an $m_X$-N-locally connected space and let $a \in X$ and $U$ be an $m_X$-open set in $X$ such that $a \in U$, as every $m_X$-open set is an $m_X$-N-open set and $X$ is an $m_X$-N-locally connected space. Then, there will be found an $m_X$-N-connected open set $V$ such that $a \in V \subseteq U$. By proposition 3.4, we conclude that $V$ is an $m_X$-connected open set in $X$. Consequently, $X$ is an $m_X$-locally connected space.

**Remark (5.4):** The opposite of the above proposition might be not valid as a rule.

**Example (5.5):** Let $X = \{a, b, c\}$, $m_X = \{\emptyset, \{b, c\}, X\}$. The $m_X$-N-open set is $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Obviously $(X, m_X)$ is an $m_X$-N-locally connected space, however $(X, m_X)$ is not an $m_X$-N-locally connected space since $a \in \{a\}$ and exist no $U$ is an $m_X$-N-connected $m_X$-open set such that $a \in U \subseteq \{a\}$.

**Remark (5.6):** If $(X, m_X)$ is $m_X$-N-locally connected space, then it needs not to be $m_X$-N-connected and if $(X, m_X)$ is an $m_X$-N-connected space, then it needs not to be an $m_X$-N-locally connected space.

**Example (5.7):** Let $X = \{a, b, c\}$, $m_X$ be discrete $m$-structure. Unmistakably, $(X, m_X)$ is $m_X$-N-locally connected, but $(X, m_X)$ is not $m_X$-N-connected space. Since $\{a\}, \{b, c\}$ are $m_X$-N-open sets in $X$ such that $X = \{a\} \cup \{b, c\}$ and $\{a\} \cap \{b, c\} = \emptyset$. Furthermore, let $N$ be the arrangement of every single natural numbers and $m_X = \{\emptyset, N\}$, then the arrangements of all $m_X$-N-open sets are $\{\emptyset, N, N / \text{finite set}\}$. Obviously, $(N, m_X)$ is $m_X$-N-connected, but it is not $m_X$-N-locally connected. Since if $M = N / \text{finite set}$, then for a piece $a \in M$, there will be found no $m_X$-N-connected open set $U$ such that $a \in U \subseteq M$.

**Proposition (5.8):** Let $m_X$, $m_X'$ be two diverse $m$-structures defined on the set $X$, such that $m_X \subseteq m_X'$. Then:

i. If $(X, m_X')$ is $m_X'$-N-locally connected space, then $(X, m_X)$ might be not $m_X$-N-locally connected space.

ii. If $(X, m_X)$ is $m_X$-N-locally connected space, then $(X, m_X')$ might be not $m_X'$-N-locally connected space.

**Examples (5.9):**

i. Let $X = \{a, b, c\}$, $m_X = \{\emptyset, \{b, c\}, X\}$, and $m_X' = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. Obviously, $(X, m_X')$ is an $m_X'$-N-locally connected space, however $(X, m_X)$ is not $m_X$-N-locally connected space.

ii. Let $X = R$ be the arrangement of all genuine number and $m_X = \{\emptyset, R / \text{finite set}, \{1, 2\}, R\}$ and $m_X = \{\emptyset, R / \text{finite}, R\}$ set. Obviously, $(X, m_X)$ is an $m_X$-N-locally connected space, yet $(X, m_X')$ is not $m_X$-N-locally space. Since $1 \in R$ and $\{1\}$ are an $m_X'$-N-open set, $1 \in \{1\}$, but there is no $U$ as an $m_X$-N-connected open set such that $1 \in U \subseteq \{1\}$.

**Proposition (5.10):** Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be a surjective $m$-N-continuous open. If $(X, m_X)$ is an $m_X$-N-locally connected space and $(Y, m_Y)$ has the property $\mathcal{B}$, then $(Y, m_Y)$ is locally connected.

