Statistical Fluctuations of Energy Spectra in the Isobar A = 68 Nuclei

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Abstract

Statistical fluctuations of nuclear energy spectra for the isobar A = 68 were examined by means of the random matrix theory together with the nuclear shell model. The isobar A = 68 nuclei are suggested to consist of an inert core of ^{56}Ni with 12 nucleons in f5p-space ($2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbitals). The nuclear excitation energies, required by this work, were obtained through performing f5p-shell model calculations using the isospin formalism f5pvh interaction with realistic single particle energies. All calculations of the present study were conducted using the OXBASH code. The calculated level densities were found to have a Gaussian shape. The distributions of level spacing $P(s)$ and Δ_3 statistic for the considered classes of states, obtained with full Hamiltonian of f5pvh (absence of the off-diagonal Hamiltonian) calculations, showed a chaotic (regular) behavior and coincided well with the distribution of Gaussian orthogonal ensemble (Poisson). Moreover, these distributions were independent of spin (J) and isospin (T).

Keywords: Random matrix theory, chaotic properties, level density, Spectral fluctuations, f5p-shell model calculations

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التقلبات الأحصائية للأطياف الطاقية في نوى الأيزوبار 68 = A

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الخلاصة

تم دراسة التقلبات الأحصائية للأطياف الطاقية في نوى الأيزوبار 68 = A بأستخدام نظرية المصفوفات العشوائية مع نظام إنموذج القشرة النووي. تحتوي نوى هذا الأيزوبار على القلب المغلق ^{56}Ni مع 12 نيوكليون تتحرك ضمن أنموذج الفضاء النووي f5p المعرف بالمدارات $2p_{3/2}$ و $1f_{5/2}$ و $2p_{1/2}$. تم الحصول على طاقات التهيج النووية ,المطلوبة في هذه الدراسة, عن طريق أجراء حسابات إنموذج القشرة f5p بواسطة البرنامج الحاسوبي OXBASH و بأستخدام التفاعل المؤثر f5pvh مع طاقات الجسيمة المنفردة الواقعية. وجدنا بأن كثافة مستويات الطاقة للحالات النووية ,قيد الدراسة, لها شكل كاوسي (Gaussian shape). أن توزيعات الأحصاء $P(s)$ والأحصاء Δ_3 الناتجة من استخدام التفاعل المؤثر f5pvh (غياب المصفوقة الغير قطرية في التفاعل المؤثر) لهما سلوك فوضوي (منتظم) وتتوافق تماما مع توزيع طاقم الحالات المتعامدة الكاوسي (بواسون) للمصفوفات العشوائية. علما ان الإحصائيين $P(s)$ و Δ_3 اظهرا عدم اعتمادهما على الأعداد الكمية J و T.

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1. Introduction

Quantum chaos was explored vastly in the past 30 years [1]. Bohigas *et al*. [2] assumed a relationship amongst disorder in a classical system and the statistical fluctuations of nuclear spectrum of identical quantum system, whereas a systematic evidence of Bohigas assumption is presented in another report [3]. At the moment, it is eminent that quantum analogs of utmost classically disordered systems depict fluctuations in energies that come to an agreement with Random Matrix Theory (RMT) [4, 5], but quantum analogs of classically ordered systems depict fluctuations in energies that come to an agreement with a Poisson limit. For invariant systems under time reversal, the proper formula of RMT is the Gaussian orthogonal ensemble (GOE). RMT was, at first, operated to illustrate the fluctuation features of neutron resonance in compound nucleus [6]. RMT was developed into a typical scheme for probing the common statistical fluctuations in disordered system [7 -10].

Mean field approximation may be employed to explore the disordered manners of single particle dynamic in nuclei. Nevertheless, the two-body residual interaction mixes various configurations in the mean field which in sequence leads to change the fluctuations properties of the nuclear spectrum and wave functions. Actually, one can investigate these fluctuations through utilizing different models. The nuclear shell model provides an attractive context for such investigations, where effective two-body residual interactions are obtainable and the basis states are designated by exact quantum numbers of *J* (total angular momentum), *T* (isospin) and π (parity). In earlier works [11-16], the context of the nuclear shell model was utilized to examine eigenvector component distributions. The basis vector amplitudes were found [14] to be in accordance with the Gaussian distribution (GOE prediction) in regions of large level density and diverged from Gaussian manners in further regions unless the computation employs degenerated single-particle energies. Another investigation [16] also recommended that computations by means of the degenerate single particle energies are disordered at lower excitation energy than that of realistic computations.

