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Soft Simply Compact Spaces

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Abstract
The aim of this research is to use the class of soft simply open set to define new types of separation axioms in soft topological spaces. We also introduce and study the concept of soft simply compactness.

Keywords: SS^M −compact, $SS^M - \mathrm{T}_0$space, $SS^M - \mathrm{T}_1$space.

الفضاءات المتراصة البسيطة الناعمة

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الخلاصة
الهدف من هذا البحث استخدام صنف من المجموعات البسيطة الناعمة لتعريف انواع جديدة من بديهيات الفصل في الفضاءات التبولوجية الناعمة كذلك قدمنا مفهوم التراص البسيط الناعم ودرسناه.

Introduction

Mathematics is established on exact notions where there is no ambiguity. In areas such as medicine, economy, engineering and sociology, the concepts are ambiguous and researchers want to predefine and study some new notions of ambiguity. They proposed several methods, such as the soft set theory, in order to overcome the uncertainty problem. In 1999, the concept of soft set theory was used for the first time as a mathematical tool by Molodtsov [1] to deal with confusion. He determined the primal upshots of this new theory and successfully applied the soft set theory in many ways, such as theory of measurement smoothness of functions, game theory, etc. In the last years, research on soft set theory was taking place rapidly. In 2003, Maji *et al.*, presented many basic notions of soft set theory, including the universe soft set and empty soft set [2]. In 2011, $Shabir$ and Naz discussed the theory of soft topological space and many fundamental concepts of soft topological spaces, including soft open, soft closed sets, soft nbd oft subspace, and soft separation axioms [3]. In 2012, $Aygünoğlu$ and $Aygün$ described the soft continuity of soft function, and they studied soft product topology, etc in soft topological spaces [4]. In 2011, Min discussed some findings on soft topological spaces [5]. In 1975, the concept of simply-open sets was introduced by $Neubrunnova$ [6]; if ($= K \cup N$ such that K is open set and N is $nowhere\ dense$ (N is $nowhere\ dense$ if ($cl(int\ N) = \emptyset$ [7])), it symbolizes by $S^M O(X)$. In 2013 El. $sayed$ and $Noamman$ presented the transformed definition of simply open set [8]; if ($O \subset (X,\tau)$ is simply open set, then $int(cl(O)) \subseteq cl(int(O))$. In 2017, $El.Sayed$ and $El.Bably$ introduced a new class of simply open sets as a generalization and modification for soft open sets, which is called soft simply open set [9]. In this study, we build on some of the results from previous works [10- 12] . The purpose of this paper is to introduce new concepts in soft topological spaces, such as SS^M −interior, SS^M −closure, SS^M −union, SS^M −intersection, SS^M −compact and soft simply separation axioms.

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1.Preliminaries

The following concepts and definition with some results are needed later on.

Definition 1.1: [1] Let U be defined as a universe set and E as a parameter set, with the power set of U is denoted by $P(U)$ and $A \subset E$. Then (F, A) is said to be a soft set, such that $F: A \to P(U)$**;** $F(a) \in P(U), \forall a \in A$.

Example 1.1: Let $U = \{2,3,4,7,9\}$ and $E = \{e_1, e_2, e_3\}$ where
e_1 ="even number " , e_2 ="odd number" , e_3 ="prime number".
Suppose that $F(e_1) = \{2,4\}$, $F(e_2) = \{3,7,9\}$, $F(e_3) = \{2,3,7\}$, then
$(F, A) = \{(e_1, \{2,4\}), (e_2, \{3,7,9\}), (e_3, \{2,3,7\})\}$ is soft set.

Definition 1.2: [2]. We say that (F, A) is a null set and it is symbolized by $\widetilde{\Phi}$, if $F(a) = \emptyset, \forall a \in A$.

Definition 1.3: [2]. We say that (F, A) is an absolute soft set and it is symbolized by $\tilde{A}, if\ F(a) = U, \forall a \in A$.

