



A Type-2 Fuzzy Somewhere Dense Set In General Type-2 Fuzzy Topological Space

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Abstract

The multiplicity of connotations in any paper does not mean that there is no main objective for that paper and certainly one of these papers is our research the main objective is to introduce a new connotation which is type-2 fuzzy somewhere dense set in general type-2 fuzzy topological space and its relationship with open sets of the connotation type-2 fuzzy set in the same space topology and theories of this connotation.

Keywords: Type-2 Fuzzy Set, Type-2 Fuzzy Somewhere Dense Set, General Type-2 Fuzzy Topological Space, Type-2 Fuzzy Open Sets.

المجموعة الضبابية نوع-2 الكثيفة نوعا ما في الفضاء التوبولوجي الضبابي نوع-2 العام

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الخلاصة

تعدد المفاهيم في أي ورقة لا يعني أنه لا يوجد هدف رئيسي لتلك الورقة وبالتأكيد واحدة من هذه الأوراق هو بحثنا والهدف الرئيسي هو إدخال مفهوم جديد والتي هي المجموعة الضبابية نوع-2 الكثيفة نوعا ما في الفضاء الطوبولوجي الضبابي نوع-2 العام وعلاقتها مع مجموعات مفتوحة ضبابية نوع-2 في نفس الفضاء التوبولوجي ونظريات هذه المفاهيم.

Introduction

The ability to provide suitable solutions to mathematical problems, including the conversion of nonlinear equations into linear equations, has made type-2 fuzzy set a wide field of scientific studies of the branches of mathematics because its appearance in the hands of L. Zada in 1975 [1] was not surprising after he introduced in 1965 fuzzy set [2] type-1 fuzzy set (fuzzy set), but this discovery became a necessary need to complete modern scientific research and the entry of fuzzy logic in its formulation. In 1976, Mizumoto and Tanaka introduced the properties of type-2 fuzzy set and the various methods of finding union and intersection [3]. Mendel and Karnik then introduced new methods in the formation of operation of type-2 fuzzy set [4] and then Mendel introduced an important connotation which is Interval type-2 fuzzy set [5] to become the study of type-2 fuzzy set into two parts the first part is interval type-2 fuzzy set and the second part general type-2 fuzzy set all these studies opened the way to study the topology spaces of each part, Zhang introduced the concept of interval type-2 fuzzy topology space [6] and Hussan and AL-Khafaji general type-2 fuzzy topology space [7]. This paper complements the topology construction path by introducing the connotation of type-2 fuzzy somewhere dense set in the general type-2 fuzzy topology space and its relationship to some type-2 fuzzy open sets.

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1. Preliminaries

Everything in this section is an important preamble of the main definitions and features that reflect the nature of type-2 fuzzy set work [1], [3], and [5]. We assume that $X \neq \emptyset$ and $I = [0,1]$ be closed unit interval.

Definition 1.1

A type-2 fuzzy set, denoted by \tilde{A} characterized by a type-2 membership function, where $x \in X \neq \emptyset$ which is universal and $\tilde{u} \in J_x \subseteq [0,1]$

$$\tilde{A} = \{((x, \tilde{u}), \mu_{\tilde{A}}(x, \tilde{u})) : 0 \leq \mu_{\tilde{A}}(x, \tilde{u}) \leq 1, \text{ where } x \in X \neq \emptyset, \tilde{u} \in J_x\} \dots\dots\dots(1)$$

We can give a new wording to \tilde{A}

$$\tilde{A} = \sum_{x \in X} \sum_{\tilde{u} \in J_x} \mu_{\tilde{A}}(x, \tilde{u}) / (x, \tilde{u}) \Rightarrow \tilde{A} = \sum_{x \in X} \sum_{\tilde{u} \in J_x} \tilde{F}_x(\tilde{u}) / \tilde{u} / x, J_x \subseteq [0,1] \dots\dots\dots(2)$$

Where $\tilde{F}_x(\tilde{u}) = \mu_{\tilde{A}}(x, \tilde{u})$ and $\sum \sum$ denotes the union in discrete sets and \sum is replaced by \int is continuous universes are set.

