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## Effects of Hematocrit on Blood Flow Through A Stenosed Human Carotid Artery

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### Abstract

In this paper, the effects of hematocrit of red blood cells on blood flow through a stenosed human carotid artery was considered by taking blood as a Newtonian fluid. The governing equations on blood flow were derived. The mathematical content involved in the equations are the variables of interest such as number of stenosis ( $n$ ), percentage of hematocrit ( $H$ ) of red blood cells in the blood, flow rate, wall shear stress, and viscosity of the blood. Guided by medical data collected on the constraint of blood flow in stenosed human carotid arteries, the governing equations were used to check the effects of pressure gradient, wall shear stress, velocity, and volumetric flow rate of blood in the human carotid arteries. Also, the one-dimensional equation for the steady and axially symmetric flow of blood through an artery was transformed using Einstein's coefficient of viscosity and hematocrit of red blood cells with the help of the boundary conditions. The effects of hematocrit on the blood flow characteristics are shown graphically and discussed briefly. It was discovered that the resistance increases as the level of hematocrit increases. Also, the wall shear stress decreases with the increase in the hematocrit level of the red blood cells.

**Keywords:** stenosis, carotid artery, wall shear stress, hematocrit, blood viscosity.

### Introduction

The study of blood flow through arteries is of considerable importance in many cardiovascular diseases. A major characteristic of the blood depends upon the hematocrit ( $H$ ) level as it is the percentage of whole blood volume occupied by the red blood cells (RBC) (Medyedev and Fomin, [1]). Flow of blood through human carotid arteries is an important physiological problem, and it is of high interest for biomedical researchers, physiologists and clinicians. One of the important aspects of this problem that requires special attention is the deviation in the flow characters of blood when the carotid arteries are exposed to stenosis. The major characteristic of the blood depends upon the hematocrit level, as it is the percentage of whole blood occupied by the red blood cells. Hematocrit level of the blood flow in the artery depends upon the diameter of the artery, and this relationship has important implications for physiological phenomena related to blood flow.

Arteriosclerosis is the abnormal growth in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions. This may be caused by unhealthy living conditions such as exposure to tobacco smoke, lack of physical activity and improper dietary habits. It is always followed by the serious changes in blood flow, pressure distribution, wall shear stress and flow resistance. A mathematical theory which could be used to explain a clinical observation in some sickle-cell anaemia patients was developed by Ayeni and Akinrele [2]. It was shown that if the energy stored in the elastic walls of the arteries of such patients is accompanied by

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thermal effects, then the existence of a local hot spot on the body of the sickle-cell anaemia patients could be due to shock formation even when the viscosity of the blood is zero. On the other hand, if the energy stored is not accompanied by thermal effects, then a jump in the temperature can only occur if the viscosity is not zero. The location of such jump in the temperature and the time of its occurrence were determined.

Hematocrit is considered as an integral part of a person's complete blood count results, along with haemoglobin concentration, white blood cell count and platelet count.

In an interesting contribution, Kaukane and Bodke [3] considered the blood flow characteristics through an artery in the presence of overlapping stenosis, where the artery was assumed to have an axisymmetric shape. The arterial wall deformability was taken to be elastic (moving wall), the radial variation of radial velocity and the shape function distribution over the length of the artery were then examined. The equations of motion were solved numerically by finite difference scheme. The effects of RBCs deformability on the dispersion of the cells was numerically investigated by Meongkeum *et al.* [4] at physiological flow rates, i.e. pseudo shear rates (PSR) of 25-1005 and a hematocrit value of 40% .

The work of Vipin and Praveen [5] analysed the characteristics of blood flow through an artery with axisymmetric multiple stenosis in series. It was discovered in their work that the condition of critical stenosis depends on the inflow rates, and that multiple stenosis developed in series in a tube may lead to complete blockage of the lumen. Demiray [6] studied the propagation of nonlinear waves in a pre-stressed thin elastic tube filled with an incompressible inviscid fluid. Considering the physiological conditions under which the arteries function, in the analysis, the tube was assumed to be subjected to uniform inner pressure and axial stretch ratio. In the course of blood flow, a dynamical displacement field was superimposed on this static deformation. By treating the blood as an incompressible inviscid fluid, the nonlinear equations of motion of both the tube and the fluid were obtained.

Onitilo and Usman [7] investigated the mathematical analysis of blood flow through a stenosed human artery. It was revealed in their work that the presence of stenosis in human arteries can damage the veins around the arteries, which can result in clotting of blood in human heart and leading to death. Mehdi *et al.* [8] studied the turbulent pulsatile blood flow through stenosed arteries considering the elastic property of the wall which was investigated numerically. During the numerical model validation, both the standard K-E model and RNG K-E mode were used. Venkateswarlu and Anand [9] worked on the unsteady blood flow through an indented tube with atherosclerosis in the presence of mild stenosis. This was studied numerically by the finite difference method. The effects of hematocrit, frequency parameter, height of stenosis, parameters determining the shape of the constriction on velocity field, volumetric flow rate, pressure gradient of the fluid in stenotic region, and wall shear stress at the surface of stenosis were obtained and shown graphically.

