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Supra Rough Membership Relations and Supra Fuzzy Digraphs on Related Topologies

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Abstract
The primary aim of this paper is to present two various standpoints to define generalized membership relations, and state the implication between them, in order to categorize the digraphs and assist for their gauge exactness and roughness. In addition, we define several kinds of fuzzy digraphs.

Keywords: *J*-surely belongs, *J*-possibly belongs, *J*-rough membership, *J*-fuzzy digraphs

علاقات الانتماء الاستقرابية العليا والمخططات المتجهة الضبابية العليا على التبولوجيات المرتبطة

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الخلاصة
الهدف الأساسي من هذا البحث هو تقديم منظورين مختلفين لتعريف علاقات الانتماء المعممة، واعطاء العلاقات بينهما، لتصنيف المخططات المتجهة والمساعدة في قياس الدقة والتقريب للمخططات المتجهة. بالإضافة الى ذلك، عرفنا عدة انواع من المخططات المتجهة الفازية.

1. Introduction

The rough set theory is a major mathematical tool for approximation reasoning for decision support that was presented by Pawlak in 1982 [1].
The indescribability of objects is taken into account in this theory.
The fuzzy set theory appeared for the first time by Zadeh in 1965 [2]. There have been many fuzzy mathematics that were created and developed. The definition of the membership grade normally depends on concepts such as fuzzy equality, fuzzy set and fuzzy subset.
The rough set and fuzzy set theories are the two major artifacts utilized in the information systems to manage incomplete and confusing information. The two theories are connected, but they are also distinct [3 - 5]. We built some results in previous articles [6-15].

2. Preliminaries

We present the basic concepts that are useful throughout our paper in this section.

Definition 2.1. [3]. Let $D = (V(D), E(D))$ be a finite digraph. The *J*-degree of r, where $r \in V(D)$, for all $J \in \{O, I, \cap, \cup, <O>, <I>, <\cap>, <\cup>\}$ is defined by

(a) $O - D(r) = \{u \in V(D);\ (r, u) \in E(D)\}$,
(b) $I - D(r) = \{u \in V(D);\ (u, r) \in E(D)\}$,
(c) $\cap - D(r) = O - D(r) \cap I - D(r)$,
(d) $\cup - D(r) = O - D(r) \cup I - D(r)$,

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(e) $< O > -D(r) = \cap_{r \in I-D(r)} O - D(r)$,
(f) $< I > -D(r) = \cap_{r \in O-D(r)} I - D(r)$,
(g) $<\cap> -D(r) = < O > -D(r) \cap < I > -D(r)$,
(h) $<\cup> -D(r) = < O > -D(r) \cup < I > -D(r)$.

Definition 2.2 [3] Let $D = (V(D), E(D))$ be a finite digraph and θ_J: $V(D) \longrightarrow P(V(D))$ be a mapping which assigns for all $r \in V(D)$ its J-degree in $P(V(D))$. The pair (D, θ_J) is namable as a J-degree space (concisely *J-DS*).

Theorem 2.3 [3] If (D, θ_J) is a *J-DS*, then the a family $\tau_J = \{V(Q) \subseteq V(D)$, for each $r \in V(Q)$, $J - D(r) \subseteq V(Q)\}$,
for all $J \in \{O, I, \cap, \cup, <O>, <I>, <\cap>, <\cup>\}$ is a topology on D.

Definition 2.4 [3] Let (D, θ_J) be a *J-DS*. The subgraph $Q \subseteq D$ is called a J-open graph if $V(Q) \in \tau_J$. While the J-open graph supplement is named a J-closed graph. The family of every J-closed graph of a *J-DS* is definable by $\Gamma_J = \{V(K) \subseteq V(D), [V(K)]^c \in \tau_J\}$.

