DOI: 10.24996/ijs.2020.SI.1.10

ISSN: 0067-2904

Multicomponent Inverse Lomax Stress-Strength Reliability

Nada S. Karam*, Shahbaa M. Yousif , Bushra J. Tawfeeq
Department of Mathematics, College of Education, Mustansiriyah University, Baghdad, Iraq

Received: 18/11/ 2019 Accepted: 15/ 3/2020

Abstract

In this article we derive two reliability mathematical expressions of two kinds of s-out of -k stress-strength model systems; $R_{1(s,k)}$ and $R_{2(s,k)}$. Both stress and strength are assumed to have an Inverse Lomax distribution with unknown shape parameters and a common known scale parameter. The increase and decrease in the real values of the two reliabilities are studied according to the increase and decrease in the distribution parameters. Two estimation methods are used to estimate the distribution parameters and the reliabilities, which are Maximum Likelihood and Regression. A comparison is made between the estimators based on a simulation study by the mean squared error criteria, which revealed that the maximum likelihood estimator works the best.

Keywords: Reliability, Multicomponent system, s-out of-k model, Invers Lomax distribution, Maximum likelihood estimation, Regression estimation.

معولية الاجهاد – المتانة متعددة المتغيرات لمعكوس لوماكس

ندى صباح كرم* , شهباء محمد يوسف , بشرى جارالله توفيق
قسم الرياضيات, كلية التربيه, الجامعه المستنصريه, بغداد, العراق

الخلاصة

في هذا البحث، تم اشتقاق تعبيرين رياضيين لمعولية نوعين من انظمة (s-out of-k) المتعددة المكونات لنموذج الاجهاد- المتانة وهما $R_{1(s,k)}$ و $R_{2(s,k)}$. حيث يتبع كل من متغيري الإجهاد والمتانة توزيع معكوس لوماكس مع معلمات شكل غير معروفة ومعلمة قياس معروفة, حيث تم دراسة الزيادة والنقصان في القيم الحقيقية لمعولية وفقًا للزيادة والنقصان في معلمات التوزيع. أيضا تم اجراء عملية تقدير لمعلمات التوزيع الغير معروفة وكذلك معولية النظامين باستخدام طريقتي تقدير مختلفتين وهما كل من طريقة الإمكان الأعظم وطريقة الانحدار واجراء مقارنة بين مقدري المعولية بناءً على دراسة محاكاة وفقًا لمعيار متوسط مربعات الخطأ ، والتي أظهرت أن مقدّر الإمكان الاعظم يعمل بشكل أفضل

1- Introduction

The Inverse Lomax distribution (ILD) is used in various fields such as stochastic modeling, economics, actuarial sciences and life testing; it is one of the notable lifetime models in statistical applications. The distribution belongs to an inverted family of distributions and found to be very flexible to analyze the situation where the non-monotonicity of the failure rate has been realized. If a random variable Y has Lomax distribution, then(X =1/Y) has an Inverse Lomax distribution [1-5].

The probability density function (pdf) and cumulative distribution function (cdf) of the random

*Email: karamns6760@gmail.com

variable following ILD are given by the following equations [6].

$$f(x,\alpha,\beta) = \begin{cases} \frac{\alpha\beta}{x^2}\,(1+\frac{\beta}{x})^{-(1+\alpha)} & x \geq 0;\ \alpha,\beta > 0 \\ 0 & ow \end{cases} \quad \ldots(1)$$

$$F(x) = (1+\frac{\beta}{x})^{-\alpha} \qquad x \geq 0;\ \alpha,\beta > 0$$

Here $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters, respectively. Now onwards, Inverse Lomax distribution (ILD) with its two parameters will be denoted by ILD (α,β). The purpose of this paper is to study two kinds of the reliability form of the multicomponent stress-strength models, based on X, Y being two independent random variables, where X~ ILD(α,β) and Y~ ILD(λ,β).

