DOI: 10.24996/ijs.2020.61.5.18

ISSN: 0067-2904

Nano Spaces Via Ĩ-*Semi*-g-*Closed Set*

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Received: 4/8/ 2019 Accepted: 30/9/2019

Abstract
In this paper, the concept of *nano-Ĩ-semi-g-closed* set (resp., *nano-Ĩ-semi-g-open*, *nano-Ĩ-semi-g-$\mathscr{f}$* and *nano-Ĩ-semi-g-φ-closed* setset) was introduced by using the concept of *ideal* in *topological spaces* and *nano topological spaces*. Some properties and examples with illustrative tables and an applied example were presented.

Keywords: *n-closed*, *n-open*, *n-semi-closed*, *n-semi-open*, *n*-Ĩ*sg-closed*, *n*-Ĩ*sg-open*, *n*-Ĩ*sg*-$\mathscr{f}$.

الفضاء النانو باستخدام مجموعة (Ĩ. SEMI. g)

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الخلاصة:
في هذا البحث يتم الاستعانة بمفهوم الفضاء التوبولوجي المثالي حيث تم ايجاد مجموعة جديدة من المجموعات شبه المفتوحة من نوع نانو وهي مجموعة نانو مغلقة من نوع
(I– semi– g) من خلال المفهوم السابق للفضاء التوبولوجي المثالي من نوع النانو حيث تم تقديم بعض الخصائص والأمثلة مع جداول توضيحية ومثال تطبيقي.

1. Introduction

In$(\mathcal{X}, \mathcal{T})$, the ideal $\tilde{I} \neq \phi$ is as a family of subsets of $\mathcal{X}$ that satisfies two prerequisites: ($\dot{A}$ *and* $\dot{B} \in \tilde{I}$ implies $\dot{A} \cup \dot{B} \in \tilde{I}$) and ($\dot{A} \subseteq \dot{B}$ *and* $\dot{B} \in \tilde{I}$ implies $\dot{A} \in \tilde{I}$) [1, 2]. By using this concept many studies have emerged which are concerned with the study of different topological properties [3- 5].

In 2013, the concept of *nano topological space* was studied using the lower and upper approximations with equivalence relations [6,7].

In this paper, by taking advantage of the above concepts, another type of near *nano closed* set is introduced, which is *nano*-Ĩ-*semi-g-closed*, and the most important characteristics of this set are clarified.

2. Preliminaries

Definition 2.1.[7] For an equivalence relation R on a set $\mathcal{X} \neq \phi$, let $\dot{A} \subseteq \mathcal{X}$:

i- The lower approximation of $\dot{A}$ via R is symbolized by $\underline{\mathrm{R}}(\dot{A})$ where $\underline{\mathrm{R}}(\dot{A}) = \cup_{\varsigma \in \dot{A}} \{\mathrm{R}(\varsigma);\ \mathrm{R}(\varsigma) \subseteq \dot{A}\}$, and $\mathrm{R}(\varsigma)$ is defined by the equivalence class of ς.

ii- The upper approximation of $\dot{A}$ via R is symbolized by $\overline{\mathrm{R}}(\dot{A})$ where $\overline{\mathrm{R}}(\dot{A}) = \cup_{\varsigma \in \dot{A}} \{\mathrm{R}(\varsigma);\ \mathrm{R}(\varsigma) \cap \dot{A} \neq \phi\}$.

iii-The boundary of $\dot{A}$ via R is symbolized by $\mathrm{R}^b(\dot{A})$ where $\mathrm{R}^b(\dot{A}) = \overline{\mathrm{R}}(\dot{A})$-$\underline{\mathrm{R}}(\dot{A})$.