The class of all type-2 fuzzy set of $x \in X \neq \emptyset$ denoted by $\tilde{F}_{T_2}(x)$.

Remark 1.2 [3], [5]:

A type-2 fuzzy universes set, denoted by \tilde{X} such that
$$\tilde{X} = \sum_{x \in X} \sum_{\tilde{u} \in [1,1]} 1 / \tilde{u} / x \dots\dots\dots(3)$$

A type-2 fuzzy empty set, denoted by $\tilde{\emptyset}$ such that
$$\tilde{\emptyset} = \sum_{x \in X} \sum_{\tilde{u} \in [0,0]} 1 / \tilde{u} / x \dots\dots\dots(4)$$

Definition 1.3 [5]:

When all the $\mu_{\tilde{A}}(x, \tilde{u}) = 1$ then type-2 fuzzy set is called interval type-2 fuzzy set.

Definition 1.3 [3]:

A normal type-2 fuzzy set \tilde{A} is one for which $\max_{x \in X} \mu_{\tilde{A}}(x, \tilde{u}) = 1$.

Operation of type-2 fuzzy set 1.3 [3]:

Consider \tilde{A} and \tilde{B} are two type-2 fuzzy sets and the membership grades of \tilde{A} and \tilde{B} respectively, we can be represented by

$$\mu_{\tilde{A}}(x) = \sum_{\tilde{u} \in J_x^{\tilde{u}}} \tilde{F}_x(\tilde{u}) / \tilde{u} \text{ and } \mu_{\tilde{B}}(x) = \sum_{\tilde{w} \in J_x^{\tilde{w}}} \tilde{K}_x(\tilde{w}) / \tilde{w} \text{ where } \tilde{F}_x(\tilde{u}), \tilde{K}_x(\tilde{w}) \in I = [0,1] \text{ and } x \in X \neq \emptyset.$$

The union of two type-2 fuzzy sets is defined as

$$\tilde{A} \cup \tilde{B} \Leftrightarrow \mu_{\tilde{A} \cup \tilde{B}}(x) = \sum_{\tilde{u} \in J_x^{\tilde{u}}} \sum_{\tilde{w} \in J_x^{\tilde{w}}} \tilde{F}_x(\tilde{u}) \wedge \tilde{K}_x(\tilde{w}) / \tilde{u} \vee \tilde{w} \dots\dots\dots(5)$$

The intersection of two type-2 fuzzy sets is defined as

$$\tilde{A} \cap \tilde{B} \Leftrightarrow \mu_{\tilde{A} \cap \tilde{B}}(x) = \sum_{\tilde{u} \in J_x^{\tilde{u}}} \sum_{\tilde{w} \in J_x^{\tilde{w}}} \tilde{F}_x(\tilde{u}) \wedge \tilde{K}_x(\tilde{w}) / \tilde{u} \wedge \tilde{w} \dots\dots\dots(6)$$

The containment type-2 fuzzy sets are defined as
$$\tilde{A} \subseteq \tilde{B} \Leftrightarrow \tilde{A} \cap \tilde{B} = \tilde{A} \dots\dots\dots(7)$$

The complement of type-2 fuzzy set defined as
$$\neg \tilde{A} = \sum_{\tilde{u} \in J_x^{\tilde{u}}} \mu_x(\tilde{u}) / (1 - \tilde{u}) \dots\dots\dots(8)$$

General type-2 fuzzy topological space 1.4 [6]:

Let \tilde{T} be the collection of type-2 fuzzy sets over $X \neq \emptyset$ then \tilde{T} is called to be general type-2 fuzzy topology on $X \neq \emptyset$

if

- i. $\tilde{X}, \tilde{\emptyset} \in \tilde{T}$
- ii. $\tilde{A} \cap \tilde{B} \in \tilde{T}$
- iii. $\bigcup_{i \in J} \tilde{A}_i \in \tilde{T}$ for $\tilde{A}_i \in \tilde{T}$, for all $i \in J$ (J is an arbitrary index set).