Definition 2.5 [3] Let (D, θ_J) be a *J-DS* and $Q \subseteq D$. The J-lower approximation of Q and the J-upper approximation of Q are defined consecutively by

$$L_J(Q) = \cup \{V(M) \in \tau_J: V(M) \subseteq V(Q)\} = J\text{-interior of } Q.$$
$$U_J(Q) = \cap \{V(M) \in \Gamma_J: V(Q) \subseteq V(M)\} = J\text{-closure of } Q.$$

Proposition 2.6 [3]. If (D, θ_J) is a *J-DS* and $M, Q \subseteq D$. Then

$(L1)$ $L_J(V(Q)) = [U_J(V(Q^c))]^c$	$(U1)$ $U_J(V(Q)) = [L_J(V(Q^c))]^c$
$(L2)$ $L_J(V(D)) = V(D)$, $L_J(\emptyset) = \emptyset$	$(U2)$ $U_J(V(D)) = V(D)$, $U_J(\emptyset) = \emptyset$
$(L3)$ If $V(M) \subseteq V(Q)$ then, $L_J(V(M)) \subseteq L_J(V(Q)$	$(U3)$ If $V(M) \subseteq V(Q)$ then, $U_J(V(M)) \subseteq U_J(V(Q))$
$(L4)$ $L_J(V(M) \cap V(Q)) = L_J(V(M)) \cap L_J(V(Q))$	$(U4)$ $U_J(V(M) \cap V(Q) \subseteq U_J(V(M)) \cap U_J(V(Q))$
$(L5)$ $L_J(V(M) \cup V(Q)) \supseteq L_J(V(M)) \cup L_J(V(Q))$	$(U5)$ $U_J(V(M) \cup V(Q)) = U_J(V(M)) \cup U_J(V(Q))$
$(L6)$ $L_J(V(Q)) \subseteq V(Q)$	$(U6)$ $V(Q) \subseteq U_J(V(Q))$
$(L7)$ $L_J(L_J(V(Q)) = L_J(V(Q))$	$(U7)$ $U_J(U_J(V(Q)) = U_J(V(Q))$

3. *J*- Rough Membership Relations, *J*-rough Membership Functions and Fuzzy Diagraphs.

In this section, we offer new types of rough membership relations, rough membership function and fuzzy digraphs. Also, we provide some properties about these concepts. In addition, we provide some solutions to adjust the rough digraphs approximations and accuracy.

Definition 3.1 Let (D, θ_J) be a *J-DS* and $Q \subseteq D$. We can say that

(a) r is a J-surely belongs to Q, (denoted by $r \underline{\in}_J V(Q)$, if $r \in L_J(V(Q))$.
(b) r is a J-possibly belongs to Q, (denoted by $r \overline{\in}_J V(Q)$, if $r \in U_J(V(Q))$.

These two membership relations are called "J-strong" and "J-weak" membership relations, respectively, for all $J \in \{O, I, <O>, <I>, \cup, \cap, <\cup>, <\cap>\}$.

Lemma 3.2 Let (D, θ_J) be a *J-DS* and $Q \subseteq D$. Then the next statements are generally satisfied

(a) If $r \underline{\in}_J V(Q)$, then $r \in V(Q)$.
(b) If $r \in V(Q)$, then $r \overline{\in}_J V(Q)$.

Proof (a) Since $r \underline{\in}_J V(Q)$, so $r \in L_J(V(Q))$, but $L_J(V(Q)) \subseteq V(Q)$, therefore $r \in V(Q)$.
(b) Since $r \in V(Q) \subseteq U_J(V(Q))$, therefore $r \overline{\in}_J V(Q)$.

Remark 3.3 In general, the converse of lemma 3.2 above is not valid, as demonstrated by the next example.

Example 3.4 Let (D, θ_J) be a *J-DS*, where $D = (V(D), E(D))$, $V(D) = \{r_1, r_2, r_3, r_4\}$ and $E(D) = \{(r_1, r_1), (r_2, r_2), (r_3, r_3), (r_3, r_2), (r_3, r_4), (r_4, r_1)\}$.

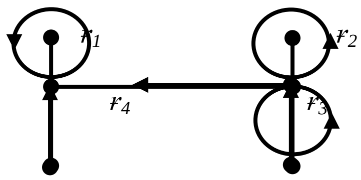


Figure 1- The digraph given in Example 3.4.

