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Modal Analysis of Vibration of Euler-Bernoulli Beam Subjected to Concentrated Moving Load

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Abstract

This paper investigates the modal analysis of vibration of Euler-Bernoulli beam subjected to concentrated load. The governing partial differential equation was analysed to determine the behaviour of the system under consideration. The series solution and numerical methods were used to solve the governing partial differential equation. The results revealed that the amplitude increases as the length of the beam increases. It was also found that the response amplitude increases as the foundation increases at fixed length of the beam.

Keywords: Euler-Bernoulli Beam, Concentrated Moving Load, Vibration, Amplitude, Moving Load.

Introduction

Beams are fundamental models for the structural elements of many engineering applications and they were extensively studied. Every structure which has some mass and elasticity is said to vibrate [1]. Research on vibrations of beams was conducted by many authors for a long period of time. So far, many authors introduced different methods to find the free vibration behaviour of shear flexible beams. Abrate [2] analysed the free vibration of non-uniform beams with general shape and arbitrary boundary conditions. Simple formulas were presented for predicting the fundamental natural frequency of non-uniform beams with various end support conditions. An earlier work [3] studied free vibrations of tapered beams with general boundary conditions. This method involves finding the ordinary differential governing equation of beams which can be solved by numerical methods. The natural frequencies are calculated by combining the Runge-Kutta method and the determinant search method. Studies were also conducted on the dynamic behaviour of beams with linearly varying cross-sections, where the equation of motion was solved using an analytical method to the initial boundary value problem described by the governing equation [4, 5, 6, 7].

- i. The aim of this research is to determine the dynamic response of a pipeline-supported beam on an elastic foundation resting on a Winkler foundation, in order to obtain the dynamic responses such as deflection and bending moments [8-11]. The objectives of this study are to present the analysis of Euler-Bernoulli beam, subjected to partially distributed moving load and to find the analytical solution of the governing partial differential equation of the beam [12-15]. We also aimed at determining the displacement of the beam as a function of time and distance $\beta(x, t)$, subject to initial and boundary conditions of the system [16-20].

Mathematical Formulation

Consider a non-prismatic Euler-Bernoulli beam of a length L , resting on a Winkler foundation and transverse by uniform partially distributed moving load [21- 24].

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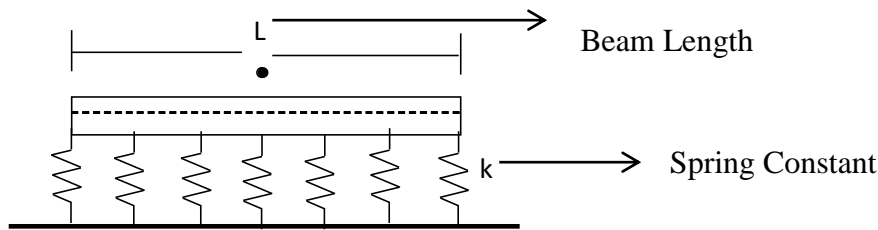


Figure 1-Geometry of the problem.

The resulting vibration behaviour of this system is described by the following partial differential equations.

A forced vibration model exists in which there is a moving load in the form of fluid on the beam. The assumed solution is in the form of Fourier series solution and the resulting vibration behaviour of the beam is described by the fourth order partial differential equation below [25, 26, 27, 28, 29, 30]:

$$EI \frac{\partial^4 \beta}{\partial x^4} + K(x)\beta + \rho A(x) \left[\frac{\partial^2 \beta}{\partial t^2} \right] = F(x, t) \tag{1.0}$$

in the case of moving force,
where,

$$F(x, t) = P\delta(x - vt) \tag{1.1}$$

where \$E\$ is Young's modulus, \$I\$ is the area of the moment of inertia about the neutral axis (\$m^4\$), \$\rho\$ is the mass density (\$kg/m^3\$), \$\beta\$ is the deflection (m) or the transverse displacement of a segment of the beam along \$x\$, \$x\$ is the horizontal space coordinate measure along the length of the beam, \$A\$ is the cross sectional area of the beam (\$m^2\$), \$t\$ is any particular instant of time in seconds, \$K\$ is spring constant per unit length, \$\delta\$ is Dirac delta function, and \$P\delta(x - vt)\$ is the applied moving force per unit mass.

