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Flat Model for Representing Contiguous UTM Coordinates over Iraq Territory

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Abstract

The process of converting coordinates is, still, considered an important and difficult issue due to the way of conversion from geographic ellipsoidal system to the projected flat system. The most common method uses contiguous UTM system as one of the most accurate systems in the conversion process, but the users of the system face problems related to contiguity, especially at the large areas that lie within more than one zone. The aim of the present research is to solve the problem related to the multiple zones coverage found in the Iraqi territory using a mathematical model based on the use of Taylor series. The most accurate conversion equation used in this paper was based on the 4^{th} order polynomial of two variables. The calculation of equations' coefficients was performed using least square criterion for the coordinate's values, i.e., either latitude, longitude) or East (E), North (N) coordinates. The two basic determinations, for the forward and backward, were applied. In the first stages, the conversion of the coordinates from Longitude/ Latitude to East/ North was determined. Then, the second conversion stage was determined, i.e., the coordinates conversion from East, North to Longitude, Latitude). For each phase, a spatial accuracy assessment was conducted. The results showed that the adopted mathematical model was successful to accomplish the conversion process. A very small error average of about 3 cm at east and less than 5 cm at north was reached using the 4th order polynomial equations.

Keywords: Geographic Coordinate System, UTM, Polynomial Representation, Taylor Series

نموذج مسطح لتمثيل إحداثيات UTM القريبة على أرض العراق

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الخلاصة

لا تزال عملية تحويل الإحداثيات من المهام الصعبة بالرغم من وجود طرق مختلفة للتحويل من النظام الجغرافي إلى النظام المسقطي (المعروف احدهم بالنظام UTM الأكثر شيوعًا)، حيث انه أكثر الأنظمة دقة في عملية التحويل، ولكن هذا النظام يواجه مشكلة في المناطق كبيرة التي تحتوي أكثر من شريحة.

يهدف هذا البحث الى حل المشكلة المتعلقة بتعدد الشرائح الموجودة في المناطق المجاورة للأراضي العراقية باستخدام نموذج رياضي يعتمد على استخدام متسلسلة تايلر ثنائية الابعاد. حيث تم الوصول إلى

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حساب المعادلة المتعددة الحدود من المرتبة الرابعة وتم حساب قيم المعاملات في هذه المعادلة وتطبيقها في عملية التحويل ومن خلال تنفيذ المرحلتين الأساسيتين – المرحلة الأولى – عملية تحويل الإحداثيات (خطوط الطول، دوائر العرض) إلى (X,Y) أي إحداثيات مترية وللتأكد من نجاح عملية التحويل، وتطبيق المراحل الثانية لتحويل الإحداثيات من (X,Y) إلى (خطوط الطول، دوائر العرض).

أظهرت النتائج أن النموذج الرياضي المستخدم كان ناجحًا في عملية التحويل. حيث تم الوصول إلى معدل خطأ صغير جدًا يبلغ متوسطة حوالي 3 سم في الشرق او(X) وأقل من 5 سم في الشمال او (Y) باستخدام معادلة متعددة الحدود من المرتبة الرابعة.

1. Introduction

The Universal Transverse Mercator (UTM) is a projection coordinate system. It is a type of plane coordinate system, also called Cartesian coordinate system [1]. The position of a point in the rectangular coordinate system is defined by its distance from the x and y axes. The two distance values are the X and Y coordinates of the point, with the use of a measurement unit such as meters, kilometers, etc. [2, 3].

A position on the Earth is given by the UTM zone number and the easting and northing planar coordinate pair in that zone. The origin point of each UTM zone is the intersection of the equator and the zone's central meridian [4]. To avoid dealing with negative numbers, the central meridian of each zone is defined to coincide with 500000 meters East. In any zone, a point that has an easting of 400000 meters is about 100 km west of the central meridian [5]. For most such points, the true distance would be slightly more than 100 km, as measured on the surface of the Earth, due to the distortion of the projection. UTM eastings range from about 167000 meters to 833000 meters at the equator [6, 7, 8].