Then we get
τ_O = {$V(D)$, $\emptyset$, {r_1}, {r_2}, {r_1, r_2}, {r_1, r_4}, {r_1, r_2, r_4}}, Γ_O = {$V(D)$, $\emptyset$, {r_2, r_3, r_4}, {r_1, r_3, r_4}, {r_3, r_4}, {r_2, r_3}, {r_3}}.
We satisfy the above remark in case of $J = O$.
Suppose that $Q \subseteq D$ such that $Q = (V(Q), E(Q))$, $V(Q) =$ {r_1, r_2, r_3}, and $E(Q) =$ {(r_1, r_1), (r_2, r_2), (r_3, r_3), (r_3, r_2)}. Then we get
$L_O(V(Q)) =$ {r_1, r_2}, $U_O(V(Q)) = V(D)$, we have $r_3 \in V(Q)$ but r_3 is not a O-surely belong to Q since $r_3 \notin L_O(V(Q))$, also $r_4 \overline{\in}_O V(Q)$ since $r_4 \in U_O(V(Q))$ but $r_4 \notin V(Q)$.
Proposition 3.5 Let (D, θ_J) be a *J-DS* and $Q, K \subseteq D$. Then we can prove the following through the use of the *J*-approximation properties in [3].
(a) Let $Q \subseteq K$, if $r \underline{\in}_J V(Q)$, then $r \underline{\in}_J V(K)$, and if $r \overline{\in}_J V(Q)$, then $r \overline{\in}_J V(K)$.
(b) $r \overline{\in}_J (V(Q) \cup V(K))$ if and only if $r \overline{\in}_J V(Q)$ or $r \overline{\in}_J V(K)$),
(c) $r \overline{\in}_J \left(V(Q) \cap V(K)\right)$ if and only if $r \overline{\in}_J V(Q)$ and $r \overline{\in}_J V(K)$,
(d) If $r \underline{\in}_J V(Q)$ or $r \underline{\in}_J V(K)$, then $r \underline{\in}_J \left(V(Q) \cup V(K)\right)$.
(e) If $r \underline{\in}_J V(Q)$ and $r \underline{\in}_J V(K)$, then $r \underline{\in}_J \left(V(Q) \cap V(K)\right)$.
(f) $r \underline{\in}_J [V(Q)]^c$ if and only if non $r \overline{\in}_J V(Q)$,
(g) $r \overline{\in}_J [V(Q)]^c$ if and only if non $r \underline{\in}_J V(Q)$.
Proof (a) Let $r \underline{\in}_J V(Q)$, then by definition $r \in L_J(V(Q))$, since $Q \subseteq K$ then by Proposition (2.6) $L_J(V(Q)) \subseteq L_J(V(K))$ which implies that $r \in L_J(V(K))$ so, $r \underline{\in}_J V(K)$.
(b) Let $r \overline{\in}_J \left(V(Q) \cup V(K)\right)$, if and only if $r \in U_J(V(Q) \cup V(K))$ and $r \in U_J(V(Q))$, or $r \in U_J(V(K))$ if and only if $r \overline{\in}_J V(Q)$ or $r \overline{\in}_J V(K)$.
(c) Let $r \overline{\in}_J \left(V(Q) \cap V(K)\right)$, then $r \in U_J(V(Q) \cap V(K)) \subseteq U_J(V(Q)) \cap U_J(V(K))$ if and only if $r \overline{\in}_J V(Q)$ and $r \overline{\in}_J V(K)$.
(d) If $r \underline{\in}_J V(Q)$ or $r \underline{\in}_J V(K)$ then $r \in L_J(V(Q))$ or $r \in L_J(V(K))$, so $r \in L_J(V(Q) \cup V(K))$, therefore $r \underline{\in}_J (V(Q) \cup V(K))$.
(e) If $r \underline{\in}_J V(Q)$ and $r \underline{\in}_J V(K)$, then $r \in L_J(V(Q))$ and $r \in L_J(V(K))$, so $r \in L_J(V(Q) \cap V(K))$, therefore $r \underline{\in}_J \left(V(Q) \cap V(K)\right)$.
(f) Let $r \underline{\in}_J [V(Q)]^c$ if and only if $r \in L_J(V(Q)^c) = [U_J(V(Q))]^c$ if and only if $r \notin U_J(V(Q))$ if and only if non $r \underline{\in}_J V(Q)$.
(g) Let $r \overline{\in}_J [V(Q)]^c$ if and only if $r \in U_J(V(Q)^c) = [L_J(V(Q))]^c$ if and only if $r \notin L_J(V(Q))$ if and only if non $r \underline{\in}_J V(Q)$.
Remark 3.6 We will redefine the *J*-approximation in [3] by depending on $\underline{\in}_J$ and $\overline{\in}_J$ as subordinates, for any $Q, K \subseteq D$ $L_J(V(Q)) = \{r \in D; r \underline{\in}_J V(Q)\}$, $U_J(V(Q)) = \{r \in D; r \overline{\in}_J V(Q)\}$