Definition 1.4: [2]. Let (F, A) and (G, B) are two soft sets, then $(F, A)\ \widetilde{\cup}\ (G,B)=(H, C)$; ($i.e$ the union of these sets is also a soft set), where $C = A\ \widetilde{\cup}\ B$ and for each $e \in C$.

$$H(e)=\begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

Definition 1.5: [2]. Let (F, A) and (G, B) are two soft sets, then $(F, A)\ \widetilde{\cap}\ (G,B)= (H, C)$; ($i.e$ the intersection of these sets is also a soft set), where $C = A\ \widetilde{\cap}\ B$ and for each $e \in C$ such that $H(e) = F(e) \cap G(e)$.

Definition 1.6: [2]. Let (F, A) and (G,B) be two soft sets over U, then $(F, A)\ \widetilde{\subset}\ (G,B)$**,** if $A \subset B$ and $F(e) \subset G(e)\ \forall\ e \in A$.

Definition 1.7: [13]. The soft topology $\tilde{\tau}$ is defined as follows:

1. $\widetilde{U}$ and $\widetilde{\emptyset} \in \tilde{\tau}$
2. The soft union of any number of soft sets in $\tilde{\tau} \in \tilde{\tau}$.
3. The soft intersection of any two soft sets in $\tilde{\tau} \in \tilde{\tau}$.

Then the triplet $(U, \tilde{\tau}, E)$ is said to be a soft topological space, the elements of $\tilde{\tau}$ are called soft open, their complements are soft closed, and we denote each of the closed soft sets by $\tilde{\mathcal{F}}$.

Definition 1.8: [13]. Assume that (F, E) is a soft set of $(U, \tilde{\tau}, E)$ which is called soft neighborhood (briefly soft nbd) subset (H, E), if $\exists\ (K, E) \tilde{\in} \tilde{\tau}$; $(H, E) \tilde{\subseteq} (K, E) \tilde{\subseteq} (F, E)$.

Definition 1.9: [13]. $(F, E)^o$ or $sint((F, E))$ is the soft interior of the set (F, E) and is defined as follows:

$sint\big((F, E)\big) = \widetilde{\cup}\ \{(G, E);\ (F, E) \tilde{\supseteq} (G, E), (G, E) \tilde{\in} \tilde{\tau}\}$.

Definition 1.10: [13]. $\overline{(F, E)}$ is a soft closure of a (F, E) and is a soft set which is defined as follows:

$scl\big((F, E)\big) = \widetilde{\cap}\ \{(G, E);\ (F, E) \tilde{\subseteq} (G, E), (G, E)^C \tilde{\in} \tilde{\tau}\}$.

Definition 1.11: [13]. We say that $(U, \tilde{\tau}, E)$ is a soft indiscrete space if $\tilde{\tau} = \{\widetilde{U}, \widetilde{\emptyset}\}$, and it is symbolized by $\tilde{\tau}_{ind}$.

Definition 1.12: [13]. We say that $(U, \tilde{\tau}, E)$ is a soft discrete space if $\tilde{\tau}$ is the family of all soft sets that can be defined over $U$and is symbolized by $\tilde{\tau}_{dis}$.

Definition 1.13: [13]. We say that $(U, \tilde{\tau}, E)$ is a soft $T_0 - space$, if for any two distinct points $a, b \tilde{\in} U$, there exist (F, E) and $(G, E) \tilde{\in} \tilde{\tau}$, such that [$a \tilde{\in} (F, E)$ and $b \tilde{\notin} (F, E)$], or [$b \tilde{\in} (G, E)$ and $a \tilde{\notin} (G, E)$].

Definition 1.14: [13]. We say that $(U, \tilde{\tau}, E)$ is a soft $T_1 - space$, if for any two distinct points $a, b \tilde{\in} U$, there exist (F, E) and $(G, E) \tilde{\in} \tilde{\tau}$, such that [$a \tilde{\in} (F, E)$, $b \tilde{\notin} (F, E)$], [$b \tilde{\in} (G, E)$ and $a \tilde{\notin} (G, E)$] .

Definition 1.15: [13] We say that $(U, \tilde{\tau}, E)$ is a soft $T_2 - space$, if for any two distinct points $a, b \tilde{\in} U$, there exist (F, E) and $(G, E) \tilde{\in} \tilde{\tau}$, such that $a \tilde{\in} (F, E)$, $b \tilde{\in} (G, E)$ and $(F, E)\ \widetilde{\cap}\ (G, E) = \widetilde{\emptyset}$.