Electromagnetic probabilities in nuclei are observables which are related to the wave function. The examination of their fluctuations would enhance the universal spectral investigation as well as assist as an extra sign of disorder in the quantum system. In the previous investigations [17-22] we adopted the context of the RMT together with the nuclear shell model to explore the physical characteristics of chaos in nuclear spectra, electromagnetic probabilities, and moments for various nuclei located in different shell model spaces. As a whole, the results were very good depicted by the GOE limit.

There has been no comprehensive analysis for the chaotic (disordered) properties in the mass region of f5p shell nuclei. Thus, in the present analysis, we look at the statistical features of excitation energies in the isobar A = 68 (such as ^{68}Se, ^{68}As and ^{68}Ge) nuclei. The present shell model computations are carried out for 12 valence nucleons in f5p-model space (with ^{56}Ni as a core) using the f5pvh interaction [23] with the realistic single particle energies (spe's). The computed results for the considered $J^{\pi}T$ classes of states exhibit Gaussian shape for the level densities and GOE distribution for the spectral fluctuations.

2. Theory

The effective shell-model Hamiltonian of many particle systems can be expressed by [11]

$$H = H_0 + H' \tag{1}$$

Here H_0 and H' are the unperturbed (one body) portion and the residual (two body) interaction of *H*, correspondingly. The one body Hamiltonian

$$H_0 = \sum_{\lambda} e_{\lambda} a_{\lambda}^+ a_{\lambda} , \tag{2}$$

defines the non-interacting nucleons in an average field of suitable core, and λ denotes the single particle orbitals. The residual interaction H' of active nucleons is given by

$$H' = \frac{1}{4} \sum V_{\lambda\mu;\nu\rho} a_{\lambda}^+ a_{\mu}^+ a_{\nu} a_{\rho}. \tag{3}$$

The nuclear wave functions of many-body, with good quantum number of J and T, are built by means of the $m-$ scheme determinants [11],

$$\left| M = J, \, T = T_3 ; m \right\rangle. \tag{4}$$

The many body Hamiltonian is given by

$$H_{kk'}^{JT} = \sum_k \langle JT; k | H | JT; k' \rangle, \tag{5}$$

where $|k\rangle$ and $|k'\rangle$ are the many-body basis states. The energies E_α as well as wave functions

$$|JT; \alpha\rangle = \sum_k C_k^\alpha |JT; k\rangle \tag{6}$$

are computed by diagonalizing the matrix elements of Eq. (5).

The chaotic properties of nuclear spectra are typically found by the level spacing $P(s)$ and Dyson-Mehta (Δ_3) statistics [4, 24]. We first built the staircase function $N(E)$ (which is the number of energy levels with energies $\leq E$), where a smooth fit to $N(E)$ is made utilizing the fit of polynomial. We second expressed the unfolded spectrum through using the mapping [25]

$$\tilde{E}_i = \tilde{N}(E_i). \tag{7}$$

The genuine spacing show forceful fluctuations but the unfolded spectrum $\tilde{E}_i$ possesses a fixed average spacing.

The distribution $P(s)$ is designated as the probability of two adjacent levels separated by a distance s. The ith spacing s_i is found via $s_i = \tilde{E}_{i+1} - \tilde{E}_i$. An ordered system is predicted to behave with the Poisson limit

$$P(s) = \exp(-s) \tag{8}$$

while the disordered system is expected to perform with the Wigner limit

$$P(s) = (\pi / 2) s \exp(-\pi s^2 / 4), \tag{9}$$

which is in agreement with the statistic of GOE.

The Δ_3 statistics are utilized to determine the rigidity of the spectrum and expressed by [4]

$$\Delta_3(\alpha, L) = \min_{A,B} \frac{1}{L} \int_\alpha^{\alpha+L} \left[N(\tilde{E}) - (A\tilde{E} + B) \right]^2 d\tilde{E}. \tag{12}$$

It determines the divergence of the function $N(\tilde{E})$ from a straight line. Here, $N(\tilde{E})$ is constructed from the unfolded spectrum of Eq. (7). It is well-known that rigid (soft) spectra have small (large) values of Δ_3. For the purpose of obtaining a smoother distribution $\overline{\Delta}_3(L)$, one can average the distribution of $\Delta_3(L)$ over a number of n_α intervals $(\alpha, \alpha + L)$

$$\overline{\Delta}_3(L) = \frac{1}{n_\alpha} \sum_\alpha \Delta_3(\alpha, L). \tag{13}$$

The successive intervals are taken to overlap by $L/2$. The Poisson distribution of the $\Delta_3(L)$ is depicted by $\Delta_3(L) = L/15$ while that of the GOE is described by $\Delta_3 \sim L/15$ for small L, and $\Delta_3 \approx \pi^{-2} \ln L$ for large L.