The next proposition is very important and provides the relation between various kinds of *J*-rough membership relations $\underline{\in}_J$ and $\overline{\in}_J$. Accordingly, we will explain the importance of utilization of the various types of membership relationships.
Proposition 3.7 Let (D, θ_J) be *J-DS* and $Q \subseteq D$. Then
(a) If $r \underline{\in}_\cap V(Q)$ implies to $r \underline{\in}_O V(Q)$ implies to $r \underline{\in}_\cup V(Q)$
(b) If $r \underline{\in}_\cap V(Q)$ implies to $r \underline{\in}_I V(Q)$ implies to $r \underline{\in}_\cup V(Q)$
(c) If $r \overline{\in}_\cup V(Q)$ implies to $r \overline{\in}_O V(Q)$ implies to $r \overline{\in}_\cap V(Q)$

(d) If $r\ \overline{\in}_{\cup}\ V(Q)$ implies to $r\ \overline{\in}_{I}\ V(Q)$ implies to $r\ \overline{\in}_{\cap}\ V(Q)$
(e) If $r\ \underline{\in}_{<\cap>}\ V(Q)$ implies to $r\ \underline{\in}_{<O>}\ V(Q)$ implies to $r\ \underline{\in}_{<\cup>}\ V(Q)$
(f) If $r\ \underline{\in}_{<\cap>}\ V(Q)$ implies to $r\ \underline{\in}_{<I>}\ V(Q)$ implies to $r\ \underline{\in}_{<\cup>}\ V(Q)$
(g) If $r\ \overline{\in}_{<\cup>}\ V(Q)$ implies to $r\ \overline{\in}_{<O>}\ V(Q)$ implies to $r\ \overline{\in}_{<\cap>}\ V(Q)$
(h) If $r\ \overline{\in}_{<\cup>}\ V(Q)$ implies to $r\ \overline{\in}_{<I>}\ V(Q)$ implies to $r\ \overline{\in}_{<\cap>}\ V(Q)$

Proof. (a) Let $r\ \underline{\in}_{\cap}\ V(Q)$, so $r \in L_{\cap}\left(V(Q)\right)$, then $r \in L_{O}\left(V(Q)\right)$. Also, if $r\ \underline{\in}_{O}\ V(Q)$, so $r \in L_{\cup}\left(V(Q)\right)$, then $r \in L_{\cup}\left(V(Q)\right)$.

We can prove (b), (c), (d), (e), (f), (g) and (h) by the similar manner.

Remark 3.8 The converse of the precedent proposition is not true generally, as shown in the next example.

Example 3.9 Let (D, θ_J) be a *J-DS*, where $D = (V(D), E(D))$, $V(D) = \{r_1, r_2, r_3, r_4\}$, and $E(D) = \{(r_1, r_1), (r_1, r_2), (r_2, r_3), (r_2, r_4), (r_3, r_1), (r_4, r_1)\}$.