Stress-strength model is a system to analyze the strength of materials on which stress is, whatever the material type is; in either part or all of the system, the system collapses if the stress applied to it exceeds the strength, which has many applications in physics and engineering topics. This model is of a special importance in reliability literature [7]. For one component, let Y be a strength random variable subjected to a stress random variable X, where X and Y are independent, then the reliability of this system is

$$R = P(Y > X) = \int_{-\infty}^{\infty} f(x)F_y(x)dx$$

Here we consider the problem of reliability estimation in a multicomponent stress-strength system (s- out of-k), which was studied by Bhattacharyya and Johnson [8]. Let the random variables X, $Y_1,\ldots,Y_k$ be independent, F(x) be the continuous distribution function of X, and F(y) be the common continuous distribution function of $Y_1,\ldots,Y_k$, , then the reliability of the multicomponent stress-strength system is given by :

$$R_{(s,k)} = Prob(at\ least\ s\ of\ \ Y_1,\ldots,Y_k \text{ exceed X}) \quad \ldots(2)$$

In 2012, Hassan and Basheikh [7] expanded the system reliability of multicomponent to more than two groups of components. Consider a system made up of k non-identical components. Out of these k components, $k_1$are of one category and their strengths $Y_1, Y_2, \ldots, Y_{k_1}$are independent and identical distribution random variables. The remaining components $k_2 = k - k_1$are of a different category and their strengths $Y_{k_1+1}, Y_{k_1+2}, \ldots, Y_k$ are independent identically distributed (iid) random variable subjected to a common stress X which is an independent random variable.

The reliability estimation for the two kinds of ILD multicomponent stress-strength systems has not received much attention in the literature. Therefore, an attempt is made here to study the reliability estimations for the two models. In section 2, we derive the mathematical expression for $R_{1(s,k)}$ and $R_{2(s,k)}$. The maximum likelihood and the regression estimators for the ILD parameters which are used to give the estimators of $R_{1(s,k)}$ and $R_{2(s,k)}$ are obtained in section 3. The comparison between the two estimators of reliabilities for different experiments and sample sizes is made through a Monte Carlo simulation study in section 4. Finally, some conclusions and comments are provided in section 5.

2. $R_{1(s,k)}$ and $R_{2(s,k)}$ for Inverse Lomax distribution (ILD)

The reliability functions are obtained by using the probability in equation (2) for $R_{1(s,k)}$ and $R_{2(s,k)}$ of two multicomponent stress-strength models. Considering the multicomponent stress-strength system given by k components, where the strength $Y_i \sim ILD(\lambda,\beta)$ with the distribution function (cdf) is given as:

$$F(y) = (1+\frac{\beta}{y})^{-\lambda} \qquad y \geq 0;\ \lambda,\beta > 0 \quad \ldots(3)$$

Under common stress $X \sim ILD(\alpha,\beta)$ with density function (pdf) given in eq. (1), then the reliability $R_{1(s,k)}$ can be obtained by (1) and (3) in (2), where:

$$R_{1(s,k)} = \sum_{i=s}^{k} C_i^k \int_0^{\infty} \left[1 - F_y(x)\right]^i [F_y(x)]^{k-i}\, \mathrm{dF(x)}$$

$$= \sum_{i=s}^{k} C_i^k \int_0^{\infty} \left[1 - (1+\frac{\beta}{x})^{-\lambda}\ \right]^i [(1+\frac{\beta}{x})^{-\lambda}\ ]^{k-i}\, \mathrm{dF(x)} \quad \ldots(4)$$

Let $u = \left(1+\frac{\beta}{x}\right)^{-\lambda}$ then $x = \frac{\beta}{\left(u^{\frac{-1}{\lambda}} - 1\right)} = \beta\left(u^{\frac{-1}{\lambda}} - 1\right)^{-1}$

The derivative of x is:

$$dx = -\beta\left(u^{\frac{-1}{\lambda}}-1\right)^{-2}\left(\frac{-1}{\lambda}u^{\frac{-1}{\lambda}-1}\right)du = \left(\frac{\beta}{\lambda}\right)\left(u^{\frac{-1}{\lambda}}-1\right)^{-2}u^{\frac{-1}{\lambda}-1}du$$

By transformation and substituting u and dx in (4), we get:

$$R_{1(s,k)} = \sum_{i=s}^{k} C_i^k \int_0^1 [1-u]^i u^{k-i} \frac{\alpha\beta}{\beta^2\left(u^{\frac{-1}{\lambda}}-1\right)^{-2}}\left(u^{\frac{-1}{\lambda}}\right)^{-(1+\alpha)}\left(\frac{\beta}{\lambda}\right)\left(u^{\frac{-1}{\lambda}}-1\right)^{-2}u^{\frac{-1}{\lambda}-1}du$$

$$= \sum_{i=s}^{k} C_i^k \frac{\alpha}{\lambda}\int_0^1 [1-u]^i u^{k-i}\left(u^{\frac{-1}{\lambda}}\right)^{-(1+\alpha)} u^{\frac{-1}{\lambda}-1}du$$