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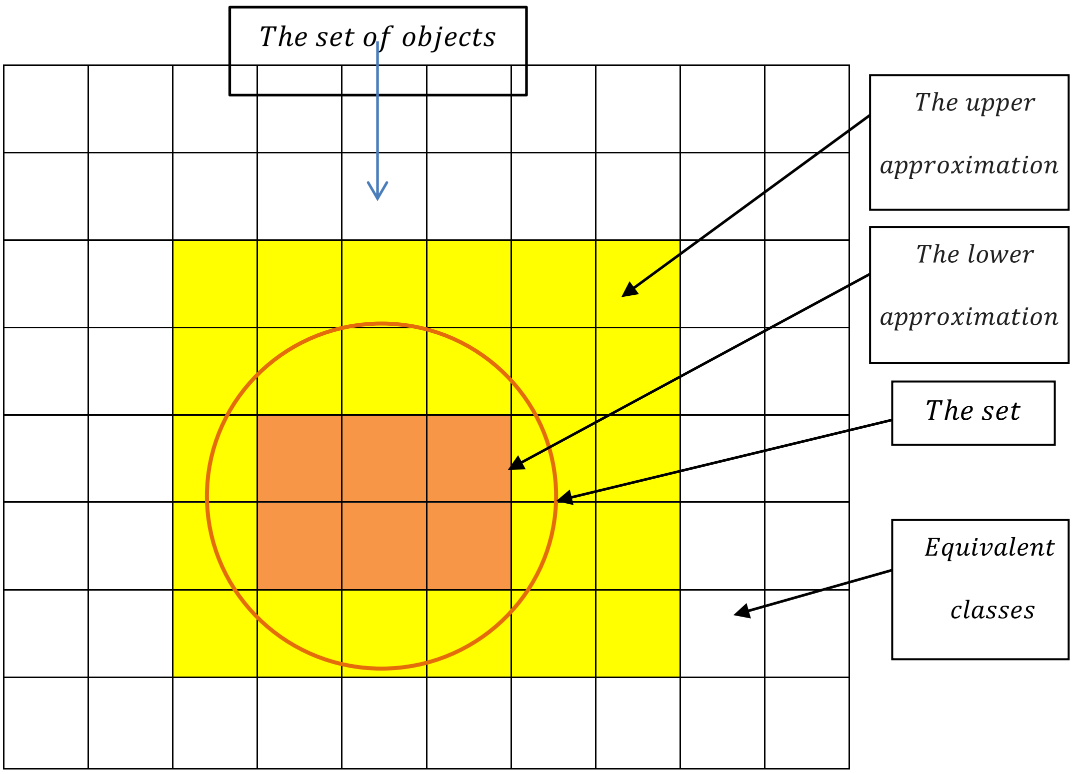

Sketch (1)

Definition 2.2.[7,8] For an equivalence relation R on a set $\mathcal{X} \neq \phi$, let $A \subseteq \mathcal{X}$ and let $\tau_R(A) = \{\mathcal{X}, \phi, \underline{R}(A), \overline{R}(A), R^b(A)\}$ is topology on $\mathcal{X}$, then $\tau_R(A)$ is namely *nano topology* with respect to A and $(\mathcal{X}, \tau_R(A))$ is namely *nano topological space* with respect to A. Every element in this previous topology is namely *nano-open* set (shortly; *n-open* set) and its complement is *nano-closed* set (shortly; *n-closed* set). The abbreviations $n\text{-}int(A)$ and $n\text{-}cl(A)$ refer to the nano-interior and the nano-closure of A, respectively.

For any ideal $\tilde{I}$, the space $(\mathcal{X}, \tau_R(A), \tilde{I})$ is a *nano ideal topological space*.

We can determine all *nano topological spaces* for any space $\mathcal{X}$ with an equivalence relation R on it, according to the given subset $A \subseteq \mathcal{X}$ to find $\tau_R(A)$. All this will be evident in table 1.

Let $\mathcal{X} = \{c_1, c_2, c_3\}$ and $R = \{(c_1, c_1), (c_2, c_2), (c_3, c_3), (c_1, c_2), (c_2, c_1)\}$ Then $P(\mathcal{X}) = \{\mathcal{X}, \phi, \{c_1\}, \{c_2\}, \{c_3\}, \{c_1, c_2\}, \{c_2, c_3\}, \{c_1, c_3\}\}$, $R(c_1) = \{c_1, c_2\} = R(c_2)$ and $R(c_3) = \{c_3\}$ then $\mathcal{X}/R = \{\{c_1, c_2\}, \{c_3\}\}$.

