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Weak and Strong Forms of ω-Perfect Mappings

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Abstract:
In this paper, we introduce weak and strong forms of ω-perfect mappings, namely the θ-ω-perfect, weakly θ-ω-perfect and strongly θ-ω-perfect mappings. Also, we investigate the fundamental properties of these mappings. Finally, we focused on studying the relationship between weakly θ-ω-perfect and strongly θ-ω-perfect mappings.

Keywords: weakly θ-ω-perfect mappings and strongly θ-ω-perfect mappings.

أشكال التطبيقات المثالية نمط ω بشكل الضعيف والقوي

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الخلاصه
في عملنا قدمنا أشكال ضعيفة وقوية من التطبيقات مثالية ، وهي (التطبيقات التامة من نمط – θ–ω ، التطبيقات التامة من نمط θ–ω الضعيفة التطبيقات التامة من نمط θ–ω القوية ، ونحن أيضا درسنا خصائصها الأساسية. وأخيرا ، لخصنا الدراسة وتطورت العلاقة بين التطبيقات التامة من نمط θ–ω ضعيفة و التطبيقات التامة من نمط θ–ω القوية.

1. Introduction

In 1943, Formin [1] introduced the concepts of θ-continuous mappings. In 1966, Bourbaki [2] defined perfect mappings. In 1968, Velicko[3] introduced the concepts of θ-open and θ-closed subsets, while in 1968 Singal [4] introduced the notion of almost continuous mappings. In 1981, Long and Herrington [5] introduced the notion of strongly continuous mappings, in 1989, Hdeib [6] introduced the concepts of ω-continuous mappings. In 1991, Chew and Tong [7] introduced the notion of weakly continuous mappings, In this work, (G, τ) and (H, σ) stand for topological spaces. For a subset K of G, the closure of K and the interior of K will be denoted by cl(K) and int(K), respectively. Let (G, τ) be a space and K be a subset of G, then a point $g \in G$ is called a condensation point of K if, for each $S \in \tau$ and $g \in S$, the set $S \cap K$ is uncountable. K is called to be ω-closed [6] if it contains all its condensation points. The complement of ω-closed set is called to be ω-open. It is well known that a subset W of a space (G, τ) is ω-open if and only if, for each $g \in W$, there exists $S \in \tau$, such that $g \in S$ and $S-W$ is countable. The family of all ω-open sets of a space (G, τ), denoted by $\tau\omega$ or $\omega O(G)$, forms a topology on G finer than τ. The ω-closure and ω-interior, that can be known in the same way as cl(K) and int(K), respectively, will be denoted by ωcl(K) and ωint(K), respectively. Several characterizations of ω-closed sets were provided in previous articles [8-16]. A point g of G is called θ-cluster point of K if $cl(S) \cap K \neq \varphi$, for all open sets S of G containing g. The set of all θ-cluster points

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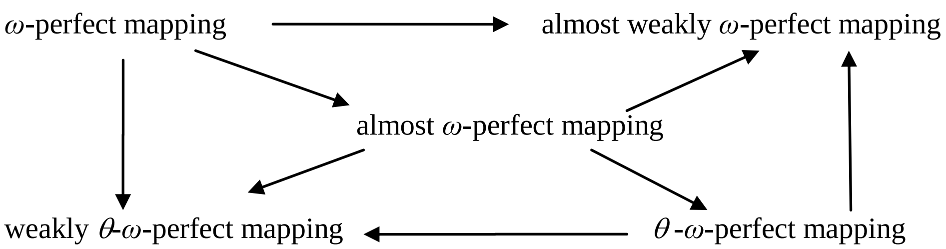
of K is called θ-closure of K and is denoted by $cl_\theta(K)$. A subset K is called θ-closed if $K = cl_\theta(K)$ [3]. The complement of θ-closed set is called θ-open. A point g of G is called an ω-θ-cluster point of K if $\omega cl(S) \cap K \neq \varphi$ for every ω-open set S of G containing g. The set of all ω-θ-cluster points of K is called ω-θ-closure of K and is denoted by $\omega Cl_\theta(K)$. A subset K is called ω-θ-closed if $K = \omega Cl_\theta(K)$. The complement of ω-θ-closed set is called ω-θ-open. The ω-θ-interior of K is defined by the union of each ω-θ-open sets contained in K and is denoted by $\omega int_\theta(K)$. A mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called ω-continuous (see [16]) (resp., almost weakly ω-continuous(see [11])) if for each $g \in G$ and each open set T of H containing $\lambda(g)$, there exists an ω-open subset S in G, such that $\lambda(S) \subseteq T$ (resp., $\lambda(S) \subseteq cl(T)$). A mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called almost ω-continuous [12] (resp., θ-ω-continuous (see [13]), strongly θ-ω-continuous (see[7])) if, for each $g \in G$ and for each regular open set T(resp., open) of H containing $\lambda(g)$, there exists an ω-open subset S in G, such that $\lambda(S) \subseteq T$ (resp., $\lambda(\omega cl(S)) \subseteq cl(T)$, $\lambda(\omega cl(S)) \subseteq T$)). A mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called θ-continuous (resp., continuous [16]), if for all an open T in H, $\lambda^{-1}(T)$ is an θ-open (resp., open) set in G. A mapping λ: $(G, \tau) \rightarrow (H ,\sigma)$ is called weakly (resp., strongly) continuous [3] if, for each $g \in G$ and all open set T of H containing $\lambda(g)$, there is an open set S of G, such that $\lambda (S)) \subseteq cl(T)$ (resp., $\lambda (cl(S)) \subseteq T$). A mapping λ: $(G, \tau) \rightarrow (H ,\sigma)$ is called almost continuous[7] if $\lambda^{-1}(T)$ is open in G for all regular open set T of H. A mapping $\lambda : (G, \tau) \rightarrow (H ,\sigma)$ is called weakly (resp.,[9] strongly) θ-continuous if, for each $g \in G$ and all open set T of H containing $\lambda(g)$, there is an open set S in G, such that $\lambda (S)) \subseteq cl(T)$ (resp., $\lambda (cl(S) \subseteq T)$. A topological space G is called a regular [14] if, for all closed set F and for each point $g \in G-F$, there exist disjoint open sets S and T such that $g \in S$ and $F \subseteq T$. A topological space G is called a semi-regular [15] if, for all point $g \in G$ and all open set S containment g, there is an open set T such that $g \in T \subseteq int(cl(T)) \subseteq S$. A topological space G is called ω-regular (resp., ω∗-regular) [12] if, for all ω-closed (resp., closed) set F and for each point $g \in G -F$, there are disjoint ω-open sets S and T such that $g \in S$ and $F \subseteq T$. Also we introduce several results and examples concerning deferent forms of ω-perfect mappings.

2. Weakly θ-ω-Perfect Mappings

In this section, we study the weakly θ-ω-perfect mappings and several related theorems.

