DOI: 10.24996/ijs.2020.SI.1.18

ISSN: 0067-2904

New Types of Totally Continuous Mappings in Topological Space

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Received: 5/12/ 2019 Accepted: 15/ 3/2020

Abstract

The goal of the research is to introduce new types of maps called semi totally Bc-continuous map and totally Bc-continuous map furthermore, study its properties. Additionally, we study the relationship of these functions and other known mappings are discussed.

Keywords: totally Bc-continuity, semi totally Bc-continuity, totally continuity.

أنواع جديد من الدوال المستمرة التامة في الفضاء التبولوجي

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الخلاصه

الهدف من هذا البحث هو تقديم نوع جديد من الدوال تسمى بالدوال المستمرة شبه التامة من النوع (B c) وكذلك الدوال المستمرة التامة من النوع (B c) ودراسة خواصهما بالاضافة الى دراسة العلاقة بينهما وبين بعض الدوال .

1. Introduction:

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. In 1963, Levine [1] studied “semi continuity in topological spaces”. Jain in 1980 [2] studied “totally- continuous function” and Benhalli ; eta studied “ semi totally continuous” in 2011[3] , and in 2013 Neeran and Hanan [4] defined Bc-continuous function .In this paper we will provide semi totally Bc-continuous mapping and totally Bc-continuous mapping and basic properties of these mappings are investigated and obtained.

2. Preliminaries:

2.1 Definition: Let X_t be a topological space, and $D \subseteq X_t$, D is called:

1. Semi closed [1] “If Int (Cl (D)) ⊂D.The complement of a semi closed set is called semi open set .The collection of all semi closed (semi open) subsets in X_t is denoted by SC(X)(SO(X))”.
2. b-open set [5] “If D⊆int(Cl(D))⋃Cl(int(D)) and b-closed set if int (Cl(D))⋂Cl(int(D))⊆D. The collection of all b-open (b-closed) subsets in X_t is denoted by BO (X) (BC (X))”.
3. Bc-open set [6]”If for all $\mathfrak{s} \in D \in BO(X)$, there exists K which is closed such that $\mathfrak{s} \in K \subset D$.The collection of any Bc-open subset in X_t is denoted by BcO(X)”
4. b-compact set [7] relative to X_t “If every cover of D denoted by b-open sets of X_t has a finite sub cover”.

2.2 Definition: A map $k: X_t \to Y_\rho$ is said to be:

1. b-continuous [8] if for each open set A of Y_ρ then k^{-1}(A) b-open in X_t .

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2. Totally-continuous [2] if for each open in Y_ρ the inverse image is to be clopen in X_t.
3. Totally semi –continuous [9] if for each open in Y_ρ the inverse image is to be semi-clopen in X_t.
4. Totally b-continues [4] If for each open in Y_ρ the inverse image is to be b-clopen in X_t.
5. Bc-continuous [4] If for each $x \in X_t$ and each open set V containing $k(x)$ such that V^c is b-compact relative to Y_ρ , there exists an open set U containing x such that $k(U) \subseteq V$".
6. Semi- continuous [1] if for each open in Y_ρ the inverse image is to be semi-open in X_t.
7. Semi totally- continuous [3] if for each semi-open in Y_ρ the inverse image is to be clopen in X_t.
8. Strongly continuous [9] if for each set in Y_ρ the inverse image is to be clopen in X_t.
9. Strongly semi continuous [9] if for each set in Y_ρ the inverse image is to be semi clopen in X_t.
10. Presemi-open [10], if the image of any semi open in X_t is semi open in Y_ρ .

2.3 Theorem :For a map $k : X_t \to Y_\rho$ the following statements are equivalent (1) k is Bc-continuous (2) If V is open in Y_ρ and V^c is b- compact relative to Y_ρ then $k^{-1}(V)$ is open in X_t (3) If H is closed in Y_ρ and b-compact relative to Y_ρ ,then $k^{-1}(H)$ is closed in X_t [8].

2.4 Theorem: Every open set is semi open [1].

2.5 Theorem: Every Bc-clopen is b-clopen [6].

3. Semi totally b-continuous:

3.1 Definition: A map $k : X_t \to Y_\rho$ is semi totally b-continuous map if for all semi open subset of the inverse image of it, is b-clopen in X_t .