3. Results and discussion

The present computations are carried out for $A = 68$ nuclei with $T = 0$ (^{68}Se), 1 (^{68}As) and 2 (^{68}Ge). The isobar of A=68 consists of the ^{56}Ni core and 12 active nucleons that move in the f5p-shell model space, defined by $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbitals. The isospin formalism interaction of f5pvh [23] is chosen as an effective two-body residual interaction with realistic single particle energies (spe's). All computations of the present work are performed using the shell model code OXBASH [26].

In Table-1, we display the dimensions for the considered $J^\pi = 0^+$, 1^+, 4^+, 7^+ and 10^+ states with $T = 0$ (^{68}Se), 1 (^{68}As) and 2 (^{68}Ge) produced with f5pvh Hamiltonian for 12 valence particles in the f5p-model space.

Table 1-Dimensions for the considered 0^+, 1^+, 4^+, 7^+ and 10^+ states with $T = 0$ (^{68}Se), 1 (^{68}As) and 2 (^{68}Ge) formed with f5pvh Hamiltonian for 12 valence particles in the f5p-model space.

J^π	$T = 0$	$T = 1$	$T = 2$
0^+	839	1372	874
1^+	2135	3985	2319
4^+	3793	6562	3700
7^+	1848	3097	1462
10^+	334	462	160

Figure-1 shows the calculated level densities $\rho(E)$ (histograms) in $A = 68$ nuclei for classes of states 0^+0 (^{68}Se), 0^+1 (^{68}As) and 0^+2 (^{68}Ge). For the purpose of comparison, we also display the distribution of the Gaussian fit [27] (red-dashed line). The corresponding fitted parameters of E_0 (the mean energy) and σ (the standard deviation) used in the fitting with the Gaussian shape [Figure-1] are presented in Table- 2. The histograms in the upper panel are calculated with the full Hamiltonian of f5pvh interaction together with realistic spe's, while those in the lower panel are calculated without the presence of the off-diagonal Hamiltonians of f5pvh. It is evident that the histograms in both panels are indistinguishable, with the exception of a shift in energy as a whole. The histograms in the lower panel reveal an enormous number of energy levels that accumulate at the mid-portion of $\rho(E)$, which in sequence leads these histograms to spread with a narrow-range of excitation energy. This behavior is due to the non-considering of the off-diagonal Hamiltonian in the computations. However, the attachment of the off-diagonal mixing interaction in the computations (upper panel) leads commonly to push up the entire set of energy levels in the direction of the higher excitation energy. Therefore, the histograms (upper panel) reveal an important drop in its mid-portion. Besides, they distribute over a wider-range of energy than that of the lower panel. It is clear from both panels that the level density abruptly evolves in conjunction with the excitation energies, attains its maximum in the mid of the spectrum, and subsequently decreases once more for the greatest energy. This behavior of the great energy, and the uneven symmetry with regard to the mid of the spectrum, are non-regular features of models with restricted Hilbert space, which is in difference to real many-body systems. It is significant to denote that the calculated $\rho(E)$ (histograms), which has a Gaussian shape, is in accordance with the expectation of Brody *et al*. [7] that is designed for systems of many-body with two body interactions.