Definition 2.1. A mapping $\lambda : (G ,\tau) \rightarrow (H ,\sigma)$ is said to be weakly θ-ω-continuous at $g \in G$ if, for every open subset T in H containing $\lambda(g)$, there exists an θ-ω-open subset S in G containing g, such that $\lambda(S) \subseteq cl(T)$. If λ is weakly θ-ω-continuous at every $g \in G$, it is said to be weakly θ-ω-continuous.

Definition 2.2. A mapping $\lambda :(G , \tau) \rightarrow (H ,\sigma)$ is said to be perfect mapping (resp., ω-perfect mapping, θ-ω-perfect mapping, almost ω-perfect mapping, weakly θ-ω-perfect mapping, almost weakly ω-perfect mapping, θ-perfect mapping) if it is continuous (resp., ω-continuous, θ-ω-continuous, almost ω-continuous, weakly θ -ω-continuous, almost weakly ω-continuous, θ-continuous), closed, and, for every $h \in H$, $\lambda^{-1}(h)$. compact. The relationships among the weakly ω-perfect mappings are given by the following figure:



In the figure above, the converses are not true, as demonstrated by the following examples.

Example 2.3. Let $\lambda : (G , \tau) \rightarrow (G , \tau)$ be a mapping such that $G = \{ K, L, M \}$, and $\tau = \{\varphi, G, \{K\},\{L\},\{K , L\}\}$ such that $\lambda(K) = \lambda(L) = \lambda(M) = M$. Then λ is θ-ω-perfect mapping but it is not almost ω-perfect mapping.

Example 2.4. Let λ: $(\Re, \tau) \rightarrow (H, \sigma)$ be a mapping such that $\Re$ be a real line with topology $\tau = \{\varphi, \Re, (0, 1)\}$. Let $H = \{u, v, w\}$ and $\sigma = \{H, \varphi, \{v\}, \{w\}, \{v, w\}\}$.

$$\lambda(g) = \begin{cases} u & ,\text{if } g \in [0,2] \\ v & ,\text{if } g \notin [0,2] \end{cases}$$

Then, λ is weakly θ-ω-perfect but it is not θ-ω-perfect.

Example 2.5. As in example 2.4, λ is weakly θ-ω-perfect mapping, but it is not ω-perfect mapping. Also, λ is weakly θ-ω-perfect mapping, but not almost ω-perfect mapping, and λ is almost weakly ω-perfect mapping, but it is not θ-ω-perfect mapping.

Example 2.6. Let $\lambda : (G, \tau) \rightarrow (G, \sigma)$ be a mapping such that $G = \{u, v, w\}$, and $\tau = \{G, \varphi, \{u, v\}\}$ and $\sigma = \{G, \varphi, \{v, w\}\}$, such that $\lambda(u) = \lambda(v) = \lambda(w) = u$. Then λ is almost ω-perfect mapping but it is not ω-perfect mapping.

Example 2.7. Let A be the upper half of a plane and B be the X-axis. Let $X = A \cup B$. If τ_{hdis} be the half disc topology on X and τ_r be the relative topology that X inherits by virtue of being a subspace of $\Re^2$. Then, the identity of the mapping $\lambda : (X, \tau_r) \rightarrow (X, \tau_{hdis})$ is that it is an almost weakly ω-perfect mapping but it is not ω-perfect mapping.

Example 2.8. Let $\lambda : (G, \tau) \rightarrow (G, \tau)$ be a mapping such that $G = \{K, L, M\}$ and $\tau = \{\varphi, G, \{K\}, \{L\}, \{K, L\}\}$, such that $\lambda(K) = \lambda(L) = \lambda(M) = M$. Then λ is almost weakly ω-perfect mapping but it is not almost ω-perfect mapping.

Lemma 2.9. [13] A topological space G is ω-regular (resp., $\omega*$-regular) if and only if, for all $S \in \omega O(G)$ (resp., $S \in O(G)$) and all point $g \in S$, there is $T \in \omega O(G, g)$; $g \in T \subseteq \omega cl(T) \subseteq S$.

Theorem 2.10. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that G be an ω-regular space. If λ is almost weakly ω-perfect mapping then it is θ-ω-perfect mapping.

Proof: Assume that λ is almost weakly ω-perfect mapping. It suffices to be demonstrated that λ is θ-ω-continuous, let $g \in G$ and T be an open set containment λ (g) in H. Because λ is almost weakly ω-continuous, there is an ω-open set S containment g, such that $\lambda(S) \subseteq cl(T)$. Since G is an ω-regular space, by Lemma 2.9, there is $W \in \omega O(G, g)$ such that $g \in W \subseteq \omega cl(W) \subseteq S$. Therefore, $\lambda(\omega cl(W)) \subseteq cl(T)$. Then λ is θ-ω-continuous, so λ is θ-ω-perfect mapping.

Corollary 2.11. Let (G, τ) be ω-regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is almost weakly ω-perfect if and only if it is θ- ω-perfect.

Theorem 2.12. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be an ω-perfect mapping, and let $\mu : (H, \sigma) \rightarrow (I, \psi)$ be almost weakly ω-perfect. Then $\mu o \lambda : (G, \tau) \rightarrow (I, \psi)$ is almost weakly ω-perfect.

Proof: Assume that $g \in G$ and W is an open set containment $(\mu o \lambda)_{(g)}$ in I. Since μ is almost weakly ω-continuous, there is an open set T containment λ (g) in H such that $\mu(T) \subseteq cl(W)$. Since λ is ω-continuous, then for each $g \in G$ and each open set T of $\lambda(g) = h$, there is an ω-open S of g in G such that $\lambda(S) \subseteq T$, so $\mu(\lambda(S)) \subseteq \mu(T)$ also $(\mu o \lambda)_{(s)} \subseteq \mu(T)$, then $(\mu o \lambda)_{(s)} \subseteq cl(W)$. Also $\mu o \lambda$ is almost weakly ω-continuous. Hence, $\mu o \lambda$ is almost weakly ω-perfect mapping.

Theorem 2.13. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping, such that G be an ω-regular space. If λ is weakly θ-ω-perfect mapping then it is ω-perfect mapping.

Proof : Let λ be a weakly θ-ω-perfect mapping. It suffices to be demonstrated that λ is ω-continuous, let $g \in G$ and T be an open set containment λ (g) in H. Since H is an ω-regular space, yond is an open set $T1$ in H such that $\lambda(g) \in T1$ and $cl(T1) \subseteq T$. Since λ is weakly θ-ω-continuous, there is an ω-open set S containment g, such that $\lambda(S) \subseteq cl(T1)$. It follows that $\lambda(S) \subseteq T$, therefore λ is ω-continuous. Henceλ is ω-perfect mapping.