The pair (X, \tilde{T}) is said to type-2 fuzzy topological space over X (Simply GT-2FTS); and the member of \tilde{T} are said to be type-2 fuzzy open sets in X and type-2 fuzzy set \tilde{A} is said type-2 fuzzy closed sets in X , if its complement $\neg \tilde{A} \in \tilde{T}$.

We must note that all the type-2 fuzzy sets is normal type-2 fuzzy sets so as to complete the topological construction and especially check identity law ($\tilde{A} \cap \tilde{\emptyset} = \tilde{\emptyset}$).

The type-2 fuzzy interior of \tilde{A} , denoted by $\text{int}(\tilde{A})$ is defined $\text{int}(\tilde{A}) = \bigcup \{ \tilde{R}_i : \tilde{R}_i \text{ type-2 fuzzy open set in } X, \tilde{R}_i \subseteq \tilde{A}, i \in J \}$

Theorem 1.4.(1):

Let (X, \tilde{T}) be a general type-2 fuzzy topology space over X , and let \tilde{A}, \tilde{B} are type-2 fuzzy sets in X , then

- 1. $\text{int}(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\text{int}(\tilde{X}) = \tilde{X}$
- 2. $\text{int}(\tilde{A}) \subseteq \tilde{A}$
- 3. $\text{int}(\text{int}(\tilde{A})) = \text{int}(\tilde{A})$
- 4. $\tilde{A} \subseteq \tilde{B} \Rightarrow \text{int}(\tilde{A}) \subseteq \text{int}(\tilde{B})$
- 5. $\text{int}(\tilde{A} \cap \tilde{B}) = \text{int}(\tilde{A}) \cap \text{int}(\tilde{B})$
- 6. \tilde{A} is a type-2 fuzzy open set if and only if $\text{int}(\tilde{A}) = \tilde{A}$

The type-2 fuzzy closure of \tilde{A} , denoted by $\text{cl}(\tilde{A})$ is defined

$\text{cl}(\tilde{A}) = \bigcap \{ \tilde{N}_i : \tilde{N}_i \text{ type-2 fuzzy closed set in } X, \tilde{A} \subseteq \tilde{N}_i, i \in J \}$

Theorem 1.4.(2):

Let (X, \tilde{T}) be a general type-2 fuzzy topology space over X , and let \tilde{A}, \tilde{B} are type-2 fuzzy sets in X , then

- 1. $\text{cl}(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\text{cl}(\tilde{X}) = \tilde{X}$
- 2. $\tilde{A} \subseteq \text{cl}(\tilde{A})$
- 3. \tilde{A} is a type-2 fuzzy closed set if and only if $\text{cl}(\tilde{A}) = \tilde{A}$
- 4. $\text{cl}(\text{cl}(\tilde{A})) = \text{cl}(\tilde{A})$
- 5. $\tilde{A} \subseteq \tilde{B} \Rightarrow \text{cl}(\tilde{A}) \subseteq \text{cl}(\tilde{B})$
- 6. $\text{cl}(\tilde{A} \cup \tilde{B}) = \text{cl}(\tilde{A}) \cup \text{cl}(\tilde{B})$

2. Some type-2 Fuzzy Open Sets in General type-2 fuzzy Topological Space.

In a concise manner all lines of this section include a presentation of open set for type-2 fuzzy set and how it was first formed in general type-2 fuzzy topological space.