3.2 Example : Assume $X=\{g_1,g_2,g_3\}$, $t=\{\varphi,X,\{g_1\},\{g_2\},\{g_1,g_2\}\}$, $BO(X) = \{\varphi,X,\{g_1\},\{g_2\},\{g_1,g_2\},\{g_1,g_3\},\{g_2,g_3\}\}$, $BC(X)=\{\varphi,X,\{g_1\},\{g_2\},\{g_3\},\{g_1,g_3\},\{g_2,g_3\}\}$, $Y= \{b_1,b_2,b_3\}$, $\rho=\{\varphi, Y,\{b_1\}\}$, $SO(Y)=\{\varphi, Y,\{b_1\},\{b_1,b_2\},\{b_1,b_3\}\}$, suppose $k: X_t \to Y_\rho$ such that $k(g_1)= k(g_3)=b_3$, $k(g_2)= b_1$. For each semi open in Y_ρ , the inverse image is to be; -clopen in X_t , thus k semi totally b-continuous .

4 . Semi totally BC-continuous

4.1 Definition :A map $k : X_t \to Y_\rho$ is semi totally Bc-continues map if for all semi open subset of Y_ρ the inverse image of it, is Bc-clopen in X_t .

4.2 Example : Assume $X = Y = \{g_1,g_2,g_3\}$, $t = \{\varphi,X,\{g_1\},\{g_2\},\{g_1,g_2\}\}$, $\rho = \{\varphi,Y,\{g_1\}\}$
Then $SO(Y) = \{\varphi ,Y,\{g_1\},\{g_1,g_2\},\{g_1,g_3\}\}$, Bc-open $=\{\varphi,X,\{g_1,g_3\},\{g_2,g_3\}\}$ Bc-closed$=\{\varphi,X,\{g_1\},\{g_2\}\}$, define $k : X_t \to Y_\rho$,$k(g_1) = k(g_2) = k(g_3)=g_1$,every semi open in Y_ρ the inverse image of it is Bc- clopen in X_t , there for k is semi totally Bc-continues.

4.3 Theorem: A map $k : X_t \to Y_\rho$ is semi totally Bc-continuous map if and only if all semi closed subset of Y_ρ their inverse image are Bc-clopen in X_t .

(Proof**)** ⟶Suppose k is totally Bc-continuous and S is semi closed set in Y_ρ then, Y-S is semi open in Y_ρ and by definition (4.1), $k^{-1}(Y-S)$ is Bc- clopen in X_t , that is $X-k^{-1}(S)$ is Bc- clopen in X_t , this leads $k^{-1}(S)$ is Bc-clopen .

⟵Now, if D is semi open in Y_ρ then Y-D is semi closed in Y_ρ , we have $k^{-1}(Y-D) = X- k^{-1}(D)$is Bc-clopen in X_t this leads $k^{-1}(D)$ is Bc- clopen in X_t thus every semi open set inY_ρ their inverse image are Bc- clopen in X_t thus k is semi totally Bc-continuous.

4.4 Theorem: Let $k : X_t \to Y_\rho$, k is semi totally Bc-continuous map if and only if for all $s \in X_t$ and for all semi open S in Y_ρ with $k(s) \in S$ there is a Bc- clopen set G in X_t such that $s \in G$ and $k(G) \subset S$

(Proof**)** let $k : X_t \to Y_\rho$ is semi totally Bc- continuous map and S is semi open in Y_ρ containing $k(s)$, so that $s \in k^{-1}(S)$, since k is semi totally Bc – continuous map and $k^{-1}(S)$ is Bc-clopen in X_t . suppose $G = k^{-1}(S)$ then G is Bc- clopen in X_t and $s \in G$,also $k(G) = k(k^{-1}(S)) \subset S$ .This leads $k(G) =S$.

On the other hand ,let S be semi open in Y_ρ and $s\in k^{-1}(S)$ be an element ,this leads $k(s) \in S$,thus, there is a Bc- clopen set $k(G_s) \subset X_t$ continuing s such that $k(G_s) \subset S$, which leads to $G_s \subset k^{-1}(S)$ that is $s \in G_s \subset k^{-1}(S)$, this implies $k^{-1}(S)$ is Bc-clopen neighborhood of s , since s is any element it implies $k^{-1}(S)$ is Bc-clopen neighborhood of any of its elements, thus it is Bc-clopen in X_t ,so k is semi totally Bc-continuous map.