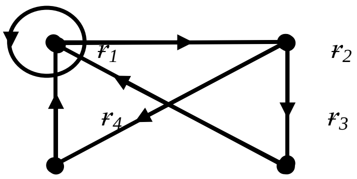
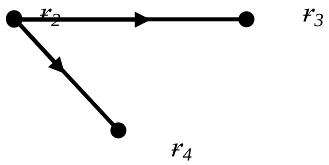


Figure 2- The digraph given in Example 3.9.

$O\text{-}D(r_1) = \{r_1, r_2\}$, $O\text{-}D(r_2) = \{r_3, r_4\}$, $O\text{-}D(r_3) = \{r_1\}$, $O\text{-}D(r_4) = \{r_1\}$.
$I\text{-}D(r_1) = \{r_1, r_3, r_4\}$, $I\text{-}D(r_2) = \{r_1\}$, $I\text{-}D(r_3) = \{r_2\}$, $I\text{-}D(r_4) = \{r_2\}$.
$\tau_{<O>} = \{V(D), \emptyset, \{r_1\}, \{r_1, r_2\}, \{r_3, r_4\}, \{r_1, r_3, r_4\}\}$, $\Gamma_{<O>} = \{V(D), \emptyset, \{r_2\}, \{r_1, r_2\}, \{r_3, r_4\}, \{r_2, r_3, r_4\}\}$.
$\tau_{<I>} = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_3, r_4\}\}$, $\Gamma_{<I>} = \{V(D), \emptyset, \{r_2\}, \{r_3, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}\}$.
$\tau_{<\cap>} = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_3, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}\}$, $\Gamma_{<\cap>} = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_3, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}\}$.
$\tau_{<\cup>} = \{V(D), \emptyset, \{r_1\}, \{r_1, r_2\}, \{r_1, r_3, r_4\}\}$, $\Gamma_{<\cup>} = \{V(D), \emptyset, \{r_2\}, \{r_3, r_4\}, \{r_2, r_3, r_4\}\}$.
Suppose that $Q \subseteq D$, where $Q = (V(Q), E(Q))$, $V(Q) = \{r_2, r_3, r_4\}$, $E(Q) = \{(r_2, r_3), (r_2, r_4)\}$. Thus we get



$L_{<O>}\left(V(Q)\right) = \{r_3, r_4\}$, $L_{<I>}\left(V(Q)\right) = \{r_2\}$, $L_{<\cap>}\left(V(Q)\right) = \{r_2, r_3, r_4\}$, $L_{<\cup>}\left(V(Q)\right) = \emptyset$.
So, $r_3\ \underline{\in}_{<O>}\ V(Q)$ and $r_2\ \underline{\in}_{<I>}\ V(Q)$ but $r_2\ \underline{\notin}_{<\cup>}\ V(Q)$ and $r_3\ \underline{\notin}_{<\cup>}\ V(Q)$, also $r_2\ \underline{\in}_{<\cap>}\ V(Q)$ and $r_3\ \underline{\in}_{\cap}\ V(Q)$, but $r_2\ \underline{\notin}_{O}\ V(Q)$ and $r_3\ \underline{\in}_{\cap}\ V(Q)$. By similar way, we can illustrate the other cases.

Definition 3.10 Let (D, θ_J) be a *J-DS* and $Q \subseteq D$. Then for all $J \in \{O, I, <O>, <I>, \cap, \cup, <\cap>, <\cup>\}$ and $Q \in D$. The *J*-rough membership functions of *J-DS* are defined as follows:
For subgraph Q, the *J*-rough membership functions on D are $\Omega_Q^J : D \rightarrow [0, 1]$,

$$\Omega_Q^J(r) = \frac{|\{J - D(r)\} \cap V(Q)|}{|\{J - D(r)\}|}$$

Where $|\{J - D(r)\}| \neq 0$ and $|A|$ denote the cardinality of A.