Definition (1.2):

A subset type-2 fuzzy set \tilde{A} of a general type-2 fuzzy topological space is called:

1. Type-2 fuzzy preopen set if $\tilde{A} \subseteq \text{int}(\text{cl}(\tilde{A}))$ (Simply T-2f $\tilde{\rho}$ s).
2. Type-2 fuzzy $\tilde{\beta}$ -open set if $\tilde{A} \subseteq \text{cl}(\text{int}(\text{cl}(\tilde{A})))$ (Simply T-2f $\tilde{\beta}$ s)
3. Type-2 fuzzy semiopen set if $\tilde{A} \subseteq \text{cl}(\text{int}(\tilde{A}))$ (Simply T-2f $\tilde{\zeta}$ s).
4. Type-2 fuzzy $\tilde{\alpha}$ -open set if $\tilde{A} \subseteq \text{int}(\text{cl}(\text{int}(\tilde{A})))$ (Simply T-2f $\tilde{\alpha}$ s).

3. Type-2 fuzzy Somewhere Dense Set in General Type-2 Fuzzy Topological Space:

The display area of this section is wider than the previous one because it provides the full definition of type-2 fuzzy somewhere dense set with the direct relationship to type-2 fuzzy open sets by theoretical theories and examples.

Definition (1.3):

Let (X, \tilde{T}) be a general type-2 fuzzy topology space over X, a type-2 fuzzy set \tilde{A} defined on X is called a type-2 fuzzy somewhere dense set if $\text{int}(\text{cl}(\tilde{A})) \neq \emptyset$ (short by T-2f \tilde{SDS}).

We can give an equivalent definition for def (1.3).

Definition (2.3):

A subset type-2 fuzzy set \tilde{A} of a general type-2 fuzzy topological space (X, \tilde{T}) is called type-2 fuzzy somewhere dense set if there exists type-2 fuzzy set $\tilde{B} \neq \emptyset \in \tilde{T}$ such that $\tilde{B} \subseteq \text{cl}(\tilde{A})$.

Definition (3.3):

The complement of type-2 fuzzy somewhere dense set subset \tilde{A} of a general type-2 fuzzy topological space (X, \tilde{T}) is called a type-2 fuzzy Co- somewhere dense set (short by Co-T-2f \tilde{SDS}).

Remark (4.3):

We denote the collection of all type-2 fuzzy somewhere dense set in a general type-2 fuzzy topology space (X, \tilde{T}) by $\tilde{SDT}-2(\tilde{T})$.

Theorem (5.3):

A non-empty T-2f $\tilde{\beta}$ s is T-2f \tilde{SDS}

Proof:

Let $\tilde{A} \neq \emptyset$ is a T-2f $\tilde{\beta}$ s then $\tilde{A} \subseteq \text{cl}(\text{int}(\text{cl}(\tilde{A}))) \Rightarrow \tilde{A} \subseteq \text{cl}(\text{cl}(\tilde{A})) = \text{cl}(\tilde{A})$, hence the set $\text{int}(\text{cl}(\tilde{A})) \neq \emptyset \in \tilde{T}$ and $\text{int}(\text{cl}(\tilde{A})) \subseteq \text{cl}(\tilde{A})$ hence \tilde{A} is T-2f \tilde{SDS} .

The converse the theorem (5.3) is not true.

Example (6.3):

Let (X, \tilde{T}) be a general type-2 fuzzy topological space, $X = \{x_1, x_2\}$ and $\tilde{T} = \{\tilde{X}, \emptyset, \tilde{A}, \tilde{B}, \tilde{C}\}$, $\mu_{\tilde{X}}(x_1, \tilde{u} = 1) = 1$, $\mu_{\tilde{X}}(x_2, \tilde{u} = 1) = 1$, $\mu_{\emptyset}(x_1, \tilde{u} = 0) = 1$, $\mu_{\emptyset}(x_2, \tilde{u} = 0) = 1$ and $\tilde{A}, \tilde{B}, \tilde{C}$, are type-2 fuzzy sets $\mu_{\tilde{A}}(x_1, \tilde{u}) = \begin{cases} 0.9 & \tilde{u} = 0.2 \\ 1 & \tilde{u} = 0.1 \end{cases}$, $\mu_{\tilde{A}}(x_2, \tilde{u} = 0.2) = 1$, $\mu_{\tilde{B}}(x_1, \tilde{u}) = \begin{cases} 0.7 & \tilde{u} = 0.7 \\ 1 & \tilde{u} = 0.8 \end{cases}$, $\mu_{\tilde{B}}(x_2, \tilde{u} = 0.2) = 1$,