4.5 Theorem : Any semi totally Bc-continuous is b-continuous.

(Proof**)** Let $k : X_t \to Y_\rho$ be semi totally Bc-continuous , and $S \subseteq Y_\rho$ such that S is open , since by theorem(2.4) we have S is semi open in Y_ρ .

We have k is semi totally Bc-continuous ,that's mean k^{-1} (S) is Bc- clopen , and every Bc-open(Bc-closed) is b- open (b-closed), we have k^{-1}(S) is b-open, this leads k is b- continuous.
But the convers is incorrect as noted in the following example.

4.6 Example:
Suppose $X = \{a_1,a_2,a_3\}$, $Y = \{b_1,b_2,b_3\}$ with the topologies $t = \{X,\varphi,\{a_1\},\{a_2\},\{a_1,a_2\}\}$ and $\rho = \{Y,\varphi,\{b_1\}\}$ on X and Y respectively , SO(Y) $= \{Y,\varphi,\{b_1\},\{b_1,b_2\},\{b_1,b_3\}\}$
BO(X) $= \{X,\varphi,\{a_1\},\{a_2\},\{a_1,a_2\},\{a_1,a_3\},\{a_2,a_3\}\}$, BcO(X) $= \{X,\varphi,\{a_1,a_3\},\{a_2,a_3\}\}$
Bc-clopen $= \{X,\varphi\}$ .Let $k : X_t \to Y_\rho$ such that ; $k(a_2) = k(a_3) = b_1$, $k(a_1) = b_2$, k is b-continuous but it is not semi totally Bc- continuous since $k^{-1}(\{b_1\})$ and $k^{-1}(\{b_1,b_3\})$ are not Bc- clopen.

4.7 Theorem : Any semi totally Bc- continuous is totally b-continuous.
(Proof **)** Let S be an open subset in Y_ρ on theorem (2.4) we have S is semi open in $Y\rho$, k is semi totally Bc-continuous(by hypothesis)thus, k^{-1}(S) is Bc- clopen , since every Bc-clopen is b- clopen that's mean k is totally b-continuous.
But in example (4 . 6) ,we note that the converse of theorem (4.7) is not always true, because k is totally b-continuous ,but k is not semi totally Bc-continuous.

4.8 Theorem : Any semi totally Bc- continuous is semi totally b – continuous
(Proof **)** Let $k : Xt \to Y\rho$ Be semi totally Bc-continuous and S be a semi open subset of Y_ρ , since k is semi totally Bc- continuous ,then k^{-1} (S) is Bc- clopen subset of X_t
Now by using theorem (2.5) we have k^{-1}(S) is b-clopen .Therefor k is semi totally b-continuous.

4.9 Remark : the converse of theorem (4.8) is not true in general.
To illustrate that the opposite is incorrect we will use the same example above (4.6),we have SO(Y) = $\{Y_\rho,\varphi,\{b_1\},\{b_1,b_2\},\{b_1,b_3\}\}$ note that k is semi totally b - continuous but not semi totally Bc-continuous.
But it is possible that the opposite becomes true if X_t is T_1-space the theorem above becomes as follows:

4.10 Theorem :Let X_t be T_1-space ,then $k : X_t \to Y_\rho$, is semi totally b-continuous if and only if k is semi totally Bc-continuous.
(Proof**)** If k is totally Bc-continuous then by using theorem (4.8) we have k is semi totally b-continuous ,on the other hand ,let S be an semi open in Y_δ ,we have k is totally b-continuous k^{-1} (S) is b- clopen , by use "If X_t is T_1 then BO(X) and BcO(X) are equals "[6] So k^{-1} (S) is Bc- clopen ,thus k is totally Bc-continuous.

4.11 Theorem: Let X_t be discrete topological space and $k : Xt \to Y\rho$ be semi totally Bc-continuous, then k is Bc-continuous.
(Proof**)**: suppose S be open subset in Y_ρ and S^c is b-compact relative to Y_ρ , to prove k^{-1} (S) is open in X_t , since S is open in Y_ρ and by (2.4) S is semi open , we have k is semi totally Bc- continuous ,then k^{-1} (S) is Bc-clopen , we have t is discrete topology this means every subset of S is open , k^{-1}(S) is open ,now by use the theorem (2.3) we have k is Bc- continuous.

