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On Soft P_c-Connected Spaces

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Abstract
In this paper, we define the concept of soft p_c-connected sets and soft p_c-connected spaces by using the notion of soft p_c-open sets in soft topological spaces. Several properties of these concepts are investigated.

Keywords: (soft p_c-open set, soft p_c-separated sets, soft p_c-connected, soft p_c-disconnected)

حول الفضاءات المتصلة الناعمة من النمط P_c

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الخلاصة:
في هذا البحث عرفنا مفهوم المجموعات المتصلة الناعمة من النمط P_c والفضاءات المتصلة الناعمة من النمط P_c وذلك باستخدام المجمعات المفتوحة الناعمة من النمط P_c . تم بحث العديد من صفات هذه المفاهيم.

1 Introduction

In topology, connectedness is used to refer to various properties meaning in some sense, (all in one piece). When a mathematical object has such a property, we say that it is connected; otherwise, it is disconnected. Connectivity occupies very important place in topology. Many authors have presented different kinds of connectivity in general, including fuzzy settings and intuitionistic fuzzy settings such as P-connectedness and semi-pre connectedness in intuitionistic fuzzy topological spaces. After the foundation of the soft set theory by D. Molodtsov [1], many researchers studied soft topological structures. In 2012, Peyhan *et al.* [2] introduced soft connectedness in soft topological spaces. In 2013, Lin [3] continued the study of soft connectedness. In 2015, Husain [4] provided more characterizations of soft connectedness in soft topological spaces. In the present paper, we introduce another type of soft separation and soft connectedness in soft topological spaces by using the concept of soft p_c-open and soft p_c-closed sets, called soft p_c-separation and soft p_c-connected space. We also describe the essential properties of these concepts along with the relations between these types and other existing types.

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Throughout this paper, X will be a nonempty initial universal set and A will be a set of parameters. A pair (F,A) is called a soft set over X, where F is a function $F: A \to P(X)$. The collection of soft sets (F,E) over a universal set X with the parameter set A is denoted by $SP(X)_A$. Any logical operation (λ) on soft sets in soft topological spaces is denoted by the usual set theoretical operations with the symbol $(\tilde{s}(\lambda))$.

2 Preliminaries

This section contains the main definitions and results in soft topological spaces which are needed in other sections. All these definitions can be found in several articles concerning soft set theory and soft topological spaces [5- 9].

Definition 2.1 [5] *A soft set (F,A) over X is said to be empty soft set denoted by $\tilde{\phi}$ if for all $e \in A$, $F(e) = \phi$ and (F,A) over X is said to be absolute soft set denoted by $\tilde{X}$ if for all $e \in A$, $F(e) = X$.*

Definition 2.2 [5] *The complement of a soft set (F,A) is denoted by $(F,A)^c$ or $\tilde{X}\backslash(F,A)$ and is defined by $(F,A)^c = (F^c, A)$ where $F^c: A \to P(X)$ is a function given by $F^c(e) = X\backslash F(e)$, for all $e \in A$.*

It is clear that $((F,A)^c)^c = (F,A)$, $\tilde{\phi}^c = \tilde{X}$ and $\tilde{X}^c = \tilde{\phi}$.

Definition 2.3 [5] *For two soft sets (F,A) and (G,B) over a common universe X, we say that (F,A) is a soft subset of (G,B), if*

1. $A \subseteq B$ and
2. for all $e \in A$, $F(e) \subseteq G(e)$

We write $(F,A) \tilde{\subseteq} (G,B)$.

Definition 2.4 [5] *The union of two soft sets of (F,A) and (G,B) over the common universe X is the soft set $(H,C) = (F,A) \tilde{\cup} (G,B)$, where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & if \quad e \in A\backslash B \\ G(e) & if \quad e \in B\backslash A \\ F(e) \cup G(e) & if \quad e \in A \cap B \end{cases}$$

In particular, $(F,A) \tilde{\cup} (G,A) = F(e) \cup G(e)$ for all $e \in A$.

Definition 2.5 [5] *The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X, denoted $(F,A) \tilde{\cap} (G,B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.*

In particular, $(F,A) \tilde{\cap} (G,A) = F(e) \cap G(e)$ for all $e \in A$.

Definition 2.6 [7] *The soft set (F,A) is called a soft point, denoted by (x_e, A) or x_e, if for the element $e \in A$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E\backslash\{e\}$.*

We say that $x_e \tilde{\in} (G,A)$ if $x \in G(e)$.

Two soft points x_e and $y_{e'}$ are distinct if either $x \neq y$ or $e \neq e'$.

Definition 2.7 [10] *Let $\tilde{\tau}$ be a collection of soft sets over a universe X with a fixed set E of parameters. Then $\tilde{\tau} \subseteq SP(X)_A$ is called a soft topology, if*

1. $\tilde{\phi}$ and $\tilde{X}$ belongs to $\tilde{\tau}$.
2. The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
3. The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(X, \tilde{\tau}, A)$ is called a soft topological space over X. The members of $\tilde{\tau}$ are called soft open sets in $\tilde{X}$ while the complements of them are called soft closed sets in $\tilde{X}$, and they are denoted by $SO(\tilde{X})$ and $SC(\tilde{X})$, respectively. Soft interior and soft closure are denoted by $\tilde{s}int$ and $\tilde{s}cl$, respectively.

Definition 2.8 [10] *Let $(X, \tilde{\tau}, A)$ be a soft topological space and let (G,A) be a soft set. Then,*

1. The soft closure of (G,A) is the soft set

$\tilde{s}cl(G,A) = \tilde{\cap}\{(K,B) \tilde{\in} SC(\tilde{X}): (G,A) \tilde{\subseteq} (K,B)\}$

2. The soft interior of (G,A) is the soft set

$\tilde{s}int(G,A) = \tilde{\cup}\{(H,B) \tilde{\in} SO(\tilde{X}): (H,B) \tilde{\subseteq} (G,A)\}$.

Definition 2.9 [11] *Let $(X, \tilde{\tau}, A)$ be a soft topological space, (G,A) is a soft set over $\tilde{X}$ and $x_e \tilde{\in} \tilde{X}$. Then (G,A) is said to be a soft neighborhood of x_e, if there exists a soft open set (H,A) such that $x_e \tilde{\in} (H,A) \tilde{\subseteq} ((G,A)$.*

Proposition 2.10 [10] *Let $(Y, \tilde{\tau}_Y, A)$ be a soft subspace of a soft topological space $(X, \tilde{\tau}, A)$ and $(F, A) \tilde{\in} SP(X)_E$. Then:*
1. If (F, A) is a soft open set in $\tilde{Y}$ and $\tilde{Y} \tilde{\in} \tilde{\tau}$, then $(F, A) \tilde{\in} \tilde{\tau}$.
2. (F, A) is a soft open set in $\tilde{Y}$ if and only if $(F, A) = \tilde{Y} \tilde{\cap} (G, A)$ for some $(G, A) \tilde{\in} \tilde{\tau}$.
3. (F, A) is a soft closed set in $\tilde{Y}$ if and only if $(F, A) = \tilde{Y} \tilde{\cap} (H, A)$ for some soft closed (H, A) in $\tilde{X}$.