In the present paper, the effects of hematocrit on blood flow through a stenosed human carotid artery are studied numerically, and the fluid characteristics were numerically and graphically solved.

### Materials and Methods

The schematic diagram of blood flow in the artery is shown in Figure-1. The problem was studied in cylindrical coordinate system  $(r, \theta, z)$ , where  $z$  axis is taken along the axis of the artery while  $r$  and  $\theta$  are along the radial and the circumferential directions, respectively.

The geometry of the stenosis in a one-dimensional form which develops symmetrically about the artery axis but non-symmetric with respect to the radial coordinate is given as

$$\frac{R(z)}{R_0} = 1 - g[(S_L)^{k-1}\{z - (d_1 + d_2 + d_3 + d_4)\} - \{z - (d_1 + d_2 + d_3 + d_4)\}^k] \\ (d_1 + d_2 + d_3 + d_4) \leq z \leq (d_1 + d_2 + d_3 + d_4) + S_L, \quad (1)$$

where  $g = \frac{\delta}{R_0(S_L)^k}$ ,  $d_1 + d_2 + d_3 + d_4$  are the distance of the stenosis region,  $z$  is the length of the artery,  $\delta$  is the maximum height of the stenosis,  $R_0$  is the radius of the vessel without stenosis and  $R(z)$  is the radius of the vessel with stenosis.

The one-dimensional equation for the steady and axially symmetric flow of blood through an artery provided with a mild stenosis under the above-mentioned assumption is

$$\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} [tr] = 0, \quad (2)$$

where  $p$  is the fluid pressure,  $\tau$  is the shear stress,  $[r, z]$  are the cylindrical polar co-ordinates with  $z$  is measured along the stenosis axis and  $r$  is measured normal to the axis of the stenosis. According to Lih [10], the Einstein coefficient of viscosity of blood was given as:

$$\mu(r) = \mu_0[1 + \alpha h(r)], \tag{3}$$

where  $\mu_0$  is the coefficient of viscosity of plasma,  $\alpha = 2.5$ , and  $h^*(r)$  is hematocrit described by the formula

$$h(r) = H \left[ 1 - \left( \frac{r}{R_0} \right)^n \right]. \tag{4}$$

The boundary conditions are:

$$\begin{aligned} u &= 0, & \text{at } r &= R(z) \\ \frac{du}{dr} &= 0, & \text{at } r &= 0. \end{aligned} \tag{5}$$

Let  $x = \frac{r}{R_0}$  (6)

be the radial coordinate, where  $R_0$  is the inner radius of the vessel and  $x$  is the arterial wall viscosity. Then by substituting equation (4) into (3), we have

$$\mu(r) = \mu_0 \left[ 1 + \alpha H - \alpha H \left( \frac{r}{R_0} \right)^n \right], \tag{7}$$

Further using of equation (6) can results in

$$\mu(r) = \mu_0 [a_1 - a_2 x^n], \tag{8}$$

where  $a_1 = 1 + \alpha H$ , and  $a_2 = \alpha H$ . (9)

The equation of wall shear stress is given as:

$$\tau = -\mu(r) \frac{du}{dr} \tag{10}$$

Substituting equation (8) into (10) gives

$$\tau = -\mu_0 [a_1 - a_2 x^n] \frac{du}{dr} \tag{11}$$

Using equation (6) in (11) gives,

$$\tau = -\mu_0 [a_1 - a_2 x^n] \frac{du}{dx} \cdot \frac{1}{R_0} \tag{12}$$

Substituting equation (12) into equation (2) yields

$$\frac{\partial p}{\partial z} = \frac{\mu_0}{R_0^2} \frac{1}{x} \frac{d}{dx} [x [a_1 - a_2 x^n]] \frac{du}{dx} \tag{13}$$

And further simplification gives:

$$\frac{R_0^2}{\mu_0} \frac{dp}{dz} = \frac{1}{x} \frac{d}{dx} [x (a_1 - a_2 x^n)] \frac{du}{dx} \tag{14}$$

The boundary condition in equation (5) becomes

$$\begin{aligned} u &= 0, & \text{at } x &= \frac{R(z)}{R_0} \\ \frac{du}{dx} &= 0, & \text{at } x &= 0 \end{aligned} \tag{15}$$

Equation (14) and the boundary condition (15) are applicable only when  $n \geq 2$ . In this paper,  $n$  is taken as 4 (i.e  $n = 4$ ).