$$= \sum_{i=s}^{k} C_i^k \frac{\alpha}{\lambda}\int_0^1 [1-u]^i u^{k-i+\frac{1}{\lambda}+\frac{\alpha}{\lambda}-\frac{1}{\lambda}-1}du$$

$$= \frac{\alpha}{\lambda}\sum_{i=s}^{k} C_i^k \int_0^1 u^{k+\frac{\alpha}{\lambda}-i-1}[1-u]^i du$$

$$= \frac{\alpha}{\lambda}\sum_{i=s}^{k} C_i^k \beta\left(\left(k+\frac{\alpha}{\lambda}-i\right),(i+1)\right)$$

$$= \frac{\alpha}{\lambda}\sum_{i=s}^{k}\frac{k!}{i!(k-i)!}\frac{\Gamma(i+1)\Gamma(k+\frac{\alpha}{\lambda}-i)}{\Gamma(k+\frac{\alpha}{\lambda}+1)}$$

Since $\Gamma(a+1) = a!$ and through the rearrangement of the boundaries, then the $R_{1(s,k)}$ is given by

$$R_{1(s,k)} = \frac{\alpha}{\lambda}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\frac{\alpha}{\lambda}-j\right)\right]^{-1} \quad \ldots(5)$$

Where s, k, i *and* j are integers.

Now to obtain the reliability $R_{2(s,k)}$, consider a system made up of k non-identical components. Let $X \sim ILD(\alpha,\beta)$ be an independent stress random variable, $Y_1,\ldots,Y_{k_1} \sim ILD(\lambda_1,\beta)$ are independent identically distributed (iid) strength random variables, and the remaining $Y_{k_1+1}, Y_{k_1+2},\ldots,Y_k \sim ILD(\lambda_2,\beta)$ are iid strength random variables with (cdf), given as

$$F(y_1) = \left(1+\frac{\beta}{y_1}\right)^{-\lambda_1} \quad , y_1 > 0; \lambda_1 \, and \, \beta > 0 \quad \ldots(6)$$

$$F(y_2) = \left(1+\frac{\beta}{y_2}\right)^{-\lambda_2} \quad , y_2 > 0; \lambda_2 \, and \, \beta > 0 \quad \ldots(7)$$

respectively, then by (1), (6) and (7), the reliability $R_{2(s,k)}$ can be given as:

$$R_{2(s,k)} = \sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2}\int_0^{\infty}\left[1-F_{y_1}(x)\right]^{i_1}[F_{y_1}(x)]^{k_1-i_1}\left[1-F_{y_2}(x)\right]^{i_2}[F_{y_2}(x)]^{k_2-i_2}dF(x)$$

Where $0 \le i_1 \le k_1, 0 \le i_2 \le k_2$ *and* $s \le i_1 + i_2 \le k$

$$R_{2(s,k)} = \sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2}\int_0^{\infty}\left[1-\left(1+\frac{\beta}{x}\right)^{-\lambda_1}\right]^{i_1}\left[\left(1+\frac{\beta}{x}\right)^{-\lambda_1}\right]^{k_1-i_1}$$
$$\left[1-\left(1+\frac{\beta}{x}\right)^{-\lambda_2}\right]^{i_2}\left[\left(1+\frac{\beta}{x}\right)^{-\lambda_2}\right]^{k_2-i_2}\frac{\alpha\beta}{x^2}\left(1+\frac{\beta}{x}\right)^{-(1+\alpha)}dx$$

By the same transformation technique, we get:

$$R_{2(s,k)} = \sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2}\int_0^1\left[1-u^{\lambda_1}\right]^{i_1}u^{\lambda_1(k_1-i_1)}\left[1-u^{\lambda_2}\right]^{i_2}u^{\lambda_2(k_2-i_2)}$$
$$\frac{\alpha\beta}{\beta^2\left(u^{-1}-1\right)^{-2}}u^{(\alpha+1)}\beta u^{-2}\left(u^{-1}-1\right)^{-2}du$$
$$= \sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2}\int_0^1\left[1-u^{\lambda_1}\right]^{i_1}\left[1-u^{\lambda_2}\right]^{i_2}\alpha\, u^{\lambda_1(k_1-i_1)+\lambda_2(k_2-i_2)+\alpha-1}du \quad \ldots(8)$$

Now using the Binomial expansion $(1-a)^h = \sum_{j=0}^{h} C_j^h(-a)^j$, we get:

$$(1-u^{\lambda_1})^{i_1} = \sum_{j_1=0}^{i_1} C_{j_1}^{i_1}(-1)^{j_1}u^{\lambda_1 j_1} \quad \ldots(9)$$

$$(1-u^{\lambda_2})^{i_2} = \sum_{j_2=0}^{i_2} C_{j_2}^{i_2}(-1)^{j_2}u^{\lambda_2 j_2} \quad \ldots(10)$$