The UTM system covers almost every surface of the earth. Only polar areas latitudes that are higher than 84° North and 80° South are excluded. The UTM system divides the Earth surface into 60 zones, each is with 6 degrees. In addition, the zone is numbered west to east from 1 to 60, starting at 180° West longitude [9, 10]. We cannot imagine the flattening of the Earth's surface without converting it to a plane form. The UTM system is the designer to provide a tool of representing each point on the Earth using a set of flat (X, Y or Easting, Northing) coordinates [11]. The advantages of representing locations on the Earth with flat coordinates are the easy planar mapping, easily derived spatial information (e.g., distances, angles, areas, etc.) from the location coordinates, and the ability to calculate the coordinates of a point based on spatial information. In UTM, all measurements are achieved in meters. Therefore, the calculation of distances, directions, and areas can be performed much more conveniently in comparison to the geographic coordinate system [12].

2. Contiguous Problem related to UTM

The UTM system, like any other system, has some problems when converting coordinates, but these problems are less than those found in other conversion systems. The principle of this system is based on the division of the world into zones, but each chip has certain characteristics that do not apply to other zones. Most states contain more than one chip and here lies the problem, so that when moving from one chip to another the results of the conversion would be unrealistic [13, 14]. For example, the United States is located within ten UTM zones and the fact that there are many tight UTM zones can produce troubles. For instance, Philadelphia city in Pennsylvania is east of the city of Pittsburgh. If the Eastings of centroids representing the two cities are compared, the result would be that the Easting of Philadelphia (almost 486 km) is less than that of Pittsburgh (almost 586 km). Despite that the two cities are both located in the state of Pennsylvania, they are existing in two various zones of the UTM system; Philadelphia is nearer to the Zone 18 than Pittsburgh which is located at the Zone 17.

over the Iraqi territory, the same type of problem exists, where Iraq is located in three zones (37, 38 and 39) within the UTM system. The zone 37 covers parts of western Iraq, the zone 38 covers the middle area which represents most of Iraq, and the zone 39 covers a small part of the city of Basra in east-south Iraq. Figure 1 illustrates the zones 37, 38 and 39 covering the territory of Iraq.

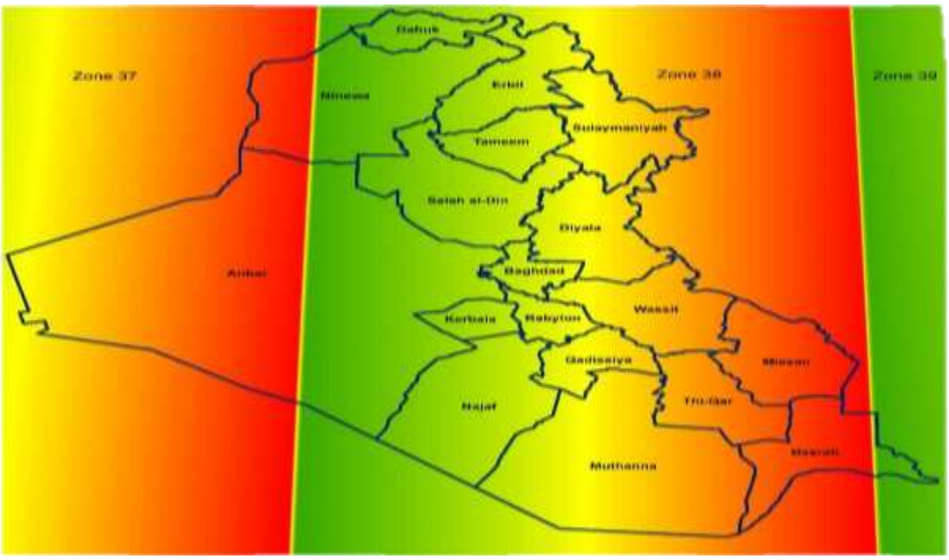

Figure 1-The 37, 38 and 39 zones covering Iraq's area.

3. Theoretical Framework

In this section, the mathematical framework of the two phases of conversion, i.e., the forward and the backward, is provided.