2.1 ASSUMPTIONS

Assumptions taken into consideration are:

- i. **Structural Assumption:**
 - (a) Initially straight beam
 - (b) Linear elastic material
 - (c) Small structural deformation
- ii. Shear deformation and rotary inertial effect are neglected (Euler-Bernoulli beam). Hence, the length ratio of the beam is small.
- iii. Weight of the moving load is larger than the mass of the beam, so we will consider only gravitational effect of load.
- iv. Load is moving at a constant speed.

2.2 Method of Solution

The deflection mode of the continuous beam with simply-supported or position-dependent boundary conditions can be derived from the equation of the beam. The deflection modes and the natural frequencies of the beam can be expressed as:

$$Y_n(x) = \sin \frac{n\pi x}{l}, \text{ and } \beta_n = \beta(x, t) = \left(\frac{n\pi}{l} \right)^2 \sqrt{\frac{EI}{\rho A}}, \text{ where } n = 1, 2, 3 \dots N \tag{1.2}$$

where \$Y_n(x)\$ is the deflection mode, \$w_n\$ is the corresponding natural frequency, and \$l\$ is the beam length.

2.3 MODAL ANALYTICAL SOLUTION

Assume a series solution of equation (1.0) in the form of a series

$$\beta(x, t) = \sum_{n=1}^N Y_n(x) q_n(t) \quad n = 1, 2, 3, \dots N \tag{1.3}$$

where \$Y_n(x)\$ is the eigen function of the beam, \$q_n(t)\$ is a function of time which must be found, and \$n\$ is the number of contributed modes.

$$Y_n(x) = \sin \frac{n\pi x}{l} \tag{1.4}$$

$$q_n(t) = \frac{2P}{\rho A l \omega_n^2} \times \frac{1}{2} (\sin \omega_n \cdot t - \omega_n \cdot t \cos \omega_n \cdot t) \tag{1.5}$$

$$\omega_n^2 = \frac{n^4 \pi^4 EI}{\rho A l^4} \tag{1.6}$$

$$\omega_n = \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \tag{1.7}$$

$$q_n(t) = 2P \div \rho A l \times \left(\frac{n\pi}{l}\right)^4 \frac{EI}{\rho A} \times \frac{1}{2} \left(\sin \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right)$$

$$q_n(t) = \frac{Pl^3}{(n\pi)^4 EI} \left(\sin \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \tag{1.8}$$

$$\beta(x, t) = \sum_{n=1}^N \frac{Pl^3}{(n\pi)^4 EI} \left(\sin \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \times \sin \frac{n\pi vt}{l} \tag{1.9}$$

Note that $\left[\sin \frac{n\pi x}{l} = \sin \frac{n\pi vt}{l} \right]_{x=vt}$

$$EI \frac{\partial^4 \beta}{\partial x^4} + K(x)\beta + \rho A(x) \left(\frac{\partial^2 \beta}{\partial t^2} \right) = F(x, t) \tag{1.10}$$

Where $F(x, t) = P\delta(x - vt)$ becomes

$$EI \frac{\partial^4 \beta}{\partial x^4} + K(x)\beta + \rho A(x) \left(\frac{\partial^2 \beta}{\partial t^2} \right) = P\delta(x - vt) \tag{1.11}$$

We substitute Eq.(1.2) together with Eq.(1.3) into Eq. (1.11) as follows;

$$EI \sum_{n=1}^N \sin \left[\frac{n\pi x}{l} \right]^4 q_n(t) + K \sum_{n=1}^N \sin \left[\frac{n\pi x}{l} \right] q_n(t) + \rho A \sum_{n=1}^N \sin \left[\frac{n\pi x}{l} \right] \ddot{q}_n(t) = P \sum_{n=1}^N \sin \left[\frac{n\pi x}{l} \right] (vt) \tag{1.12}$$

Using simplified subscript for differentiation in Eq.(1.12) gives

$$EI \left[\frac{n\pi}{l} \right]^4 \sum_{n=1}^N Y_n(x) q_n(t) + K \sum_{n=1}^N Y_n(x) q_n(t) + \rho A \sum_{n=1}^N Y_n(x) \ddot{q}_n(t) = P \sum_{n=1}^N Y_n(vt) \tag{1.13}$$

Multiplying both sides of equation (1.13) by $Y_p(x)$ results in

$$EI \left[\frac{n\pi}{l} \right]^4 \sum_{n=1}^N Y_n(x) q_n(t) Y_p(x) + K \sum_{n=1}^N Y_n(x) q_n(t) Y_p(x) + \rho A \sum_{n=1}^N Y_n(x) \ddot{q}_n(t) Y_p(x) = P \sum_{n=1}^N Y_n(vt) Y_p(vt) \tag{1.14}$$