Actually, Figure-1 also provides an opportunity to examine the effect of altering the isospin T on the computed $\rho(E)$ (histograms). It is evident from Figure-1 that the computed level densities have no dependency on the isospin T. Similar arguments are obtained for other classes of states J^π with isospin $T = 0, 1, 2$.
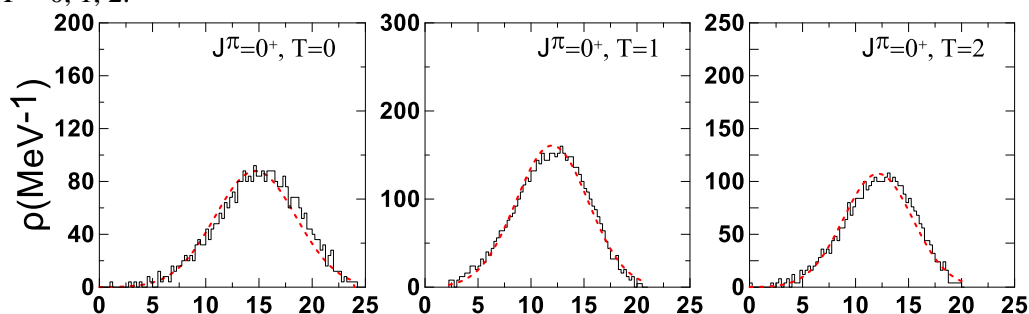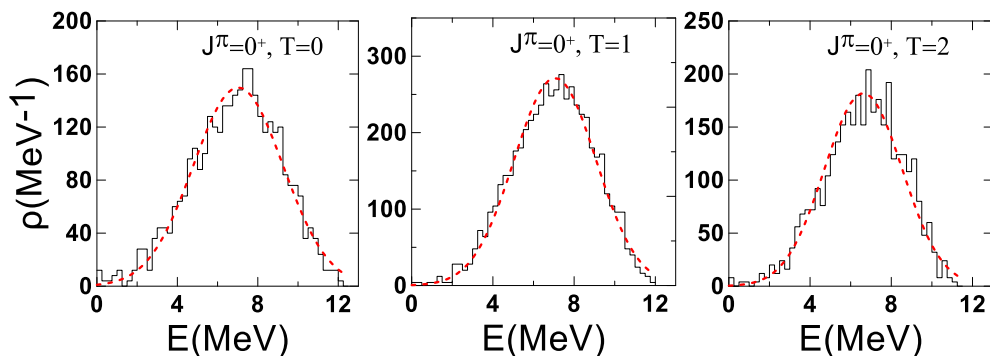
Figure 1-The calculated level density $\rho(E)$ (histograms) and the Gaussian fit (red dashed line) for 0^+0 (^{68}Se), 0^+1 (^{68}As) and 0^+2 (^{68}Ge) states. Upper panel corresponds to the results obtained with full Hamiltonian of f5pvh together with realistic spe's while the lower panel corresponds to those of the absence of the off-diagonal Hamiltonian.

Table 2-Gaussian fit parameters to the level densities of Figure- 1 for $J^\pi = 0$ with $T = 0$, 1 and 2.

Type of calculations	J^π	$T = 0$ (^{68}Se)		$T = 1$ (^{68}As)		$T = 2$ (^{68}Ge)	
		E_0(MeV)	σ	E_0(MeV)	σ	E_0(MeV)	σ
Full Hamiltonian	0^+	14.669	3.806	11.992	3.403	12.120	3.246
Absence of the off-diagonal Hamiltonian	0^+	6.774	2.219	6.589	2.002	6.497	1.901

To examine the influence of varying the spin J^π on the computed $\rho(E)$, we replicate the computations in Figure-2 exactly as those of Figure-1 (upper panel), but this time we consider different classes of states $J^\pi = 1^+$, 4^+, 7^+ and 10^+ with $T = 0$. The Gaussian fit parameters of E_0 and σ utilized in Figure- 2 are displayed in Table-3. Figure-2 shows that the computed level densities $\rho(E)$ [for low spin (1^+), medium spin (4^+ and 7^+), and high spin (10^+)] are in coincident with the Gaussian shape, i.e., they demonstrate independency on J^π. Similar results are found for $T = 1$ or 2 with different J^π states.
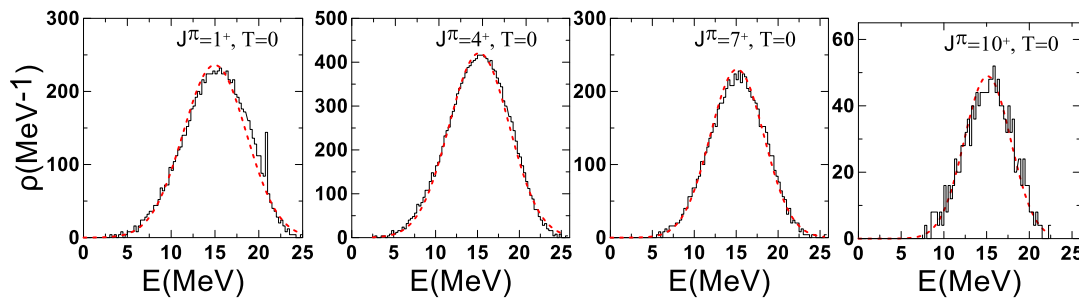

Figure 2-Same as in Figure-1 (upper panel) but for various classes of states 1^+0, 4^+0, 7^+0 and 10^+0.