Definition 2.11 [12] *A soft subset (F, A) of a soft space $\tilde{X}$ is said to be soft pre-open if $(F, A) \tilde{\subseteq} \tilde{s}int\tilde{s}cl(F, A)$. The complement of soft pre-open set is said to be soft pre-closed. The family of soft pre-open (soft pre-closed) set is denoted by $\tilde{s}PO(X)$ and $\tilde{s}PC(X)$ respectively.*

Lemma 2.1 [12] *Arbitrary union of soft pre-open sets is a soft pre-open set.*

Definition 2.12 [13] *Let $(X, \tilde{\tau}, A)$ be a soft topological space and let (G, A) be a soft set. Then*
1. The soft pre-closure of (G, A) is the soft set

$\tilde{s}pcl(G, A) = \tilde{\cap} \{(K, B) \tilde{\in} \tilde{s}PC(\tilde{X}) : (G, A) \tilde{\subseteq} (K, B)\}$

2. The soft pre-interior of (G, A) is the soft set

$\tilde{s}pint(G, A) = \tilde{\cup} \{(H, B) \tilde{\in} \tilde{s}PO(\tilde{X}) : (H, B) \tilde{\subseteq} (G, A)\}$.

In [14], Bayramov and Aras defined a soft T_1-space as:

Definition 2.13 [14] *A soft topological space $(X, \tilde{\tau}, A)$ is said to be soft T_1, if for each pair of distinct soft points x_e , $y_{e'} \tilde{\in} SP(X)_A$, there exist two soft open sets (F, A) and (G, A) such that $x_e \tilde{\in} (F, A)$ but $y_{e'} \tilde{\notin} (F, A)$ and $y_{e'} \tilde{\in} (G, A)$ but $x_e \tilde{\notin} (G, A)$.*

Proposition 2.14 [14] *A soft topological space $(X, \tilde{\tau}, A)$ is soft T_1 if and only if each soft point is soft closed.*

Definition 2.15 [15] A soft pre-open set (F, A) in a soft topological space $(X, \tilde{\tau}, A)$ is called soft p_c-open if, for each $x_e \tilde{\in} (F, A)$, there exists a soft closed set (K, A) such that $x_e \tilde{\in} (K, A) \tilde{\subseteq} (F, A)$. The soft complement of each soft p_c-open set is called soft p_c-closed set.

The family of all soft p_c-open (resp., soft p_c-closed) sets in a soft topological space $(X, \tilde{\tau}, A)$ is denoted by $\tilde{s}p_cO(X, \tilde{\tau}, A)$ (resp., $\tilde{s}p_cC(X, \tilde{\tau}, A)$) or $\tilde{s}p_cO(X)$ (resp., $\tilde{s}p_cC(X)$).

Lemma 2.2 [15] *A soft set (F, A) in a soft topological space $(X, \tilde{\tau}, A)$ is soft p_c-open if and only if for each $x_e \tilde{\in} (F, A)$, there exists a soft p_c-open set (K, A) such that $x_e \tilde{\in} (K, A) \tilde{\subseteq} (F, A)$.*

Definition 2.16 [16] *Let $(X, \tilde{\tau}, A)$ be a soft topological space and let (G, A) be a soft set. Then,*
1. A soft point $x_e \tilde{\in} \tilde{X}$ is said to be a soft p_c-limit soft point of a soft set (F, A), if for every soft p_c-open set (G, A) containing x_e, $(G, A) \tilde{\cap} [(F, A) \setminus \{x_e\}] \neq \tilde{\phi}$.

The set of all soft p_c-limit soft points of (F, A) is called the soft p_c-derived set of (F, A) and is denoted by $\tilde{s}p_cD(F, A)$.

2. The soft p_c-closure of (G, A) is the soft set

$\tilde{s}p_ccl(G, A) = \tilde{\cap} \{(K, B) \tilde{\in} \tilde{s}P_cC(\tilde{X}) : (G, A) \tilde{\subseteq} (K, B)\}$.

3. The soft p_c-interior of (G, A) is the soft set

$\tilde{s}p_cint(G, A) = \tilde{\cup} \{(H, B) \tilde{\in} \tilde{s}P_O(\tilde{X}) : (H, B) \tilde{\subseteq} (G, A)\}$.

Lemma 2.3 [15] *Let $(X, \tilde{\tau}, A)$ be a soft topological space and let (G, A) be a soft set. Then, $\tilde{s}p_ccl(G, A) = (G, A) \tilde{\cup} \tilde{s}p_cD(G, A)$.*

Lemma 2.4 [15] *If $(F, A) \tilde{\subseteq} \tilde{Y} \tilde{\subseteq} \tilde{X}$ and $\tilde{Y}$ is soft clopen, then, $(F, A) \tilde{\in} \tilde{s}p_cO(Y)$ if and only if $(F, A) \tilde{\in} \tilde{s}p_cO(X)$.*

Lemma 2.5 [15] *Let (F, A) , $\tilde{Y} \tilde{\subseteq} \tilde{X}$ and $\tilde{Y}$ be soft clopen. If $(F, A) \tilde{\in} \tilde{s}p_cO(X)$, then $(F, A) \tilde{\cap} \tilde{Y} \tilde{\in} \tilde{s}p_cO(Y)$.*

Lemma 2.6 [16] *Let $(F, A) \tilde{\subseteq} \tilde{Y} \tilde{\subseteq} \tilde{X}$. If $\tilde{Y}$ is soft clopen, then $\tilde{s}p_ccl_Y(F, A) = \tilde{s}p_ccl_X(F, A) \tilde{\cap} \tilde{Y}$.*

Definition 2.17 [17] *Let $SP(X)_A$ and $SP(Y)_B$ be families of soft sets. Let $u: X \to Y$ and $p: E \to B$ be functions. Then, a function $f_{pu}: SP(X)_A \to SP(Y)_B$ is defined as follows:*
1. If (F, A) is a soft set in $SP(X)_A$, then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SP(Y)_B$ such that

$$f_{pu}(F)(e') = \begin{cases} \bigcup_{e \in p^{-1}(e') \cap E} u(F(e)) & if \quad p^{-1}(e') \cap E \neq \phi \\ \phi & if \quad p^{-1}(e') \cap E = \phi \end{cases}$$

for all $e' \in B$.

2. If (G,B) is a soft set in $SP(Y)_B$, then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SP(X)_A$ such that

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(e))) & if \quad p(e) \in B \\ \phi & otherwise \end{cases}$$

for all $e \in A$.