Method of Solution

Equation (14) is solved with the help of boundary conditions (15), which gives:

$$\frac{d}{dx} [x (a_1 - a_2 x^n)] \frac{du}{dx} = \frac{x R_0^2}{\mu} \frac{dp}{dz} \tag{16}$$

On integrating equation (16), we have

$$x (a_1 - a_2 x^n) \frac{du}{dx} = \frac{x^2 R_0^2}{2\mu} \frac{dp}{dz} + C, \tag{17}$$

where  $C$  is the constant of integration.

Noting that  $\frac{du}{dx} = 0$ , at  $x = 0$ , then  $C = 0$ . (18)

Therefore, equation (17) becomes

$$x (a_1 - a_2 x^n) \frac{du}{dx} = \frac{x^2 R_0^2}{2\mu} \frac{dp}{dz}. \tag{19}$$

By solving equation (19) with the boundary condition in equation (15) analytically, we have

$$u(x) = \frac{a_1}{2a_2} \left[ \left( \frac{R(z)}{R_0} \right)^{-2} - \frac{1}{x^2} \right] + \frac{R_0^2}{2\mu_0 a_2} \frac{dp}{dz} \left[ \frac{1}{x} - \left( \frac{R(z)}{R_0} \right)^{-1} \right], \tag{20}$$

which is the velocity of the blood.

The volumetric flow rate  $Q$  is given by,

$$Q = 2\pi R_0 \int_0^{R/R_0} x u(x) dx. \tag{21}$$

Hence, substituting equation (20) into (21) yields,

$$Q = \frac{\pi R_0 a_1}{2a_2} - \frac{\pi R_0 a_1}{a_2} \log\left(\frac{R(z)}{R_0}\right) + \frac{\pi R_0^2}{\mu_0 a_2} R(z) \frac{dp}{dz} \tag{22}$$

The flow rate of plasma fluid in the unconstricted tube is given as,

$$Q_0 = -\frac{\pi R_0^4}{8\mu} \left(\frac{dp}{dz}\right)_0 \tag{23}$$

By equating equations (22) and (23), we obtain

$$\frac{\pi R_0^2}{\mu_0 a_2} R(z) \frac{dp}{dz} = \frac{\pi R_0 a_1}{a_2} \log\left(\frac{R(z)}{R_0}\right) - \frac{\pi R_0 a_1}{2a_2} - \frac{\pi R_0^4}{8\mu} \left(\frac{dp}{dz}\right)_0 \tag{24}$$

By putting  $S_L = 1$ ,  $k = 2$ ,  $d_1 = 1/2$ ,  $d_2 = 1/4$ ,  $d_3 = 1/8$ ,  $d_4 = 1/16$ ,  $g = 1/2$  into equation (1), we have

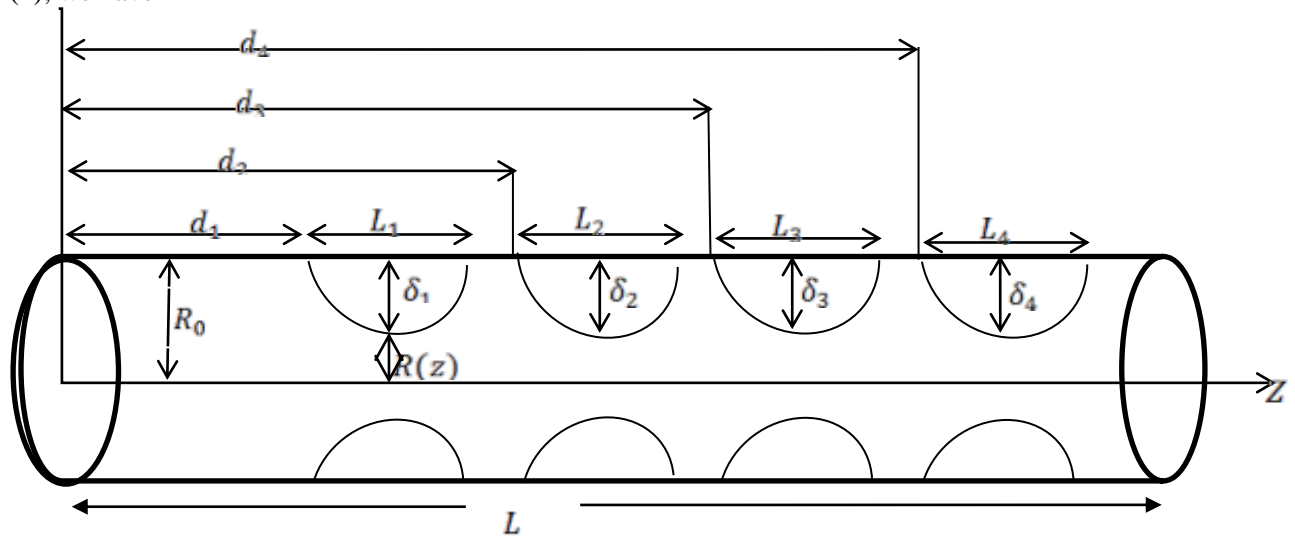


Figure 1-Schematic Diagram of Blood Vessels with Stenosis

$$\frac{R(z)}{R_0} = 1 - \frac{1}{2} \left[ 1 \left\{ z - \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right) \right\} - \left\{ z - \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right) \right\}^2 \right] \tag{25}$$