Then by substitution (9) and (10) in (8), we get:

$$R_{2(s,k)} = \sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2}\sum_{j_1=0}^{i_1} C_{j_1}^{i_1}(-1)^{j_1}\sum_{j_2=0}^{i_2} C_{j_2}^{i_2}(-1)^{j_2}\alpha\int_0^1 u^{\lambda_1(k_1-i_1+j_1)+\lambda_2(k_2-i_2+j_2)+\alpha-1}du$$

So

$$R_{2(s,k)} = \alpha\sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2}\sum_{j_1=0}^{i_1} C_{j_1}^{i_1}(-1)^{j_1}\sum_{j_2=0}^{i_2} C_{j_2}^{i_2}(-1)^{j_2}\left(\lambda_1(k_1-i_1+j_1)+\lambda_2(k_2-i_2+j_2)+\alpha\right)^{-1} \quad (11)$$

Where s, k, i *and* j are integers.

3. Estimation of the Inverse Lomax Distribution (ILD)

Estimation the reliability functions $R_{1(s,k)}$ and $R_{2(s,k)}$ is performed by using the maximum likelihood and the regression methods.

3.1 Maximum likelihood Estimation (MLE)

Let the $y_1, y_2, \ldots, y_n$ strength sample has $ILD(\lambda, \beta)$ distribution with a sample size n, where λ is an unknown parameter, then the likelihood function L can be written as follows [9]:

$L(\lambda, \beta) = \prod_{i=1}^{n} f(y_i\,; \lambda, \beta)$

Then the likelihood function using the equation (1) is given as:-

$$L(y_1, y_2, \ldots, y_n; \lambda, \beta) = \lambda^n \beta^n \prod_{i=1}^{n} y_i^{-2} \prod_{i=1}^{n} (1 + \frac{\beta}{y_i})^{-(1+\lambda)} \quad \ldots(12)$$

Then the natural logarithm function for the equation (12) can be written as;

$$\ln L = n\ln\lambda + n\ln\beta - 2\sum_{i=1}^{n} \ln y_i - (1+\lambda)\sum_{i=1}^{n} \ln(1 + \frac{\beta}{y_i}) \quad \ldots(13)$$

By taking the partial derivative with respect to the unknown parameterλ, we get:

$\frac{\partial \ln L}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} \ln(1 + \frac{\beta}{y_i})$

By equating the partial derivative to zero, so:

$\frac{n}{\hat{\lambda}} - \sum_{i=1}^{n} \ln(1 + \frac{\beta}{y_i}) = 0$

Then the ML estimator for λ is given by:

$$\hat{\lambda}_{MLE} = \frac{n}{\sum_{i=1}^{n} \ln(1 + \frac{\beta}{y_i})} \quad \ldots (14)$$

In the same way, let X stress random variable has $ILD(\alpha, \beta)$ with a sample size m where α is an unknown parameter, then the ML estimator for α from equation (13) can be derived as:

$$\hat{\alpha}_{MLE} = \frac{m}{\sum_{j=1}^{m} \ln(1 + \frac{\beta}{x_j})} \quad \ldots (15)$$

By the substitution of the equations (14) and (15) in the equation (5), the ML estimator for $R_{1(s,k)}$ is $\hat{R}_{1ML}$ can be obtained as:

$$\hat{R}_{1ML} = \frac{\hat{\alpha}_{MLE}}{\hat{\lambda}_{MLE}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i} \left(k + \frac{\hat{\alpha}_{MLE}}{\hat{\lambda}_{MLE}} - j\right)]^{-1} \quad \ldots(16)$$

Now, for the second multicomponent reliability function, let$Y_{1_{i_1}}$; $i_1 = 1,2, \ldots, n_1$ and $Y_{2_{i_2}}$; $i_2 = 1,2, \ldots, n_2$ be strength random samples from $ILD(\lambda_1, \beta)$ and $ILD(\lambda_2, \beta)$, respectively, and let X be a stress random variable with a sample size m from $ILD(\alpha, \beta)$, then the M estimators for the unknown parameters λ_1 , λ_2 and α for $R_{2(s,k)}$, are:

$$\hat{\lambda}_{\xi MLE} = \frac{n_\xi}{\sum_{i=1}^{n_\xi} \ln(1 + \frac{\beta}{y_{i_\xi}})}, \qquad \xi = 1,2 \quad \ldots(17)$$

$$\hat{\alpha}_{MLE} = \frac{m}{\sum_{j=1}^{m} \ln(1 + \frac{\beta}{x_j})} \quad \ldots(18)$$