The *J*-rough membership function represents a conditional probability that Q includes r given that $E(D)$ and it can be interpreted as a degree that r belongs to Q in consideration of the information presented by $E(D)$ about r. Furthermore, in the situation of infinite digraph, the precedent membership function Ω_Q^J can be used for spaces, which have locally finite minimal degrees for every vertex.

Remark 3.11 To define the *J*-approximations of a digraph Q, the *J*-rough membership functions can be utilized as explicated below

$$L_J(V(Q)) = \{r \in D; \Omega_Q^J(r) = 1\}$$
$$U_J(V(Q)) = \{r \in D; \Omega_Q^J(r) > 0\}$$

The following implications indicate the essential properties of the *J*-rough membership functions referred to above.

Proposition 3.12. Let (D, θ_J) be a *J-DS* and $Q, K \subseteq D$. Then

(a) $\Omega_Q^J(r) = 1$ if and only if $r \underline{\in}_J (V(Q))$,

(b) $\Omega_Q^J(r) = 0$ if and only if $r \underline{\in}_J V(D) - V(Q)$,

(c) $0 < \Omega_Q^J(r) < 1$ if and only if $r \in B(Q)$,

(d) $\Omega_{D-Q}^J(r) = 1 - \Omega_Q^J(r)$ for all $r \in D$,

(e) $\Omega_{Q \cup K}^J(r) \geq \max\{\Omega_Q^J(r), \Omega_K^J(r)\}$ for all $r \in D$,

(f) $\Omega_{Q \cap K}^J(r) \leq \min\{\Omega_Q^J(r), \Omega_K^J(r)\}$ for all $r \in D$.

Proof. (a) Let $\Omega_H^J(r) = 1$ if and only if $r \in V(Q)$, $J - D(r) \subseteq V(Q)$ if and only if $r \in L_J(V(Q))$ then $r \underline{\in}_J V(Q)$.

(b) Let $\Omega_Q^J(r) = 0$ if and only if $J - D(r) \subseteq [V(Q)]^c$ if and only if $r \in L_J [V(Q)]^c$ then $r \underline{\in}_J [V(Q)]^c$.

(c) Let $0 < \Omega_Q^J(r) < 1$, implies to $\Omega_Q^J(r) > 0$ and $\Omega_Q^J(r) < 1$, so by Remark (3. 11) $r \in U_J(V(Q)$ and $r \notin L_J(V(Q))$, therefore $r \in U_J(V(Q)) - L_J(V(Q))$, which means $r \in B_J(V(Q)$. Conversely, $r \in B_J(V(Q))$, so $r \in U_J(V(Q)$ and $r \notin L_J(V(Q))$, therefore $\Omega_Q^J(r) > 0$ and $\Omega_Q^J(r) \neq 1$, so $0 < \Omega_Q^J(r) < 1$.

(d) $\Omega_{D-Q}^J(r) = \frac{|J-D(r) \cap V(D)-V(Q)|}{|J-D(r)|} = \frac{|J-D(r)|}{|J-D(r)|} - \frac{|J-D(r) \cap V(Q)|}{|J-D(r)|} = 1 - \Omega_Q^J(r)$.

(e) Since $|V(Q) \cup V(K)| \geq |V(Q)|$ then $|\{J - D(r)\} \cap V(Q) \cup V(K)| \geq |J - D(r) \cap V(Q)|$, so $\Omega_{Q \cup K}^J(r) \geq \Omega_Q^J(r)$, and by the same way we get $\Omega_{Q \cup K}^J(r) \geq \Omega_K^J(r)$, therefore $\Omega_{Q \cup K}^J(r) \geq \max\{\Omega_Q^J(r), \Omega_K^J(r)\}$.

(f) Since $|V(Q) \cap V(K)| \leq |V(Q)|$, then $|\{J - D(r)\} \cap V(Q) \cap V(K)| \leq |J - D(r) \cap V(Q)|$, so $\Omega_{Q \cap K}^J(r) \leq \Omega_Q^J(r)$, and by the same way we get $\Omega_{Q \cap K}^J(r) \leq \Omega_K^J(r)$, therefore $\Omega_{Q \leq K}^J(r) \leq \min\{\Omega_Q^J(r), \Omega_K^J(r)\}$.