Table 3-Gaussian fit parameters to the level densities of Figure-2 for the considered J^π and $T = 0$ states.

Type of calculations	J^π	T	E_0(MeV)	σ
Full Hamiltonian	1^+	0	14.854	3.604
	4^+	0	14.893	3.581
	7^+	0	14.857	3.201
	10^+	0	14.765	2.729

Figure-3 reveals the computed nearest-neighbors level spacing distributions $P(s)$ (histograms) in $A = 68$ nuclei for the unfolded 0^+0 (^{68}Se), 0^+1 (^{68}As) and 0^+2 (^{68}Ge) states. The histograms in the upper panel are computed with the full Hamiltonian of f5pvh interaction together with realistic spe's, whereas those in the lower panel are computed without the off-diagonal Hamiltonians of f5pvh. The distribution of GOE (green-solid line) defines systems of disordered dynamic, while that of Poisson (blue-dashed line) defines systems of ordered dynamic. The computed histograms in the upper panel demonstrate chaotic manners, where they are in an astonishing agreement with GOE limit. Besides, the repulsion of levels at small spacing and the Gaussian tail (which are unique properties of disordered level statistics), formed due to the mixing by the off-diagonal residual interaction, is evidently noticed in the computed histograms. While the computed histograms displayed in the lower panel exhibit regular performance (where they are in remarkable agreement with Poisson limit) as a result of the nonexistence of repulsion and mixing among levels.

Again, Figure-3 offers the chance of investigating the influence of varying the isospin T on the computed distributions of $P(s)$. It is apparent that these distributions (histograms) have independency on T. Similar points of view are obtained for further classes of states J^π with isospin $T = 1$ or 2.
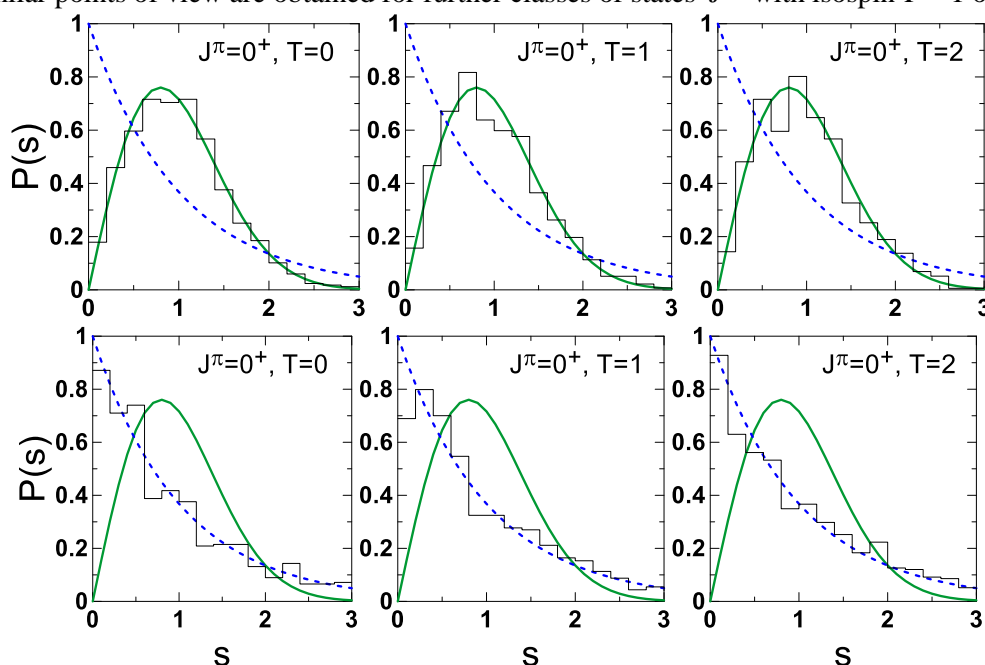

Figure 3-The spacing distributions $P(s)$ for unfolded 0^+0 (^{68}Se), 0^+1 (^{68}As) and 0^+2 (^{68}Ge) states. The calculated results are displayed by histograms. The GOE limit (green-solid line) and Poisson limit (blue-dashed line) are also displayed for comparison. Upper and lower panels correspond to the results obtained with full Hamiltonian and nonexistence of the off-diagonal Hamiltonian, respectively.