The soft function f_{pu} is surjective, if p and u are surjective and it is injective if p and u are injective.

Definition 2.18 *Let $(X,\tilde{\tau},A)$ and $(Y,\tilde{\mu},B)$ be two soft topological spaces. A soft function $f_{pu}:\tilde{X} \to \tilde{Y}$ is called soft continuous* [18] *(resp., $\tilde{s}p_c$ -continuous* [15]*), if $f_{pu}^{-1}((G,B)) \tilde{\in} \tilde{\tau}$ (resp., $f_{pu}^{-1}((G,B)) \tilde{\in} \tilde{s}P_cO(X)$) for all $(G,B) \tilde{\in} \tilde{\mu}$.*

Lemma 2.7 [15] *A soft function $f_{pu}:\tilde{X} \to \tilde{Y}$ is $\tilde{s}p_c$-continuous if and only if the inverse image of each soft open (soft closed) set is $\tilde{s}P_c$-open ($\tilde{s}P_c$-closed).*

Definition 2.19 [4] *A soft subset (F,A) of a soft topological space (X,τ,A) is said to be soft connected, if it does not have a soft separation, otherwise it is called soft disconnected.*

Definition 2.20 [4] *A soft topological space (X,τ,A) is said to be a soft connected space, if it does not have a soft separation, otherwise it is called soft disconnected .*

3 $\tilde{s}p_c$-separated and $\tilde{s}p_c$-connected sets

In this section, we introduce the concept of soft p_c- separated sets and soft p_c- connected sets in a soft topological space. Also, we discuss some of the main properties based on these concepts.

Definition 3.1 *Let (X,τ,A) be a soft topological space. Two non empty soft subsets (F,A) and (G,A) of $SP(X)_A$ are said to be soft p_c -separated ($\tilde{s}p_c$ -separated) sets over $\tilde{X}$, if $\tilde{s}p_c cl(F,A) \tilde{\cap} (G,A) = \tilde{s}p_c cl(G,A) \tilde{\cap} (F,A) = \tilde{\phi}$.*

Remark 3.1 *Two soft sets (F,A) and (G,A) are $\tilde{s}p_c$-separated if and only if (F,A) and (G,A) are disjoint and neither of them contains $\tilde{s}p_c$-limit points of the other.*

Therefore we have, if $(F,A) \tilde{\cap} \tilde{s}p_c cl(G,A) = \phi$ then $(F,A) \tilde{\cap} ((G,A) \tilde{\cup} \tilde{s}p_c D(G,A)) = [(F,A) \tilde{\cap} (G,A)] \tilde{\cup} [(F,A) \tilde{\cap} \tilde{s}p_c D(G,A)] = \phi$ so that $(F,A) \tilde{\cap} \tilde{s}p_c D(G,A) = \tilde{\phi}$. Therefore, (F,A) contains no $\tilde{s}p_c$-limit points of (G,A).

Theorem 3.2 *Let (X,τ,A) be a soft topological space. Then, the following are equivalent:*

1. The only $\tilde{s}p_c$-clopen set in (X,τ,A) is $\tilde{X}$ and $\tilde{\phi}$.
2. $\tilde{X}$ is not the union of two disjoint non-empty $\tilde{s}p_c$-open sets.
3. $\tilde{X}$ is not the union of two disjoint non-empty$\tilde{s}p_c$-closed sets.
4. $\tilde{X}$ is not the union of two disjoint non-empty $\tilde{s}p_c$-separated sets.

Proof. (1) → (2). If $\tilde{X} = (F,A) \tilde{\cup} (G,A)$, where (F,A) and (G,A) are disjoint non-empty $\tilde{s}p_c$-open sets, then, $\tilde{X} \backslash (F,A) = (G,A)$is a non-empty $\tilde{s}p_c$-closed set. Hence, (G,A) is a non-empty proper $\tilde{s}p_c$-clopen set. This contradicts (1), hence (2) is proved.

(2) → (3). Assume that $\tilde{X}$ is not the soft union of two soft disjoint non-empty $\tilde{s}p_c$-open sets. Suppose that $\tilde{X} = (K,A) \tilde{\cup} (L,A)$ where (K,A) and (L,A) are two disjoint non empty $\tilde{s}p_c$-closed sets. Now, (K,A) and (L,A) being respectively the complement of each other. Therefore, (K,A) and (L,A) are $\tilde{s}p_c$-open sets which contradicts (2). Hence, we obtain (3).

(3) → (4). Suppose that $\tilde{X} = (F,A) \tilde{\cup} (G,A)$, where (F,A) and (G,A) are non-empty $\tilde{s}p_c$-separated sets. Since $(F,A) \tilde{\cap} \tilde{s}p_c cl\ (G,A) = \tilde{\phi}$, we get $\tilde{s}p_c cl\ (G,A) \tilde{\subseteq} \tilde{X} \backslash (F,A)$, hence (G,A) is $\tilde{s}p_c$-closed set. Similarly, (F,A) must be $\tilde{s}p_c$-closed set. This contradicts (3) and hence the proof is finished.

(4) → (1). Suppose that (F,A) is a non-empty proper $\tilde{s}p_c$-clopen subset of $\tilde{X}$. Then, $(G,A) = \tilde{X} \backslash (F,A)$ is a non-empty proper $\tilde{s}p_c$-clopen subset of $\tilde{X}$. Since $\tilde{X} = (F,A) \tilde{\cup} (G,A)$, so (F,A) and (G,A) are $\tilde{s}p_c$-separated sets, which shows that (4) is false. Therefore, (1) is proved.

Definition 3.3 *A soft p_c-separation ($\tilde{s}p_c$-separation) of a soft topological space (X,τ,A) is a pair of $\tilde{s}p_c$-separated sets (F,A) and (G,A) whose union is $\tilde{X}$.*

The following example illustrates a non-trivial $\tilde{s}p_c$-separation of a soft topological space (X,τ,A).

Example 3.1 *Let* $X = \{x, y\}$, $A = \{e_1, e_2\}$. *Let* (X, τ, A) *be the soft topological space where* $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ *where*
$(F_1, A) = \{(e_1, \{x, y\}), (e_2, \phi)\}$, $(F_2, A) = \{(e_1, \{x\}), (e_2, \{x\})\}$,
$(F_3, A) = \{(e_1, \phi), (e_2, \{x, y\})\}$, $(F_4, A) = \{(e_1, \phi), (e_2, \{x\})\}$.
Then, it is easy to check that $\tilde{s}p_cO(X) = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_3, A)$ and $\tilde{s}p_ccl(F_1, A) \tilde{\cap} (F_3, A) = \tilde{s}p_ccl(F_3, A) \tilde{\cap} (F_1, A) = \tilde{\phi}$. Therefore, (F_1, A) and (F_3, A) are $\tilde{s}p_c$-separated sets. Hence (F_1, A) and (F_3, A) form an $\tilde{s}p_c$-separation of $\tilde{X}$.