By substituting (17) and (18) in the equation (11), the ML estimator for $R_{2(s,k)}$ is $\hat{R}_{2ML}$ can be obtained as:

$$\hat{R}_{2ML} = \hat{\alpha}_{MLE} \sum_{i_1=s_1}^{k_1} \sum_{i_2=s_2}^{k_2} C_{i_1}^{k_1} C_{i_2}^{k_2} \sum_{j_1=0}^{i_1} C_{j_1}^{i_1} (-1)^{j_1} \sum_{j_2=0}^{i_2} C_{j_2}^{i_2} (-1)^{j_2} \; [\hat{\lambda}_{1MLE}(k_1 - i_1 + j_1) + \hat{\lambda}_{2MLE}\; (k_2 - i_2 + j_2) + \hat{\alpha}_{MLE}]^{-1} \quad \ldots.(19)$$

3.2 Regression Method (RM)

We used the regression method (RM) to estimate $R_{1(s,k)}$ and $R_{2(s,k)}$, with the following standard regression equation:-

$$w_i = a + bu_i + e_i \quad \ldots(20)$$

Where w_i is a dependent variable, u_i is an independent variable, and e_i is the error random variable independent. Taking natural logarithm to the (cdf) of the Inverse Lomax Distribution, we obtain:

$$\ln F(y_i) = -\lambda \ln(1 + \frac{\beta}{y_i}) \quad \ldots(21)$$

$y_i\,; i = 1,2, \ldots, n$ denotes the order sample, and by changing $F(y_i)$ by the plotting position P_i, where $P_i = \frac{i}{n+1}$ so $i = 1,2, \ldots, n$; then

$$lnP_i = -\lambda \ln\left(1+\frac{\beta}{y_i}\right) \quad \ldots(22)$$

By comparing equation (22) with (20), we get:

$w_i = lnP_i\ , a = 0, b = \lambda\ and\ u_i = -\ln(1+\frac{\beta}{y_i})$

Where b can be estimated by minimizing the summation of the squared error with respect to b, then we get:

$$\hat{b} = \frac{n\sum_{i=1}^{n} w_i u_i - \sum_{i}^{n} w_i \sum_{i=1}^{n} u_i}{n\sum_{i=1}^{n}[u_i]^2 - [\sum_{i=1}^{n} u_i]^2} \quad \ldots(23)$$

By substitution, then the RM estimator forλ is:

$$\hat{\lambda}_{RM} = \frac{-n\sum_{i=1}^{n} lnP_i \ln\left(1+\frac{\beta}{y_i}\right) + \sum_{i}^{n} lnP_i \sum_{i=1}^{n} \ln\left(1+\frac{\beta}{y_i}\right)}{n\sum_{i=1}^{n}[\ln\left(1+\frac{\beta}{y_i}\right)]^2 - [\sum_{i=1}^{n}\ln\left(1+\frac{\beta}{y_i}\right)]^2} \quad \ldots.(24)$$

And the estimated formula for the unknown stress parameter αcan be formulated as:

$$\hat{\alpha}_{RM} = \frac{-n\sum_{i=1}^{n} lnP_i \ln\left(1+\frac{\beta}{x_i}\right) + \sum_{i}^{n} lnP_i \sum_{i=1}^{n} \ln\left(1+\frac{\beta}{x_i}\right)}{n\sum_{i=1}^{n}[\ln\left(1+\frac{\beta}{x_i}\right)]^2 - [\sum_{i=1}^{n}\ln\left(1+\frac{\beta}{x_i}\right)]^2} \quad \ldots(25)$$

By substituting the equation (24) and (25) in the equation (5), the approximate RM estimator for $R_{1(s,k)}$, can be obtained as:

$$\hat{R}_{1RM} = \frac{\hat{\alpha}_{RM}}{\hat{\lambda}_{RM}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i}(k + \frac{\hat{\alpha}_{RM}}{\hat{\lambda}_{RM}} - j)]^{-1} \quad \ldots(26)$$