3.1 Forward Representation

The coordinate value of East and North (X and Y) can be represented mathematically as

$$X = F(\varphi,\lambda)\ ,\ \ Y = G(\varphi,\lambda) \tag{1}$$

These functions are continuous over most of the regions of globe, except the poles. Thus, the Taylor series [15] of both functions near the point (φ_o, λ_o) are

$$X = F(\varphi_o,\lambda_o) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \left.\frac{\partial F^{m+n}(\varphi,\lambda)}{\partial^m\varphi\,\partial^n\lambda}\right|_{\varphi_o,\lambda_o} \frac{(\varphi-\varphi_o)^m}{m!}\frac{(\lambda-\lambda_o)^n}{n!},\quad \text{where } n+m>0 \tag{2}$$

$$Y = G(\varphi_o,\lambda_o) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \left.\frac{\partial G^{m+n}(\varphi,\lambda)}{\partial^m\varphi\,\partial^n\lambda}\right|_{\varphi_o,\lambda_o} \frac{(\varphi-\varphi_o)^m}{m!}\frac{(\lambda-\lambda_o)^n}{n!},\quad \text{where } n+m>0 \tag{3}$$

In the case that the neighborhood of (φ,λ) is bounded to an area extended to few degrees far from $\varphi o, \lambda o$, then the values of m and n can be bounded to be small, i.e., less than 3 or 4. In such case, the above equations can be approximated to be:

$$X = \sum_{i=0}^{Order}\sum_{j=0}^{i} a_{n(i)+j}(\Delta\varphi)^{i-j}(\Delta\lambda)^j \tag{4a}$$

$$Y = \sum_{i=0}^{Order}\sum_{j=0}^{i} b_{n(i)+j}(\Delta\varphi)^{i-j}(\Delta\lambda)^j \tag{4b}$$

where, $n(i) = \frac{1}{2}i(i+1)\,,\quad \Delta\varphi = \varphi - \varphi_o,\quad \Delta\lambda = \lambda - \lambda_o$

For the 2nd Order Polynomials

$$X = a_0 + a_1\Delta\varphi + a_2\Delta\lambda + a_3(\Delta\varphi)^2 + a_4\Delta\varphi\Delta\lambda + a_5(\Delta\lambda)^2 \tag{5a}$$

$$Y = b_0 + b_1\Delta\varphi + b_2\Delta\lambda + b_3(\Delta\varphi)^2 + b_4\Delta\varphi\Delta\lambda + b_5(\Delta\lambda)^2 \tag{5b}$$

For the 3rd Order Polynomials

$$X = a_0 + a_1\Delta\varphi + a_2\Delta\lambda + a_3(\Delta\varphi)^2 + a_4\Delta\varphi\Delta\lambda + a_5(\Delta\lambda)^2 + a_6(\Delta\varphi)^3 + a_7(\Delta\varphi)^2\Delta\lambda + a_8\Delta\varphi(\Delta\lambda)^2 + a_9(\Delta\lambda)^3 \quad (6a)$$

$$Y = b_0 + b_1\Delta\varphi + b_2\Delta\lambda + b_3(\Delta\varphi)^2 + b_4\Delta\varphi\Delta\lambda + b_5(\Delta\lambda)^2 + a_6(\Delta\varphi)^3 + a_7(\Delta\varphi)^2\Delta\lambda + a_8\Delta\varphi(\Delta\lambda)^2 + a_9(\Delta\lambda)^3 \quad (6b)$$

For the 4th Order Polynomials

$$X = a_0 + a_1\Delta\varphi + a_2\Delta\lambda + a_3(\Delta\varphi)^2 + a_4\Delta\varphi\Delta\lambda + a_5(\Delta\lambda)^2 + a_6(\Delta\varphi)^3 + a_7(\Delta\varphi)^2\Delta\lambda + a_8\Delta\varphi(\Delta\lambda)^2 + a_9(\Delta\lambda)^3 + a_{10}(\Delta\varphi)^4 + a_{11}(\Delta\varphi)^3\Delta\lambda + a_{12}(\Delta\varphi)^2(\Delta\lambda)^2 + a_{13}\Delta\varphi(\Delta\lambda)^3 + a_{14}(\Delta\lambda)^4 \quad (7a)$$