Definition 1.16: [13] We say that $(U, \tilde{\tau}, E)$is a soft $regular\ space$, if for all $(H, E) \tilde{\in} \tilde{\tau}^C$ ($i.e(H, E)$ is soft closed in U) and any points $a \tilde{\in} U$ such that $a \tilde{\notin} (H, E)$, then there exist (F, E) and $(G, E) \tilde{\in} \tilde{\tau}$, such that [$a \tilde{\in} (F, E)$, $(H, E) \tilde{\subset} (G, E)$ and $(F, E)\ \widetilde{\cap}\ (G, E) = \widetilde{\emptyset}$].

Definition 1.17: [13] We say that $(U,\tilde{\tau},E)$ is a soft $normal\ space$, if for each $(H,E),\ (K,E)\ \tilde{\in}\ \tilde{\tau}^{C}$ ($i.e(H,E)\ and\ (K,E)$ are soft closed in U) such that $(H,E)\ \tilde{\cap}\ (K,E)=\tilde{\emptyset}$, then there exist (F,E) and $(G,E)\ \tilde{\in}\ \tilde{\tau}$, such that [$(H,E)\ \tilde{\subset}\ (F,E),\ (K,E)\ \tilde{\subset}\ (G,E)$ and $(F,E)\ \tilde{\cap}\ (G,E)=\tilde{\emptyset}$].
Definition 1.18: [13]. We say that $(U,\tilde{\tau},E)$ is a soft $T_3-space$, if U is both soft $regular\ space$ and soft $T_1-space$.
Definition 1.19: [13]. We say that $(U,\tilde{\tau},E)$ is a soft $T_4-space$, if U is both soft $normal\ space$ and soft $T_1-space$.
Definition 1.20: [4]. A family δ of soft sets is called a cover of a soft set (F,E) if $(F,E)\ \tilde{\subseteq}\ \tilde{\cup}\{(F_i,E);\ (F_i,E)\ \tilde{\in}\ \delta\ ;i\in I\}$. δ is said to be a soft open cover if every members of δ is a soft open set.
Definition 1.21: [4]. We say that $(U,\tilde{\tau},E)$ is a soft compact, if every soft open cover has a finite sub cover $(U,\tilde{\tau},E)$.
Definition 1.22: [8]. A soft subset (F,A) of soft topological space $(U,\tilde{\tau},E)$ is called Soft simply-open (for short SS^M-open) set, if $sint\big(scl((F,A))\big)\ \tilde{\subseteq}\ scl(sint((F,A)))$. It is symbolized by $SS^MO(U)$. The complement of a soft simply open set is a soft simply closed set (for short, $SS^M-closed$) and it is symbolized by $SS^MC(U)$.
Remark 1.2: [8]. Every open soft set is a soft simply-open set but the converse is not true for the example:
Example 1.2: Let $U=\{a,b,c,d\}$, E=$\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{U},\tilde{\emptyset},(F_1,E)\ ,(F_2,E)\ ,(F_3,E)\ ,(F_4,E)\}$ where:
$(F_1,E)=\{(e_1,\{a,b\}),(e_2,\{a,c,d\})\}$
$(F_2,E)=\{(e_1,\{a,b,d\}),\big(e_2,\tilde{\emptyset}\big)\}$
$(F_3,E)=\{(e_1,\{a,b\}),\big(e_2,\tilde{\emptyset}\big)\}$
$(F_4,E)=\{(e_1,\{a,b,d\}),(e_2,\{a,c,d\})\}$.
Let$(G,E)=\{(e_1,\{a,b\}),(e_2,\{c,d\})\}$, then
$(F_4,E)=sint\big(scl(G,E)\big)\ \tilde{\subseteq}\ scl\big(sint(G,E)\big)=U$
Thus (G,E) is SS^M-open set, but it is not open soft set .
Theorem 1.1: [8]. The following statement is introduced on $(U,\tilde{\tau},E)$. Then
1. Let $(F_1,E)\ and\ (F_2,E)$ are SS^M-open sets, then $(F_1,E)\ \tilde{\cup}\ (F_2,E)=(F_3,E)$, where (F_3,E) is SS^M-open set.
2. Let $(F_1,E),(F_2,E),\dots,(F_n,E)$ are SS^M-open sets, then $(F_1,E)\ \tilde{\cap}\ (F_2,E)\ \tilde{\cap}\ \dots\tilde{\cap}\ (F_n,E)$ is also SS^M-open set.
Definition 1.23: [14]. We say that $(U,\tilde{\tau},E)$ is a soft $lindel\ddot{o}f$, if every cover of U has a countable sub cover.