Table-1

A	$\underline{R}(A)$	$\overline{R}(A)$	$R^b(A)$	$\tau_R(A)$
ϕ	ϕ	ϕ	ϕ	$\{\mathcal{X}, \phi\}$
$\mathcal{X}$	$\mathcal{X}$	$\mathcal{X}$	ϕ	$\{\mathcal{X}, \phi\}$
$\{c_1\}$	ϕ	$\{c_1, c_2\}$	$\{c_1, c_2\}$	$\{\mathcal{X}, \phi, \{c_1, c_2\}\}$
$\{c_2\}$	ϕ	$\{c_1, c_2\}$	$\{c_1, c_2\}$	$\{\mathcal{X}, \phi, \{c_1, c_2\}\}$
$\{c_3\}$	$\{c_3\}$	$\{c_3\}$	ϕ	$\{\mathcal{X}, \phi, \{c_3\}\}$
$\{c_1, c_2\}$	$\{c_1, c_2\}$	$\{c_1, c_2\}$	ϕ	$\{\mathcal{X}, \phi, \{c_1, c_2\}\}$
$\{c_2, c_3\}$	$\{c_3\}$	$\mathcal{X}$	$\{c_1, c_2\}$	$\{\mathcal{X}, \phi, \{c_1, c_2\}, \{c_3\}\}$
$\{c_1, c_3\}$	$\{c_3\}$	$\mathcal{X}$	$\{c_1, c_2\}$	$\{\mathcal{X}, \phi, \{c_1, c_2\}, \{c_3\}\}$

Definition 2.3.[7],[8] For $(\mathcal{X}, Ƭ_R(Ạ))$, the set $Ḅ \subseteq \mathcal{X}$ is *nano-semi-open* (briefly, *n-semi-open*) whenever $Ḅ \subseteq n\text{-}cl(n\text{-}int(Ḅ))$, where its complement is a *nano-semi-closed* (briefly, *n-semi-closed*) set. The shortcut $n\text{-}SC(\mathcal{X})$ (respectively, $n\text{-}SO(\mathcal{X})$) is used for the family of all *n-semi-closed* (respectively, *n-semi-open*) sets.

Definition 2.4.[9] In $(\mathcal{X}, Ƭ_R(Ạ))$, if $Ḅ \subseteq \mathcal{X}$, then $n\text{-}\mathcal{K}er(Ḅ) = \bigcap\{Ụ' ;\ Ḅ \subseteq Ụ',\ Ụ' \in Ƭ_R(Ạ)\}$ which is denoted by the *nano-kernal* of Ḅ.

3. *Nano*-Ĩ- *Semi- g- Closed* Set

The concepts of ideal and equivalence relations will be used to define new notions and then clarify the relationships with the concept of *ideal topological space*.

Definition 3.1. In $(\mathcal{X}, Ƭ_R(Ạ), Ĩ)$, the subset $Ḅ \subseteq \mathcal{X}$ is a *nano*-Ĩ-*semi-g-closed* set (shortly, *n*-Ĩ*sg-closed*), if Ḅ-Ụ' ∈ Ĩ and Ụ' is *n-semi-open*, then $cl(Ḅ)$-Ụ' ∈ Ĩ. The complement of Ḅ is a *nano*-Ĩ-*semi-g-open* set (shortly, *n*-Ĩ*sg-open*). The shortcuts n-Ĩsg-$C(\mathcal{X})$ and n-Ĩsg-$O(\mathcal{X})$ refer to the families of all *n*-Ĩ*sg-closed* and *n*-Ĩ*sg-open* sets, respectively.

From Table-1, we can determine the family of all *n-semi-closed* (respectively, *n-semi-open*) sets , according to the given $Ƭ_R(Ạ)$ in the previous table, as explained in Table-2.

Table-2

Ạ	$Ƭ_R(Ạ)$	***n*-*SO*($\mathcal{X}$)**	***n*-*SC*($\mathcal{X}$)**
ϕ	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi\}$
$\mathcal{X}$	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi\}$
$\{ɕ_1\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}\}$
$\{ɕ_2\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}\}$
$\{ɕ_3\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$
$\{ɕ_1, ɕ_2\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}\}$
$\{ɕ_2, ɕ_3\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$
$\{ɕ_1, ɕ_3\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$

From Table-2, let $Ĩ = \{\phi, \{ɕ_1\}, \{ɕ_2\}, \{ɕ_1, ɕ_2\}\}$ be the ideal. We can determine the family of all *n*-Ĩ*sg-closed* (respectively, *n*-Ĩ*sg-open*) sets, according to the given $Ƭ_R(Ạ)$ and $n\text{-}SO(\mathcal{X})$ in the previous table, as described in Table-3.