Further simplification of equation (25) gives

$$\frac{R(z)}{R_0} = 1 - \left[ \frac{256z^2 - 736z + 977}{512} \right], \tag{26}$$

$$\text{where } \gamma = 256z^2 - 736z + 977. \tag{27}$$

We now take the logarithm of both sides to have

$$\log\left(\frac{R(z)}{R_0}\right) = \log\left(\frac{\gamma}{512}\right). \tag{28}$$

Substituting equations (26) and (28) into equation (24) gives

$$\frac{dp/dz}{\left(\frac{dp}{dz}\right)_0} = \frac{64R_0 a_2}{\gamma Q_0} \left[ -\frac{\pi R_0 \mu_0 a_1}{\mu a_2} \log\left(\frac{\gamma}{512}\right) + \frac{\pi \mu_0 R_0 a_1}{2\mu a_2} - \frac{\mu_0}{\mu} Q_0 \right]. \tag{29}$$

Putting  $Q = Q_0$  and  $\mu = \mu_0$  yields

$$\frac{dp/dz}{\left(\frac{dp}{dz}\right)_0} = \frac{64R_0 a_2}{\gamma Q_0} \left[ \frac{\pi R_0 a_1}{2a_2} - Q - \frac{\pi R_0 a_1}{a_2} \log\left(\frac{\gamma}{512}\right) \right], \tag{30}$$

where equation (32) is the blood pressure gradient.

The wall shear stress of artery is defined by

$$\tau_w = \mu \left(\frac{du}{dr}\right)_{r=R(z)}. \tag{31}$$

$$\text{If } u = \frac{a_1}{2a_2} \left[ \left(\frac{R(z)}{R_0}\right)^{-2} - \frac{1}{x^2} \right] + \frac{R_0^2}{2\mu_0 a_2} \frac{dp}{dz} \left[ \frac{1}{x} - \left(\frac{R(z)}{R_0}\right)^{-1} \right], \tag{32}$$

then

$$x = \frac{r}{R_0} . \tag{33}$$

Let us observe that equation (32) can be written as

$$u = \frac{a_1}{2a_2} \left[ \frac{r^2}{x^2(R(z))^2} - \frac{1}{x^2} \right] + \frac{R_0^2}{2\mu_0 a_2} \frac{dp}{dz} \left[ \frac{1}{x} - \frac{r}{xR(z)} \right] \tag{34}$$

Then by differentiating it, we obtain

$$\frac{du}{dr} = \frac{a_1 r}{a_2 x^2 (R(z))^2} + \frac{R_0^2}{2\mu_0 a_2} \frac{dp}{dz} \left( \frac{-1}{xR(z)} \right) . \tag{35}$$

Since  $r = R(z)$ , we have

$$\tau_w = \mu \left[ \frac{a_1}{a_2 x^2 r} - \frac{R_0^2}{2\mu_0 a_2} \frac{dp}{dz} \left( \frac{1}{xr} \right) \right] \tag{36}$$

From equation (8),  $\mu = \mu_0(a_1 - a_2 x^n)$

Substituting  $n = 4$  into equation (8) gives

$$\mu = \mu_0(a_1 - a_2 x^4) \tag{37}$$

Substituting equation (37) into equation (36), yields

$$\tau_w = \frac{\mu_0}{x^3} (a_1 - a_2 x^3) \left[ \frac{a_1 x}{a_2 r} - \frac{R_0^2}{2\mu_0 a_2} \cdot \frac{x^2}{r} \frac{dp}{dz} \right] . \tag{38}$$

But  $\frac{1}{R_0} = \frac{x}{r}$ , therefore equation (38) becomes

$$\tau_w = \frac{\mu_0}{x^3} (a_1 - a_2 x^4) \left[ \frac{a_1}{a_2 R_0} - \frac{x R_0}{2\mu_0 a_2} \frac{dp}{dz} \right] . \tag{39}$$

If  $\tau_N$  is the shear stress of plasma fluid at the normal artery wall, then it is defined as

$$\tau_N = \left( -\frac{R_0}{2} \right) \left( \frac{dp}{dz} \right)_0 . \tag{40}$$