Theorem 3.4 *Let* (F, A) *and* (G, A) *be two non empty soft sets in a space* $\tilde{X}$. *Then, the following statements are true:*

1. If (F, A) and (G, A) are $\tilde{s}p_c$-separated and $(F_1, A) \tilde{\subseteq} (F, A)$, $(G_1, A) \tilde{\subseteq} (G, A)$, then (F_1, A) and (G_1, A) are also $\tilde{s}p_c$-separated.
2. If $(F, A) \tilde{\cap} (G, A) = \phi$ such that each of (F, A) and (G, A) are both $\tilde{s}p_c$-closed (if they are $\tilde{s}p_c$-open and their union is $\tilde{X}$), then (F, A) and (G, A) are $\tilde{s}p_c$-separated.
3. If each of (F, A) and (G, A) are both $\tilde{s}p_c$-closed ($\tilde{s}p_c$-open) and if $(U, A) = (F, A) \tilde{\cap} (\tilde{X} \backslash (G, A))$, $(V, A) = (G, A) \tilde{\cap} (\tilde{X} \backslash (F, A))$, then (U, A) and (V, A) are $\tilde{s}p_c$-separated.

Proof. (1) Since $(F_1, A) \tilde{\subseteq} (F, A)$, then $\tilde{s}p_ccl(F_1, A) \tilde{\subseteq} \tilde{s}p_ccl(F, A$. So, $\tilde{s}p_ccl(F, A) \tilde{\cap} (G, A) = \phi$ implies $\tilde{s}p_ccl(F_1, A) \tilde{\cap} (G, A) = \phi$ and $\tilde{s}p_ccl(F_1, A) \tilde{\cap} (G_1, A) = \phi$. Similarly $(F_1, A) \tilde{\cap} \tilde{s}p_ccl(G_1, A) = \phi$. Hence, (F_1, A) and (G_1, A) are $\tilde{s}p_c$-separated.
(2) Since $(F, A) = \tilde{s}p_ccl(F, A)$, $(G, A) = \tilde{s}p_ccl(G, A)$, and $(F, A) \tilde{\cap} (G, A) = \phi$, then $\tilde{s}p_ccl(F, A) \tilde{\cap} (G, A) = \tilde{\phi}$ and $\tilde{s}p_ccl(G, A) \tilde{\cap} (F, A) = \tilde{\phi}$. Hence (F, A) and (G, A) are $\tilde{s}p_c$-separated. If (F, A) and (G, A) are $\tilde{s}p_c$-open and their union is $\tilde{X}$, then their complements are $\tilde{s}p_c$-closed with empty intersection.
(3) If (F, A) and (G, A) are $\tilde{s}p_c$-open, then $\tilde{X} \backslash (F, A)$ and $\tilde{X} \backslash (G, A)$ are $\tilde{s}p_c$- closed. Since $(U, A) \tilde{\subseteq} (\tilde{X} \backslash (G, A))$, $\tilde{s}p_ccl(U, A) \tilde{\subseteq} \tilde{s}p_ccl(\tilde{X} \backslash (G, A)) = (\tilde{X} \backslash (G, A))$ and so $\tilde{s}p_ccl(U, A) \tilde{\cap} (G, A) = \tilde{\phi}$. Thus, $\tilde{s}p_ccl(U, A) \tilde{\cap} (V, A) = \tilde{\phi}$. Similarly, $(U, A) \tilde{\cap} \tilde{s}p_ccl(V, A) = \tilde{\phi}$. Therefore, (U, A) and (V, A) are $\tilde{s}p_c$-separated.

Theorem 3.5 *The soft sets* (F, A) *and* (G, A) *of a space* $\tilde{X}$ *are* $\tilde{s}p_c$*-separated if and only if there exist* $\tilde{s}p_c$*-open sets* (U, A) *and* (V, A) *such that* $(F, A) \tilde{\subseteq} (U, A)$, $(G, A) \tilde{\subseteq} (V, A)$ *and* $(V, A) \tilde{\cap} (F, A) = \tilde{\phi}$, $(G, A) \tilde{\cap} (U, A) = \tilde{\phi}$.

Proof. Let (F, A) and (G, A) be $\tilde{s}p_c$-separated sets. Then, the sets $(V, A) = \tilde{X} \backslash \tilde{s}p_ccl(F, A)$ and $(U, A) = \tilde{X} \backslash \tilde{s}p_ccl(G, A)$ are $\tilde{s}p_c$-open sets such that $(F, A) \tilde{\subseteq} (U, A)$, $(G, A) \tilde{\subseteq} (V, A)$ and $(V, A) \tilde{\cap} (F, A) = \tilde{\phi}$, $(G, A) \tilde{\cap} (U, A) = \tilde{\phi}$.

Conversely, let (U, A), $(V, A) \in \tilde{s}p_cO(X)$ such that $(F, A) \tilde{\subseteq} (U, A)$, $(G, A) \tilde{\subseteq} (V, A)$ and $(V, A) \tilde{\cap} (F, A) = \tilde{\phi}$, $(G, A) \tilde{\cap} (U, A) = \tilde{\phi}$. Since $\tilde{X} \backslash (V, A)$ and $\tilde{X} \backslash (U, A)$ are $\tilde{s}p_c$-closed, then $\tilde{s}p_ccl(F, A) \tilde{\subseteq} \tilde{X} \backslash (V, A) \tilde{\subseteq} \tilde{X} \backslash (G, A)$ and $\tilde{s}p_ccl(G, A) \tilde{\subseteq} \tilde{X} \backslash (U, A) \tilde{\subseteq} \tilde{X} \backslash (F$. Thus, $\tilde{s}p_ccl(F, A) \tilde{\cap} (G, A) = \tilde{\phi}$ and $\tilde{s}p_ccl(G, A) \tilde{\cap} (F, A) = \tilde{\phi}$ and hence the proof is finished.

Theorem 3.6 *Let* (F, A) *and* (G, A) *be nonempty soft disjoint subsets of a space* $\tilde{X}$ *and* $(V, A) = (G, A) \tilde{\cup} (F, A)$. *Then* (F, A) *and* (G, A) *are* $\tilde{s}p_c$*-separated if and only if each of* (F, A) *and* (G, A) *is* $\tilde{s}p_c$*-clopen in* (V, A).

Proof. Let (F, A) and (G, A) be $\tilde{s}p_c$-separated sets. By Remark 3.1, (F, A) contains no $\tilde{s}p_c$-limit points of (G, A). Then, (G, A) contains all $\tilde{s}p_c$-limit points of (G, A) which implies that (G, A) is $\tilde{s}p_c$-closed in $(G, A) \tilde{\cup} (F, A) = (V, A)$. Similarly (F, A) is $\tilde{s}p_c$–closed in (V, A).
The converse part is obvious.