Table-3

0.	$Ƭ_R(Ạ)$	***n*-*SO*($\mathcal{X}$)**	***n*-Ĩ*sg*-*C*($\mathcal{X}$)**	***n*-Ĩ*sg*-*O*($\mathcal{X}$)**
ϕ	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$
$\mathcal{X}$	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$
$\{ɕ_1\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$
$\{ɕ_2\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$
$\{ɕ_3\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$P(\mathcal{X})$	$P(\mathcal{X})$
$\{ɕ_1, ɕ_2\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$	$\{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$
$\{ɕ_2, ɕ_3\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$P(\mathcal{X})$	$P(\mathcal{X})$
$\{ɕ_1, ɕ_3\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_3\}\}$	$P(\mathcal{X})$	$P(\mathcal{X})$

Remark 3.2

i. Every *n-closed* set in $(\mathcal{X}, \tau_R(Ạ))$ is n-$\tilde{I}sg$-*closed* in $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$.

ii. Every *n-open* set in $(\mathcal{X}, \tau_R(Ạ))$ is n-$\tilde{I}sg$-*open* in $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$.

The converse of neither parts of the above remark is true. By Table-3, if we suggest the set $Ạ = \{ɕ_1, ɕ_2\}$ then $\tau_R(Ạ) = \{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$, $n\text{-}\tilde{I}sg\text{-}O(\mathcal{X}) = \{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\}$ and $n\text{-}\tilde{I}sg\text{-}C(\mathcal{X}) = \{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$. The set $\{ɕ_3\}$ is n-$\tilde{I}sg$-*closed* in $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$ but not *n-closed* set in $(\mathcal{X}, \tau_R(Ạ))$ and the set $\{ɕ_1\}$ is n-$\tilde{I}sg$-*open* in $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$ but not *n-open* set in $(\mathcal{X}, \tau_R(Ạ))$.

4. Nano-$\tilde{I}$- Semi- g- Kernal of Set

The connotation of the definition of *nano-kernal* of the set [9] can be generalized by using the notion of *nano*-$\tilde{I}$-*semi-g-open* set, as defined below:

Definition 4.1. In $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, if $Ḅ \subseteq \mathcal{X}$, then $n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ) = \cap\{Ư\,;\ Ḅ \subseteq Ư,\ Ư \in n\text{-}\tilde{I}sg\text{-}O(\mathcal{X})\}$ which is a shortcut for *nano*-$\tilde{I}$-*semi-g-kernal* of Ḅ. It is clear that, if $Ḅ \subseteq \mathcal{X}$ is n-$\tilde{I}sg$-*open* set, then $Ḅ = n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ)$.

From table 3, if we suggest the set $Ạ = \{ɕ_1, ɕ_2\}$ then $\tau_R(Ạ) = \{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$ and $n\text{-}\tilde{I}sg\text{-}O(\mathcal{X}) = \left\{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}, \{ɕ_1\}, \{ɕ_2\}\right\}$. According to the given $Ḅ \subseteq \mathcal{X}$, we can determine $n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ)$, as described in Table-4.

Table 4-

Ḅ	n-$\mathcal{K}er(Ḅ)$	n-$\tilde{I}sg$-$\mathcal{K}er(Ḅ)$
ϕ	ϕ	ϕ
$\mathcal{X}$	$\mathcal{X}$	$\mathcal{X}$
$\{ɕ_1\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_1\}$
$\{ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_2\}$
$\{ɕ_3\}$	$\mathcal{X}$	$\mathcal{X}$
$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2\}$
$\{ɕ_2, ɕ_3\}$	$\mathcal{X}$	$\mathcal{X}$
$\{ɕ_1, ɕ_3\}$	$\mathcal{X}$	$\mathcal{X}$

Remark 4.2. In $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, if $Ḅ \subseteq \mathcal{X}$, then $n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ) \subseteq n\text{-}\mathcal{K}er(Ḅ)$.