4.12 Theorem: Let the set of all b-open subset of X_t is a topology on X_t , $k : X_t \to Y_\rho$ is semi totally Bc-continuous and H is Bc- clopen subset of X_t , then the restriction function $k/_H : H \to Y_\rho$ is semi totally Bc-continuous .
(Proof**)** Let $k/_H : H \to Y_\rho$ and S semi open in Y_ρ ,since k is semi totally Bc-continuous , k^{-1}(S) is Bc- clopen in X_t ,since H and k^{-1}(S) are Bc – open then H and k^{-1}(S) are b- open , we have BO(X) is a topology on X_t (by hypothesis) so : H⋂ k^{-1}(S) is b- open , suppose x∈ H⋂ k^{-1}(S) then x∈H , x∈ k^{-1}(S) so exists N and M such that x∈ N ⊂ H and x ∈ M⊂ k^{-1}(S), thus N ⋂ M is closed and H⋂ k^{-1}(S) is Bc- open , now H is Bc-clopen this leads x ∈ N ⋂ M ⊂H ⋂ k^{-1}(S) , since the intersection of any closed sets is closed that leads to H is Bc- closed and since k^{-1}(S) is Bc- clopen thus k^{-1}(S) is Bc- closed since the {⋂Bα :αΔ} Bc-closed [6],thus k^{-1}(S)⋂ K is Bc- closed and $(k/_H)^{-1}$ (S) = H⋂k^{-1}(S) is Bc- clopen in H, it follows $(k/_H)^{-1}$(S) is Bc-clopen in K thus $k/_H$ is semi totally Bc- continuous .

4.13 Theorem: If $k : Xt \to Y\rho$ is semi totally Bc-continuous and $j: Y\rho \to Z_\delta$ is irresolute then $j\circ k : Xt \to Z_\delta$ is semi totally Bc- continuous.
(Proof**)** suppose S is semi open set in Z_δ , since j is irresolute ,j^{-1}(S) is semi open set in Y_ρ , since k is semi totally Bc-continuous $k^{-1}(j^{-1}(S))=(j\circ k)^{-1}(S)$ is Bc-clopen in X_t hence $j\circ k$ is semi totally Bc- continuous .

4. 14 Definition: A mapping $\mathscr{k} : X_t \to Y_\rho$ is semi totally Bc-open if every semi open set in X_t their image are Bc-clopen in Y_ρ .

4.15 Theorem :If a map $\mathscr{k} : X_t \to Y_\rho$ is a bijective semi totally Bc-open then the image of each semi closed set in X_t is Bc-clopen in Y_ρ .

(Proof) suppose S is semi closed in X_t then Y-S is semi open in X_t since $\mathscr{k}$ is semi totally Bc-open map , $\mathscr{k}$ (X-S) = Y - $\mathscr{k}$(S) is Bc-clopen in Y_ρ . This leads $\mathscr{k}$(S)is Bc-clopen in Y_ρ .

Now we will provide the relationships between semi totally Bc-continuous function and semi totally Bc-open function.

4.16 Theorem: If $\mathscr{k} : X_t \to Y_\rho$ is bijective function then ,the Inverse of $\mathscr{k}$ is semi totally Bc- continuous if and only if , $\mathscr{k}$ is semi totally Bc-open.

(Proof): suppose S is semi open in X_t . On assumption $(\mathscr{k}^{-1})^{-1}$(S) =$\mathscr{k}$(S) is Bc-clopen in Y_ρ , thus f is semi totally Bc-open

If H semi open in X_t , then $\mathscr{k}$(H) is Bc-clopen in Y_ρ , that's mean $(\mathscr{k}^{-1})^{-1}$ (H) is Bc-clopen in Y_ρ so $\mathscr{k}^{-1}$ is semi totally Bc-continuous .

4.17 Theorem : If $\mathscr{k} : X_t \to Y_\rho$ is presemi open map and $\mathscr{j} : Y_\rho \to Z_\delta$ semi totally Bc-open, then $\mathscr{j} \circ \mathscr{k} : X_t \to Z_\delta$ semi totally Bc-open map .

(Proof) suppose S is semi open in X_t , since $\mathscr{k}$ is presemi open $\mathscr{k}$(S) is semi open in Y_ρ ,also $\mathscr{j}$ is semi totally Bc-open map,thus $\mathscr{j}(\mathscr{k}$(S)) is Bc- clopen in Z_δ . That is, $(\mathscr{j}\circ\mathscr{k})$(S) is Bc-clopen in Z_δ . thus $\mathscr{j}\circ\mathscr{k}$ is semi totally Bc-open .