2. Main results

In this section, we introduce the soft simply compact and define a new concepts in soft topological spaces like SS^M −interior, SS^M −closure, SS^M −union, SS^M −intersection and soft simply separation axioms.
Definition 2.1: Let $(F,A)^{\ M}$ and $(G,B)^M$ be two soft simply sets then $(F,A)^{\ M}\ \tilde{\cup}^M\ (G,B)^M$=$(H,C)^M$, where $C=A\cup B$ and for each $e\in C$

$$H(e)=\begin{cases}F(e) & if\ e\in A-B\\ G(e) & if\ e\ \in B-A\\ F(e)\cup G(e) & if\ e\in A\cap B\end{cases}$$

Definition 2.2: Let $(F,A)^{\ M}$ and $(G,B)^M$ be two soft simply sets over U, then the soft simply intersection of them (for short $SS^M-intersection$) is the soft simply set $(H,C)^M$, where $C=A\cap B$ and for all $e\in C$ such that $H(e)=F(e)\cap G(e)$ and denoted $(F,A)^{\ M}\ \tilde{\cap}^M\ (G,B)^M$=$(H,C)^M$.
Definition 2.3: Let $(F,A)^{\ M}$ and $(G,B)^M$ be two soft simply sets over U, then $(F,A)^{\ M}$ is said to be soft simply subset of $(G,B)^M$ (for short $SS^M-subset$) and denoted $(F,A)^{\ M}\ \tilde{\subset}^M\ (G,B)^M$**,** if $A\subset B$ and $F(e)\subset G(e)$ for all $e\in A$.
Definition 2.4: $(F,A)^{\ C\ M}$ is a soft simply complement of $(F,A)^{\ M}$ (for short $SS^M-complement$) which is defined as $F^C(a)=U\setminus F(a),\forall\ a\ \in A$.
Definition 2.5: $SS^M(int(F,E)^M)$ is a soft simply interior of a soft simply set $(F,E)^{\ M}$ (for short $SS^M-interior$), which is a soft simply set defined as follows

$SS^M(int(F,E)^M) = \tilde{\cup}^M \{(G,E)^M ; (G,A)^M \tilde{\subseteq}^M (F,E)^M, (G,E)^M \in \tilde{\tau}\}$.

Definition 2.6: $SS^M(cl(F,E)^M)$ is a soft simply closure of a soft simply set $(F,E)^M$ (for short $SS^M - closure$), which is a soft simply set defined as follows

$SS^M(cl(F,E)^M) = \tilde{\cap}^M \{(G,E)^M ; (F,E)^M \tilde{\subseteq}^M (G,E)^M; (G,E)^{C^M} \in \tilde{\tau}\}$.

Definition 2.7: We say that $(U,\tilde{\tau},E)$ is a soft simply $T_0 - space$ (for short $SS^M - T_0\ space$) if for any two distinct points $a, b \in U$, then $\exists\ (F,E)^M$ and $(G,E)^M \in \tilde{\tau}$, such that $[a \in (F,E)^M, b \notin (F,E)^M] \vee [b \in (G,E)^M$ and $a \notin (G,E)^M]$.

Definition 2.8: We say that $(U,\tilde{\tau},E)$ is a soft simply $T_1 - space$ (for short $SS^M - T_1\ space$) if for any two distinct points $a, b \in U$, then there exist $(F,E)^M$ and $(G,E)^M \in \tilde{\tau}$ such that $[a \in (F,E)^M, b \notin (F,E)^M]$, $[b \in (G,E)^M$ and $a \notin (G,E)^M]$.

Definition 2.9: We say that $(U,\tilde{\tau},E)$ is a soft simply $T_2 - space$ (for short $SS^M - T_2\ space$) if for any two distinct points $a, b \in U$, then $\exists\ (F,E)^M\ and\ (G,E)^M \in \tilde{\tau}$, such that $a \in (F,E)^M$, $b \in (G,E)^M\ and\ (F,E)^M\ \tilde{\cap}^M\ (G,E)^M = \emptyset$.