Therefore, the shear stress is

$$\tau = \frac{\tau_w}{\tau_N} . \tag{41}$$

Substituting equations (39) and (40) into equation (41) gives

$$\tau = \frac{2\mu_0}{R_0^2} \left\{ \left( m_2 x - \frac{m_1}{x^3} \right) \left[ \frac{m_1}{m_2 \left( \frac{dp}{dz} \right)_0} - \frac{R_0^2 x}{2\mu_0 m_2} \frac{dp/dz}{\left( \frac{dp}{dz} \right)_0} \right] \right\} . \tag{42}$$

But  $r = R(z)$ , and  $x = \frac{r}{R_0} = \frac{R(z)}{R_0} = \frac{\gamma}{512}$ . (43)

Substituting equation (43) into equation (42) gives

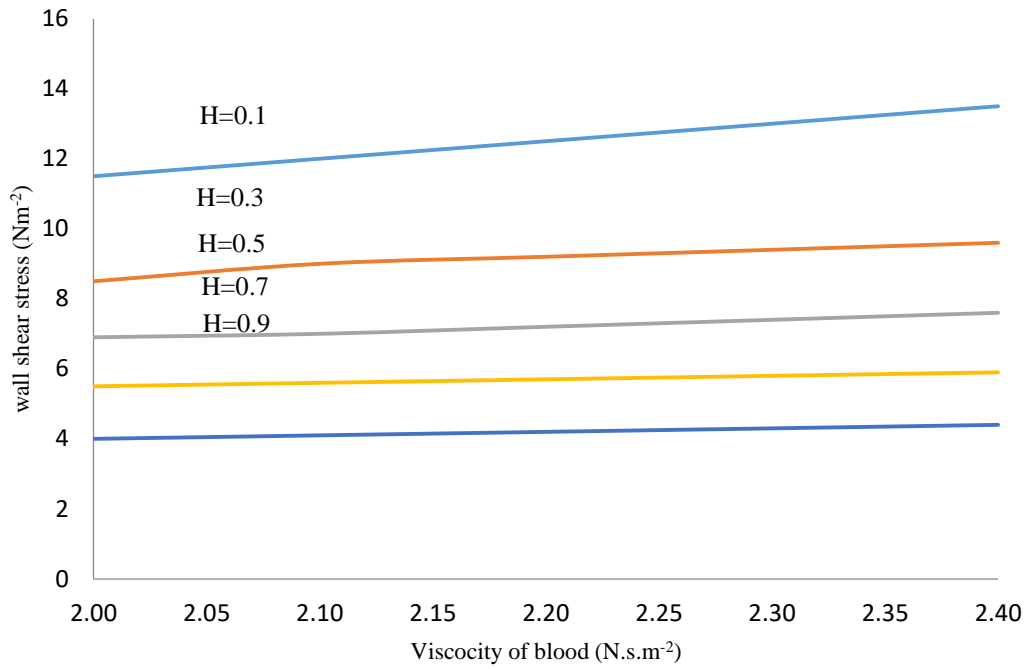
$$\tau = \frac{2\mu_0}{R_0^2} \left( \frac{\gamma}{512} a_2 - \left( \frac{512}{\gamma} \right)^3 a_1 \right) \left[ \frac{a_1}{a_2 \left( \frac{dp}{dz} \right)_0} - \frac{\gamma R_0^2}{1024 \mu_0 a_2} \cdot \frac{dp/dz}{\left( \frac{dp}{dz} \right)_0} \right] \tag{44}$$

which is the arterial wall shear stress.