Theorem 3.7 *If* (F, A) *is an* $\tilde{s}p_c$*-connected set and* $(F, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}p_ccl(F, A)$, *then* (G, A) *is* $\tilde{s}p_c$*-connected.*

Proof. Let $(F, A) \in SP(X)_A$ be an $\tilde{s}p_c$-connected set and (G, A) be any soft subset of $\tilde{X}$ such that $(F, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}p_ccl(F, A)$. We have to show that (G, A) is an $\tilde{s}p_c$-connected set. Suppose that (G, A) is not $\tilde{s}p_c$-connected. Then, there exists a pair of $\tilde{s}p_c$-separated sets (F_1, A) and (F_2, A) such that $(G, A) = (F_1, A) \tilde{\cup} (F_2, A)$. Since $(F, A) \tilde{\subseteq} (G, A)$, then $(F, A) \tilde{\subseteq} (F_1, A) \tilde{\cup} (F_2, A)$. If $(F, A) \tilde{\cap} (F_1, A) \neq \tilde{\emptyset}$ and $(F, A) \tilde{\cap} (F_2, A) \neq \tilde{\emptyset}$, then $(F, A) = ((F, A) \tilde{\cap} (F_1, A)) \tilde{\cup} ((F, A) \tilde{\cap} (F_2, A)$. But $(F, A) \tilde{\cap} (F_1, A)$ and $(F, A) \tilde{\cap} (F_2, A)$ are $\tilde{s}p_c$-separated sets. This is a contradiction to the $\tilde{s}p_c$-connectivity of (F, A). Hence, either $(F, A) \tilde{\subseteq} (F_1, A)$ or $(F, A) \tilde{\subseteq} (F_2, A)$. Suppose that

$(F,A) \tilde{\subseteq} (F_1,A)$, then $\tilde{s}p_c cl(F,A) \tilde{\subseteq} \tilde{s}p_c cl(F_1,A)$. Since (F_1,A) and (F_2,A) are $\tilde{s}p_c$-separated sets, then $\tilde{s}p_c cl(F_1,A) \tilde{\cap} (F_2,A) = \tilde{\emptyset}$. Therefore, $\tilde{s}p_c cl(F,A) \tilde{\cap} (F_2,A) = \tilde{\emptyset}$ but $(F_2,A) \tilde{\subseteq} (G,A)$. Then, by the hypothesis, we have $(F_2,A) \tilde{\subseteq} (G,A) \tilde{\subseteq} \tilde{s}p_c cl(F,A)$. Therefore, $\tilde{s}p_c cl(F,A) \tilde{\cap} (F_2,A) = (F_2,A)$. Thus, $\tilde{s}p_c cl(F,A) \tilde{\cap} (F_2,A) = \tilde{\emptyset}$ and $\tilde{s}p_c cl(F,A) \tilde{\cap} (F_2,A) = (F_2,A)$. Hence, $(F_2,A) = \tilde{\emptyset}$, which is a contradiction. Similarly, if $(F,A) \tilde{\subseteq} (F_2,A)$, then we obtain that $(F_1,A) = \tilde{\emptyset}$, which is a contradiction. Therefore, there does not exist an $\tilde{s}p_c$-separation of (G,A). Hence, (G,A) is $\tilde{s}p_c$-connected.

Theorem 3.8 *Let (Y,τ_Y,A) be a soft subspace of a soft topological space (X,τ,A) and (F,A), $(G,A) \tilde{\subseteq} \tilde{Y} \tilde{\subseteq} \tilde{X}$. Then, (F,A), (G,A) are soft $\tilde{s}p_c$-separated in $\tilde{Y}$ if and only if (F,A), (G,A) are soft $\tilde{s}p_c$-separated in $\tilde{X}$.*

Proof. (F,A) *and* (G,A) are $\tilde{s}p_c$-separated in $\tilde{Y}$ $\Leftrightarrow$ $\tilde{s}p_c cl_Y(F,A) \tilde{\cap} (G,A) = (F,A) \tilde{\cap} \tilde{s}p_c cl_Y(G,A) = \tilde{\phi}$ $\Leftrightarrow$ $[\tilde{s}p_c cl_X(F,A) \tilde{\cap} \tilde{Y}] \tilde{\cap} (G,A) = \tilde{s}p_c cl_X(F,A) \tilde{\cap} (G,A) = \tilde{\phi}$ and $\tilde{s}p_c cl_Y(G,A) \tilde{\cap} (F,A) = [\tilde{s}p_c cl_X(G,A) \tilde{\cap} \tilde{Y}] \tilde{\cap} (F,A) = \tilde{s}p_c cl_X(G,A) \tilde{\cap} (F,A) = \tilde{\phi}$. $\Leftrightarrow$ (F,A) and (G,A) are $\tilde{s}p_c$-separated in $\tilde{X}$.

4 $\tilde{s}p_c$-connectedness

In this section, we introduce the concept of $\tilde{s}p_c$-connected spaces by using $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed sets in soft topological spaces. Basic properties of this space are obtained.

Definition 4.1 *Let (X,τ,A) be a soft topological space. A soft set $(F,A) \in SP(X)_A$ is said to be $\tilde{s}p_c$-connected if it does not have an $\tilde{s}p_c$-separation, otherwise it is called $\tilde{s}p_c$-disconnected.*

Remark 4.2 *The following statements are obvious.*

1. The singleton soft set in a soft topological space is an $\tilde{s}p_c$-connected set.
2. All soft subsets in the soft indiscrete topological space are $\tilde{s}p_c$-connected.

Definition 4.3 *A soft subset (F,A) of a soft topological space (X,τ,A) is said to be soft pre-connected if it does not have a soft pre-separation, otherwise it is called soft pre-disconnected.*

Proposition 4.4 *Every soft pre-connected set is an $\tilde{s}p_c$-connected set.*

Proof. Let (F,A) be a soft pre-connected set in the soft topological space (X,τ,A). Hence there does not exist a soft pre-separated set of (F,A). Since every $\tilde{s}p_c$- open set is a soft pre-open set, hence there does not exist an $\tilde{s}p_c$-separated set of (F,A). So, (F,A) is $\tilde{s}p_c$-connected.

If the soft topological space is a soft T_1, then by Proposition 2.14, the converse of Proposition 3.9 is also true.