Integrating along the beam length and employing the orthogonality property among the normal modes gives

$$EI \left[\frac{n\pi}{l} \right]^4 \sum_{n=1}^N q_n(t) \int_0^l Y_n(x) Y_p(x) dx + K \sum_{n=1}^N q_n(t) \int_0^l Y_n(x) Y_p(x) dx + \rho A \sum_{n=1}^N \ddot{q}_n(t) \int_0^l Y_n(x) Y_p(x) dx = P \sum_{n=1}^N Y_n(vt) Y_p(vt) dt \tag{1.15}$$

Which gives

$$EI \left[\frac{n\pi}{l} \right]^4 \sum_{n=1}^N q_n(t) \int_0^l [Y_n(x)]^2 dx + K \sum_{n=1}^N q_n(t) \int_0^l [Y_n(x)]^2 dx + \rho A \sum_{n=1}^N \ddot{q}_n(t) \int_0^l [Y_n(x)]^2 dx = P \sum_{n=1}^N Y_n(vt) Y_p(vt) dt \tag{1.16}$$

The definition of function of orthogonality is applied to give the LHS of equation (1.16). The definition is as follows:

(1) The two non-zero functions $f(x)$ and $g(x)$ are said to be **orthogonal** on $a \leq x \leq b$ if

$$\int_a^b f(x)g(x)dx = 0 \tag{1.17}$$

(2) A set of non-zero function $\{f_i(x)\}$ is said to be **mutually orthogonal** or **orthogonal set** on $a \leq x \leq b$ if $f_i(x)$ and $f_j(x)$ are orthogonal for every $i \neq j$. In other words

$$\int_a^b f_i(x)f_j(x)dx = \begin{cases} 0 & \text{if } i \neq j \\ c > 0 & \text{if } i = j \end{cases} \tag{1.18}$$

Note that in the case of $i = j$ for the second definition, we know that we will get a positive value from the integral, because

$$\int_a^b f_i(x)f_i(x)dx = \int_a^b [f_i(x)]^2 dx > 0 \tag{1.19}$$

Recall that when we integrate a positive function, we know that the result will be positive as well. Similarly, the non-zero requirement is important because otherwise the integral would be trivially zero regardless of the function.

In the case of equation (1.15) above, $n = p$, which gives equation (1.16). After integration and some rearrangements, equation (1.16) gives

$$\frac{EI}{\rho A} \left(\frac{n\pi}{l} \right)^4 q_n(t) + \ddot{q}_n(t) + \frac{K}{\rho A} q_n(t) = \frac{2P}{\rho Al} \sum_{n=1}^N Y_n(vt) Y_p(vt) \tag{1.20}$$

For $n = 1, 2, 3, \dots, N$.

Since $n = p$, equation (17) becomes

$$\frac{EI}{\rho A} \left(\frac{n\pi}{l} \right)^4 q_n(t) + \ddot{q}_n(t) + \frac{K}{\rho A} q_n(t) = \frac{2P}{\rho Al} \sum_{n=1}^N (Y_n(vt))^2 \tag{1.21}$$

Equation (1.21) is an ordinary differential equation, and a numerical procedure shall be employed to solve it. It can be also simplified further by substituting the unknown functions in the equation. The unknown functions are:

$$q_n(t), \ddot{q}_n(t) \text{ and } Y_n(vt)$$

From equation (1.3) and (1.8), $Y_n(x) = Y_n(vt) = \sin \frac{n\pi x}{l}$ and

$$q_n(t) = \frac{Pl^3}{(n\pi)^4 EI} \left(\sin \left(\frac{n\pi}{l} \right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l} \right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos \left(\frac{n\pi}{l} \right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \tag{1.22}$$