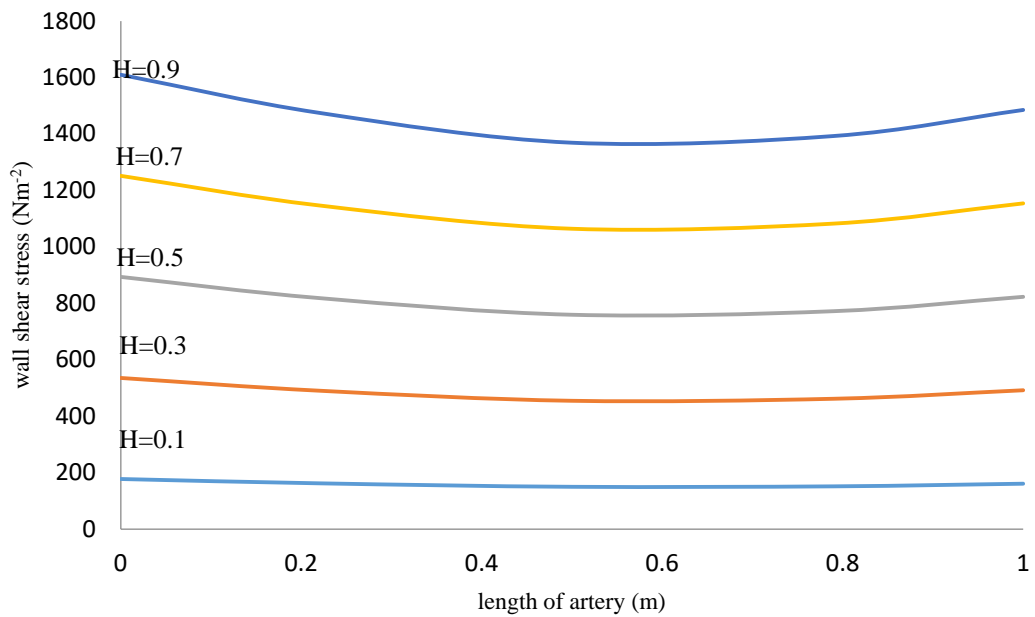
**Results and Discussion**

With the following values collected medically, the following results were derived.

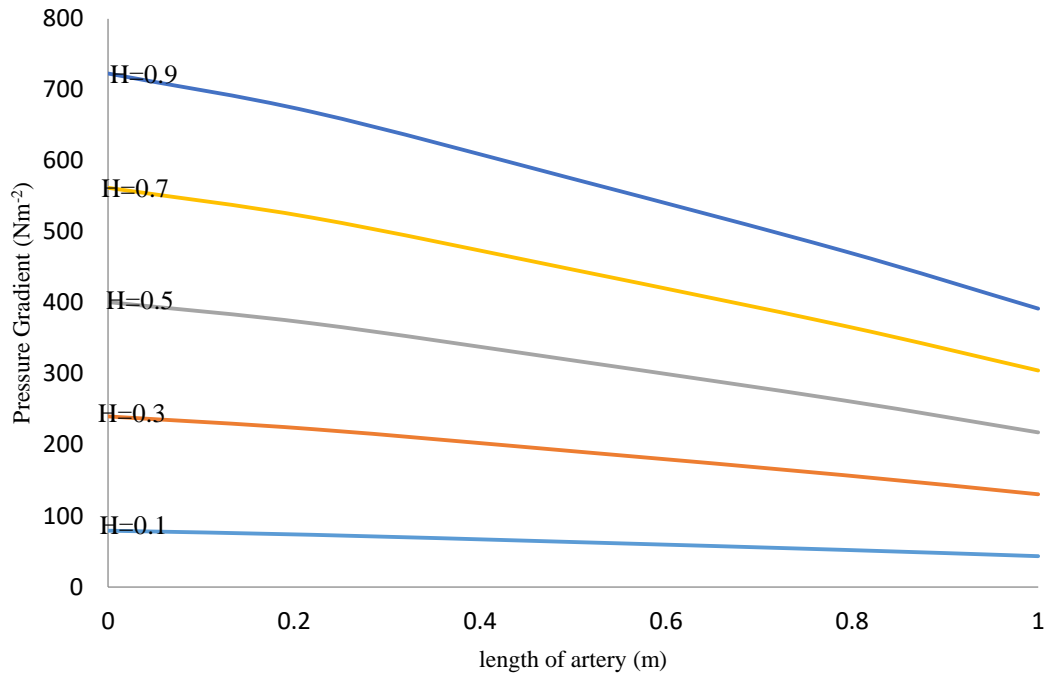
$\alpha = 2.5$ ,  $S_L = 1$ ,  $k = 2$ ,  $d_1 = \frac{1}{2}$ ,  $d_2 = \frac{1}{4}$ ,  $d_3 = \frac{1}{8}$ ,  $d_4 = \frac{1}{16}$ ,  $g = \frac{1}{2}$ ,  $z = 1$  and  $n = 4$ . The hematocrit values were taken as  $H = 0.1, 0.3, 0.5, 0.7$  and  $0.9$ .