Remark 3.13 The *J*- rough membership functions can be divided by the digraph *D* depending on the *J*-positive, *J*-negative and *J*-boundary areas of $Q \subseteq D$, consecutively, as in the following

$$POS_J(Q) = \{r \in D; \Omega_Q^J(r) = 1\}, NEG_J(Q) = \{r \in D; \Omega_Q^J(r) = 0\},$$
$$B_J = \{r \in D;\ 0 < \Omega_Q^J(r) < 1\},$$

Lemma 3.14 Let (D, θ_J) be a *J*-DS and $Q \subseteq D$. Then for each $r \in D$

(a) If $\Omega_Q^{\cup}(r) = 1$ implies to $\Omega_Q^{O}(r) = 1$ implies to $\Omega_Q^{\cap}(r) = 1$,

(b) If $\Omega_Q^{\cup}(r) = 1$ implies to $\Omega_Q^{I}(r) = 1$ implies to $\Omega_Q^{\cap}(r) = 1$,

(c) If $\Omega_Q^{<\cup>}(r) = 1$ implies to $\Omega_Q^{<O>}(r) = 1$ implies to $\Omega_Q^{<\cap>}(r) = 1$,

(d) If $Q(r) = 1$ implies to $\Omega_Q^{<I>}(r) = 1$ implies to $\Omega_Q^{<\cap>}(r) = 1$.

Proof. (a). If $\Omega_Q^{\cup}(r) = 1$, then $r \underline{\in}_{\cup} (V(Q))$, so $r \underline{\in}_O (V(Q))$, thus $\Omega_Q^O(r) = 1$. Also, if $\Omega_Q^O(r) = 1$, then $r \underline{\in}_O (V(Q))$, so $r \underline{\in}_{\cap} (V(Q))$, thus $\Omega_Q^{\cap}(r) = 1$.

Similarly, we can proof (b), (c) and (d).

Lemma 3.15 Let (D, θ_J) be a *J-DS* and $Q \subseteq D$. Then for each $r \in D$

(a) If $\Omega_Q^{\cup}(r) = 0$ implies to $\Omega_Q^{O}(r) = 0$ implies to $\Omega_Q^{\cap}(r) = 0$,

(b) If $\Omega_Q^{\cup}(r) = 0$ implies to $\Omega_Q^{I}(r) = 0$ implies to $\Omega_Q^{\cap}(r) = 0$,

(c) If $\Omega_Q^{<\cup>}(r) = 0$ implies to $\Omega_Q^{<O>}(r) = 0$ implies to $\Omega_Q^{<\cap>}(r) = 0$,

(d) If $\Omega_Q^{<\cup>}(r) = 0$ implies to $\Omega_Q^{<I>}(r) = 0$ implies to $\Omega_Q^{<\cap>}(r) = 0$.

Proof. (a) If $\Omega_Q^{\cup}(r) = 0$, then $\cup -D(r) \cap V(Q) = \emptyset$, so $O - D(r) \cap V(Q) = \emptyset$, thus $\Omega_Q^O(r) = 0$. Also, If $\Omega_Q^O(r) = 0$, then $O - D(r) \cap V(Q) = \emptyset$, so $\cap -D(r) \cap V(Q) = \emptyset$, thus $\Omega_Q^{\cap}(r) = 0$.

Similarly, we can proof (b), (c) and (d).

Remark 3.16

(a) We can prove that Ω_Q^I is more accurate than the other types depending on the above implication and by utilizing Proposition 3.7, this means that

(1) If $r \in V(Q)$ then $\Omega_Q^{\cup}(r) \leq \Omega_Q^{O}(r) \leq \Omega_Q^{\cap}(r)$ and if $r \in V(Q)$ then $\Omega_Q^{\cup}(r) \leq \Omega_Q^{I}(r) \leq \Omega_Q^{\cap}(r)$.