$\mu_{\tilde{C}}^{\tilde{u}}(x_1, \tilde{u}) = \begin{cases} 0.8 & \tilde{u}=0.1 \\ 1 & \tilde{u}=0.2 \end{cases}$, $\mu_{\tilde{C}}^{\tilde{u}}(x_2, \tilde{u} = 0.2) = 1$, and we have \tilde{K} type-2 fuzzy set in (X, \tilde{T}) such that $\mu_{\tilde{K}}^{\tilde{u}}(x_1, \tilde{u}) = \begin{cases} 0.9 & \tilde{u}=0.8 \\ 1 & \tilde{u}=0.9 \end{cases}$, $\mu_{\tilde{K}}^{\tilde{u}}(x_2, \tilde{u} = 0.8) = 1$ then \tilde{K} is T-2fSDS and is not T-2fβs.

Theorem (7.3):

A non-empty T-2f αs is T-2fSDS.

Proof:

Let $\tilde{A} \neq \tilde{\emptyset}$ is a T-2f αs then $\tilde{A} \subseteq \text{int}(\text{cl}(\text{int}(\tilde{A})))$, and by definition T-2fSDS we have $\text{int}(\text{cl}(\tilde{A})) \neq \tilde{\emptyset}$ suppose that $\text{int}(\text{cl}(\tilde{A})) = \tilde{\emptyset}$ such that $\tilde{A} \subseteq \text{int}(\text{cl}(\text{int}(\tilde{A}))) \subseteq \text{int}(\text{cl}(\tilde{A})) = \tilde{\emptyset}$ that is contradiction therefore \tilde{A} is T-2fSDS.

The converse the theorem (7.3) is not true.

Example (8.3):

Let (X, \tilde{T}) be a general type-2 fuzzy topological space, $X = \{x_1, x_2\}$ and $\tilde{T} = \{\tilde{X}, \tilde{\emptyset}, \tilde{A}\}$, $\mu_{\tilde{X}}^{\tilde{u}}(x_1, \tilde{u} = 1) = 1$, $\mu_{\tilde{X}}^{\tilde{u}}(x_2, \tilde{u} = 1) = 1$, $\mu_{\tilde{\emptyset}}^{\tilde{u}}(x_1, \tilde{u} = 0) = 1$, $\mu_{\tilde{\emptyset}}^{\tilde{u}}(x_2, \tilde{u} = 0) = 1$, let \tilde{A} a type-2 fuzzy sets $\mu_{\tilde{A}}^{\tilde{u}}(x_1, \tilde{u}) = \begin{cases} 0.8 & \tilde{u}=0.3 \\ 1 & \tilde{u}=0.2 \end{cases}$, $\mu_{\tilde{A}}^{\tilde{u}}(x_2, \tilde{u} = 0.1) = 1$, and \tilde{B} type-2 fuzzy set in (X, \tilde{T}) such that $\mu_{\tilde{B}}^{\tilde{u}}(x_1, \tilde{u}) = \begin{cases} 0.3 & \tilde{u}=0.7 \\ 1 & \tilde{u}=0.6 \end{cases}$, $\mu_{\tilde{B}}^{\tilde{u}}(x_2, \tilde{u} = 0.5) = 1$ therefore \tilde{B} is T-2fSDS but not a T-2f αs.

Theorem(9.3):

A non-empty T-2f ζs is T-2fSDS.

Proof:

Let $\tilde{A} \neq \tilde{\emptyset}$ is a T-2f ζs then $\tilde{A} \subseteq \text{cl}(\text{int}(\tilde{A}))$, and by defined T-2fSDS we have $\text{int}(\text{cl}(\tilde{A})) \neq \tilde{\emptyset}$ suppos that $\text{int}(\text{cl}(\tilde{A})) = \tilde{\emptyset}$ such that $\tilde{A} \subseteq \text{cl}(\text{int}(\tilde{A})) \subseteq \text{cl}(\text{int}(\text{cl}(\tilde{A}))) \Rightarrow \tilde{A} \subseteq \text{cl}(\tilde{\emptyset}) = \tilde{\emptyset}$

That is contradiction then $\tilde{A} \neq \tilde{\emptyset}$ is a T-2fSDS.