Proof: Let $ɕ \notin n\text{-}\mathcal{K}er(Ḅ)$. Then $ɕ \notin \cap\{Ư\,;\ Ḅ \subseteq Ư,\ Ư \in \tau_R(Ạ)\}$, so $\exists\, Ư \in \tau_R(Ạ)$, $Ḅ \subseteq Ư$; $ɕ \notin Ư$. Since every *n-open* set in $(\mathcal{X}, \tau_R(Ạ))$ is n-$\tilde{I}sg$-*open* in $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, then $\exists\, Ư \in n\text{-}\tilde{I}sg\text{-}O(\mathcal{X})$, $Ḅ \subseteq Ư$; $ɕ \notin Ư$, then $ɕ \notin \cap\{Ư\,;\ Ḅ \subseteq Ư,\ Ư, Ḅ \in n\text{-}\tilde{I}sg\text{-}O(\mathcal{X})\}$. Hence, $ɕ \notin n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ)$.

The phrase $(n\text{-}\mathcal{K}er(Ḅ) \subseteq n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ))$ is not true by Table 4 . If we suggest the set $Ḅ = \{ɕ_1\}$, then $n\text{-}\mathcal{K}er(Ḅ) = \{ɕ_1, ɕ_2\}$, but $n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ) = \{ɕ_1\}$, then $n\text{-}\mathcal{K}er(Ḅ) \nsubseteq n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ)$.

For $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, if $\mathcal{X}$ is a finite space, then $Ḅ \subseteq \mathcal{X}$ is an n-$\tilde{I}sg$-*open* set, if and only if $Ḅ = n\text{-}\tilde{I}sg\text{-}\mathcal{K}er(Ḅ)$.

Definition 4.3. In $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, if $Ḅ = n\text{-}\tilde{I}sg\text{-}\boldsymbol{\mathcal{K}}er(Ḅ)$, where $Ḅ \subseteq \mathcal{X}$, then Ḅ is said to be *nano*-$\tilde{I}$-*semi-g*-$\mathcal{S}$ set, shortly n-$\tilde{I}sg$-$\mathcal{S}$ set.

From Table 4, the sets $\phi, \mathcal{X}, \{ɕ_1\}, \{ɕ_2\}$ and $\{ɕ_1, ɕ_2\}$ are n-$\tilde{I}sg$-$\mathcal{S}$ sets.

Remark 4.4. For $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, $Ḅ \subseteq \mathcal{X}$, if Ḅ is an n-$\tilde{I}sg$-*open* set, then Ḅ is an n-$\tilde{I}sg$-$\mathcal{S}$ set.

Definition 4.5. In $(\mathcal{X}, \tau_R(Ạ), \tilde{I})$, if $Ư = Ḥ \cap Ḅ$, where $Ư \subseteq \mathcal{X}$, Ḥ is n-$\tilde{I}sg$-*closed* set and Ḅ is n-$\tilde{I}sg$-$\mathcal{S}$ set, then Ư is said to be *nano*-$\tilde{I}$-*semi-g*-φ-*closed* set (briefly, n-$\tilde{I}sg$-φ-*closed* set).

From Table 4, where $Ạ = \{ɕ_1, ɕ_2\}$ then $\tau_R(Ạ) = \{\mathcal{X}, \phi, \{ɕ_1, ɕ_2\}\}$ and $n\text{-}\tilde{I}sg\text{-}C(\mathcal{X}) = \{\mathcal{X}, \phi, \{ɕ_3\}, \{ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_3\}\}$. So, every subset Ư of $\mathcal{X}$ is n-$\tilde{I}sg$-φ- *closed* since $Ư = Ḥ \cap Ḅ$, such that Ḥ is n-$\tilde{I}sg$-*closed* set and Ḅ is n-$\tilde{I}sg$-$\mathcal{S}$ set.

Proposition 4.6.

i. Every n-$\tilde{I}sg$-$\mathcal{S}$ set is n-$\tilde{I}sg$-φ-*closed* set.

ii. Every n-$\tilde{I}sg$-*open* set is n-$\tilde{I}sg$-φ-*closed* set.

iii. Every n-$\tilde{I}sg$-*closed* set is n-$\tilde{I}sg$-φ-*closed* set.