Definition 2.10: We say $(U,\tilde{\tau},E)$ is a soft simply $regular\ space$ (for short $SS^M - regular\ space$) if for all $(H,E)^M \in \tilde{\tau}^C$ ($i.e$: $(H,E)^M$ is soft simply closed in U) and any points $a \in U$ such that $a \notin (H,E)^M$, then $\exists (F,E)^M and\ (G,E)^M \in \tilde{\tau}$, such that $[a \in (F,E)^M, (H,E)^M \tilde{\subseteq}^M (G,E)^M$, and $(F,E)^M\ \tilde{\cap}^M\ (G,E)^M = \emptyset]$.

Definition 2.11: We say that $(U,\tilde{\tau},E)$ is a soft simply $normal\ space$ (for short $SS^M - normal\ space$) if for all $(H,E)^M$, $(K,E)^M \in \tilde{\tau}^C$ ($i.e$: $(H,E)^M$ and $(K,E)^M$ are soft simply closed in U) such that $(H,E)^M\ \tilde{\cap}^M\ (K,E)^M = \emptyset$, then $\exists\ (F,E)^M$ and $(G,E)^M \in \tilde{\tau}$, such that $[(H,E)^M \tilde{\subseteq}^M (F,E)^M$, $(K,E)^M \tilde{\subseteq}^M (G,E)^M\ and\ (F,E)^M\ \tilde{\cap}^M\ (G,E)^M = \emptyset]$.

Definition 2.12: We say that $(U,\tilde{\tau},E)$ is a soft simply $T_3 - space$ (for short $SS^M - T_3\ space$) if U is both soft simply $regular\ space$ and soft simply $T_1 - space$.

Definition 2.13: We say that $(U,\tilde{\tau},E)$ is a soft simply $T_4 - space$ (for short SS^M–T_4 space) if $U\ is$ both soft simply $normal\ space$ and soft simply $T_1 - space$.

$$SS^M\text{–}T_4\ space \longrightarrow SS^M - T_3\ space \longrightarrow SS^M - T_2\ space \longrightarrow SS^M\text{–}T_1\ space$$

Diagram 2.1

Definition 2.14: A collection η of soft simply sets is said to be soft simply cover of a soft simply set $(F,E)^M$, if $(F,E)^M \tilde{\subseteq}^M\ U\{(F_i,E)^M; (F_i,E)^M \in \eta\ ; i \in I\}$.

Definition2.15: A soft simply compact (for short $SS^M - compact$), if every soft simply open cover that has a finite sub soft simply covers.

Definition 2.16: A soft simply $lindelö\ f$ (for short $SS^M - lindelöf$), if each cover of U by soft simply open sets has a countable sub cover.

Remark 2.1: Any $SS^M - compactt$ is $SS^M - lindelöf$.

Proposition 2.1: A soft simply $regular$ and $SS^M - lindelöf$ are $SS^M - normal$.

Definition 2.17: If $(F,E)^M$ is a soft simply set of $(U,\tilde{\tau},E)$, then we say that $(F,E)^M$ is a soft simply neighborhood (for short $S^M - nbd$) of the soft simply set $(H,E)^M$if $\exists (K,E)^M \in \tilde{\tau}$ such that $(H,E)^M \subseteq (K,E)^M \subseteq (F,E)^M$.

Theorem 2.1: Any $SS^M - compact$ sub set of a $SS^M - T_2\ space$ is $SS^M - closed$.

Theorem 2.2: A soft simply closed subset of a soft simply compact space $(U,\tilde{\tau},E)$ is a soft simply compact.

Proof: Let $(F,E)^M \in SS^M C(U)$, then $U \setminus (F,E)^M \in SS^M O(U)$. Suppose that $\{V_i : i \in I\} \tilde{\subseteq}^M SS^M O(U)$ be a cover of $(F,E)^M$, because U is $SS^M - compact$, then there exists a finite subsets I_0 of I such that $U = U \setminus (F,E)^M\ \tilde{\cup}^M\ (\tilde{\cup}^M{}_{i \in I_0} V_i) \in SS^M O(U)$. Hence $(F,E)^M \tilde{\subseteq}^M (\tilde{\cup}^M{}_{i \in I_0} V_i)$ and $(F,E)^M$ are a $SS^M - compact$ sub set of U.