To investigate the effect of changing the spin J^π on the calculated distributions of $P(s)$, we replicate the computations in Figure- 4 precisely as those of Figure-3 (upper panel), but now we choose various classes of states $J^\pi = 1^+$, 4^+, 7^+ and 10^+ with $T = 0$. Figure-4 illustrates that the calculated nearest-neighbors level spacing distributions $P(s)$ [for low spin (1^+), medium spin (4^+ and 7^+), and high spin (10^+)], are in agreement with the GOE distribution, i.e., they demonstrate no dependence on the spin J^π. The same outcomes are gained for $T = 1$ or 2 with various J^π states.
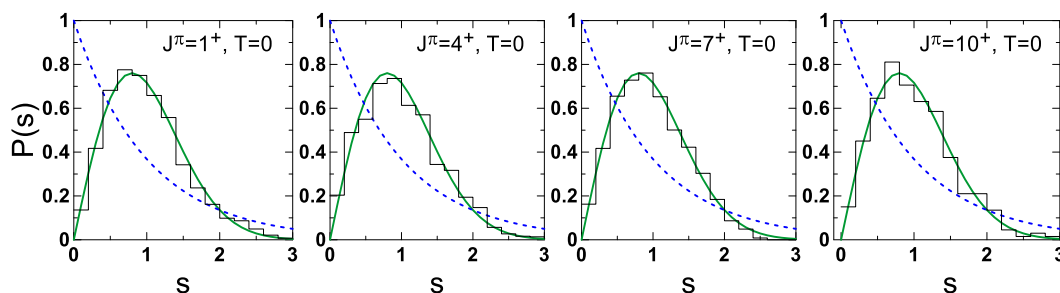
Figure 4-Same as in Fig. 3 (upper panel) but for various classes of states 1^+0, 4^+0, 7^+0 and 10^+0.

Figure-5 displays the Dyson's Δ_3 statistics (the spectral rigidity) in $A = 68$ nuclei. The computed average $\Delta_3(L)$ distribution (open circle symbols) is plotted versus L for the unfolded 0^+0 (^{68}Se), 0^+1 (^{68}As) and 0^+2 (^{68}Ge) classes. Poisson limit (blue-dashed line) and GOE limit (green- solid line) are as well presented. Open circle symbols distributions displayed in the upper panel are obtained with the full Hamiltonian of f5pvh interaction together with realistic spe's, but those in the lower panel are obtained without considering the off-diagonal Hamiltonians of f5pvh. The computed $\Delta_3(L)$ statistics displayed in the upper panel exhibit chaotic presentation (in very well agreement with the GOE limit), whereas those displayed in the lower panel exhibit regular presentation (in very good accordance with the Poisson limit).

Once more, Figure-5 gives the occasion of examining the influence of changing the isospin T on the calculated distributions of $\Delta_3(L)$ statistics. It is clear that the calculated $\Delta_3(L)$ statistics have no dependency on the isospin T. Similar arguments are found for other classes of states J^π with isospin $T = 0, 1, 2$.
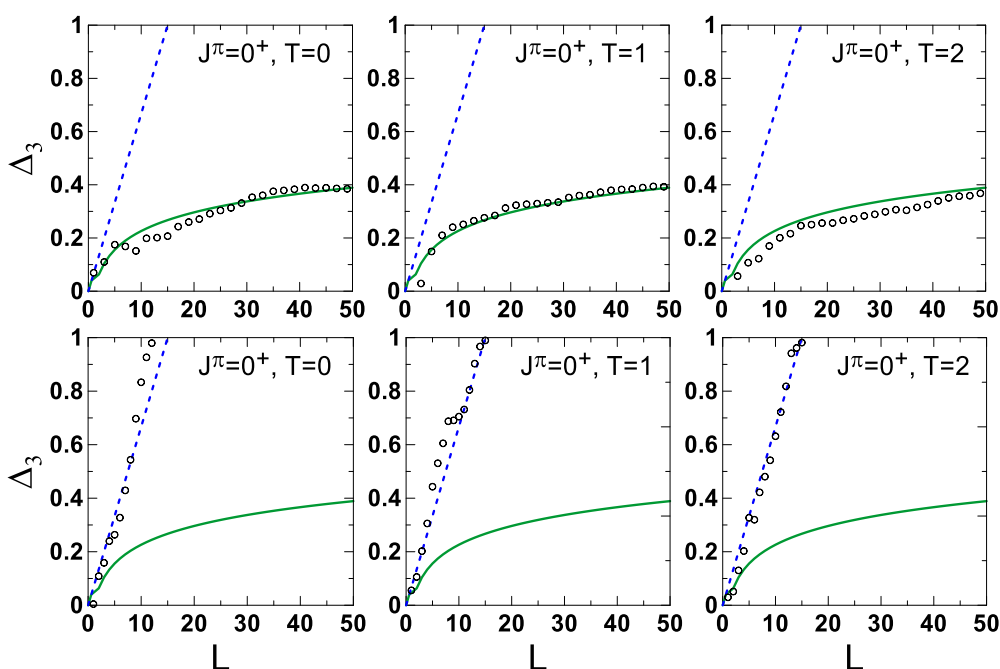