Proof

i. Let $Ḅ$ be an n-$\tilde{I}sg$-$\mathcal{S}$ set. Since $\mathcal{X} \in n$-$\tilde{I}sg$-$C(\mathcal{X})$ such that $Ḅ = Ḅ \cap \mathcal{X}$, then $Ḅ$ is n-$\tilde{I}sg$-φ-$closed$ set.

ii. Since $Ḅ \subseteq \mathcal{X}$ is n-$\tilde{I}sg$-$open$ set, then $Ḅ = n$-$\tilde{I}sg$-$\mathcal{K}er(Ḅ)$, then $Ḅ$ is n-$\tilde{I}sg$-$\mathcal{S}$ set, then $Ḅ$ is n-$\tilde{I}sg$-φ-$closed$ set by (i).

iii. Let $Ḅ \in n$-$\tilde{I}sg$-$C(\mathcal{X})$. Since $\mathcal{X}$ is n-$\tilde{I}sg$-$\mathcal{S}$ set such that $Ḅ = Ḅ \cap \mathcal{X}$, then $Ḅ$ is n-$\tilde{I}sg$-φ-$closed$ set

The opposite of Proposition 4.6, is not true.

Example 4.7. From Table ,, if we suggest that $Ḅ = \{\varsigma_3\}$ where $Ạ = \{\varsigma_1, \varsigma_2\}$, then $\mathcal{T}_R(Ạ) = \{\mathcal{X}, \phi, \{\varsigma_1, \varsigma_2\}\}$. And n-$\tilde{I}sg$-$C(\mathcal{X}) = \{\mathcal{X}, \{\phi\}, \{\varsigma_3\}, \{\varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_3\}\}$. Hence, n-$\tilde{I}sg$-$\mathcal{K}er(Ḅ) = \mathcal{X}$, then $Ḅ$ is not n-$\tilde{I}sg$-$\mathcal{S}$ set and not n-$\tilde{I}sg$-$open$ set, but $Ḅ$ is n-$\tilde{I}sg$-φ-$closed$ set, since $Ḅ = Ḅ \cap \mathcal{X}$. If we suggest that $Ḅ = \{\varsigma_1, \varsigma_2\}$ with the same set $Ạ$, then n-$\tilde{I}sg$-$\mathcal{K}er(Ḅ) = \{\varsigma_1, \varsigma_2\}$, then $Ḅ$ is n-$\tilde{I}sg$-$\mathcal{S}$ set and $Ḅ$ is n-$\tilde{I}sg$-φ-$closed$ set, but $Ḅ$ is not n-$\tilde{I}sg$-$closed$ set.

Proposition 4.8. In $(\mathcal{X}, \mathcal{T}_R(Ạ), \tilde{I})$, if $\mathcal{X}$ is a finite set and $Ự \subseteq \mathcal{X}$; $Ự$ is n-$\tilde{I}sg$-φ-$closed$ set, then $Ự = n$-$\tilde{I}sg$-$\mathcal{K}er(Ự) \cap Ḥ$, where $Ḥ$ is n-$\tilde{I}sg$-$closed$ set.

Proof: Since $Ự$ is n-$\tilde{I}sg$-φ-$closed$ set, then $Ự = Ḥ \cap Ḅ$ such that $Ḥ$ is a n-$\tilde{I}sg$- $closed$ set and $Ḅ$ is n-$\tilde{I}sg$-$\mathcal{S}$ set. This implies that $Ự \subseteq n$-$\tilde{I}sg$-$\mathcal{K}er(Ḅ) = Ḅ$, $Ự \subseteq Ḥ$ and $Ự \subseteq n$-$\tilde{I}sg$-$\mathcal{K}er(Ự)$. Which implies that $Ự \subseteq n$-$\tilde{I}sg$-$\mathcal{K}er(Ự) \cap Ḥ \subseteq n$-$\tilde{I}sg$-$\mathcal{K}er(Ḅ) \cap Ḥ = Ḅ \cap Ḥ = Ự$.