Substituting $q_n(t)$ and $Y_n(vt)$ into equation (1.21), gives

$$\begin{aligned} & \frac{EI}{\rho A} \left(\frac{n\pi}{l}\right)^4 \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) + \ddot{q}_n(t) \\ & + \frac{K}{\rho A} \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \\ & = \frac{2P}{\rho Al} \sum_{n=1}^N \left(\sin\left(\frac{n\pi x}{l}\right)\right)^2 \end{aligned} \tag{1.23}$$

$$\ddot{q}_n(t) + \omega_n^2 q_n(t) = \left(\frac{2P}{\rho Al}\right) \sin \bar{\omega}_n \cdot t \tag{1.24}$$

$$\ddot{q}_n(t) = \left(\frac{2P}{\rho Al}\right) \sin \bar{\omega}_n \cdot t - \omega_n^2 q_n(t) \tag{1.25}$$

where

$$\omega_n^2 = \frac{n^4 \pi^4 EI}{\rho Al^4}, \bar{\omega}_n = \frac{n\pi v}{l} \tag{1.26}$$

and

$$q_n(t) = \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \tag{1.27}$$

Substituting equation (1.26) and (1.27) into equation (1.25) gives

$$\begin{aligned} \ddot{q}_n(t) & = \left(\frac{2P}{\rho Al}\right) \sin \frac{n\pi v}{l} \cdot t \\ & - \frac{n^4 \pi^4 EI}{\rho Al^4} \cdot \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \end{aligned} \tag{1.28}$$

Simplifying equation (1.28) gives

$$\ddot{q}_n(t) = \left(\frac{2P}{\rho Al}\right) \sin \frac{n\pi v}{l} \cdot t - \frac{P}{\rho Al} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \tag{1.29}$$

$$\ddot{q}_n(t) = \frac{P}{\rho Al} \left(2 \left(\sin \frac{n\pi v}{l} \cdot t\right) - 1 \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \right) \tag{1.30}$$

By substituting equation (1.30) into equation (1.23), it becomes

$$\begin{aligned} & \frac{EI}{\rho A} \left(\frac{n\pi}{l}\right)^4 \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \\ & + \frac{P}{\rho Al} \left(2 \left(\sin \frac{n\pi v}{l} \cdot t\right) \right. \\ & \left. - 1 \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \right) \\ & + \frac{K}{\rho A} \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \\ & = \frac{2P}{\rho Al} \sum_{n=1}^N \left(\sin\left(\frac{n\pi x}{l}\right)\right)^2 \end{aligned} \tag{1.31}$$

Simplifying equation (1.31) gives

$$\begin{aligned} & \frac{Pl^3}{\rho A(n\pi)^4 EI} \left(\left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \left[EI \left(\frac{n\pi}{l}\right)^4 + K \right] \right. \\ & \quad \left. + \frac{P}{\rho Al} \left(2 \left(\sin\frac{n\pi v}{l} \cdot t \right) \right. \right. \\ & \quad \left. \left. - 1 \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \right) \right) \\ & = \frac{2P}{\rho Al} \sum_{n=1}^N \left(\sin\left(\frac{n\pi x}{l}\right) \right)^2 \end{aligned} \tag{1.32}$$

Multiply equation (1.32) by ρA , gives

$$\begin{aligned} & \frac{Pl^3}{(n\pi)^4 EI} \left(\left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \left[EI \left(\frac{n\pi}{l}\right)^4 + K \right] \right. \\ & \quad \left. + \frac{P}{l} \left(2 \left(\sin\frac{n\pi v}{l} \cdot t \right) - 1 \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \right) \right) \\ & = \frac{2P}{l} \sum_{n=1}^N \left(\sin\left(\frac{n\pi x}{l}\right) \right)^2 \end{aligned} \tag{1.33}$$

Equation (1.33) above is the simplified form of equation (1.23) above, i.e.,

$$\frac{EI}{\rho A} \left(\frac{n\pi}{l}\right)^4 q_n(t) + \ddot{q}_n(t) + \frac{K}{\rho A} q_n(t) = \frac{2P}{\rho Al} \sum_{n=1}^N (Y_n(vt))^2 \tag{1.34}$$

While, the modal deflection of the beam in equation (1.11) is given as:

$$w(x, t) = \sum_{n=1}^N \frac{Pl^3}{(n\pi)^4 EI} \left(\sin\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t - \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \times \cos\left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \cdot t \right) \times \sin\frac{n\pi vt}{l} \tag{1.35}$$

Result and Discussion

The beam was subjected to a moving load; the equation considered a fourth order Partial Differential Equation (PDE). The PDE was transformed to an ordinary differential equation. The equation was solved with an assumed solution in form of series. A software package was used to analyse the solution obtained graphically with the parameters. Graphs are presented in Figures (2-7) below. The dynamic responses, such as deflection and bending moment of the beam, were obtained under different velocities (v), contributed mode (n), and time (t) domain.