In the same manner, we obtain the estimate for the unknown shape parameters $\lambda_1,\ \lambda_2\ and\ \alpha$ for $R_{2(s,k)}$, then the result will be :

$$\hat{\lambda}_{\xi RM} = \frac{-n_\xi \sum_{i_\xi=1}^{n_\xi} lnP_{i_\xi} \ln\left(1+\frac{\beta}{y_{i_\xi}}\right) + \sum_{i_\xi=1}^{n_\xi} lnP_{i_\xi} \sum_{i_\xi=1}^{n_\xi} \ln\left(1+\frac{\beta}{y_{i_\xi}}\right)}{n_\xi \sum_{i_\xi=1}^{n_\xi}[\ln\left(1+\frac{\beta}{y_{i_\xi}}\right)]^2 - [\sum_{i_\xi=1}^{n_\xi}\ln\left(1+\frac{\beta}{y_{i_\xi}}\right)]^2}\ , \xi = 1,2 \quad \ldots(27)$$

And

$$\hat{\alpha}_{RM} = \frac{-m\sum_{j=1}^{m} lnP_j \ln\left(1+\frac{\beta}{x_j}\right) + \sum_{j}^{m} lnP_j \sum_{j=1}^{m} \ln\left(1+\frac{\beta}{x_j}\right)}{m\sum_{j=1}^{m}[\ln\left(1+\frac{\beta}{x_j}\right)]^2 - [\sum_{j=1}^{m}\ln\left(1+\frac{\beta}{x_j}\right)]^2} \quad \ldots(28)$$

By Substituting the equations (27) and (28) in the equation (11) of the RM estimator for $R_{2(s,k)}$, $\hat{R}_{2RM}$, we obtain:

$$\hat{R}_{2RM} = \hat{\alpha}_{RM} \sum_{i_1=s_1}^{k_1} C_{i_1}^{k_1} \sum_{i_2=s_2}^{k_2} C_{i_2}^{k_2} \sum_{j_1=0}^{i_1} C_{j_1}^{i_1} (-1)^{j_1} \sum_{j_2=0}^{i_2} C_{j_2}^{i_2} (-1)^{j_2} (\hat{\lambda}_{1RM}(k_1-i_1+j_1) + \hat{\lambda}_{2RM}(k_2-i_2+j_2) + \hat{\alpha}_{RM})^{-1} \quad \ldots(29)$$

4. Simulation study

In this section, we found the real values of the reliability in the two models discussed previously in section 3, at different values for the distribution parameters of the variables and for the case of two different experiments for s and k. The results are recorded in Tables-(1 and 2). It is observed from those two tables that for fixed strength parameters, the values of the reliabilities are increased by increasing the values of the stress parameter, and at the same value of stress parameter, the values of the two reliabilities are decreased by increasing the strength parameter.

Table 1-Real values of R_1 .

s=2, k=3, for α =.2						
λ	0.4000	0.5100	0.6000	0.9000	1.5000	2.9000
R1	0.6857	0.7394	0.7714	0.8379	0.8976	0.9449
for λ =.2						
α	0.4000	0.5100	0.6000	0.9000	1.5000	2.9000
R1	0.3000	0.2376	0.2000	0.1231	0.0602	0.0208
s=3, k=4, for α =.2						
λ	0.4000	0.5100	0.6000	0.9000	1.5000	2.9000
R1	0.6095	0.6734	0.7121	0.7938	0.8687	0.9289

for λ =.2						
α	0.4000	0.5100	0.6000	0.9000	1.5000	2.9000
R1	0.2000	0.1451	0.1143	0.0579	0.0209	0.0045

Table 2-Real values of R_2

s1=1, k1=2, s2=1, k2=3, for α =.2						
λ1 λ2	0.4000 0.3800	0.5100 0.4700	0.6000 0.4800	0.6900 0.8400	0.9400 1.3000	1.3000 2.5000
R2	0.7442	0.7879	0.8056	0.8487	0.8885	0.9220
For λ1 =.6, λ2=.48						
α	0.3800	0.4700	0.6000	0.8000	1.5000	3.0000
R2	0.6765	0.6236	0.5577	0.4754	0.2966	0.1420
s1=2, k1=3, s2=3, k2=4, for α =.2						
λ1 λ 2	0.4000 0.3800	0.5100 0.4700	0.6000 0.4800	0.6900 0.8400	0.9400 1.3000	1.3000 2.5000
R2	0.5354	0.6021	0.6203	0.7231	0.7978	0.8646
For λ1 =.6, λ2=.48						
α	0.3800	0.4700	0.6000	0.8000	1.5000	3.0000
R2	0.4217	0.3522	0.2750	0.1928	0.0670	0.0125

For some experiments, a simulation study was performed to compare the performance of the two estimation methods; the maximum likelihood and regression estimator for each one of $R_{1(s,k)}$ *and* $R_{2(s,k)}$ with different sample sizes ((n=m) = 15,30,90) and β =0.7, as well as for different parameter values and for two different S out of K systems.