5. The Application in n-$\tilde{I}sg$-Closed set

Example 5.1. Smallpox is a disease that affects children and adults, where symptoms are manifested by the emergence of liquid blisters or pills in the skin, with itching as well as high temperatures. Smallpox is caused by a type of virus which can be spread by direct contact by touching, with the emergence of a rash. It may also spread with diarrhea and the emaciation of an infected person.

In this example, according to table 5 below, we present data of 4 patients some of whom are infected with smallpox and the others are not. Among the five symptoms stated above, we determine the effective factors for diagnosing this disease.

Table 5-

Patients	**Skin rash (with Liquid pills) (S)**	**Diarrhea (D)**	**Itching (H)**	**Emaciation (E)**	**Temperature (T)**	**Smallpox**
v_1	T	T	T	T	$Very\ high$	T
v_2	T	F	T	F	$High$	T
v_3	F	F	T	F	$High$	F
v_4	F	F	F	F	$Very\ high$	F

In this table, let $\mathcal{X} = \{v_1, v_2, v_3, v_4\}$, $\tilde{I} = \{\phi, \{v_1\}\}$ and $Ạ = \{v_1, v_3\}$, with an equivalence relation $Ɽ$ on $\mathcal{X}$, where $Ɽ = \{(v_i, v_j);\ v_i, v_j \in \mathcal{X}\}$ such that v_i and v_j have the same symptoms. From Table 5, $\mathcal{X}/Ɽ = \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}$, therefore $\mathcal{T}_R(Ạ) = \{\mathcal{X}, \phi, \{v_1, v_3\}\}$, then n-$SO(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_1, v_3\}, \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}\}$ and n - $\tilde{I}sg$ -$C(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_2, v_4\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}\}$. If we cancel the emaciation column, we get $\mathcal{X}/Ɽ(E) = \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\} = \mathcal{X}/Ɽ$, therefore $\mathcal{T}_R(Ạ) = \{\mathcal{X}, \phi, \{v_1, v_3\}\}$, n-$SO(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_1, v_3\}, \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}\}$ and n-$\tilde{I}sg$-$C(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_2, v_4\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}\}$ which is equal to n-$\tilde{I}sgC(\mathcal{X})$ with respect to $\mathcal{T}_R(Ạ)$. If we cancel the itching column, we get $\mathcal{X}/Ɽ(H) = \{\{v_1\}, \{v_2\}, \{v_3, v_4\}\}$, therefore $\mathcal{T}_{R(H)}(Ạ) = \{\mathcal{X}, \phi, \{v_1\}, \{v_3, v_4\}, \{v_1, v_3, v_4\}\}$, then $\mathcal{T}_{R(H)}(Ạ) \neq \mathcal{T}_R(Ạ)$, then n-$SO(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_1\}, \{v_3, v_4\}, \{v_1, v_3, v_4\}, \{v_1, v_2\}, \{v_2, v_3, v_4\}\}$ and n-$\tilde{I}sg$-$C(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_2\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}\}$ which is differ from n-$\tilde{I}sg$- -$C(\mathcal{X})$ with respect to $\mathcal{T}_R(Ạ)$. If we cancel the skin rash column, we get $\mathcal{X}/Ɽ(S) = \{\{v_1\}, \{v_4\}, \{v_2, v_3\}\}$, therefore $\mathcal{T}_{R(S)}(Ạ) = \{\mathcal{X}, \phi, \{v_1\}, \{v_2, v_3\}, \{v_1, v_2, v_3\}\}$, then $\mathcal{T}_{R(S)}(Ạ) \neq \mathcal{T}_R(Ạ)$. Hence, n-$SO(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_1\}, \{v_2, v_3\}, \{v_1, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}\}$ and n-$\tilde{I}sg$-$C(\mathcal{X}) = \{\mathcal{X}, \phi, \{v_4\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$ which differs from $n\text{-}\tilde{I}sg\text{-}C(\mathcal{X})$ with respect to $\mathcal{T}_R(A)$. If we cancel the diarrhea column, we get $\mathcal{X}/R(D) = \{\{v_1\},\{v_2\},\{v_3\},\{v_4\}\} = \mathcal{X}/R$, therefore $\mathcal{T}_R(A) = \{\mathcal{X},\phi,\{v_1,v_3\}\}$, then $n\text{-}SO(\mathcal{X}) = \{\mathcal{X},\phi,\{v_1,v_3\},\{v_1,v_2,v_3\},\{v_1,v_3,v_4\}\}$ and $n\text{-}\tilde{I}sg\text{-}C(\mathcal{X}) = \{\mathcal{X},\phi,\{v_2,v_4\},\{v_1,v_2,v_4\},\{v_2,v_3,v_4\}\}$ which is equal to $n\text{-}\tilde{I}sg\text{-}C(\mathcal{X})$ with respect to $\mathcal{T}_R(A)$. From all that the above, we get $core(R) = \{H, S\}$. This result implies that itching and skin rash are the necessary and sufficient symptoms to diagnose patients developing smallpox.