Corollary 2.14. Let (G, τ) be ω-regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is weakly θ-ω-perfect if and only if it is ω-perfect.

Theorem 2.15. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping, such that G be an ω-regular space. If λ is θ-ω-perfect mapping then it is almost ω-perfect mapping.

Proof: Let λ be a θ-ω-perfect mapping. It suffices to be demonstrated that λ is almost ω-continuous, let $g \in G$ and T be an open set containing λ (g) in H. Because λ is θ-ω-continuous, yond is an ω-open set S containing g, such that $\lambda(\omega cl(S)) \subseteq cl(T))$. Because $int(cl(T)) \subseteq cl(T)$, then $\lambda(\omega cl(S)) \subseteq int(cl(T)) \subseteq cl(T)$, then $\lambda(\omega cl(S)) \subseteq cl(T)$. Also G is ω-regular space, and there is an ω-open set $S1$ in

G, such that g $\in$ *S1* and cl(*S1*) $\subseteq$ *S*, so λ(ωcl(S1)) $\subseteq$ λ(S) and int(cl(*T*)) $\subseteq$ cl(*T*). It follows that λ (S) $\subseteq$ int(cl(*T*)). So λ is almost *ω*-continuous. Hence, consider that λ is almost *ω*-perfect mapping.

Corollary 2.16. Let (G , τ) be a *ω*-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is *θ*-*ω*-perfect if and only if it is almost *ω*-perfect.

Theorem 2.17. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that *H* be an *ω*-regular space. If λ is almost weakly *ω*-perfect mapping on *G*, then it is *ω*-perfect mapping on *G*.

Proof : Let λ be almost weakly *ω*-perfect mapping. It suffices to be demonstrated that λ is *ω*-continuous, let g $\in$ *G* and *T* be an open set containing λ (g) in *H*. Since *H* is an *ω*-regular space, there is an open set *T1* in *H* such that λ (g) $\in$*T1* and cl(*T1*) $\subseteq$ *T*. Since λ is almost weakly *ω*-continuous, there is an *ω*-open set *S* containment g , such that λ (S) $\subseteq$ cl(*T1*). It follows that λ (S) $\subseteq$ *T* , therefore λ is *ω*-continuous on *G*. Henceλ is *ω*-perfect mapping on *G*.

Corollary 2.18. Let (H , τ) be *ω*-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is almost weakly *ω*-perfect if and only if it is *ω*-perfect.

Theorem 2.19. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that *G* be an *ω*-regular space. If λ is weakly *θ*-*ω*-perfect mapping then it is *θ*-*ω*-perfect mapping.

Proof : Let λ be weakly *θ*-*ω*-perfect mapping. It suffices to be demonstrated that λ is *θ*-*ω*-continuous, let g $\in$ *G* and *T* be an open set containment λ (g) in *H*. Since *G* is an *ω*-regular space, there is an open set *T1* in *H* such that λ (g) $\in$*T1* and cl(*S1*) $\subseteq$ *S*. Since λ is weakly *θ*-*ω*-continuous, there is an *ω*-open set *S* containment *g*, such that λ (*S*) $\subseteq$ cl(*T*). It follows that λ(*ω*cl(S)) $\subseteq$ cl(*T*), therefore λ is *θ*-*ω*-continuous. Hence, λ is *θ*-*ω*-perfect mapping.

Corollary 2.20. Let (G , τ) be *ω*-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is weakly *θ*-*ω*-perfect if and only if it is *θ*-*ω*-perfect.

Theorem 2.21. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that *H* be an *ω*-regular space. If λ is almost *ω*-perfect mapping then it is *ω*-perfect mapping.

Proof: Let λ be an almost *ω*-perfect mapping. It suffices to be demonstrated that λ is *ω*-continuous, let g $\in$ *G* and *T* be an open set containment λ (g) in *H*. Because λ is almost *ω*-continuous, there is an *ω*-open set *S* containment g , such that λ (*S*) $\subseteq$ int(cl(*T*)). Because int(cl(*T*)) $\subseteq$ cl(*T*), then λ (*S*) $\subseteq$ int(cl(*T*)) $\subseteq$ cl(*T*). Then λ (*S*) $\subseteq$ cl(*T*), *H* is *ω*-regular space, and there is an *ω*-open set *S1* in *G*, such that g $\in$ *S1* and cl(*T1*) $\subseteq$ *T*, so λ(S) $\subseteq$ cl(*T1*) $\subseteq$ *T* . It follows that λ (S) $\subseteq$ *T*. So λ is *ω*-continuous. Hence, consider that λ is *ω*-perfect mapping.

Corollary 2.22. Let (G , τ) be *ω*-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is almost *ω*-perfect if and only if it is almost *ω*-perfect.

Theorem 2.23. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that *H* be an *ω*-regular space. If λ is weakly *θ*-*ω*-perfect mapping then it is almost *ω*-perfect mapping.

Proof : Let λ be weakly *θ*-*ω*-perfect mapping. It suffices to be demonstrated that λ is almost *ω*-continuous, let g $\in$ *G* and *T* be an open set containment λ (g) in *H*. Since *H* is an *ω*-regular space then it is an open set *T1* in *H* such that λ (g) $\in$*T1* and cl(*T1*) $\subseteq$ *T*. Since λ is weakly*θ*-*ω*-continuous, there is an *ω*-open set *S* containment g, such that λ(S) $\subseteq$ cl(*T1*). Also, int(cl(*T*)) $\subseteq$ cl(*T*). It follows that λ (S) $\subseteq$ int(cl(*T*)) $\subseteq$ cl(*T*) , therefore λ (S) $\subseteq$ int(cl(*T*)). So λ is almost *ω*-continuous on *G*. Hence λ is almost *ω*-perfect mapping on *G*.

Corollary 2.24. Let (H , τ) be *ω*-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is weakly *θ*-*ω*-perfect if and only if it is almost *ω*-perfect.

Theorem 2.25. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping and $\mu : G \to G \times H$ be the graph mapping of λ defined by $\mu (g) = (g, \lambda(g))$ for every $g \in G$. Then μ is *θ*-*ω*-perfect if and only if λ is *θ*-*ω*-perfect.

Proof : Necessity. Assume that μ is *θ*-*ω*-perfect mapping. It suffices to be demonstrated that λ is *θ*-*ω*-continuous, let g $\in$ *G* and *T* be an open set containment λ (g). Then $G \times T$ is an open set of $G \times H$ containment μ (g). Because μ is *θ*-*ω*-continuous, there is $S \in \omega O(G, g)$ such that μ (*ω*cl(*S*)) $\subseteq$ cl($G \times T$) = $G \times$ cl(*T*). Therefore, λ(*ω*cl(S)) $\subseteq$ cl(*T*), then λ is *θ*-*ω*-continuous. So λ is *θ*-*ω*-perfect mapping.