Figure 5- Same as in Fig. 3 but for the Dyson's $\Delta_3(L)$ statistics.

To explore the effect of altering the spin J^π on the evaluated distributions of $\Delta_3(L)$ statistics, we imitate the evaluation in Figure-6 exactly as those of Figure- 5 (upper panel), but this time we select different classes of states $J^\pi = 1^+$, 4^+, 7^+ and 10^+ with $T = 0$. Figure-6 exhibits that the evaluated distributions of $\Delta_3(L)$ statistics [for low spin (1^+), medium spin (4^+ and 7^+), and high spin (10^+)] are

in accordance with the GOE limit, i.e., they are independent on J^{π}. An equivalent point of view is obtained for $T = 1$ or 2 with various J^{π} states.
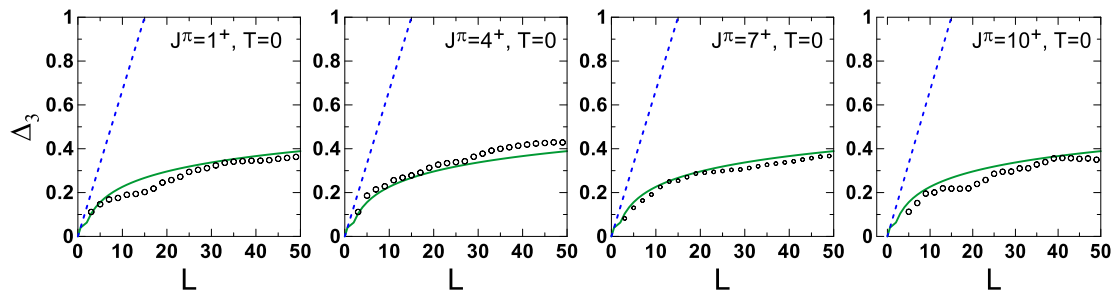

Figure 6-Same as in Figure-5 (upper panel) but for different classes of states 1^+0, 4^+0, 7^+0 and 10^+0.

4. Conclusions

The present results for the spectral fluctuations in the isobar $A = 68$ are computed for various $J^{\pi}T$ classes with full Hamiltonian (absence of the off-diagonal Hamiltonian). The level density distributions $\rho(E)$, which have a Gaussian shape, are found to have a spreading over a wider (narrower) range of excitation energy due to accumulating a smaller (larger) number of energy levels at the middle part of their distributions. The distributions of level spacing $P(s)$ and Δ_3 statistic are found to have a chaotic (regular) behavior and coincide well with the GOE (Poisson) distribution. Besides, the distributions of $\rho(E)$, $P(s)$ and Δ_3 statistic are found to have no dependency on J^{π} and T.