We obtain the following by combining theorems (2.1) and (2.2):

Corollary 2.1: A subset of a $SS^M - compact$ and SS^M–T_2 space is $SS^M - compact$ if and only if it is $SS^M - closed$.

Corollary 2.2: A $SS^M - compact$ and $SS^M - T_2\ space$ is SS^M–$T_3\ space$.

Proof : Let $(F,E)^M \in U$, by Theorem (2.1) $(F,E)^M$, is $SS^M - closed$, since there exists (F_1,E), such that $(F_1,E)^M$ and $(F_2,E)^M \in SS^M - open$, such that $e \in (F_1,E)^M$, $(F,E)^M \tilde{\subseteq}^M (F_2,E)^M$and

$(F_1, E)^M \tilde{\cap}^M (F_2, E)^M = \emptyset$ (by proof from Theorem (2.1)). Hence, U is $SS^M - regular$. Since U is $SS^M - T_2$ space, by Diagram (2.1), then U is $SS^M - T_1$ space. Hence, U is $SS^M - T_3$space.

Corollary 2.3: An $SS^M - compact$ and $SS^M - T_2$space are $SS^M - T_4$ $space$.

Proof : Since U is $SS^M - compact$ then U is $SS^M - lindelöf$ (by Remark 2.1), and any $SS^M - compact$ and $SS^M - T_2$ $space$ are $SS^M - T_3$ $space$ (by Corollary 2.2), then U is $SS^M - T_3$ $space$. Hence, U is $SS^M - regular\ space$. Then U is $SS^M - normal$ (by Proposition 2.1). Hence, U is $SS^M - T_4$ $space$.

Definition 2.18:[8] A function $f: (U, \tilde{\tau}, E) \to (V, \tilde{\sigma}, E)$ is said to be $SS^M - irresolute$ such that $(U, \tilde{\tau}, E)$ and $(V, \tilde{\sigma}, E)$ are two soft topological spaces, if for each $(F, E)^M \in SS^M O(V)$, $f^{-1}(F, E)^M \in SS^M O(U)$.

Theorem 2.3: Let $f : (U, \tilde{\tau}, E) \to (V, \tilde{\sigma}, E)$ is a $SS^M - irresolute$, where $(U, \tilde{\tau}, E)$ be a $SS^M - compact$, then the image of a $SS^M - compact$ is $SS^M - compact$.

Proof : Let $(F, E)^M$, by a $SS^M - compact$, be a subset of $(U, \tilde{\tau}, E)$, and η is the collection of $SS^M - open$ sets cover of $(f(F, E)^M)$ in $(V, \tilde{\sigma}, E)$, such that $(f(F, E)^M) \tilde{\subseteq}^M \tilde{\cup}^M \{(F_i, E)^M; (F_i, E)^M \in \eta ; i \in I\}$. Thus, $(F, E)^M \tilde{\subseteq}^M f^{-1}(f(F, E)^M)) \tilde{\subseteq}^M f^{-1}[\ \tilde{\cup}^M \{(F_i, E)^M;\ i \in I\}] \tilde{\subseteq}^M \tilde{\cup}^M \{f^{-1}(F_i, E)^M; i \in I\}$, $(F, E)^M \tilde{\subseteq}^M \tilde{\cup}^M \{f^{-1}(F_i, E)^M; i \in I\}$. Hence, $f^{-1}(F, E)^M$ is $SS^M - open$ in $(U, \tilde{\tau}, E)$ (since f is $SS^M - irresolute$ function). Since $(F, E)^M$ is an $SS^M - compact$ and $\{f^{-1}(F_i, E)^M; i \in I\}$ is an $SS^M - open$ cover of $(F, E)^M$, hence, $\exists$ is a finite subset I_0 of I ; $(F, E)^M$ $\tilde{\cup}^M \{f^{-1}(F_i, E)^M; i \in I_0\}$. Hence, $f(F, E)^M \tilde{\subseteq}^M \tilde{\cup}^M \{f f^{-1}(F_i, E)^M; i \in I_0\} \tilde{\subseteq}^M \tilde{\cup}^M \{(F_i, E)^M; i \in I_0\}$. Then $f(F, E)^M$ is $SS^M - compact$.