An $\tilde{s}p_c$-connected set needs not be a soft pre-connected set as it is shown in the following example:

Example 4.1 *Any subset in the indiscrete soft topological space is $\tilde{s}p_c$-connected and since the family of soft pre-open sets forms the discrete soft topology, then any soft subset containing more than one soft point is not soft pre-connected.*

Theorem 4.5 *Let (X,τ,A) be a soft topological space and (F,A) be an $\tilde{s}p_c$-connected set. Let (F_1,A) and (F_2,A) be $\tilde{s}p_c$-separated sets. If $(F,A) \tilde{\subseteq} (F_1,A) \tilde{\cup} (F_2,A)$, then either $(F,A) \tilde{\subseteq} (F_1,A)$ or $(F,A) \tilde{\subseteq} (F_2,A)$.*

Proof. If $(F,A) \tilde{\not\subseteq} (F_1,A)$ and $(F,A) \tilde{\not\subseteq} (F_2,A)$, then $(G,A) = (F,A) \tilde{\cap} (F_1,A) \neq \tilde{\emptyset}$, $(H,A) = (F,A) \tilde{\cap} (F_2,A) \neq \tilde{\emptyset}$, and $(F,A) = (G,A) \tilde{\cup} (H,A)$. Since $(G,A) \tilde{\subseteq} (F_1,A)$, implies that $\tilde{s}p_c cl(G,A) \tilde{\subseteq} \tilde{s}p_c cl(F_1,A)$. Since (F_1,A), (F_2,A) are $\tilde{s}p_c$-separated sets, we have $\tilde{s}p_c cl(F_1,A) \tilde{\cap} (F_2,A) = \tilde{\emptyset}$. Therefore, $\tilde{\emptyset} = \tilde{s}p_c cl(F_1,A) \tilde{\cap} (F_2,A) \tilde{\supseteq} \tilde{s}p_c cl(G,A) \tilde{\cap} (F_2,A) \tilde{\supseteq} \tilde{s}p_c cl(G,A) \tilde{\cap} (H,A)$. Hence, $\tilde{s}p_c cl(G,A) \tilde{\cap} (H,A) = \tilde{\emptyset}$. By the same way, we can show that $(G,A) \tilde{\cap} \tilde{s}p_c cl(H,A) = \tilde{\emptyset}$, but $(F,A) = (G,A) \tilde{\cup} (H,A)$. Hence, there exists an $\tilde{s}p_c$-separation of (F,A). Therefore, (F,A) is not an $\tilde{s}p_c$-connected set, which is a contradiction. Hence, either $(F,A) \tilde{\subseteq} (F_1,A)$ or $(F,A) \tilde{\subseteq} (F_2,A)$.

Corollary 4.6 *If (F,A) is an $\tilde{s}p_c$-connected set, then $\tilde{s}p_c cl(F,A)$ is $\tilde{s}p_c$-connected.*

Proof. The proof follows directly from Theorem 3.7 .

Theorem 4.7 *The soft union (F,A) of any family $\{(F_i,A): i \in I\}$ of $\tilde{s}p_c$-connected sets having a nonempty soft intersection is an $\tilde{s}p_c$-connected set.*

Proof. Let (F,A) be a soft union of any family of $\tilde{s}p_c$-connected sets having a non-empty soft intersection. Suppose that $(F,A) = (H_1,A) \tilde{\cup} (H_2,A)$, where (H_1,A) and (H_2,A) form an $\tilde{s}p_c$-separation of (F,A). By hypothesis, we may choose a soft point

$$x_\alpha \in \bigcap_{i \in I} (F_i,A).$$

Then, $x_\alpha \in (F_i, A)$ for all $i \in I$. So $x_\alpha \in (F, A)$, and then either $x_\alpha \in (H_1, A)$ or $x_\alpha \in (H_2, A)$. Since (H_1, A) and (H_2, A) are soft disjoint, we must have $(F_i, A) \tilde{\subseteq} (H_1, A)$ (say), since (F_i, A) is $\tilde{s}p_c$-connected and it is true for all $i \in I$, and so $(F, A) \tilde{\subseteq} (H_1, A)$. From this we obtain that $(H_2, A) = \emptyset$, which is a contradiction.

Definition 4.8 *A soft topological space (X, τ, A) is said to be a $\tilde{s}p_c$-connected space if it does not have an $\tilde{s}p_c$-separation, otherwise it is called $\tilde{s}p_c$-disconnected.*

It is clear that every soft indiscrete topological space with at least two soft points is an $\tilde{s}p_c$-connected space and is soft pre-disconnected.

Definition 4.9 *A soft topological space (X, τ, A) is said to be a soft pre-connected space if it does not have a soft pre-separation, otherwise it is called soft pre-disconnected.*

Proposition 4.10 *Every soft pre-connected space is an $\tilde{s}p_c$-connected. Furthermore, the converse is also true if the space is soft T_1.*

Proof. Similar to the proof of Proposition 4.4 and the converse follows from Proposition 2.14.

Corollary 4.11 *A soft topological space (X, τ, A) is $\tilde{s}p_c$-disconnected if and only if there exists a non-empty proper soft subset of $\tilde{X}$ which is $\tilde{s}p_c$-clopen.*

Proof. It follows directly from (1) and (4) of Theorem 3.2.

Proposition 4.12 *If (X, τ, A) is a soft disconnected space, then it is $\tilde{s}p_c$-disconnected.*

Proof. If (X, τ, A) is a soft disconnected space, then it contains a non-empty proper soft clopen set (F, A), so (F, A) is both $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed. Hence, by Corollary 4.11, (X, τ, A) is $\tilde{s}p_c$-disconnected.

Proposition 4.13 *Let (X, τ, A) be a finite soft topological space. If (X, τ, A) contains a non-empty proper $\tilde{s}p_c$-open set, then (X, τ, A) is a soft disconnected space and hence, $\tilde{s}p_c$-disconnected.*

Proof. In a finite space (X, τ, A), if (F, A) is a non-empty proper $\tilde{s}p_c$-open set, then (F, A) is a union of soft closed sets and hence it is closed. Also (F, A) is pre-open, implies that $(F, A) \tilde{\subseteq} \tilde{s}int\tilde{s}cl(F, A)$, so $(F, A) \tilde{\subseteq} \tilde{s}int(F, A)$ which implies that (F, A) is soft open. Therefore, (X, τ, A) is soft disconnected and by Proposition 4.12 it is $\tilde{s}p_c$-disconnected.

Corollary 4.14 A finite soft space (X, τ, A) is $\tilde{s}p_c$-disconnected ($\tilde{s}p_c$-connected) if and only if it is soft disconnected (soft connected).

Proof. Follows from Proposition 4.12 and Proposition 4.13.

The following example shows that a soft connected space may not be $\tilde{s}p_c$-connected.