$$Y = b_0 + b_1\Delta\varphi + b_2\Delta\lambda + b_3(\Delta\varphi)^2 + b_4\Delta\varphi\Delta\lambda + b_5(\Delta\lambda)^2 + a_6(\Delta\varphi)^3 + a_7(\Delta\varphi)^2\Delta\lambda + a_8\Delta\varphi(\Delta\lambda)^2 + a_9(\Delta\lambda)^3 + b_{10}(\Delta\varphi)^4 + b_{11}(\Delta\varphi)^3\Delta\lambda + b_{12}(\Delta\varphi)^2(\Delta\lambda)^2 + b_{13}\Delta\varphi(\Delta\lambda)^3 + b_{14}(\Delta\lambda)^4 \quad (7b)$$

3.2 Backward Representation

Also, the inverse mapping equations can be approximately written in the form:

$$\varphi = H(X,Y)\ ,\ \ \lambda = Q(X,Y) \quad (8)$$

Also, the equation is continuous in the region on interest (i.e., the Iraqi territory), so the Taylor series [16, 17] for both equations no. 2 and 3 can be written as in equations 9 and 10 below:

$$\varphi = H(X_o,Y_o) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \left.\frac{\partial H(X,Y)}{\partial^m X\,\partial^n Y}\right|_{X_o,Y_o} \frac{(X-X_o)^m}{m!}\frac{(Y-Y_o)^n}{n!},\quad \text{where } n+m>0 \quad (9)$$

$$\lambda = Q(X_o,Y_o) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \left.\frac{\partial Q^{m+n}(X,Y)}{\partial^m X\,\partial^n Y}\right|_{X_o,\lambda_o} \frac{(X-X_o)^m}{m!}\frac{(Y-Y_o)^n}{n!},\quad \text{where } n+m>0 \quad (10)$$

The same mathematical manipulation can be performed to rewrite the above equations in terms of 2D polynomials:

$$\varphi = \sum_{i=0}^{Order}\sum_{j=0}^{i} c_{n(i)+j}(\Delta X)^{i-j}(\Delta Y)^j \quad (11a)$$

$$\lambda = \sum_{i=0}^{Order}\sum_{j=0}^{i} d_{n(i)+j}(\Delta X)^{i-j}(\Delta Y)^j \quad (11b)$$

where, $n(i) = \frac{1}{2}i(i+1)\,,\quad \Delta x = x - x_o \qquad \Delta y = y - y_o$

For the 2nd Order Polynomials

$$\varphi = c_0 + c_1\Delta x + c_2\Delta y + c_3(\Delta x)^2 + c_4\Delta x\Delta y + c_5(\Delta y)^2 \quad (12a)$$

$$\lambda = d_0 + d_1\Delta x + d_2\Delta y + d_3(\Delta x)^2 + d_4\Delta x\Delta y + d_5(\Delta y)^2 \quad (12b)$$

For the 3rd Order Polynomial

$$\varphi = c_0 + c_1\Delta x + c_2\Delta y + c_3(\Delta x)^2 + c_4\Delta x\Delta y + c_5(\Delta y)^2 + c_6(\Delta x)^3 + c_7(\Delta x)^2\Delta y + c_8\Delta x(\Delta y)^2 + c_9(\Delta y)^3 \quad (13a)$$

$$\lambda = d_0 + d_1\Delta x + d_2\Delta y + d_3(\Delta x)^2 + d_4\Delta x\Delta y + d_5(\Delta y)^2 + d_6(\Delta x)^3 + d_7(\Delta x)^2\Delta y + d_8\Delta x(\Delta y)^2 + d_9(\Delta y)^3 \quad (13b)$$