From Figure-2, it is observed that the deflection is somehow symmetric at a minimum point. The figure also shows that the maximum dynamic deflection for both upward and downward directions increases and decreases, respectively, with the increase in velocity. The maximum bending moment occurs at 0.1851, 0.2637 and 0.3672 while the minimum bending moment takes place at -0.0991, -0.1406 and -0.1945, respectively, for the velocities.

Figure-3 demonstrates the deflection tilted towards the positive side rather than the negative side of the beam, with minimum bending moments of -0.0661, -0.0938 and -0.1297 and maximum deflections at 0.1092, 0.1470 and 0.1831, respectively. The figure further explains that the deflection is wider upward and narrows at the downward part of the beam.

In Figure-4a and 4b, the same parameter were used in each part of the figure; [fig.4a](#) was time-partitioned along the different point of the beam while [fig. 4b](#) was not partitioned. It is observed that

both graphs pick their maximum deflection towards the end of the beam, with varying minimum bending moment values for different velocities, almost at the same point of the beam.

Figure-5 shows that the minimum bending moment of the deflection is approximately symmetric at a point for all the selected velocities, and the maximum bending moment occurs just at the beginning of the beam, with the successive crest of the maximum velocity being almost the same. The maximum deflection occurs at 1.0505, 1.4138 and 1.7618 while the minimum deflection occurs at -0.7633, -1.0743 and -1.4637, respectively, for all the velocities, as shown in the figure.

Figure-6 illustrates that both the deflection and bending moments are more visible as they progress along the beam length, with more deflection noticed at the downward part of the beam. The minimum and maximum deflections for the maximum velocity are 0.58773 and -0.6342, respectively.

In Figure-7, it is observed that the visible maximum deflection occurs at the upper part of the beam, while the deflection damps almost from the middle to the end of the beam, with maximum and minimum bending moments of 1.4836 and -0.4039, respectively, for the maximum velocity. It shows the analysis under the same condition of contributed modes, with different time domains.

Table 1-The parameters used in this study along with their respective values

Parameters	Value(s)
ρ	25kg/m ³
EI	2.1528x10 ⁵ N/m ²
P	9.936 x 10N
l	10m
A	2.45m ²
V	7m/s, 10m/s, 14m/s
$n(3.1)$	1, 2, 3
$k(3.2)$	1, 2, 3
t	1s, 2s, 3s
m	10kg, 15kg, 20kg
h	0.5, 1.0, 1.5
c_1	0.1, 0.2, 0.3