5 .Totally Bc- continuous

5.1 Definition : A map $\mathscr{k} : X_t \to Y_\rho$ is totally Bc- continuous if the inverse image of every open subset of Y_ρ it is Bc- clopen in X_t .

5.2 Example : Consider X= $\{a_1,a_2,a_3\}$with the topology t ={φ,X,$\{a_1,a_2\}$,$\{a_1\}$,$\{a_2\}$},and Y=$\{b_1,b_2,b_3\}$, with the topology **ρ**= {φ,Y, $\{b_1\}$},let $\mathscr{k} : X_t \to Y_\rho$ such that $\mathscr{k}(a_1)=\mathscr{k}(a_2)=\mathscr{k}(a_3)=b_1$, BcO(X)={φ,X,$\{a_3,a_1\}$,$\{a_2,a_3\}$} ,BcC(X)= {φ,X,$\{a_2\}$,$\{a_1\}$} , Bc –clopen = {φ,X}, so $\mathscr{k}$ is totally Bc-continuous because every open subset of Y_ρ their inverse image are Bc- clopen in X_t .

5.3 Theorem: A map $\mathscr{k} : X_t \to Y_\rho$ is totally Bc-continuous , if and only if the inverse image of every closed subset in Y_ρ is Bc- clopen.

(Proof) ⟶suppose S is any subset closed of Y_ρ then Y-S is open in Y_ρ , by definition (5.1) $\mathscr{k}^{-1}$ (Y-S) is Bc-clopen in X_t that is X- $\mathscr{k}^{-1}$(S) is Bc-clopen in X_t this implies $\mathscr{k}^{-1}$(S) is Bc- clopen , ⟵ if D is open in Y_ρ then Y- D is closed in Y_ρ ,we have $\mathscr{k}^{-1}$(Y-D) = X – $\mathscr{k}^{-1}$(D) is Bc- clopen in X_t which leads $\mathscr{k}^{-1}$ (D) is Bc – clopen in X_t thus for any open set in Y_ρ the inverse image of it is Bc- clopen in X_t therefore $\mathscr{k}$ is totally Bc- continuous.

5.4 Theorem: Let $\mathscr{k} : X_t \to Y_\rho$, $\mathscr{k}$ is totally Bc- continuous if and only if for each element $\mathfrak{s}$ in X_t and each open S in with $\mathscr{k}(\mathfrak{s})\in S$ there is a Bc- clopen set G in X_t such that $\mathfrak{s} \in G$ and $\mathscr{k}$ (G) ⊂ S.

(Proof) See Theorem (4 .4).

5.5 Theorem : Totally Bc –continuity, is b- continuity.

(Proof) suppose $\mathscr{k} : X_t \to Y_\rho$ is totally Bc-continuous and S an open subset in Y_ρ since $\mathscr{k}$ is totally Bc- continuous , $\mathscr{k}^{-1}$ (S) is Bc- clopen Then since $\mathscr{k}^{-1}$(S) is Bc- open in X_t since "any Bc- open is b-open " then $\mathscr{k}$ is b-continuous.

The following example illustrate that the converse of Theorem (5.5) is incorrect

5.6 Example: Consider X=$\{a_1,a_2,a_3\}$with the topology t ={X ,φ,$\{a_2\}$,$\{a_1\}$,$\{a_1,a_2\}$} ,and Y=$\{b_1,b_2,b_3\}$, with the topology **ρ**= {φ,Y, $\{b_2,b_3\}$},let $\mathscr{k} : X_t \to Y_\rho$ such that $\mathscr{k}(a_1)=b_1$,$\mathscr{k}(a_2)= b_2$,$\mathscr{k}(a_3)=b_3$, BO(X)={φ,X,$\{a_1\}$,$\{a_2\}$,$\{a_1,a_2\}$,$\{a_1,a_3\}$,$\{a_2,a_3\}$} and Bc –clopen = {φ,X} and $\mathscr{k}$ is b-continuous but $\mathscr{k}$ is not Bc-continuous because $\mathscr{k}^{-1}$ ($\{b_2,b_3\}$) = $\{a_2,a_3\}$ which is not Bc-clopen in X_t .