For the 4th Order Polynomials

$$\varphi = c_0 + c_1\Delta x + c_2\Delta y + c_3(\Delta x)^2 + c_4\Delta x\Delta y + c_5(\Delta y)^2 + c_6(\Delta x)^3 + c_7(\Delta x)^2\Delta y + c_8\Delta x(\Delta y)^2 + c_9(\Delta y)^3 + c_{10}(\Delta x)^4 + c_{11}(\Delta x)^3\Delta y + c_{12}(\Delta x)^2(\Delta y)^2 + c_{13}\Delta x(\Delta y)^3 + c_{14}(\Delta y)^4 \quad (14a)$$

$$\lambda = d_0 + d_1\Delta x + d_2\Delta y + d_3(\Delta x)^2 + d_4\Delta x\Delta y + d_5(\Delta y)^2 + d_6(\Delta x)^3 + d_7(\Delta x)^2\Delta y + d_8\Delta x(\Delta y)^2 + d_9(\Delta y)^3 + d_{10}(\Delta x)^4 + d_{11}(\Delta x)^3\Delta y + d_{12}(\Delta x)^2(\Delta y)^2 + d_{13}\Delta x(\Delta y)^3 + d_{14}(\Delta y)^4 \quad (14b)$$

4. Results and data sets

4.1 Datasets

The datasets represent grid points of the coordinates of longitude, latitude covering the Iraqi territory and some adjacent areas; so that the distance between each two points in the grid is 0.5 degrees. These coordinates are then converted to X, Y coordinates using UTM system conversion software in the conversion process, but we will determine this program on zone 38 and calculate X, Y coordinates values, as shown in Table-1 and Figure-3.

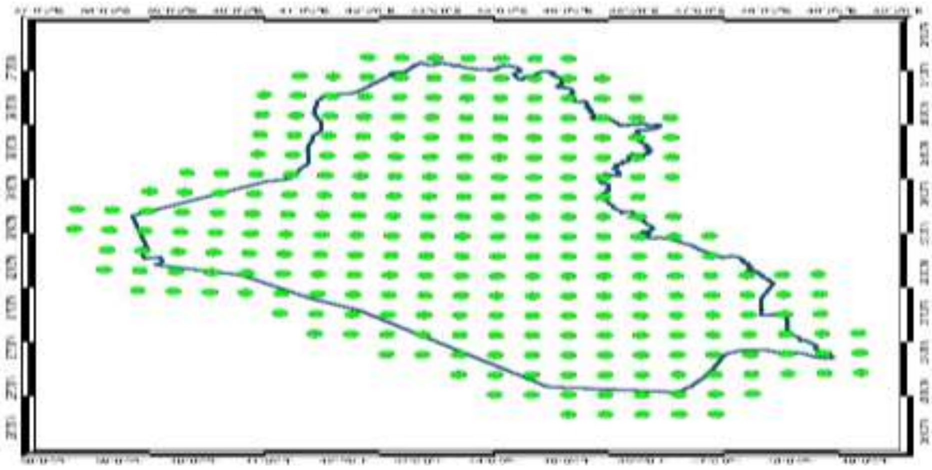

Figure 2-Dataset (Grid Points).

4.2. Results

4.2.1 For forward representation

This step represents the calculation rate of error for the first test at X east and Y north using the 2nd, 3rd, and 4th order polynomial equations, depending on normalization conversion process.

1. Tables-(1 and 2) represent the values of coefficients rate error, respectively, at X east and y north using the 2nd order polynomial equations 5a and 5b. Also, the error distributions could be drawn as shown in Figures-(4 and 5).

Table 1-Values of coefficients using the 2nd order polynomial equations (5a & 5b)

	Coefficients		Coefficients
a_0	0.056233205	b_0	0.000481156
a_1	10.67302884	b_1	-0.010988732
a_2	0.093370086	b_2	1.002562804
a_3	-0.097019663	b_3	0.232457867
a_4	-4.023212612	b_4	-0.005665115
a_5	0.012356132	b_5	0.001717335

Table 2-The error rate values using the 2nd order polynomial equations (5a & 5b)

	rate error at X easting (m)	rate error at Y northing (m)
Maximum	479.237	303.906
Minimum	0.639	0.0794
Average	111.704	57.995