**Proof:** Let the data of the proposition be achieved. To demonstrate that $(Y, m_Y)$ is $m_Y$-locally connected, let $b \in Y$ and $V$ be an $m_Y$-open set in $Y$ such that $b \in V$. Since $f$ is onto, then there will be found $a \in X$ such that $f(a) = b$. Since $f$ is $m$-N-continuous, then $f^{-1}(V)$ is $m_X$-N-open set in $X$ such that $a \in f^{-1}(V)$. Since $X$ is an $m_X$-N-locally connected space, then there will be found $U$ as an $m_X$-N-
connected open set in \( X \) such that \( a \in U \subseteq f^{-1}(V) \). Hence, \( b = f(a) \in f(U) \subseteq V \). Since \( f \) is an m-open function, then \( f(U) \) is an \( m_Y \)-open set, and by theorem 3.7, then \( f(U) \) is \( m_Y \)-connected. Therefore, \( Y \) is \( m_Y \)-locally connected.

**Remark (5.11):** The above Proposition is also true if we change the property m-N-continuous to m-continuous.

**Remark (5.12):** If \( f: (X, m_X) \to (Y, m_Y) \) be an m-N-continuous or m-continuous or m-N**-continuous image of \( m_X \)-N-locally connected space need not be \( m_Y \)-N-locally connected space.

**Example (5.13):** Let \( X = \{a, b, c\} \), \( Y = \{1, 2, 3\} \), \( m_X = \{\emptyset, \{a\}, \{b\}, \{c\}, X\} \), and \( m_Y = \{\emptyset, \{1\}, Y\} \).

Define \( f: (X, m_X) \to (Y, m_Y) \) such that \( f(a) = 1, f(b) = 2, f(c) = 3 \). It is obvious that \( f \) is m-N-continuous, m-continuous and m-N**-continuous and \( (X, m_X) \) is an \( m_X \)-N-locally connected space.

Yet \( (Y, m_Y) \) is not \( m_Y \)-N-locally connected space, Since \( \{2, 3\} \) is an \( m_Y \)-open set, \( 3 \in \{2, 3\} \), and there exists no \( m_Y \)-N-connected open set \( U \) to such an extent that \( 3 \in U \subseteq \{2, 3\} \).

**Proposition (5.14):** Let \( f: (X, m_X) \to (Y, m_Y) \) be a surjective m-N**-continuous, m-open. If \( (X, m_X) \) is an \( m_X \)-N-locally connected space, then \( (Y, m_Y) \) is an \( m_Y \)-N-locally connected space.

**Proof:** Let \( f: (X, m_X) \to (Y, m_Y) \) be an m-N**-continuous, m-open, and onto function and \( (X, m_X) \) is an \( m_X \)-N-locally connected space. To demonstrate that \( (Y, m_Y) \) is \( m_Y \)-N-locally connected, let \( b \in Y \) and \( U \) is an \( m_Y \)-N-open set in \( Y \) such that \( b \in U \). Since \( f \) is onto, then there will be found \( a \in X \) such that \( f(a) = b \).

Since \( f \) is an m-N**-continuous, then \( f^{-1}(U) \) is an \( m_X \)-N-open set in \( X \), such that \( a \in f^{-1}(U) \). Since \( (X, m_X) \) is an \( m_X \)-N-locally connected space, then there will be found \( V \) as an \( m_X \)-N-connected open set in \( X \) such that \( a \in V \subseteq f^{-1}(U) \). Hence, \( b = f(a) \in f(V) \subseteq U \). Since \( f \) is m-open, then \( f(V) \) is an \( m_Y \)-open set, and by Proposition 3.8, \( f(V) \) is \( m_Y \)-N-connected. Thus, \( Y \) is an \( m_Y \)-N-locally connected space.

**Remark (5.15):** The above Proposition is also true if we change the property m-N**-continuous to m-N*-continuous.

**Reference**