As in the following steps:

Step 1- Using MATLAB 2017 by generating random values of the random variables and by applying the inverse function as follows:

$$y = \beta\,(F^{\frac{-1}{\lambda}} - 1)^{-1}$$

Step 2- By computing the mean by the equation:

$$\text{Mean} = \frac{\sum_{i=1}^{N}\hat{R}_i}{N}$$

Step 3- The comparison between the estimation methods by using MSE criteria:

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(\hat{R}_i - R)^2$$

Where N is The number of replications for each experiment, which is 500.

The results of comparison are shown as in Tables- 3, 4, 5, and 6. From those tables, we found that for all experiments and for each of the small (15), moderate (30) and large (90) sample sizes, the preference is for the maximum likelihood estimators

Table 3-Estimation of R_1 (s=2; k=3).

α= 0.2, λ=0.4, R1= 0.45714				
N		ML	RE	Best
15	Mean	0.4635	0.4593	
	MSE	0.0121	0.0190	ML
30	Mean	0.4482	0.4429	
	MSE	0.0057	0.0109	ML
90	Mean	0.4581	0.4571	
	MSE	0.0021	0.0043	ML
α= 1.3, λ=0.9, R1= 0.16034				
15	Mean	0.1703	0.1753	
	MSE	0.0063	0.0103	ML
30	Mean	0.1669	0.1697	
	MSE	0.0030	0.0054	ML
90	Mean	0.1627	0.1606	

	MSE	0.0011	0.0019	ML
α= 2, λ=1.5, R1= 0.17802				
15	Mean	0.1884	0.1952	
	MSE	0.0075	0.0130	ML
30	Mean	0.1836	0.1887	
	MSE	0.0038	0.0070	ML
90	Mean	0.1794	0.1826	
	MSE	0.0010	0.0021	ML
α= 2.2, λ=3, R1=0.33922				
15	Mean	0.3423	0.3474	
	MSE	0.0109	0.0187	ML
30	Mean	0.3352	0.3362	
	MSE	0.0054	0.0095	ML
90	Mean	0.3386	0.3441	
	MSE	0.0020	0.0041	ML

Table 4-Estimation of R_1 (s=3; k=4).

α= 0.2, λ=0.4, R1=0.40635				
N		ML	RE	Best
15	Mean	0.4075	0.4118	
	MSE	0.0133	0.0213	ML
30	Mean	0.4067	0.4044	
	MSE	0.0061	0.0115	ML
90	Mean	0.4053	0.4025	
	MSE	0.0023	0.0042	ML
α= 1.3, λ=0.9, R1=0.1178				
15	Mean	0.1349	0.1406	
	MSE	0.0054	0.0095	ML
30	Mean	0.1249	0.1345	
	MSE	0.0026	0.0052	ML
90	Mean	0.1192	0.1194	
	MSE	0.0008	0.0017	ML
α= 2, λ=1.5, R1=0.13352				
15	Mean	0.1430	0.1513	
	MSE	0.0059	0.0115	ML
30	Mean	0.1387	0.1414	
	MSE	0.0028	0.0050	ML
90	Mean	0.1344	0.1366	
	MSE	0.0010	0.0017	ML
α= 2.2, λ=3, R1=0.28666				
15	Mean	0.2988	0.2988	
	MSE	0.0105	0.0175	ML
30	Mean	0.2909	0.2927	
	MSE	0.0058	0.0114	ML
90	Mean	0.2854	0.2880	
	MSE	0.0019	0.0037	ML

Table 5-Estimation of R_2 ((s1=1, k1=2, s2=1, k2=3).