5 .7 Theorem : Any totally Bc – continuous is totally b- continuous .

(Proof) Suppose $\mathscr{k} : X_t \to Y_\rho$ is totally Bc- continuous and S an open subset in Y_ρ ,since $\mathscr{k}$ is totally Bc-continuous $\mathscr{k}^{-1}$ (S) is Bc- clopen subset of X_t , now by using Theorem 2.5, we have $\mathscr{k}^{-1}$(S) is b- clopen therefore $\mathscr{k}$ is totally b-continuous .

Example 5.8 illustrate that the converse of theorem 5.7 is not true :

5.8 Example :Consider X = $\{a_1,a_2,a_3\}$with the topology t ={φ,X,$\{a_2\}$,$\{a_1\}$,$\{a_1,a_2\}$} , and Y=$\{b_1,b_2,b_3\}$, with the topology **ρ**= {φ,Y, $\{b_1\}$},let $\mathscr{k} : X_t \to Y_\rho$ such that $\mathscr{k}(a_1)= \mathscr{k}(a_3)=b_1$, $\mathscr{k}(a_2)= b_2$ then BO(X)={φ,X,$\{a_1\}$,$\{a_2\}$,$\{a_1,a_2\}$,$\{a_1,a_3\}$,$\{a_2,a_3\}$},BC(X)= {φ,X,$\{a_1\}$,$\{a_2\}$,$\{a_3\}$,$\{a_2,a_3\}$,$\{a_1,a_3\}$}, B-clopen={φ,X,$\{a_2,a_3\}$,$\{a_1\}$,$\{a_1,a_3\}$,$\{a_2\}$} BcO(X) ={φ,X,$\{a_1,a_3\}$,$\{a_2,a_3\}$} , BcC(X)= {φ,X,$\{a_2\}$,$\{a_1\}$} , Bc

–clopen = {φ,X} so 𝓀 is totally b-continuous but; it is not totally Bc-continuous because $𝓀^{-1}(\{b_1\})=\{a_1,a_3\}$ which is not Bc-clopen in X_t .

5. 9 Theorem : Let 𝓀 be a map from T_1 – space X_t to any space Y_ρ then 𝓀 is totally Bc-continuous; if and only if 𝓀 is totally b-continuous .

(Proof) On Theorem (5.7) any totally Bc-continuous is totally b-continuous ,it remained to prove that any totally b-continuous is totally Bc- continuous if X_t is T_1-space ,Let $𝓀 : X_t \to Y_\rho$ be a totally b-continuous and S be an open subset of Y_ρ since 𝓀 is totally b-continues then $𝓀^{-1}$ (S)is b-clopen "If X_t is T_1 space then BO(X) is equal to BcO(X)" [6] so $𝓀^{-1}$(S) is Bc- clopen , thus 𝓀 is totally Bc-continuous.

5 .10 Theorem : Let X_t be a T_1 discrete space and $𝓀 : X_t \to Y_\rho$ is Bc- continuous function then 𝓀 is totally Bc- continuous .

(Proof) Suppose S be any open subset of Y_ρ is Bc- continuous that's mean $𝓀^{-1}$(S) is open . since t is discrete topology, this means all open set is closed .now using "Every open (closed) is b-open (b-closed)" $𝓀^{-1}$ (S) is b-clopen ,now since X_t is T_1–space this leads every b-clopen is BC- clopen that is mean $𝓀^{-1}$ (S) is Bc- clopen thus 𝓀 is totally Bc- continuous.

5 .11 Theorem: If X_t is T_1- space then any semi totally continuous map is totally Bc-continuous map .

(Proof) Suppose S is an open subset of Y_ρ and : $X_t \to Y_\rho$, By theorem (2.4), S is semi open in Y_ρ . since 𝓀 is semi totally continuous . Hence $𝓀^{-1}$(S) is clopen set , since "any open (closed) is b- open (b-closed)" then $𝓀^{-1}$(S) is b-clopen , since X_t is T_1 –space then every b-clopen = Bc-clopen, thus $𝓀^{-1}$ (S) is Bc-clopen that's mean 𝓀 is totally Bc-continuous.

5 .12 Theorem: Any semi totally Bc-continuous is totally Bc-continuous

(Proof) Let $𝓀 : X_t \to Y_\rho$ semi totally Bc-continuous and S open subset of Y_ρ , since by Theorem (2.4), S is a semi open in Y_ρ and since 𝓀 semi totally Bc- continuous that leads $𝓀^{-1}$(S) is Bc-clopen in X_t therefore the inverse image of all open in Y_ρ is Bc- clopen in X_t , thus the map 𝓀 is totally Bc-continuous.