Acknowledgment

References
1. Haak, F. **2001**. *Quantum signature of chaos*, 2nd enlarged ed. Springer-Verlag, Berlin.
2. Bohigas.O, Giannoni M. J. and Schmit.C.**1984**.Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws, *Phys. Rev. Lett.* **52**, 1.
3. Heusler S., Muller S., Altland A., Braun P., and Haak F. **2007**. Periodic-Orbit Theory of Level Correlations,*Phys. Rev. Lett.* **98**: 044103.
4. Mehta M. L. **2004**. *Random Matrices*, 3^{nnd} ed. Academic, New York.
5. Papenbrock T. and Weidenmuller H. A. **2007**. Colloquium: Random matrices and chaos in nuclear spectra, *Rev. Mod. Phys.*, **79** 3:)997.
6. Porter C. E. **1965**. *Statistical Theories of Spectra*: Fluctuations, Academic, New York.
7. Brody T. A., Flores J., French J. B., Mello P. A., Pandey A and Wong S. S. M. **1981**. Random matrix, *Rev. Mod. Phys.* **53**: 385.
8. Guhr T., M$\ddot{u}$ller-Groeling A. and Weidenm$\ddot{u}$ller1H. A. **1997**. Random Matrix Theories in Quantum Physics: Common Concepts, *Phys. Rep.* **299**: 189.
9. Alhassid Y. **2000**. The statistical theory of quantum dots. *Rev. Mod. Phys.* **72**: 895.
10. Gutzwiller M. C. **1990**. *Chaos in Classical and Quantum Mechanics*. Springer-Verlag, New York.
11. Zelevinsky V., Brown. B. A., frazier N. and Horoi. M.**1996**. The nuclear shell model as a testing ground for many-body quantum chaos, *Phys. Rep.* **276**: 85.
12. R.R.Whitehead, A.Watt, D.Kelvin and A.Conkie. **1978**. On the distribution of shell-model eigenvector components, *Phys. Lett. B* **76**: 149.
13. Verbaarschot. J. J. M and Brussaard P. J. **1979**. A statistical study of shell-model eigenvectors, *Phys. Lett. B* **87**: 155.
14. Brown B. A. and Bertsch G. **1984**. Gaussian distributions of shell-model eigenvector components, *Phys. Lett. B* **148**: 5.
15. H. Dias, M. S. Hussein, N. A. de Oliveira and B. H. Wildenthal **1989**. Chaotic behaviour of the nuclear shell-model Hamiltonian, *Phys. G* **15**: L79.

16. Zelevinsky V., Horoi M. and Brown B. A. **1995**. Information entropy, chaos and complexity of the shell model eigenvectors, *Phys. Lett. B* **350**: 141-146.

17. Hamoudi A., Nazmitdinov R. G., Shahaliev E. and Alhassid Y. **2002**. Statistical fluctuations of electromagnetic transition intensities and electromagnetic moments in pf-shell nuclei *Phys. Rev. C* **65**: 064311-5.

18. Hamoudi A. **2011**. Spectral fluctuations and the statistics of electromagnetic transition intensities and electromagnetic moments in ^{136}Xe using the framework of the shell model, *Nucl. Phys. A* **849**: 27-34.

19. Hamoudi A. and Al-Rahmani A. A. **2012**. Regular–chaos transition of the energy spectrum and electromagnetic transition intensities in ^{44}V nucleus using the framework of the nuclear shell model, *Nucl. Phys. A* **892**: 21-33.

20. Hamoudi A. and Abdul Hussein Th. **2017**. Spectral Fluctuations in A=32 Nuclei Using the Framework of the Nuclear Shell Model, *American Journal of Physics and Applications* **5**(3): 35-40.

21. Hamoudi Adel K.. and Murad Shatha F. **2018**. Statistical study of nuclear energy levels for 6 particles in the nuclear *N82* model space, *Iraqi J. of Science* **59**(4C): 2225-2233

22. Hamoudi Adel K. and Murad Shatha F. **2019**. The effect of the one-body Hamiltonian on the statistical fluctuations of the nuclear energy spectrum in ^{138}Ba using the context of the nuclear shell model, *Indian J. Phys.* **93**(9): 1203-1209.

23. Glaudemans P. W. M, Brussaard. P. J and Wildenthal R. H. **1967**. Two-body matrix elements from a modified surface delta interaction, *Nucl. Phys. A* **102**: 593-601.

24. Stephens F. S., Deleplanque M. A., Lee. I.Y., Macchiavelli A. O., Ward D., Fallon P., Cromaz M., Clark R. M., Descovich M., Diamond R. M., and Rodriguez-Vieitez E. **2004**. Searching for E(5) behavior in nuclei, *Phys. Rev. Lett*. **94**: 042501-18.

25. Alhassid Y. and Novoselsky A. **1992**. Chaos in the low-lying collective states of even-even nuclei: Quantal fluctuations, *Phys. Rev. C* **45**: 1677-1687.

26. Brown B. A., Etchegoyen A., Godin N. S., Rae W. D. M., Richter W. A., Ormand W. E., Warburton E. K., Winfield J. S., Zhao L. and Zimmermam C. H. **1984**. The computer code oxbash. MSU-NSCL Report Number 524.

27. Ribeiro M. I., Instituto Superior Tecnico, Av. Rovisco Pais. **2004**. 1; 1049-0.001 Lisboa Portugal, Februeary.