We can show the previous information as described in Table-6.

Table 6-

The collection of equivalent classes	The *nano topology*	$nSO(\mathcal{X})$	$\tilde{I}sgC(\mathcal{X})$
$\mathcal{X}/R = \{\{v_1\},\{v_2\},\{v_3\},\{v_4\}\}$	$\mathcal{T}_R(A) = \{\mathcal{X},\phi,\{v_1,v_3\}\}$	$\{\mathcal{X},\phi,\{v_1,v_3\},\{v_1,v_2,v_3\},\{v_1,v_3,v_4\}$	$\{\mathcal{X},\phi,\{v_2,v_4\},\{v_1,v_2,v_4\},\{v_2,v_3,v_4\}\}$
$\mathcal{X}/R(E) = \{\{v_1\},\{v_2\},\{v_3\},\{v_4\}\}$	$\mathcal{T}_{R(E)}(A) = \{\mathcal{X},\phi,\{v_1,v_3\}\}$	$\{\mathcal{X},\phi,\{v_1,v_3\},\{v_1,v_2,v_3\},\{v_1,v_3,v_4\}$	$\{\mathcal{X},\phi,\{v_2,v_4\},\{v_1,v_2,v_4\},\{v_2,v_3,v_4\}\}$
$\mathcal{X}/R(H) = \{\{v_1\},\{v_2\},\{v_3,v_4\}\}$	$\mathcal{T}_{R(H)}(A) = \{\mathcal{X},\phi,\{v_1\},\{v_3,v_4\},\{v_1,v_3,v_4\}\}$	$\{\mathcal{X},\phi,\{v_1\},\{v_3,v_4\},\{v_1,v_3,v_4\},\{v_1,v_2\},\{v_2,v_3,v_4\}\}$	$\{\mathcal{X},\phi,\{v_2\},\{v_1,v_2\},\{v_2,v_3\},\{v_2,v_4\},\{v_2,v_3,v_4\},\{v_1,v_2,v_3\},\{v_1,v_2,v_4\}\}$
$\mathcal{X}/R(S) = \{\{v_1\},\{v_4\},\{v_2,v_3\}\}$	$\mathcal{T}_{R(S)}(A) = \{\mathcal{X},\phi,\{v_1\},\{v_2,v_3\},\{v_1,v_2,v_3\}\}$	$\{\mathcal{X},\phi,\{v_1\},\{v_2,v_3\},\{v_1,v_4\},\{v_1,v_2,v_3\},\{v_2,v_3,v_4\}\}$	$\{\mathcal{X},\phi,\{v_4\},\{v_1,v_4\},\{v_2,v_4\},\{v_3,v_4\},\{v_2,v_3,v_4\},\{v_1,v_2,v_4\},\{v_1,v_3,v_4\}\}$
$\mathcal{X}/R(D) = \{\{v_1\},\{v_2\},\{v_3\},\{v_4\}\}$	$\mathcal{T}_{R(D)}(A) = \{\mathcal{X},\phi,\{v_1,v_3\}\}$	$\{\mathcal{X},\phi,\{v_1,v_3\},\{v_1,v_2,v_3\},\{v_1,v_3,v_4\}$	$\{\mathcal{X},\phi,\{v_2,v_4\},\{v_1,v_2,v_4\},\{v_2,v_3,v_4\}\}$

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