The converse the theorem (9.3) is not true.

Example (10.3):

Let (X, \tilde{T}) be a general type-2 fuzzy topological space, $X = \{x_1, x_2\}$ and $\tilde{T} = \{\tilde{X}, \tilde{\emptyset}, \tilde{W}\}$, $\mu_{\tilde{X}}^{\tilde{u}}(x_1, \tilde{u} = 1) = 1$, $\mu_{\tilde{X}}^{\tilde{u}}(x_2, \tilde{u} = 1) = 1$, $\mu_{\tilde{\emptyset}}^{\tilde{u}}(x_1, \tilde{u} = 0) = 1$, $\mu_{\tilde{\emptyset}}^{\tilde{u}}(x_2, \tilde{u} = 0) = 1$, let \tilde{W} a type-2 fuzzy set $\mu_{\tilde{W}}^{\tilde{u}}(x_2, \tilde{u}) = \begin{cases} 0.7 & \tilde{u}=0.3 \\ 1 & \tilde{u}=0.1 \end{cases}$, $\mu_{\tilde{W}}^{\tilde{u}}(x_2, \tilde{u} = 0.2) = 1$ and \tilde{L} a type-2 fuzzy set in (X, \tilde{T}) , $\mu_{\tilde{L}}^{\tilde{u}}(x_2, \tilde{u}) = \begin{cases} 0.8 & \tilde{u}=0.6 \\ 1 & \tilde{u}=0.9 \end{cases}$, $\mu_{\tilde{L}}^{\tilde{u}}(x_1, \tilde{u} = 0.7) = 1$ therefore \tilde{L} is not T-2f ζs but is a T-2fSDS.

Theorem (11.3):

A non-empty T-2f ϕs is T-2fSDS.

Proof:

Let $\tilde{A} \neq \tilde{\emptyset}$ is a T-2f $\tilde{\wp}$ s then $\tilde{A} \subseteq \text{int}(\text{cl}(\tilde{A}))$, and by defined T-2fSDS we have $\text{int}(\text{cl}(\tilde{A})) \neq \tilde{\emptyset}$ suppos that $\text{int}(\text{cl}(\tilde{A})) = \tilde{\emptyset}$ such that $\tilde{A} \subseteq \text{int}(\text{cl}(\tilde{A})) = \tilde{\emptyset}$ that is contradiction therefore \tilde{A} is T-2fSDS.

The converse the theorem (11.3) is not true.

Example (12.3):

Let (X, \tilde{T}) be a general type-2 fuzzy topological space, $X = \{x_1, x_2\}$ and $\tilde{T} = \{\tilde{X}, \tilde{\emptyset}, \tilde{Y}\}$, $\mu_{\tilde{X}}(x_1, \tilde{u} = 1) = 1$, $\mu_{\tilde{X}}(x_2, \tilde{u} = 1) = 1$, $\mu_{\tilde{\emptyset}}(x_1, \tilde{u} = 0) = 1$, $\mu_{\tilde{\emptyset}}(x_2, \tilde{u} = 0) = 1$,

\tilde{Y} type-2 fuzzy sets $\mu_{\tilde{Y}}(x_1, \tilde{u}) = \begin{cases} 0.4 & \tilde{u} = 0.4 \\ \tilde{u} = 0.3 & \end{cases}$, $\mu_{\tilde{Y}}(x_2, \tilde{u} = 0.1) = 1$, and let \tilde{N} type-2 fuzzy set in (X, \tilde{T}) , $\mu_{\tilde{N}}(x_2, \tilde{u}) = \begin{cases} 0.3 & \tilde{u} = 0.8 \\ \tilde{u} = 0.7 & \end{cases}$, $\mu_{\tilde{N}}(x_2, \tilde{u} = 0.6) = 0.4$ thererore \tilde{N} is not a T-2f $\tilde{\wp}$ s but is a T-2fSDS.