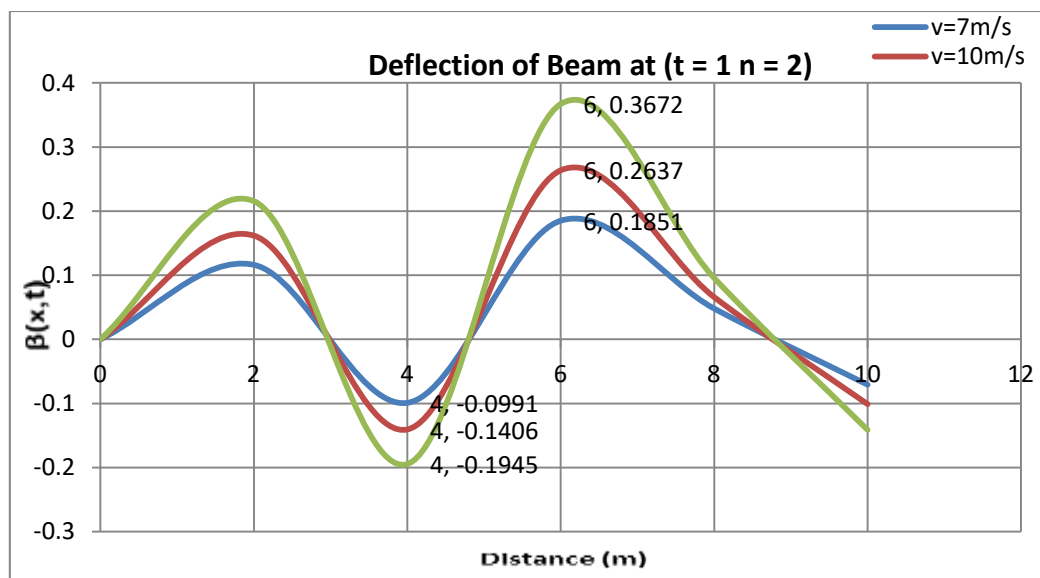


Figure 2-Graph of deflection at t=1s (NP) and n = 2

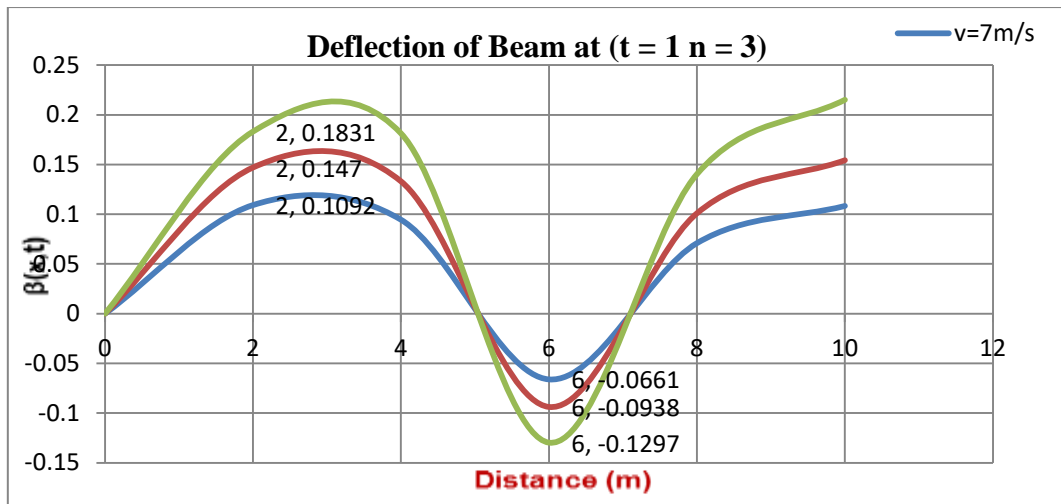


Figure 3-Graph of deflection at t = 1s (NP) and n = 3

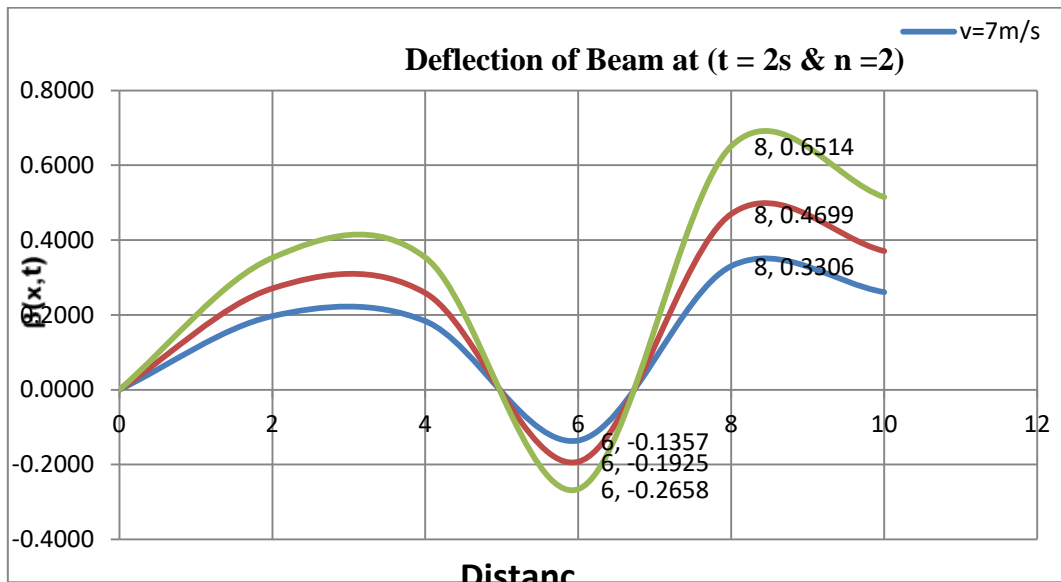


Figure 4a-Graph of deflection at t = 2s (partitioned) and n = 2

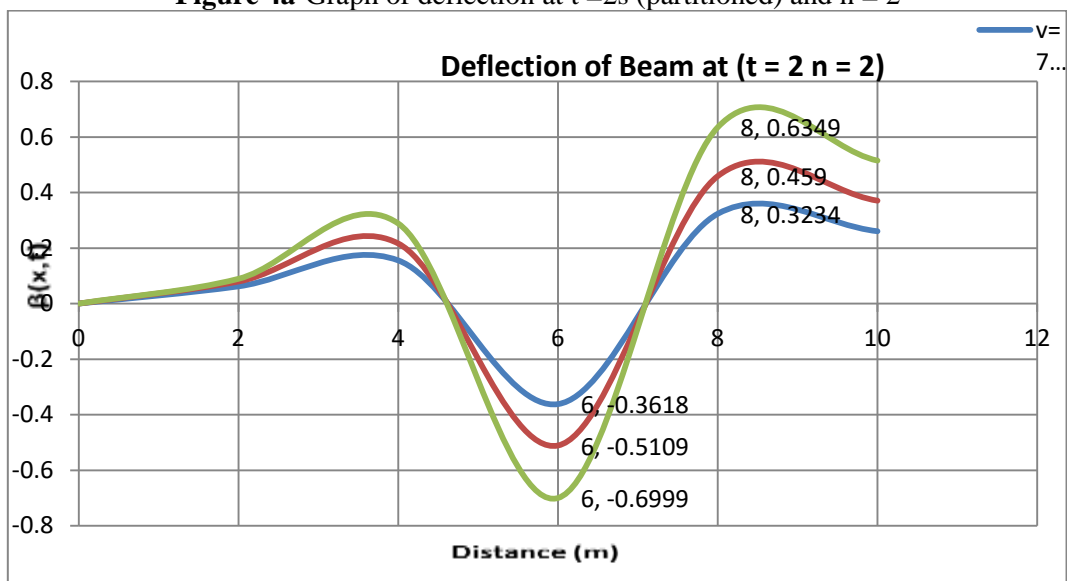


Figure 4b-Graph of deflection at t = 2s (NP) and n = 2

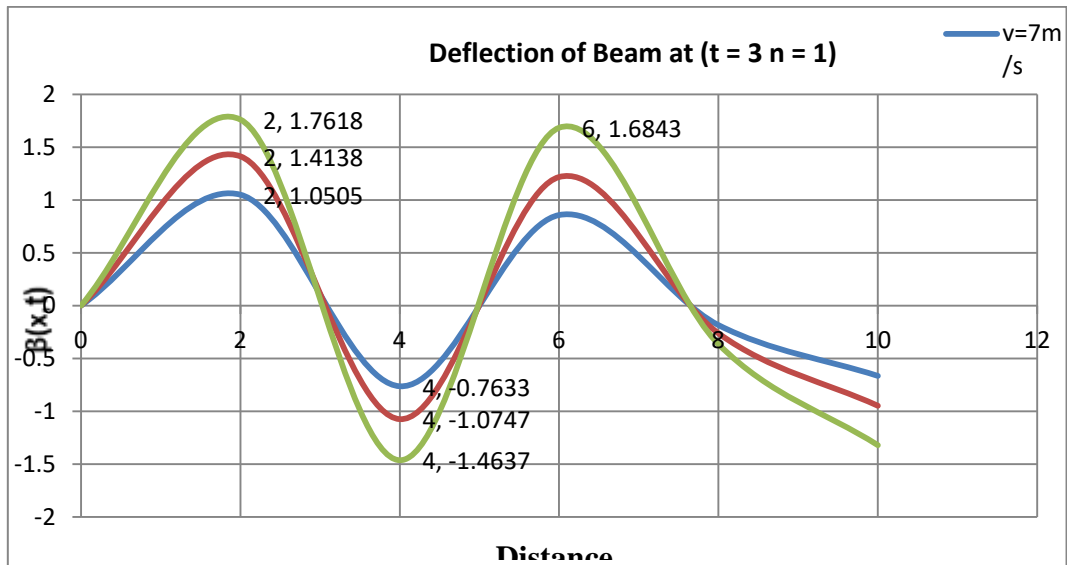


Figure 5-Graph of deflection at t =3s (NP) and n = 1

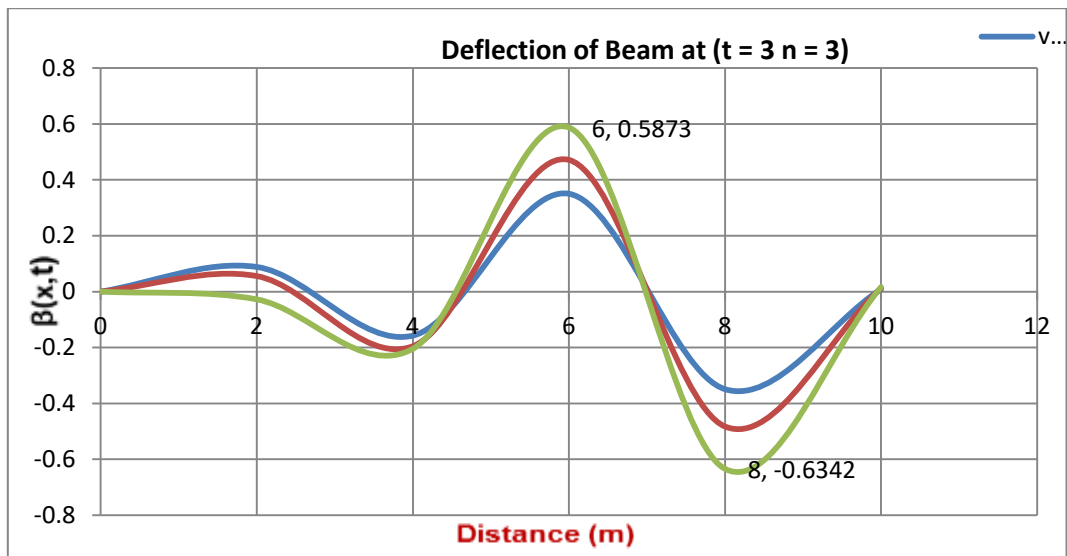


Figure 6-Graph of deflection at t = 3s (NP) and n = 3