Definition 2.19: Let $(U, \tilde{\tau}, E)$ and $(V, \tilde{\sigma}, E)$ be two soft topological spaces, then $(U, \tilde{\tau}, E)$ is said to be $SS^M - homeomorphic$ to $(V, \tilde{\sigma}, E)$ iff $\exists$ a $SS^M - homeomorphism$ function (f is bij , f is $SS^M - irresolute$, f^{-1} is $SS^M - irresolute$) from $(U, \tilde{\tau}, E)$ onto $(V, \tilde{\sigma}, E)$, which we denoted as $(U, \tilde{\tau}, E) \cong^M (V, \tilde{\sigma}, E)$.

Proposition 2.2: Let $f : (U, \tilde{\tau}, E) \to (V, \tilde{\sigma}, E)$ be an $SS^M - irresolute$ function from a compact topological space $(U, \tilde{\tau}, E)$ to a $SS^M - T_2$ $space$ $(V, \tilde{\sigma}, E)$. Then $f\ ((F, E)^M)$ is closed in $(V, \tilde{\sigma}, E)$ for every $(F, E)^M$ is closed set $(U, \tilde{\tau}, E)$.

Theorem 2.4: The $SS^M - irresolute$ and bijectione $f: (U, \tilde{\tau}, E) \to (V, \tilde{\sigma}, E)$ from a $SS^M - compact$ $(U, \tilde{\tau}, E)$ to $SS^M - T_2$space $(V, \tilde{\sigma}, E)$ are $SS^M - homeomorphism$.

Proof: Let g: $(V, \tilde{\sigma}, E) \to (U, \tilde{\tau}, E)$ be the $SS^M - inverse$ of the bijection $f: (U, \tilde{\tau}, E) \to (V, \tilde{\sigma}, E)$. If $(F, E)^M$ is $SS^M - open$ in $(U, \tilde{\tau}, E)$, then $(F, E)^{M^C}$ is $SS^M - closed$ in $(U, \tilde{\tau}, E)$, and hence $f(F, E)^{M^C}$ is $SS^M - closed$ in $(V, \tilde{\sigma}, E)$ (by Proposition 2.2). But $f(F, E)^{M^C} = g^{-1}\left((F, E)^{M^C}\right) = V \setminus g^{-1}((F, E)^M)$. Hence, $g^{-1}((F, E)^M)$ is $SS^M - open$ in $(V, \tilde{\sigma}, E)$ for every $SS^M -$open set $(F, E)^M$ in $(U, \tilde{\tau}, E)$. Therefore, g: $(V, \tilde{\sigma}, E) \to (U, \tilde{\tau}, E)$ is $SS^M - irresolute$ and thus f is $SS^M - homeomorphism$ function.

Definition 2.20: A family δ of subsets of a soft topological space $(U, \tilde{\tau}, E)$ has a $SS^M - finite\ intersection$ property (for short $SS^M - fip$) if each finite sub family of δ has non empty intersection .

Theorem 2.5 : A soft topological space $(U, \tilde{\tau}, E)$ is a $SS^M - compact$ if and only if any given collection of $SS^M - closed$ subsets of U with the $SS^M - fip$ has non empty intersection.

Proof: Let $(U, \tilde{\tau}, E)$ be $SS^M - compact$, and start with $\delta = \{(F_i, E_i)^M; i \in I\}$ being a family of a $SS^M - closed$ sets of U that has a $SS^M - fip$. Then we want to prove that $\tilde{\cap}^M_{i \in I} (F_i, E_i)^M \neq \emptyset$. Suppose that $\tilde{\cap}^M_{i \in I} (F_i, E_i)^M = \emptyset$, then by the DeMorgan's law, $\left(\tilde{\cap}^M_{i \in I} (F_i, E_i)^M\right)^C = \tilde{\cup}^M_{i \in I} (F_i, E_i)^{M^C} = U$. This implies that $\{(F_i, E_i)^{M^C}; i \in I\}$ is a collection of $SS^M - open$ cover of , then $\exists\ I_\alpha \in I$ (soft simply sub set) such that $U = \tilde{\cup}^M_{i \in I} (F_i, E_i)^{M^C}$ then $U^C = \left(\tilde{\cup}^M_{i \in I} (F_i, E_i)^{M^C}\right)^C = \tilde{\cap}^M_{i \in I_\alpha} (F_i, E_i)^M$. Therefore, $\tilde{\cap}^M_{i \in I_\alpha} (F_i, E_i)^M = \emptyset$, which is a contradiction. Then $\tilde{\cap}^M_{i \in I} (F_i, E_i)^M \neq \emptyset$.