5.13 Theorem: Let the sets of all b-open subset of a space X_t is a topology on X_t, $𝓀 : X_t \to Y_\rho$ is totally Bc-continuous and K is Bc- clopen subset of X_t then the restriction map $𝓀/_K : K \to Y_\rho$ is totally Bc-continuous .

(Proof) suppose the map $𝓀/_K : K \to Y_\rho$ and S be an open subset of Y_ρ since 𝓀 is totally Bc-continuous , $𝓀^{-1}$(S) is Bc- clopen subset of X_t . since K and $𝓀^{-1}$(S) are two Bc – open sets .hence K and $𝓀^{-1}$(S) are b- open sets , we have BO(X) is a topology on X_t (by hypothesis) so ; $K \cap 𝓀^{-1}(S)$ is b-open , let $\mathfrak{s} \in K \cap 𝓀^{-1}(S)$ then $\mathfrak{s} \in K$ and $\mathfrak{s} \in 𝓀^{-1}(S)$. so there exists closed sets G and H such that $\mathfrak{s} \in G \subset K$ and $\mathfrak{s} \in H \subset 𝓀^{-1}(S)$, thus $G \cap H$ is closed and $K \cap 𝓀^{-1}(S)$ is Bc- open , now K is Bc-clopen this leads $\mathfrak{s} \in G \cap H \subset K \cap 𝓀^{-1}(S)$, since the intersection of any closed sets is closed ; that leads to :K is Bc- closed and since $𝓀^{-1}$(S) is Bc- clopen thus $𝓀^{-1}$(S) is Bc- closed since the {⋂Bα :αΔ} Bc-closed [6],thus $𝓀^{-1}(S) \cap K$ is Bc- closed $(𝓀/_K)^{-1}(S) = K \cap 𝓀^{-1}(S)$ is Bc- clopen in K it follows $(𝓀/_K)^{-1}(S)$ is Bc-clopen in K hence $𝓀/_K$ is totally Bc- continuous

5 .14 Theorem :If $𝓀 : X_t \to Y_\rho$ is semi totally Bc-continuous and $𝒿 : Y_\rho \to Z_\delta$ is semi- continuous (semi totally -continuous) then $𝒿 \circ 𝓀 : X_t \to Z_\delta$ is totally Bc-continuous.

(Proof)suppose S is open in Z_δ By (2.4) S is semi open; and since 𝒿 is semi- continuous(totally semi-continuous)then $𝒿^{-1}$(S) is semi open (semi clopen), 𝓀 is semi totally Bc-continuous this lead to $𝓀^{-1}(𝒿^{-1}(S))$ is Bc- clopen , from this we conclude that $𝒿 \circ 𝓀$ is totally Bc-continuous.

5.15 Theorem : If $𝓀 : X_t \to Y_\rho$ is semi totally Bc-continuous and $𝒿 : Y_\rho \to Z_\delta$ is strongly semi-continuous then $𝒿 \circ 𝓀 : X_t \to Z_\delta$ is semi totally Bc-continuous (totally Bc-continuous).

(Proof) suppose S is any subset of Z_δ since 𝒿 is strongly semi continuous thus $𝒿^{-1}$(S) is semi clopen , and 𝓀 is semi totally Bc- continuous it follows $𝓀^{-1}(𝒿^{-1}(S))$ is Bc –clopen , therefore $𝒿 \circ 𝓀$ is semi totally Bc-continuous ,if S is semi open and $𝒿 \circ 𝓀$ is totally Bc- continuous if S is open .

5.16 Theorem: If $𝓀 : X_t \to Y_\rho$ is totally Bc-continuous and $𝒿 : Y_\rho \to Z_\delta$ is strongly continuous then $𝒿 \circ 𝓀 : X_t \to Z_\delta$ is semi totally Bc-continuous (totally Bc-continuous).

(Proof) let S be any subset of Z_δ since 𝒿 is strongly continuous then $𝒿^{-1}$(S)is clopen so $𝒿^{-1}$(S) is open . since 𝓀 is totally Bc- continuous this lead to $𝓀^{-1}(𝒿^{-1}(S))$ is Bc-clopen , from this we conclude that $𝒿 \circ 𝓀$ is semi totally Bc-continuous if S is semi open and 𝓀 is totally Bc- continuous if S is open .