Sufficiency. Assume that λ is θ-ω-perfect mapping. It suffices to be demonstrated that λ is θ-ω-continuous, let g $\in G$ and W be an open set of $G \times H$ containment μ (g). There are the open sets $S1 \subseteq G$ and $T \subseteq H$ such that $\mu(g) = (g, \lambda(g)) \in S1 \times T \subseteq W$. Because λ is θ-ω-continuous, there is $S2 \in \omega O(G, g)$ such that $\lambda(\omega\text{cl}(S2)) \subseteq \text{cl}(T)$. Assume that $S = S1 \cap S2$, then $S \in \omega O(G, g)$. Therefore, $\mu(\omega\text{cl}(S)) \subseteq \text{cl}(S1) \times \lambda(\omega\text{cl}(S2)) \subseteq \text{cl}(S1) \times \text{cl}(T) \subseteq \text{cl}(W)$. Then μ is θ-ω-continuous. So μ is θ-ω-perfect mapping.

Theorem 2.26. For a mapping $\lambda : G \rightarrow H$ and H is regular, the following properties are equivalent.

(a) λ is weakly θ-ω-perfect.
(b) λ is ω-perfect.
(c) λ is almost ω-perfect.
(d) λ is θ-ω-perfect .
(e) λ is almost ω-perfect.

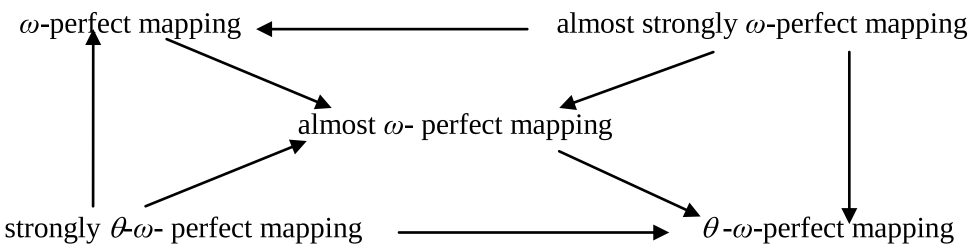
3. Strongly θ-ω-Perfect Mappings

In this section we study the strongly θ-ω-perfect mappings and some of their theorems.

Definition 3.1. A mapping λ: $(G, \tau) \rightarrow (H, \sigma)$ is said to be almost strongly ω-continuous if, for each $g \in G$ and each regular open set T of H containing $\lambda(g)$, there exists an ω-open subset S in G, such that $\lambda(\text{cl}(S)) \subseteq T$.

Definition 3.2. A mapping λ: $(G, \tau) \rightarrow (H, \sigma)$ is said to be strongly θ-ω-perfect mapping (resp., almost strongly ω-perfect mapping) if it is strongly θ-ω-continuous (resp., almost strongly ω-continuous), closed, and, for every $h \in H$, $\lambda^{-1}(h)$,compact.

The relationships among the strongly ω-perfect mappings are given by the following figure:



In the figure above, the converses are not to be right, as demonstrated by the following examples*:*

Example 3.3. Let $\lambda : (G, \tau) \rightarrow (G, \tau)$ be a mapping such that $G = \{K, L, M\}$, and $\tau = \{\varphi, G, \{K\},\{L\},\{K, L\}\}$such that $\lambda(K) = \lambda(L) = K$, $\lambda(M) = M$. Then λ is ω-perfect mapping but is not strongly θ-ω-perfect mapping.

Theorem 3.4. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that H be an regular space. If λ is ω-perfect mapping then it is strongly θ-ω-perfect mapping.

Proof : Let λ be an ω-perfect mapping. It suffices to demonstrate that λ is strongly θ-ω-continuous. Let g $\in G$ and T be an open set containment λ (g) in H. Because of H is an regular space, there is an open set W such that $\lambda(g) \in W \subseteq \text{cl}(W) \subseteq T$. Since λ is ω-continuous, then, $\lambda^{-1}(W)$ is an ω-open set and $\lambda^{-1}(\text{cl}(W))$ is an ω-closed. Assume that $S = \lambda^{-1}(W)$, then $g \in \lambda^{-1}(W) \subseteq \lambda^{-1}(\text{cl}(W))$, $S \in \omega\text{O}(G, g)$ and $\omega\text{cl}(S) \subseteq \lambda^{-1}(\text{cl}(W))$. We have $\lambda(\omega\text{cl}(S)) \subseteq \text{cl}(W) \subseteq T$, therefore λ is strongly θ-ω-continuous. Hence λ is strongly θ-ω-perfect mapping.

Corollary 3.5. Let (H, τ) be regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is ω-perfect if and only if it is strongly θ-ω-perfect.

Example 3.6. Let $\lambda : (\Re, \tau) \rightarrow (\Re, \tau)$ be a mapping where $\lambda(g) = g$, and let $(\Re, \tau)$ where τ is the topology with a basis whose members are of the form (a, b) and (a, b) -N such that $N = \{1\backslash n ; n \in Z^+\}$. Then $(\Re, \tau)$ is a Hausdorff but not ω-regular. Then λ is ω-perfect but not almost strongly ω-perfect mapping.

Theorem 3.7. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that G be an ω-regular space. If λ is ω-perfect mapping then it is almost strongly ω-perfect mapping.

Proof : Let λ be an ω-perfect mapping. It suffices to demonstrate that λ is almost strongly ω-continuous. Let g $\in G$ and T be an open set containment λ (g) in H. Since λ is ω-continuous, there is an ω-open set S containment g in G such that $\lambda(S) \subseteq T$ and $T \subseteq$ cl(T), then $\lambda(S) \subseteq$ cl(T). Since G is ω-regular, there is an ω-open set $S1$ in G such that $g \in S1$ and cl($S1$) $\subseteq S$, so λ(cl($S1$)) $\subseteq \lambda(S)$, $\lambda(S) \subseteq$ cl(T) and int(cl((T)) $\subseteq$ cl(T) . It follows that λ(cl($S1$)) $\subseteq$ int(cl((T)), therefore λ is almost strongly ω-continuous. Hence λ is almost strongly ω-perfect mapping.

Corollary 3.8. Let (G , τ) be ω-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is ω-perfect if and only if it is almost strongly ω-perfect.

Example 3.9. Let $G = \{ u ,v, w \}$ and $\lambda :(G , \tau) \to (G, \sigma)$, such that $\tau = \{G , \varphi , \{u, v\} \}$, $\sigma =\{G , \varphi , \{v, w\}\}$, and $\lambda (u) = \lambda (w) = w$, $\lambda (v) = v$. Then λ is θ-ω-perfect mapping but not strongly θ-ω-perfect mapping.

Theorem 3.10. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that H be an regular space. If λ is θ-ω-perfect mapping then it is strongly θ-ω-perfect mapping.