Theorem (13.3):

Every a type-2 fuzzy subset of a general type-2 fuzzy topology space is T-2fSDS or Co-T-2fSDS.

Proof:

Let $\tilde{A} \neq \tilde{\emptyset}$ is a type-2 fuzzy subset of X and not T-2fSDS, that is $\text{int}(\text{cl}(\tilde{A})) = \tilde{\emptyset}$ such that can not have $\text{cl}(\tilde{A}) = \tilde{X}$ therefore $\neg(\text{cl}(\tilde{A})) \neq \tilde{\emptyset} \in \tilde{T} \subseteq \neg\tilde{A}$, $\neg\tilde{A}$ is T-2fSDS and \tilde{A} is Co-T-2fSDS.

Theorem (14.3):

Let $\{\tilde{A}_k : k \in J\}$ be a class of a fuzzy subset of a general type-2 fuzzy topology space (X, \tilde{T}) then $\bigcup_{k \in J} \tilde{A}_k$ is T-2fSDS if and only if $\bigcup_{k \in J} \neg\tilde{A}_k$ is T-2fSDS.

Proof:

Suppose that $\bigcup_{k \in J} \tilde{A}_k$ is T-2fSDS we must prove that $\bigcup_{k \in J} \neg\tilde{A}_k$ is T-2fSDS,

we have $\bigcup_{k \in J} \tilde{A}_k$ is T-2fSDS such that $\text{int}(\text{cl}(\bigcup_{k \in J} \tilde{A}_k)) \neq \tilde{\emptyset}$ and but by

$$\neg(\bigcup_{k \in J} \neg\tilde{A}_k) = \neg(\bigcup_{k \in J} \tilde{A}_k), \text{ such that}$$

$$\text{int}[\text{cl}(\neg(\bigcup_{k \in J} \tilde{A}_k))] \neq \tilde{\emptyset}$$

$$\text{int}[\text{cl}(\neg\bigcup_{k \in J} \neg\tilde{A}_k)] \neq \tilde{\emptyset}$$

$$\text{int}[\text{cl}(\neg\bigcup_{k \in J} \tilde{A}_k)] \neq \tilde{\emptyset}$$

$$\text{int}[\text{cl}(\bigcup_{k \in J} \neg\tilde{A}_k)] \neq \tilde{\emptyset}$$

therefore $\bigcup_{k \in J} \tilde{A}_k$ is T-2fSDS.

Proposition (15.3):

Let $\{\tilde{K}_r : r \in J\}$ be a family of T-2fSDS then $\bigcup_{r \in J} \tilde{K}_r$ is T-2fSDS.

proof:

Let $\{\tilde{K}_r : r \in J\}$ be a family of T-2fSDS and by definition of T-2fSDS then there exist $\tilde{M}_{r_0} \neq \emptyset \subseteq \text{cl}(\tilde{K}_{r_0})$, but $\text{cl}(\tilde{K}_{r_0}) \subseteq \text{cl}(\bigcup_{r \in J} \tilde{K}_r)$, therefore $\bigcup_{r \in J} \tilde{K}_r$ is T-2fSDS.

Proposition (16.3):

Let $\{\tilde{B}_r : r \in J\}$ be a family of Co-T-2fSDS then $\bigcap_{r \in J} \tilde{B}_r$ is Co-T-2fSDS.

Proof:

Let $\{\tilde{B}_r : r \in J\}$ be a family of Co-T-2fSDS then $\{\neg \tilde{B}_r : r \in J\}$ be a family of T-2fSDS and by proposition(15.3) $\bigcup_{r \in J} \neg \tilde{B}_r$ is T-2fSDS therefore $\bigcap_{r \in J} \tilde{B}_r$ is Co-T-2fSDS.

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