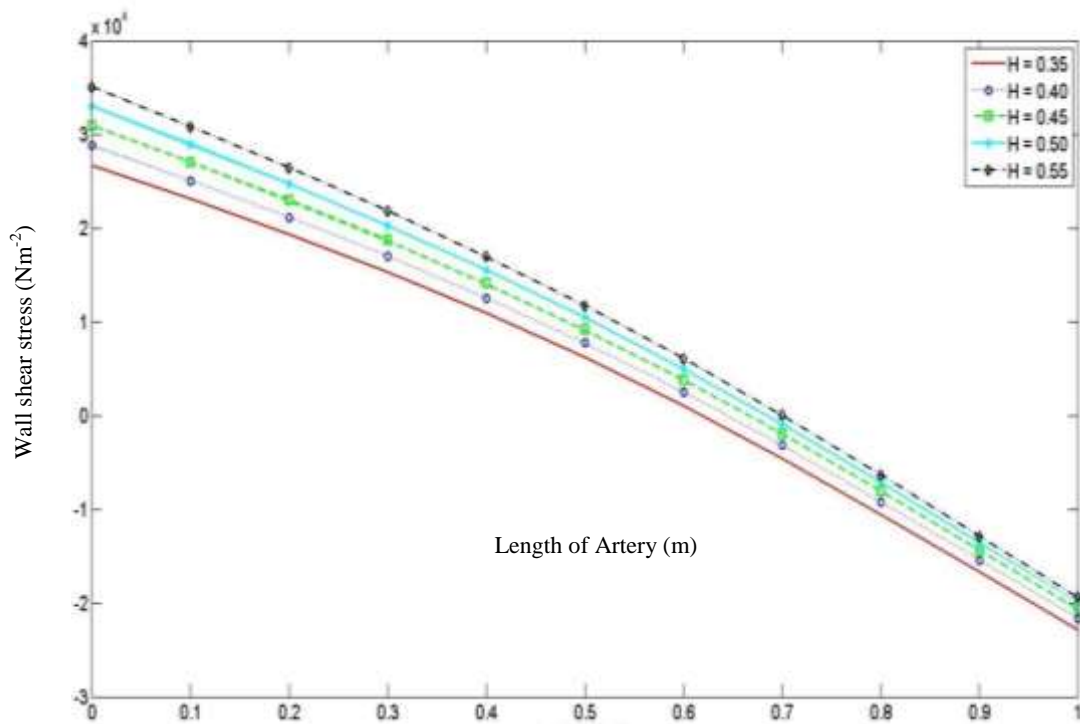
**Figure 2**-The variation of the wall shear stress along blood viscosity ( $\mu$ ) for different values of hematocrit ( $H$ ) with the length of the artery  $Z = 1$ , when the number of stenosis is 4 ( $n = 4$ ).



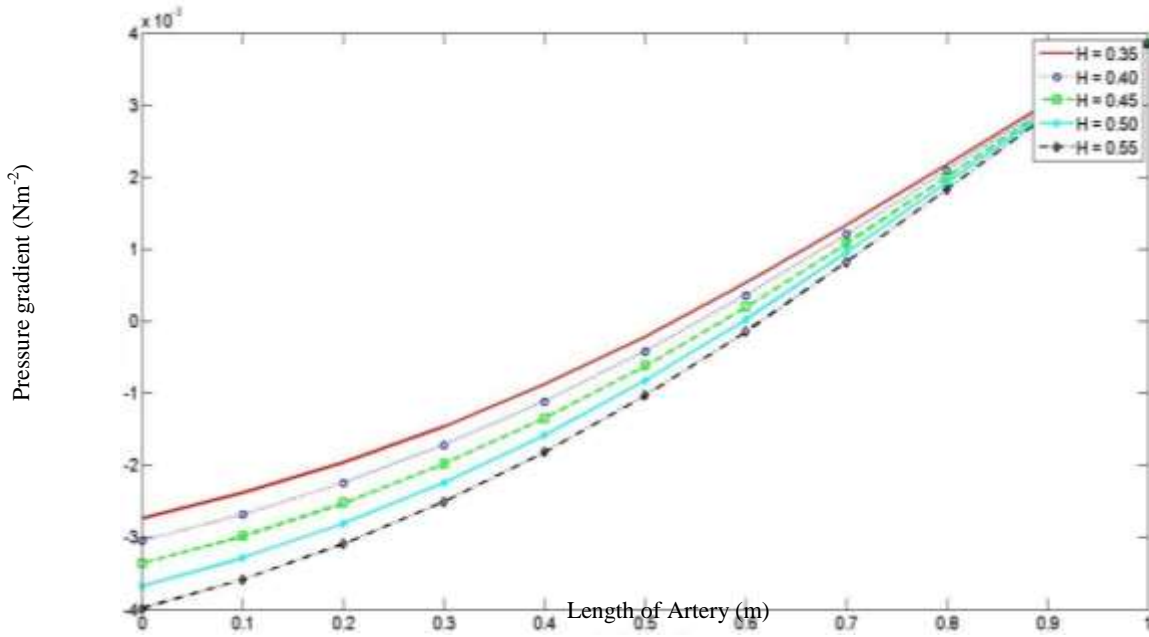
**Figure 3**-The variation of the wall shear stress along the length of artery for different values of hematocrit ( $H$ ) with viscosity ( $\mu$ ) = 2, when the number of stenosis is 4 ( $n = 4$ ).