5.17 Theorem: If $\mathcal{k} : X_t \rightarrow Y_\rho$ is semi totally Bc-continuous and $\mathcal{j} : Y_\rho \rightarrow Z_\delta$ is totally semi-continuous then $\mathcal{k} \circ \mathcal{j} : X_t \rightarrow Z_\delta$ is totally Bc-continuous .

(Proof) let S be any open subset of Z_δ since $\mathcal{j}$ is totally semi continuous then $\mathcal{j}^{-1}(S)$ is semi-clopen that's mean $\mathcal{j}^{-1}(S)$ is semi-open since $\mathcal{k}$ is semi totally Bc- continuous this lead to $\mathcal{k}^{-1}(\mathcal{j}^{-1}(S))$ is Bc-clopen , from this we conclude that $\mathcal{j} \circ \mathcal{k}$ is totally Bc-continuous .

5.18 Definition: A map $\mathcal{k} : Xt \rightarrow Y\rho$ is totally Bc-open map if the image of any open subset of X_t is Bc-clopen in Y_ρ .

5.19 Theorem: If a map $\mathcal{k} : Xt \rightarrow Y\rho$ is totally Bc-open and bijective then the image of any closed subset of X_t is Bc-clopen in Y_ρ .

(Proof) Suppose S is closed subset of X_t then Y-S is open in X_t . since $\mathcal{k}$ is totally Bc-open map ,so $\mathcal{k}$(X-S) = Y- $\mathcal{k}$(S) is Bc-clopen in Y_ρ ; this implies $\mathcal{k}$(S)is clopen in Y_ρ .

Now we will provide a relationship between totally Bc-continuous function and totally Bc-open function ;

5.20 Theorem: If $\mathcal{k} : Xt \rightarrow Y\rho$ is bijective function then the inverse of $\mathcal{k}$ is totally Bc-continuous if and only if $\mathcal{k}$ is totally Bc-open.

(Proof):⟶ Let S be any open set in X_t ; By assumption $(\mathcal{k}^{-1})^{-1}(S) = \mathcal{k}(S)$ is Bc-clopen in Y_ρ , thus $\mathcal{k}$ is totally Bc-continuous

⟵Now let K be open in X_t , then $\mathcal{k}$(K) is Bc-clopen in Y_ρ .hence $(\mathcal{k}^{-1})^{-1}(K)$ is Bc-clopen in Y_ρ .Therefore $\mathcal{k}^{-1}$ is totally Bc-continuous .

5.21 Theorem: If $\mathcal{k} : X_t \rightarrow Y_\rho$ is preopen map and $\mathcal{j} : Y_\rho \rightarrow Z_\delta$ is totally Bc-open then $\mathcal{j} \circ \mathcal{k} : X_t \rightarrow Z_\delta$ is totally Bc-open map .

(Proof) suppose C is open subset of X_t ,since $\mathcal{k}$ is preopen .so $\mathcal{k}$(C) is open in Y_ρ . Since $\mathcal{j}$ is totally Bc-open map . Then ($\mathcal{j}$(C)) is Bc- clopen in Z_δ that is $(\mathcal{k} \circ \mathcal{j})$(K) is Bc-clopen in Z_δ ,hence $\mathcal{j} \circ \mathcal{k}$ is totally Bc-open.

5.22 Theorem: If $\mathcal{k} : X_t \rightarrow Y_\rho$ is presemi -open map and $\mathcal{j} : Y_\rho \rightarrow Z_\delta$ is semi totally Bc-open then $\mathcal{j} \circ \mathcal{k} : X_t \rightarrow Z_\delta$ is totally Bc-open map .

(Proof) Let C be any open set in X_t ,that's mean C is semi open , since $\mathcal{k}$ is pre semi-open map $\mathcal{k}$(K) is semi open in Y_ρ .since $\mathcal{j}$ is semi totally Bc-open function $\mathcal{j}(\mathcal{k}(K))$ is Bc- clopen in Z_δ that is $(\mathcal{j} \circ \mathcal{k})$(K) is Bc-clopen in Z_δ ,hence $\mathcal{j} \circ \mathcal{k}$ is semi totally Bc-open map.

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