α= 0.2, λ1=0.4, λ2=0.38, R2=0.7442				
N		ML	RE	Best
15	Mean	0.7364	0.7271	
	MSE	0.0040	0.0083	ML
30	Mean	0.7366	0.7309	
	MSE	0.0021	0.0040	ML
90	Mean	0.7431	0.7415	
	MSE	0.0007	0.0014	ML
α= 0.2, λ1=1.3, λ2=2.5, R2=0.92201				
15	Mean	0.9164	0.9106	
	MSE	0.0008	0.0018	ML
30	Mean	0.9204	0.9177	
	MSE	0.0004	0.0007	ML
90	Mean	0.9218	0.9212	
	MSE	9.6348e-05	0.0002	ML
α= 0.5, λ1=0.6, λ2=0.48, R2=0.60739				
15	Mean	0.5975	0.5881	
	MSE	0.0072	0.0115	ML
30	Mean	0.6036	0.6015	
	MSE	0.0032	0.0059	ML
90	Mean	0.6081	0.6065	
	MSE	0.0012	0.0026	ML
α= 3, λ1=0.6, λ2=0.48, R2=0.14195				
15	Mean	0.1496	0.1553	
	MSE	0.0030	0.0060	ML
30	Mean	0.1450	0.1438	
	MSE	0.0014	0.0024	ML
90	Mean	0.1432	0.1422	
	MSE	0.0005	0.0009	ML

Table 6-Estimation of R_2 (s1=2, k1=3, s2=3, k2=4).

α= 0.2, λ1=0.4, λ2=0.38, R2= 0.53543				
N		ML	RE	Best
15	Mean	0.5192	0.5079	
	MSE	0.0104	0.0185	ML
30	Mean	0.5273	0.5222	
	MSE	0.0046	0.0090	ML
90	Mean	0.5331	0.5321	
	MSE	0.0016	0.0031	ML
α= 0.2, λ1=1.3, λ2=2.5, R2= 0.86464				
15	Mean	0.8565	0.8492	
	MSE	0.0019	0.0034	ML
30	Mean	0.8588	0.8542	
	MSE	0.0009	0.0017	ML
90	Mean	0.8638	0.8640	
	MSE	0.0002	0.0005	ML
α= 0.5, λ1=0.6, λ2=0.48, R2= 0.3322				
15	Mean	0.3305	0.3292	
	MSE	0.0105	0.0179	ML
30	Mean	0.3334	0.3339	
	MSE	0.0054	0.0092	ML
90	Mean	0.3317	0.3310	

	MSE	0.0019	0.0039	ML
α= 3, λ1=0.6, λ2=0.48, R2= 0.012485				
15	Mean	0.0170	0.0201	
	MSE	0.0002	0.0006	ML
30	Mean	0.0146	0.0168	
	MSE	8.6858e-05	0.0002	ML
90	Mean	0.0132	0.0139	
	MSE	2.7428e-05	6.3914e-05	ML

5. Conclusions

In this paper, we study the multicomponent stress-strength reliability for the ILD with different shape parameters. After the implementation of simulation experiments, we found that the best estimation MSE for the maximum likelihood estimator works quite well for all experiments and for all sizes of samples, while the regression estimation was not performing well in the ILD of stress and strength.

References

1. Jan U. and Ahmad S.P. **2017**. "Bayesian Analysis of Inverse Lomax Distribution Using Approximation Techniques", *Mathematical Theory and Modeling*, **7**(7): 1-12.
2. Karam N.S. and Ali H.M. **2014**. "Estimation of Two Component Parallel System Reliability Stress-Strength using Lomax Distribution", Magistra No. 88.
3. Karam N.S., Yousif SH.M. and Tawfeeq B.J. **2019**. "Stress-Strength Reliability for Inverse Lomax Distribution", *College of Education Journal*, **Special Edition**(1): 109-120.
4. Rahman J., Aslam M. and Ali S. **2013**. "Estimation and Prediction of Inverse Lomax Model via Bayesian Approach ", *Caspian Journal of Applied Sciences Research*, **2**(3): 43-56.
5. Reyad H.M. and Othman S.A. **2018**. "E-Bayesian Estimation of Two-Component Mixture of Inverse Lomax Distribution Based on Type-I Censoring Scheme", *Journal of Advances in Mathematics and Computer Science*, **26**(2): 1-22.
6. Yadav, A. S., Singh, S. K. and Singh, U. **2016**. "On Hybrid Censored Inverse Lomax Distribution" , Application to the Survival data. *Statistica*, **76**(2): 185-203.
7. Hassan A.S. and Basheikh H. M. **2012**. "Reliability Estimation of Stress-Strength Model with Non-Identical Component Strength: The Exponentiated Pareto Case ", *International Journal of Engineering Research and Application*, **2**(3): 2774-2781.
8. Bhattacharyya G.K. and Johnson R.A. **1974**. " Estimation of Reliability in Multicomponent Stress-Strength Models", *Journal of American Statistical Association, Theory Methods Section*, **69**(348).
9. Shawky A.I. and AL-Kashkari F.H. **2007**. "On a Stress-Strength Model in Burr of Type III", METRON- *International Journal of Statistics*, **LXV**(3): 371-385.