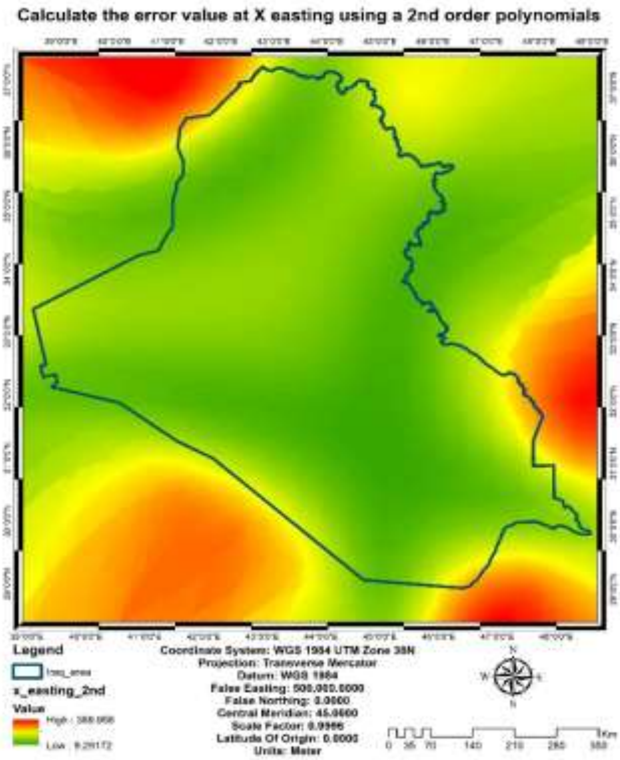

Figure 3-present the distribution of error values at X easting using a 2nd order polynomials

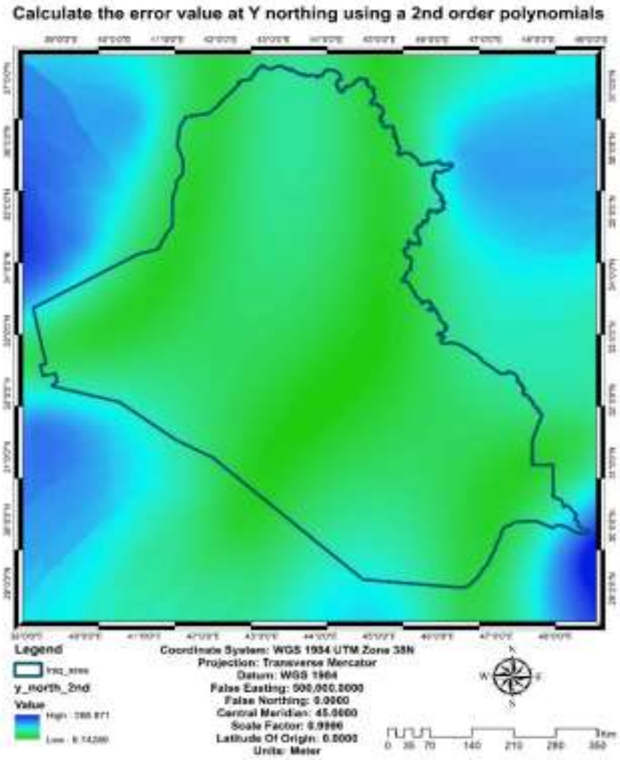

Figure 4- present the distribution of error values at Y northing using a 2nd order polynomials

2. Tables-(3 and 4) represent the value of coefficients and rate error, respectively, at X east and y north using the 3rd order polynomial equations (6a and 6b. Also, the error distributions were drawn as shown in Figures-(6 and 7).