Proof : Let λ be an θ-ω-perfect mapping. It suffices to demonstrate that λ is a strongly θ-ω-continuous, let g $\in G$ also T be an open set containment λ (g) in H. Since λ is θ-ω-continuous, there is an ω-open set S containment g in G such that λ(ωcl(S)) $\subseteq$ cl(T). Since H is regular, there is an open set W such that λ (g) $\in W \subseteq$ cl(W) $\subseteq T$, then $\lambda(\omega$cl(S)) $\subseteq$ cl(W) $\subseteq T$, therefore λ(cl(S)) $\subseteq T$. So λ is strongly θ-ω-continuous. Hence λ is strongly θ-ω-perfect mapping.

Corollary 3.11. Let (H , τ) be regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is θ-ω-perfect if and only if it is strongly θ-ω-perfect.

Theorem 3.12. A space G is ω∗-regular if and only if, for any space H, any perfect mapping $\lambda : (G , \tau) \to (H , \sigma)$ is strongly θ-ω-perfect.

Proof : Sufficiency. Let $\lambda : G \to G$ be the identity mapping. Then λ is continuous and strongly θ-ω-continuous by our hypothesis. For any open set S of G and for any point g of S, we have $\lambda(g) = g \in S$. Also, there is $T\in \omega$O(G, g) such that $\lambda(\omega$cl(T)) $\subseteq S$, therefore $g \in T \subseteq \omega$cl(T) $\subseteq S$. It follows from Lemma 2.9 that G is ω∗-regular.

Necessity. Assume that $\lambda : G \to H$ is continuous and G is ω∗-regular. For any g $\in G$ and any open neighborhood T of λ (g), $\lambda^{-1}(T)$ is an open set of G containing g. Since G is ω∗-regular, there is $S \in \omega$O(G) such that $g \in S \subseteq \omega$cl(S) $\subseteq \lambda^{-1}(T)$ by Lemma 2.9. Therefore, $\lambda(\omega$cl(S)) $\subseteq T$. Hence λ is strongly θ-ω-perfect.

Example 3.13. Let $\lambda : (G , \tau) \to (G , \tau)$ be a mapping such that $G = \{ K, L, M \}$ and $\tau = \{\varphi, G, \{K\},\{L\},\{K , L\}\}$, such that $\lambda(K) = \lambda(L) = \lambda(M) = M$. Then λ is θ-ω-perfect mapping, but not almost strongly ω-perfect mapping.

Theorem 3.14. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that H be an ω-regular space. If λ is θ-ω-perfect mapping then it is almost strongly ω-perfect mapping.

Proof : Let λ be an θ-ω-perfect mapping. It suffices to demonstrate that λ is almost strongly ω-continuous, let g $\in G$ and T be an open set containment λ (g) in H. Since λ is θ-ω-continuous, there is an ω-open set S containment g in G such that λ(ωcl(S)) $\subseteq$ cl(T). Since H is an ω-regular, there is an ω-open set $T1$ in H such that λ (g) $\in T1$, also cl($T1$) $\subseteq T$ and int(cl($T1$) $\subseteq$ cl($T1$). It follows that λ(cl(S)) $\subseteq$ int(cl($T1$), therefore λ is almost strongly ω-continuous. So λ is almost strongly ω-perfect mapping.

Corollary 3.15. Let (G , τ) be ω-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is ω-perfect if and only if it is almost strongly ω-perfect.

Example 3.16. Let $\lambda :(G , \tau) \to (H, \sigma)$ such that $G = \{ u ,v, w \}$, $H =\{ a ,b, c \}$, $\tau = \{G , \varphi , \{u\},\{v\},\{u, v\} \}$ and $\sigma =\{H , \varphi , \{a\},\{b\},\{c\},\{a, b\}, \{a, c\},\{b, c\} \}$, such that $\lambda (u) = b, \lambda (v) = \lambda (w) = a$. Then λ is almost ω-perfect mapping, but not almost strongly ω-perfect mapping.

Theorem 3.17. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that G be an ω-regular space. If λ is almost ω-perfect mapping then it is almost strongly ω-perfect mapping.

Proof : Let λ be almost ω-perfect mapping. It suffices to demonstrate that λ is almost strongly ω-continuous, let g $\in G$ and T be an open set containment λ (g) in H. Since λ is almost ω-continuous, there is an ω-open set S containment g in G such that $\lambda(S) \subseteq$ int(cl(T)). Since G is ω-regular, there is an ω-open set $S1$ in G such that $g \in S1$**,** also cl($S1$) $\subseteq S$, so λ(cl($S1$)) $\subseteq \lambda(S)$, then λ(cl($S1$)) $\subseteq \lambda(S) \subseteq$

int(cl(T)). It follows that λ(cl($S1$)) ⊆ int(cl(T)), therefore λ is almost strongly ω-continuous. So λ is almost strongly ω-perfect mapping.

Corollary 3.18. Let (G , τ) be ω-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is almost ω-perfect if and only if it is almost strongly ω-perfect.

Lemma 3.19. Let a mapping $\lambda : G \to H$ be strongly θ-ω-perfect and $\mu : H \to$ L be perfect. Then $\mu o \lambda$ is strongly θ-ω-perfect.

Theorem 3.20. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping and $\mu : G \to G \times H$ the graph mapping of λ defined by $\mu(g) = (g, \lambda(g))$ for each $g \in G$. Then $\mu : G \to G \times H$ is strongly θ-ω-perfect if and only if $\lambda : (G , \tau) \to (H , \sigma)$ is strongly θ-ω-perfect and G is an ω-regular.

Proof : By Lemma 3.19, λ is strongly θ-ω-perfect if the graph mapping μ is strongly θ-ω-perfect. Also it follows that G is regular. To prove the converse, assume that λ is strongly θ-ω-perfect. Let g $\in G$ and W be an open set of $G \times H$ containment μ (g). There are the open sets $S1 \subseteq G$ and $T \subseteq H$ such that $\mu(g) = (g, \lambda(g)) \in S1 \times T \subseteq W$. Since λ is strongly θ-ω-continuous, there is $S2 \in \omega O(G, g)$ such that $\lambda(\omega$cl$(S2)) \subseteq T$. Because G is an ω-regular and $S1 \cap S2 \in \omega O(G, g)$, there is $S \in \omega O(G, g)$ such that g $\in S \subseteq \omega$cl$(S) \subseteq S1 \cap S2$ (by Lemma 2.9). Therefore, $\mu(\omega$cl$(S)) \subseteq S1 \times \lambda(\omega$cl$(S2)) \subseteq S1 \times T \subseteq W$. Then μ is strongly θ-ω-continuous. So μ is strongly θ-ω-perfect mapping.