Example 4.2 Let $\mathbb{R}$ be the set of real numbers and $A = \{e_1, e_2\}$ and let $\tilde{\tau}$ be a family consisting of $\tilde{\phi}$ and all soft subsets (F, A) such that $F(e_1) = B$ where $\mathbb{R}\backslash B$ is finite and $F(e_2) = D$ where $D \subseteq \mathbb{R}$. Then, this space is soft connected but it is $\tilde{s}p_c$-disconnected because the soft sets (G, A) such that $G(e_1) = \mathbb{Q}$, $F(e_2) = D$, where $D \subseteq R$ and (H, A) such that $H(e_1) = \mathbb{R}\backslash\mathbb{Q}$ and $F(e_2) = D^c$, where $D \subseteq \mathbb{R}$, are disjoint $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed and their union is $\widetilde{\mathbb{R}}$.

In Proposition 4.13, the condition of the soft topological space to be finite is necessary. The following example shows that there exists a non-empty proper $\tilde{s}p_c$-open set in the space but the space is neither soft disconnected nor $\tilde{s}p_c$-disconnected.

Example 4.3 Let X be any infinite set, A consists of infinite parameters, P_e is a fixed soft point in $\tilde{X}$, and let $\tilde{\tau}$ be the family of all soft subsets (F, A) such that either $P_e \tilde{\notin} (F, A)$ or, if $P_e \tilde{\in} (F, A)$, then $\bigcup_{e \in E} X\backslash F(e)$ is finite. Then, this space contains a non-empty proper $\tilde{s}p_c$-open set but it is neither soft disconnected nor $\tilde{s}p_c$-disconnected .

In the following example we show that it is not necessarily true that if the space is $\tilde{s}p_c$-connected then every subspace must be $\tilde{s}p_c$-connected.

Example 4.4 *Let $X = \{x, y\}$ and $A = \{e_1, e_2\}$. Let*
$\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ be the soft topology where
$(F_1, A) = \{(e_1, \{x\}), (e_2, \phi)\}$, $(F_2, A) = \{(e_1, \{x\}), (e_2, \{y\})\}$,
$(F_3, A) = \{(e_1, X), (e_2, \{y\})\}$, $(F_4, A) = \{(e_1, \phi), (e_2, \{y\})\}$.
This space is both soft connected and $\tilde{s}p_c$-connected. Consider the soft subset $(F_2, A) = \{(e_1, \{x\}), (e_2, \{y\})\}$. Then the soft relative topology induced on the soft set (F_2, A) is $\tau_{(F_2,A)} = \{\widetilde{\emptyset}, (F_2, A), (F_1, A), (F_4, A)\}$. Then, $\tilde{s}p_cO((F_5, A)) = \tau_{(F_2,A)}$. Therefore, the soft set (F_2, A) has an $\tilde{s}p_c$-separation in the soft relative topology induced on the soft subset (F_2, A). Hence, (F_2, A) is an $\tilde{s}p_c$-disconnected subset of a soft topological space (X, τ, A) .

Corollary 4.15 *A soft topological space (X,τ,A) is an $\tilde{s}p_c$-disconnected space if and only if any one of the following statements is satisfied.*

1. $\tilde{X}$ is a soft union of two non empty disjoint $\tilde{s}p_c$-open sets.
2. $\tilde{X}$ is a soft union of non empty disjoint $\tilde{s}p_c$-closed sets.

Proof. It follows directly from parts 1,2, and 3 of Theorem 3.2.

Theorem 4.16 *Let (X,τ_1,A) be a $\tilde{s}p_c$-connected space and $\tau_2 \tilde{\subseteq} \tau_1$. Then, (X,τ_2,A) is $\tilde{s}p_c$-connected.*

Proof. Suppose on the contrary that (X,τ_2,A) is not $\tilde{s}p_c$-connected, and let (H,A) and (G,A) be an $\tilde{s}p_c$-separation of (X,τ_2,A). Since $\tilde{\tau}_2 \tilde{\subseteq} \tilde{\tau}_1$ then (H,A) and (G,A) are $\tilde{s}p_c$-separation of (X,τ_1,A), which is a contradiction. Therefore, (X,τ_2,A) is $\tilde{s}p_c$-connected.

Theorem 4.17 *Let (X,τ_1,A) and (Y,τ_2,B) be two soft topological spaces and $u: X \to Y$ and $p: A \to B$ be functions. Also a soft function $f_{pu}: SP(X)_A \to SP(Y)_B$ is $\tilde{s}p_c$-continuous and onto. If (X,τ_1,A) is $\tilde{s}p_c$-connected, then the soft image (Y,τ_2,B) is soft connected*

Proof. Let a soft function $f_{pu}: SP(X)_A \to SP(Y)_B$ be $\tilde{s}p_c$-continuous and onto. On the contrary, suppose that (Y,τ_2,B) is soft disconnected and let (H,B) be a soft non-empty proper subset of (Y,τ_2,B) which is both soft open and soft closed. Since $f_{pu}: SP(X)_A \to SP(Y)_B$ is soft $\tilde{s}p_c$-continuous. Therefore, by Lemma 2.7, $f_{pu}^{-1}(H,B)$ is both $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed in (X,τ_1,A), which is a contradiction. Hence (Y,τ_2,B) is soft connected.

Definition 4.18 *Let (X,τ,A) be a soft topological space and $x_\alpha \in (F,A) \in SP(X)_A$. The $\tilde{s}p_c$-component of (F,A) corresponding to x_α is the union of all $\tilde{s}p_c$-connected subsets in (F,A) containing x_α.*

From Theorem 4.7, we obtain that the $\tilde{s}p_c$-component of X is $\tilde{s}p_c$-connected.

Theorem 4.19 *For a topological space (X,τ,A), the following properties hold:*

1. Each $\tilde{s}p_c$-component of X is a maximal $\tilde{s}p_c$-connected subset of X.
2. The set of all distinct $\tilde{s}p_c$-components of X forms a partition of X.
3. Each $\tilde{s}p_c$-component of X is $\tilde{s}p_c$-closed in X.

Proof. (1) Obvious.

(2) Each soft point $x_\alpha \in SP(X)_A$ is contained in an $\tilde{s}p_c$-component of X containing x_α. Suppose that (F_1,A) and (F_2,A) are two distinct $\tilde{s}p_c$-components of X. If (F_1,A) and (F_2,A) intersect, then by Theorem 3.13, $(F_1,A) \tilde{\cup} (F_2,A)$ is $\tilde{s}p_c$-connected. Thus, either (F_1,A) is not maximal or (F_2,A) is not maximal, which is a contradiction. Therefore, (F_1,A) and (F_2,A) are disjoint.

(3) Let (F,A) be any $\tilde{s}p_c$-component of X containing x_α. By Corollary 4.6, $\tilde{s}p_c cl(F,A)$ is $\tilde{s}p_c$-connected set containing x_α. Since (F,A) is maximal $\tilde{s}p_c$-connected set containing x_α, so $\tilde{s}p_c cl(F,A) \tilde{\subseteq} (F,A)$. Thus, (F,A) is $\tilde{s}p_c$-closed in $\tilde{X}$.