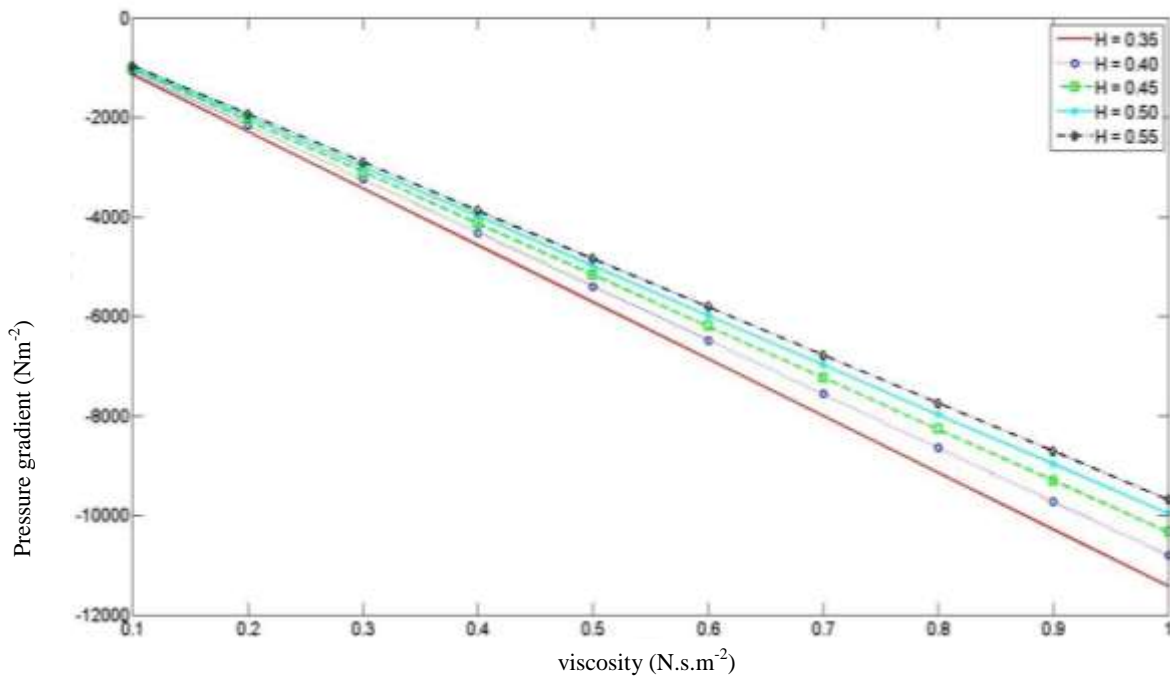
**Figure 4**-The variation of pressure gradient along the length of artery for different values of hematocrit ( $H$ ) of red blood cell, when the number of stenosis is 4 ( $n = 4$ ).



**Figure 5**-The variation of wall shear stress along length of artery ( $Z$ ), when the number of stenosis is 4 ( $n = 4$ ).



**Figure 6**-The variation of pressure gradient along length of artery ( $Z$ ), when the number of stenosis is 4 ( $n = 4$ ).



**Figure 7**-The variation of pressure gradient along viscosity ( $\mu$ ), when the number of stenosis is 4 ( $n = 4$ ).

Figure-2 shows the variation of wall shear stress and viscosity of blood ( $\mu = 4$ ) for different values of hematocrit of red blood cells. It is clearly observed that for a particular value of blood viscosity, the increase in the wall shear stress is significant for hematocrit of red blood cells. The graph shows that the wall shear stress of human artery decreases when the viscosity increases with the increase in the hematocrit level. This is because the blood flow in the obstructed human carotid artery is reduced and the other part of the human body receives low oxygenated blood, which is dangerous to human health. In the same way, Figures- 3 and 5 indicate that the wall shear stress decreases when the hematocrit increases with the increase in the length of human artery. Figures- 4, 6, and 7 show the variation of



pressure gradient along the length of human artery for different hematocrit values and increased blood viscosity ( $\mu = 4$ ), and they indicate an increase in relative pressure gradient with the increase in hematocrit.

### Conclusions

In this paper, it was discovered that increasing the number of stenosis, fluid characteristics (the wall shear stress, pressure gradient, and viscosity of the blood), and hematocrit of red blood cells are dangerous to human heart. This can result into bleeding and blood clotting of the vessels, which could damage the human heart and results in heart attack or stroke.

This paper focused on the effects of hematocrit on flow parameters of blood such as resistance, stenosis in the artery and fluid characteristics, taking blood as non-Newtonian fluid. It was found that resistance increased with increasing hematocrit level and the height and number of stenosis. It is also shown that with the increase in hematocrit of red blood cells, the pressure gradient and viscosity of the blood increase. The results also indicate that the increase in the height and number of stenosis in the artery could result in high blood pressure which can reach a critical region that is beyond control. This can cause bleeding or clotting of the blood which is very dangerous to human heart/brain as it can result in heart failure or stroke.

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