Example 3.21. Let $\lambda : (G , \tau) \to (H , \sigma)$, such that $G = H = \{u, v, w\}$ and $\tau = \{ \varphi, G, \{u\},\{v\}, \{u, v\} \}$, $\sigma = \{ \varphi, H, \{w\} \}$, defined by $\lambda(u) = \lambda(v) = \lambda(w) = w$ .Then λ is strongly θ-ω-perfect doesn't the mappings μ of the λ. Then, $\mu(g) = (g, \lambda(g))$, then it is not strongly θ-ω-perfect mapping at u and v .

Example 3.22. from in Example 3.9, λ is almost ω-perfect mapping, but not strongly θ-ω-perfect mapping.

Theorem 3.23. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that H be an ω-regular space. If λ is almost ω-perfect mapping then it is strongly θ-ω-perfect mapping.

Proof: Let λ be almost ω-perfect mapping. It suffices to demonstrate that λ is stronglyθ-ω-continuous, let g $\in G$ and T be an open set containment λ (g) in H. Since λ is almost ω-continuous, there is an ω-open set S containment g in G such that $\lambda(S) \subseteq$ int(cl(T)). Since G is ω-regular, there is an ω-open set $S1$ in G such that $g \in S1$ and cl$(S1) \subseteq S$. So λ(cl($S1$)) ⊆ $\lambda(S)$, also int(cl(T)) ⊆ cl(T). It follows that λ(cl($S1$)) ⊆ T, therefore λ is strongly θ-ω-continuous. So λ is strongly θ-ω-perfect mapping.

Corollary 3.24. Let (G , τ) be a ω-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is almost ω-perfect if and only if it is strongly θ-ω-perfect

Theorem 3.25. For a mapping $\lambda :(G, \tau) \to (H ,\sigma)$ and H is regular space, the following properties are equivalent :

(a) λ is almost strongly θ-ω-perfect.
(b) λ is ω-perfect.
(c) λ is almost ω-perfect.
(d) λ is θ-ω-perfect .

4. Relationship between Weak and Strong Forms of ω-Perfect Mappings

In this section, we study the relationship between weaklyθ-ω-perfect mappings and strongly θ-ω-perfect mappings and some theorems concerning them.

Definition 4.1. A mapping λ: $(G, \tau) \to (H ,\sigma)$ is said to be super (resp., weakly, strongly) ω-continuous if for each $g \in G$ and each open neighborhood (resp., open set) T of H containing $\lambda(g)$, there exists an ω-open neighborhood (resp., ω-open set) S of G, such that λ (int (cl(S)) $\subseteq T$ (resp., $\lambda(S)) \subseteq$ cl(T) , λ ((cl(S)) $\subseteq T$)) .

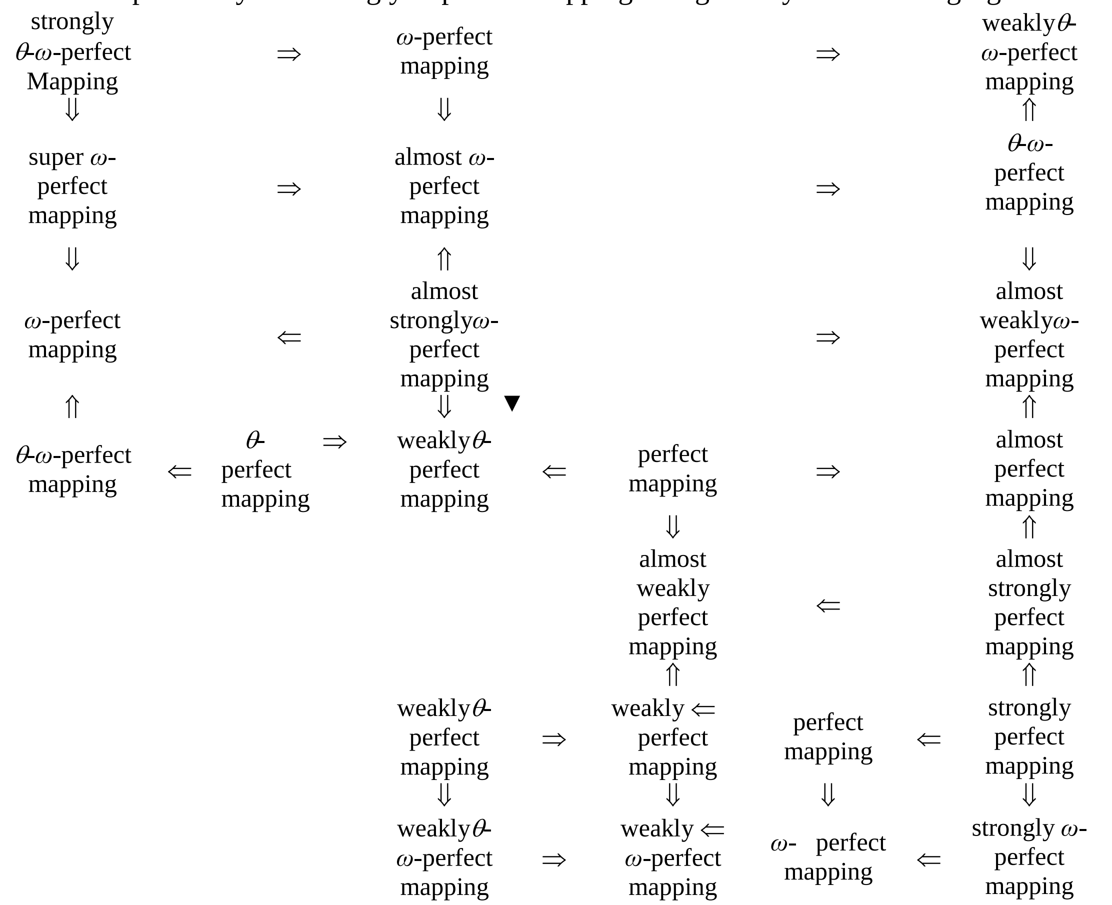
Definition 4.2. A mapping λ: $(G, \tau) \to (H ,\sigma)$ is said to be almost weakly (resp., almost strongly) continuous if for each $g \in G$ and each open (resp., regular open) set T of H containing $\lambda(g)$, there exists an open set S in G, such that $\lambda(S)) \subseteq$ cl(T) (resp., λ (cl(S) $\subseteq T$).

Definition 4.3. A mapping λ: $(G, \tau) \to (H ,\sigma)$ is said to be weakly θ-continuous if for each $g \in G$ and each open set T of H containing $\lambda(g)$, there exists an open set S in G, such that $\lambda(S)) \subseteq$ cl(T).

Definition 4.4. A mapping λ: $(G, \tau) \to (H ,\sigma)$ is called to be super ω-perfect mapping (resp., weakly ω-perfect mapping, strongly ω-perfect mapping , almost weakly perfect mapping, almost strongly perfect mapping, weakly θ-perfect mapping) if it is super ω-continuous (resp., weakly ω-continuous

,strongly ω-continuous , almost weakly continuous, almost strongly continuous, weakly θ-continuous), closed, and, for every $h \in H$, $\lambda^{-1}(h)$,compact.