Theorem 4.20 *A soft topological space (X,τ,A) is $\tilde{s}p_c$-disconnected if and only if there is a non-empty proper subset which has an empty soft $\tilde{s}p_c$-boundary.*

Proof. Let (X,τ,A) be soft $\tilde{s}p_c$-disconnected. Then by Corollary 4.11, (X,τ,A) has a proper $\tilde{s}p_c$-clopen soft set (F,A). Thus, $\tilde{s}p_c cl(F,A) = (F,A) = \tilde{s}p_c int(F,A) = \tilde{X} \backslash \{\tilde{s}p_c cl(\tilde{X} \backslash (F,A))\}$. Hence, $\tilde{s}p_c bd(F,A) = \tilde{s}p_c cl((F,A)) \tilde{\cap} \tilde{s}p_c cl(\tilde{X} \backslash (F,A)) = \tilde{\phi}$. Therefore, (F,A) has an empty soft $\tilde{s}p_c$-boundary.

Conversely, suppose that there is a non-empty proper soft subset (F,A) that has an empty soft $\tilde{s}p_c$ -boundary. Then, $\tilde{s}p_c bd(F,A) = \tilde{s}p_c cl(F,A) \tilde{\cap} \tilde{s}p_c cl(\tilde{X} \backslash (F,A)) = \tilde{\phi}$. Consequently, $\tilde{s}p_c cl(F,A) \tilde{\subseteq} [\tilde{X} \backslash \tilde{s}p_c cl(\tilde{X} \backslash (F,A))] = \tilde{s}p_c int(F,A)$, and thus $(F,A) \tilde{\subseteq} \tilde{s}p_c cl(F,A) \tilde{\subseteq} \tilde{s}p_c int(F,A) \tilde{\subseteq} (F,A)$. Hence, (F,A) is a proper $\tilde{s}p_c$-clopen soft set and by Corollary 4.11 , (X,τ,A) is soft $\tilde{s}p_c$-disconnected.

Definition 4.21 *A space (X,τ,A) is called locally $\tilde{s}p_c$-connected at $x_\alpha \in SP(X)_A$. if for each $\tilde{s}p_c$-open set (G,A) containing x_α, there is an $\tilde{s}p_c$-connected $\tilde{s}p_c$-open set (H,A) such that $x_\alpha \in (H,A) \tilde{\subseteq} (G,A)$. The space X is locally $\tilde{s}p_c$-connected if it is locally $\tilde{s}p_c$-connected at each of its soft points.*

It is clear that the discrete soft space is locally $\tilde{s}p_c$-connected but it is not $\tilde{s}p_c$-connected.

Theorem 4.22 *A space (X,τ,A) is locally $\tilde{s}p_c$-connected if and only if the $\tilde{s}p_c$-components of each $\tilde{s}p_c$-open subset of X are $\tilde{s}p_c$-open.*

Proof. Suppose that $\tilde{X}$ is locally $\tilde{s}p_c$-connected , (G,A) *is an* $\tilde{s}p_c$*-open subset of* $\tilde{X}$ *and* (F,A) is an $\tilde{s}p_c$-component of the soft subset (G,A) corresponding to the soft point x_α. Then, by definition, there is an $\tilde{s}p_c$-connected $\tilde{s}p_c$-open set $(H,A) \tilde{\subseteq} \tilde{X}$ such that $x_\alpha \in (H,A) \tilde{\subseteq} (G,A)$. Since (F,A) is an $\tilde{s}p_c$-component of (G,A), so we get $x_\alpha \in (H,A) \tilde{\subseteq} (F,A)$. Thus, by Lemma 2.2, (F,A) is an $\tilde{s}p_c$-open set.

Conversely, let $(G,A) \tilde{\subseteq} \tilde{X}$ be an $\tilde{s}p_c$ -open set and $x_\alpha \in (G,A)$. By the hypothesis, the $\tilde{s}p_c$-component (H,A) of (G,A) containing x_α is $\tilde{s}p_c$-open, so X is locally $\tilde{s}p_c$-connected at x_α.

Theorem 4.23 *A soft topological space (X,τ,A) is locally $\tilde{s}p_c$ connected if and only if, given any soft point $x_\alpha \in SP(X)_A$ and a $\tilde{s}p_c$-open set (G,A) containing x_α, there is a soft open set (H,A) containing x_α such that (H,A) is contained in a single $\tilde{s}p_c$-component of (G,A).*

Proof. Let X be locally $\tilde{s}p_c$-connected, $x_\alpha \in SP(X)_A$ and (G,A) be an $\tilde{s}p_c$-open set containing x_α. Let (F,A) be the $\tilde{s}p_c$ component of (G,A) that contains x_α. Since X is locally $\tilde{s}p_c$ connected and (G,A) is $\tilde{s}p_c$ open, there is an $\tilde{s}p_c$ connected $\tilde{s}p_c$ -open set (H,A) such that $x_\alpha \in (H,A) \tilde{\subseteq} (G,A)$. By Theorem 4.14, (F,A) is the maximal $\tilde{s}p_c$ connected set containing x_α and so $x_\alpha \in (H,A) \tilde{\subseteq} (F,A) \tilde{\subseteq} (G,A)$. Since $\tilde{s}p_c$ components are disjoint sets, it follows that (H,A) is not contained in any other $\tilde{s}p_c$ component of (G,A).

Conversely, we suppose that, given any soft point $x_\alpha \in SP(X)_A$ and any $\tilde{s}p_c$-open set (G,A) containing x_α, there is a soft open set (H,A) containing x_α which is contained in a single $\tilde{s}p_c$-component (F_1,A) of (G,A). Then $x_\alpha \in (H,A) \tilde{\subseteq} (F_1,A) \tilde{\subseteq} (G,A)$. Let $x_{\alpha 1} \in (F_1,A)$, then $x_{\alpha 1} \in (G,A)$. Thus there is a soft open set (F_2,A) such that $x_{\alpha 1} \in (F_2,A)$ and (F_2,A) is contained in a single $\tilde{s}p_c$-component of (G,A). As the $\tilde{s}p_c$ components are disjoint soft sets and $x_{\alpha 1} \in (F_1,A)$, hence $x_{\alpha 1} \in (F_2,A) \tilde{\subseteq} (F_1,A)$. Thus, (F_1,A) is soft open. Hence, for every $x_\alpha \in SP(X)_A$ and for every $\tilde{s}p_c$-open set (G,A) containing x_α, there is an $\tilde{s}p_c$-connected $\tilde{s}p_c$-open set (F_1,A) such that $x_\alpha \in (F_1,A) \tilde{\subseteq} (G,A)$. Thus, X is locally $\tilde{s}p_c$-connected at x_α. Since $x_\alpha \in SP(X)_A$ is arbitrary, so X is locally $\tilde{s}p_c$-connected.

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