Conversely, suppose that $\{(F_i, E_i)^M; i \in I\}$ is a collection of an $SS^M - open$ cover of U. Let for all any finite subset $I_\alpha \in I$, we have $\tilde{\cup}^M_{i \in I} (F_i, E_i)^M \neq U$, then $\left(\tilde{\cap}^M_{i \in I_\alpha} (F_i, E_i)^M\right)^C \neq \emptyset$. Therefore, $\{(F_i, E_i)^{M^C}; i \in I\}$ has an $SS^M - fip$, we have $\left(\tilde{\cap}^M_{i \in I_\alpha} (F_i, E_i)^M\right)^C \neq \emptyset$, which implies

$\tilde{\cap}^M_{i\in I_\alpha}(F_i,E_i)^M \neq U$, and this contradicts that $\{(F_i,E_i)^M; i\in I\}$ is $SS^M - open$ cover of U, then U is $SS^M - compact$.

Definition 2.21: [8]. A function $f : (U,\tilde{\tau},E) \to (V,\tilde{\sigma},E)$ is said to be $SS^M - continuous$ such that $(U,\tilde{\tau},E)$ and $(V,\tilde{\sigma},E)$ are two soft topological spaces, if every $(F,E)^M$ is soft open set of V, then $f^{-1}(F,E)^M$ is $SS^M - open$ set of U.

Theorem 2.6 : Let $f : (U,\tilde{\tau},E) \to (V,\tilde{\sigma},E)$ between two soft topological spaces. Then f is $SS^M - continuous$ if and only if the inverse image of each soft closed set in $(V,\tilde{\sigma},E)$ is $SS^M - closed$ in $(U,\tilde{\tau},E)$.

Theorem 2.7: A soft continuous function is SS^M −continuous function.

Proof : Consider $f: (U,\tilde{\tau},E) \to (V,\tilde{\sigma},E)$ be a soft continuous function. Let (G,E) be a soft open set in V, then $f^{-1}(G,E)$ is soft open in U, and so $f^{-1}(G,E)$ is $SS^M - open$ set in U. Therefore, f is $SS^M - continuous$ function.

Theorem 2.8: Let $f : (U,\tilde{\tau},E) \to (V,\tilde{\sigma},E)$ and $g : (V,\tilde{\sigma},E) \to (W,\tilde{\rho},E)$ be two functions. Then $g \circ f : (U,\tilde{\tau},E) \to (W,\tilde{\rho},E)$ is $SS^M - continuous$ if f is $SS^M - continuous$ and g is soft continuous.

Proof : Consider $(H,E)^M$ be a soft closed set in W, because $g : (V,\tilde{\sigma},E) \to (W,\tilde{\rho},E)$ is soft continuous, then $g^{-1}(H,E)^M$ is soft closed set of V. Now $f : (U,\tilde{\tau},E) \to (V,\tilde{\sigma},E)$ is SS^M −continuous and $g^{-1}(H,E)^M$ is soft closed set of V, so (by Definition 2.25) $f^{-1}\left(g^{-1}(H,E)^M\right) = (g \circ f)^{-1}(H,E)^M$ is SS^M −closed in U. Hence, $g \circ f : (U,\tilde{\tau},E) \to (W,\tilde{\rho},E)$ is SS^M −continuous.

Conclusion

In this paper, we introduced a new concept in soft topological spaces such as SS^M −interior, SS^M −closure, SS^M −union, and SS^M −intersection. We also introduced the SS^M −compactness , studied some of its properties, and defined the soft simply separation axioms .

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