The relationships weakly and strongly ω-perfect mappings are given by the following figure:



In the figure above, the converses are not to be right as demonstrated by the following examples*:*

Example 4.5. Let λ: $(G, \tau) \to (G, \tau)$, such that $G = \{ u, v, w \}$ and $\tau = \{ \varphi, G, \{u\},\{v\}, \{u, v\} \}$ defined by $\lambda(u) = u$, $\lambda(v) = v$, $\lambda(w) = w$. Then λ is super ω-perfect mapping but it is not strongly θ-ω-perfect mapping.

Theorem 4.6. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that G be a regular space. If λ is super ω-perfect mapping then it is strongly θ-ω-perfect mapping.

Proof: Let λ be a super ω-perfect mapping. It suffices to demonstrate that λ is strongly θ-ω-continuous, let g $\in G$ and T be an open set containment λ (g) in H. Because of λ is a super ω-continuous, there is a regular open set S containment g , such that $\lambda(S) \subseteq T$. Because int(cl(T)) $\subseteq$ cl(T), then $\lambda(S) \subseteq$ int(cl(T)) $\subseteq$ cl(T), then $\lambda(S) \subseteq$ cl(T). Also G is a regular space, there is an open set W such that g $\in W \subseteq$ cl(W) $\subseteq S$, so λ(cl(W)) $\subseteq T$. Therefore λ is strongly θ-ω-continuous. Hence consider that λ is strongly θ-ω-perfect mapping.

Corollary 4.7. Let (G , τ) be regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is super ω-perfect if and only if it is strongly θ-ω-perfect.

Example 4.8. Let λ: $(G, \tau) \to (H, \sigma)$ be a mapping, such that $G = \{ u, v, w \}$, $H = \{a, b\}$, and $\tau = \{ \varphi, G, \{u\},\{v\}, \{u, v\}, \{v, w\}\}$, $\sigma = \{H , \varphi, \{a\}\}$ defined by $\lambda(u) = \lambda(w) =$ b , $\lambda(v) = a$. Then, λ is ω-perfect but it is not super ω-perfect.

Theorem 4.9. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that *G* be a regular space. If λ is ω-perfect mapping then it is super ω-perfect mapping.

Proof: Let λ be ω-perfect mapping. It suffices to demonstrate that λ is super ω-continuous, let g $\in G$ and *T* be an open set containment λ (g) in *H*. Because of λ is ω-continuous, there is $S \in \omega O(G, g)$, such that $\lambda (S) \subseteq T$. Also, int(cl(*S*)) ⊆ cl(*S*), then λ (int(cl(*S*)) ⊆ λ (cl(*S*)). Also *G* is a regular space, there is an open set *S1* such that g ∈ *S1* ⊆ cl(*S1*) ⊆ *S*, so λ(int(cl(*S*)) ⊆ λ (cl(*S1*)) also λ (*S*) ⊆ *T*. So λ(int(cl(*S*)) ⊆ *T*, then λ is super ω-continuous. Hence consider that λ is super ω-perfect mapping.

Corollary 4.10. Let (G , τ) be regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is ω-perfect if and only if it is super ω-perfect.

Example 4.11. Let $\lambda : (\Re , \tau) \to (\Re , \tau)$ be a mapping, such that $\lambda(g) = g$, and let $(\Re , \tau)$ where τ is the topology with a basis whose members are of the form (a, b) and (a, b) -*N*, such that *N* = {1\n ; n $\in Z^+$ }. Then $(\Re , \tau)$ is a Hausdorff but not ω-regular. Then λ is perfect but it is not strongly perfect mapping.

Theorem 4.12. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that *G* be an regular space. If λ is perfect mapping then it is strongly perfect mapping.

Proof: Let λ be perfect mapping. It suffices to demonstrate that λ is strongly continuous, let g ∈ *G* and *T* be an open set containment λ (g) in *H*. Since λ is continuous, there is an open set *S* containment g in *G* such that $\lambda(S) \subseteq T$. Since *G* is regular space, there is an open set *S1* in *G* such that $g \in S1$ and cl(*S1*) ⊆ *S*, so λ(cl(*S1*)) ⊆ λ(*S*). Then λ(cl(*S1*)) ⊆ *T*, therefore λ is strongly continuous. So λ is strongly perfect mapping.

Corollary 4.13. Let (G , τ) be regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is perfect if and only if it is strongly perfect.

Theorem 4.14. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping such that *H* be a regular space. If λ is weakly perfect mapping then it is perfect mapping.

Proof: Let λ be weakly perfect mapping. It suffices to demonstrate that λ is continuous, let g ∈ *G* and *T* be an open set containment λ (g) in *H*. Since *H* is regular, there is an open set *T1* in *H* such that $\lambda(g) \in T1$ and cl(*T1*) ⊆ *T*. Since λ is weakly continuous, there is an open set *S* containment g in *G*, such that λ(*S*) ⊆ cl(*T1*), then λ(*S*) ⊆ *T*. It follows that λ is continuous. So λ is perfect mapping.

Corollary 4.15. Let (G , τ) be regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is weakly perfect if and only if it is perfect.

Example 4.16. A mapping $\lambda : (G , \tau) \to (H , \sigma)$ such that $G = \{u , v , w\}$, $H = \{a , b\}$, $\tau = \{G , \varphi , \{u\} , \{v\} , \{u , v\}, \{v , w\} \}$, $\sigma = \{ H , \varphi , \{a\}\}$, defined by $\lambda (u) = \lambda (v) = \lambda(w) = b$. The mapping λ is almost ω-perfect mapping but it is not super ω-perfect mapping.

Theorem 4.17. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that *G* and *H* are semi-regular spaces. If λ is almost ω-perfect mapping then it is super ω-perfect mapping.

Proof: Let λ be an almost ω-perfect mapping. It suffices to demonstrate that λ is super ω-continuous, let g ∈ *G* and let *T* be an open set containment λ (g) in *H*. Because of λ is almost ω-continuous, there is an ω-open set *S* containment g, for each regular open set *T* of *H* containment λ(g) such that λ (*S*) ⊆ *T*. So λ (*S*) ⊆ int(cl(*T*)). Because the space *G* is semi-regular space, there is an open set *S*1 in G such that $g \in S1$ and *T* ⊆ int(cl(*T*)) ⊆ *S*, so λ (*T*) ⊆ λ (int(cl(*T*))) ⊆ λ (*S*). Also λ (*S*) ⊆ int(cl(*T*)). Then λ (int(cl(*T*))) ⊆ λ (*S*) ⊆ int(cl(*T*)). Also the space *H* is semi-regular space, there is an open set *T*1 in *H* such that $\lambda (g) \in T1$, and *S* ⊆ int(cl(*S*)) ⊆ *T*, so λ (*S*) ⊆ λ (int(cl(*S*))) . It follows that λ (int(cl(*S*))) ⊆ *T*. Then λ is super ω-continuous. Hence λ is super ω- perfect mapping.