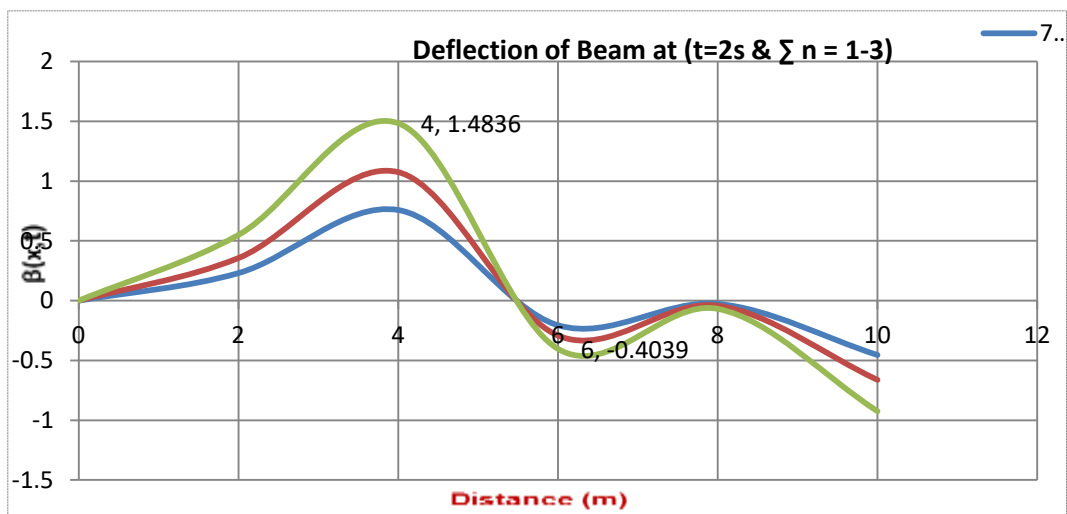


Figure 7-Graph of deflection at t = 2s (NP) and $\sum_{n=1}^3$.

Conclusions

This study demonstrates the vibration analysis of Euler-Bernoulli beam resting on a Winkler foundation subjected to a concentrated distributed moving load. Fourier transform method was employed to find the exact solution of the governing partial differential equation of the beam of length L on both cases. Modal analysis was also applied to convert the P.D.E to O.D.E. in equation (1.0). Numerical calculations were performed to analyse the deflection and bending moment responses of the beam on a Winkler foundation subjected to a concentrated load with different velocity and a free load with mass of the beam.

The results show that the increment in velocity leads to increase in both the displacement and bending moments of the beam. It further shows that the deflection and the bending moment decreases as the number of the contributed mode increases.

Finally, it can be concluded that the deflection of the beam increases as the velocity of the load increases. Bending moment also increases in response to the increase in velocity and time.

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