(2) If $r \notin V(Q)$ then $\Omega_Q^{\cap}(r) \leq \Omega_Q^{O}(r) \leq \Omega_Q^{\cup}(r)$ and if $r \notin V(Q)$ then $\Omega_Q^{\cap}(r) \leq \Omega_Q^{I}(r) \leq \Omega_Q^{\cup}(r)$.

(3) If $r \in V(Q)$ then $\Omega_Q^{<\cup>}(r) \leq \Omega_Q^{<O>}(r) \leq \Omega_Q^{<\cap>}(r)$ and if $r \in V(Q)$ then $\Omega_Q^{<\cup>}(r) \leq \Omega_Q^{<I>}(r) \leq \Omega_Q^{<\cap>}(r)$.

(4) If $r \notin V(Q)$ then $\Omega_Q^{<\cap>}(r) \leq \Omega_Q^{<O>}(r) \leq \Omega_Q^{<\cup>}(r)$ and if $r \notin V(Q)$ then $\Omega_Q^{<\cap>}(r) \leq \Omega_Q^{<I>}(r) \leq \Omega_Q^{<\cup>}(r)$.

(b) Generally, the converse of the above lemma is not true.

We will illustrate Remark 3.16 in the following example.

Example 3.17 According to Example 3.9, consider the subgraph $Q \subseteq D$, where $Q = (V(Q), E(Q))$, $V(Q) = \{r_2, r_3, r_4\}$, $E(Q) = \{(r_2, r_3), (r_2, r_4)\}$.

Then we get:

$\Omega_Q^{<O>} r_1 = 0, \Omega_Q^{<O>} r_2 = \frac{1}{2}, \Omega_Q^{<O>} r_3 = 1, \Omega_Q^{<O>} r_4 = 1.$

$\Omega_Q^{<I>} r_1 = 0, \Omega_Q^{<I>} r_2 = 1, \Omega_Q^{<I>} r_3 = \frac{2}{3}, \Omega_Q^{<I>} r_4 = \frac{2}{3}.$

$\Omega_Q^{<\cap>} r_1 = 0, \Omega_Q^{<\cap>} r_2 = 1, \Omega_Q^{<\cap>} r_3 = 1, \Omega_Q^{<\cap>} r_4 = 1.$

$\Omega_Q^{<\cup>} r_1 = 0, \Omega_Q^{<\cup>} r_2 = \frac{1}{2}, \Omega_Q^{<\cup>} r_3 = \frac{2}{3}, \Omega_Q^{<\cup>} r_4 = \frac{2}{3}.$

Definition 3.19 Let (D, θ_J) be *J-DS* and $Q \subseteq D$. Then for each $r \in D$. Then the *J*-fuzzy digraph in D is a digraph of order pairs:

$$\tilde{Q}_J = \{(r, \Omega_Q^J(r)); r \in D\}$$

Example 3. 20 According to Example 3.9, consider the sub digraph $Q \subseteq D$. Then we get

$\tilde{Q}_{<O>} = \{(r_1, 0), (r_2, \frac{1}{2}), (r_3, 1), (r_4, 1)\}$,

$\tilde{Q}_{<I>} = \{(r_1, 0), (r_2, 1), (r_3, \frac{2}{3}), (r_4, \frac{2}{3})\}$,

$\tilde{Q}_{<\cap>} = \{(r_1, 0), (r_2, 1), (r_3, 1), (r_4, 1)\}$,

$\tilde{Q}_{<\cup>} = \{(r_1, 0), (r_2, \frac{1}{2}), (r_3, \frac{2}{3}), (r_4, \frac{2}{3})\}$.

Conclusions

By using the *J*-rough membership functions, we defined the *J*-lower and *J*-upper approximations. Depending on these functions, the digraph *D* could be divided into three areas; *J*-positive, *J*-negative and *J*-boundary areas. Also by using *J*-rough membership functions, we introduced many kinds of fuzzy digraphs.

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