Corollary 4.18. Let (G , τ) and (H , σ) be semi-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is almost ω-perfect if and only if it is super ω-perfect.

Example 4.19. A mapping $\lambda : (G , \tau) \to (G , \tau)$ such that $G = \{ u,v, w \}$ $\tau = \{G , \varphi , \{u\} ,\{v\} , \{u, v \}\}$, $\lambda (u) = \lambda (v) = u$, and $\lambda (w) = w$, then λ is almost weakly perfect mapping but it is not almost strongly perfect mapping

Theorem 4.20. Let $\lambda : (G , \tau) \to (H , \sigma)$ be a mapping, such that *G* is a regular space. If λ is almost weakly perfect mapping then it is almost strongly perfect mapping.

Proof: Let λ be almost weakly perfect mapping. It suffices to demonstrate that λ is almost strongly continuous, let g ∈ *G* and let *T* be an open set containment λ (g) in *H*. Because of λ is almost weakly continuous and g $\in G$ for each open set T of H containment λ(g), there is an open set S containment g, such that $\lambda(S) \subseteq$ cl(T). Because the space G is a regular space, there is an open set $S1$ in G such that $g \in S1$ also cl($S1$) $\subseteq S$, so λ (cl($S1$)) $\subseteq \lambda(S)$. Also $\lambda(S) \subseteq$ cl(T). Then λ (cl($S1$)) $\subseteq$ cl(T) and int(cl($T1$)) $\subseteq$ cl($T1$). Then λ (cl($S1$)) $\subseteq$ int(cl($T1$)). It follows that λ is almost strongly continuous. Hence λ is almost strongly perfect mapping.

Corollary 4.21. Let (G, τ) and (H, σ) are regular spaces. The mapping $\lambda : (G, \tau) \to (H, \sigma)$ is almost weakly perfect if and only if it is almost strongly perfect.

Theorem 4.22. Let $\lambda : (G, \tau) \to (H, \sigma)$ and (H, σ) be regular spaces, then the following properties are equivalent :

(a) λ is strongly perfect.
(b) λ is perfect.
(c) λ is weakly perfect.

Theorem 4.23. Let $\lambda : (G, \tau) \to (H, \sigma)$ be a mapping with a regular space and $\mu : G \to G \times H$. where the λ defined by $\mu(g) = (g, \lambda(g))$ for each $g \in G$. If $\lambda : (G, \tau) \to (H, \sigma)$ is strongly perfect , then $\mu : G \to G \times H$ is strongly perfect.

Proof : Assume that λ is strongly perfect, let g $\in G$ and W be an open set of $G \times H$ containment μ(g). Yond represents open sets $S1 \subseteq G$ and $T \subseteq H$ such that $\mu(g) = (g, \lambda(g)) \in S1 \times T \subseteq W$. Since λ is strongly continuous and G is a regular space. an open set S containing g in G such that cl(S) $\subseteq S1$ and λ (cl(S) $\subseteq T$. Therefore μ (cl(S)) $\subseteq S1 \times T \subseteq W$, then μ is strongly continuous. So the mapping $\mu = id_x \Delta \lambda : G \to G \times H$ maps G homeomorphically onto the graph $\mu(g)$ which is a closed subset of $G \times H$. So μ is perfect, and because G is regular, then $G \times H$ is regular by theorem 4.22. Hence $\mu : G \to G \times H$ is strongly perfect.

Theorem 4.24. For a mapping $\lambda : (G, \tau) \to (H, \sigma)$ and since H is a regular space, the following properties are equivalent :

(a) λ is almost strongly θ-ω-perfect.
(b) λ is ω-perfect.
(c) λ is almost ω-perfect.
(d) λ is θ-ω-perfect .
(e) λ is almost weakly ω-perfect.

References

1. Formin, S. **1943**. ″ Extension of topological spaces,″ *Annals of Mathematics*. Second Series, **44**: 471-480, 1943.
2. Bourbaki, N. **1966**. *General Topology*, Part I, Addison-Wesly, Reding, Mass.
3. Velicko, N.V. **1968**. H-closed topological spaces, *Amer. Math. Soc. Transl*., **78**: 103-118. Current address: Selcuk University Faculty of Science and Arts Department of Mathematics.
4. Singal, M.K. and Singal, A.R. **1968**. Almost- continuous mappings,*Yokohama . Math. J*., **16**: 63-73.
5. Long, P.E. and Herrington, L.L. **1981**. Strongly θ-continuous functions, *J.of the Korean Math. Soc*., **18**(1): 21-28.
6. Hdeib, H.Z. **1989**. ″ ω-continuous functions″ *Dirasat*, **16**(2): 136-142.
7. Chew, J. and Tong, J. **1991**. Some Remarks on Weak continuity, *American Mathematical Monthly*, **98**: 931-934.
8. Noiri, T., Al-Omari, A. and Noorani, M.S.M. **2009**. "Weak forms of ω -open sets and decomposition of continuity ", *E.J.P.A.M*. **2**(1): 73-84.
9. Noiri, T. **1980**. On δ-continuous functions. *J. Korean Math. Soc*., **16**: 161-166.
10. Noiri, T. **1989**. ″ On almost continuous function,″ *Indian Journal of pure and Applied Mathematics*, **20**(6): 571-576.
11. AI-Omari, A. and Noorani, M.S.M. **2007**. ″ Contra-ω-continuous and almost contra-ω-continuous,″ *International Journal of Mathematics and Mathematical Sciences*, VoI. 2007, Article ID 40469 , 13 pages.

12. AI-Omari, A. and Noorani, M.S.M. **2007**. ″Regular generalized *ω*-closed sets,″*International Journal of Mathematics and Mathematical, Sciences*, VoI. Article ID 16292, 2007, 11 pages, 2007.
13. AI-Omari, A. and Noorani, M.S.M. 2009. ″Weak and Strong form of ω-continuous″*International Journal of Mathematics and Mathematical Sciences*,VoI. Article ID 174042, 12 pages.
14. N. Bourbaki, N. **1989**. "*Regular Space. " in Elements of Mathematics: General Topology*. Berlin: Springer- Verlag, pp. 80-81.
15. Stone, M.H. **1937**. Applications of the theory boolean rings to General Topology. *Trans. Am. Math, Soc*., **41**: 375-481.
16. Devi, R., Balachan dran, K. and Maki, H. **1995**. on Generalized α-continuous maps, *Far.East J. Math*., **16**: 35-48.