Table 3-Values of coefficients using the 3rd order polynomial equations (6a & 6b)

	Coefficients		Coefficients
a_0	0.056052815	b_0	0.000474652
a_1	10.67656151	b_1	-0.010679744
a_2	0.090270214	b_2	1.002351623
a_3	-0.025197051	b_3	0.235597899
a_4	-3.979903972	b_4	-0.005187345
a_5	0.039522455	b_5	0.002347984
a_6	0.477670176	b_6	-0.008588967
a_7	0.133050544	b_7	0.121444681
a_8	-1.730962083	b_8	0.005931998
a_9	0.004883848	b_9	0.001189638

Table 4-The error rate values using the 3rd order polynomial equations (6a & 6b)

	rate error at X easting (m)	rate error at Y northing (m)
Maximum	21.369	25.593
Minimum	0.027	0.025
Average	3.609	4.604

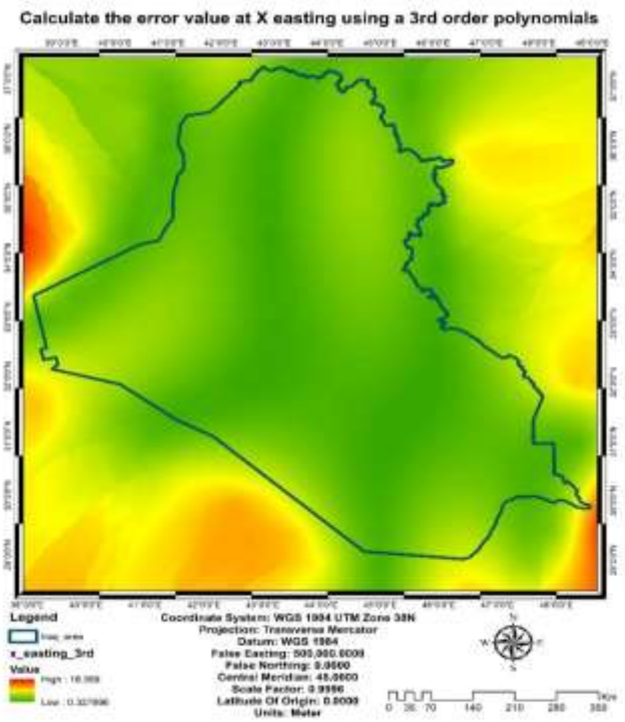

Figure 5-present the distribution of error values at X easting using a 3rd order polynomials

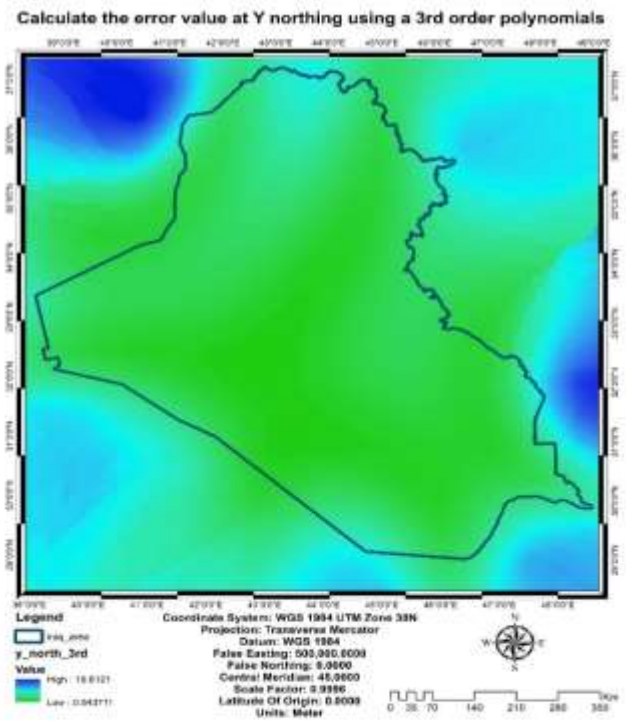

Figure 6-present the distribution of error values at Y northing using a 3rd order polynomials

3.Tables-(5 and 6) represent the values of coefficients and rate error, respectively, at X east and y north using the 4th order polynomial equations (7a and 7b. In addition, the error distributions are shown in Figures-(8 and 9).

Table 5-Values of coefficients using the 4th order polynomial equations (7a & 7b)

	Coefficients		Coefficients
a_0	0.056057752	b_0	0.000474249
a_1	10.67676232	b_1	-0.010706578
a_2	0.090342133	b_2	1.002359723
a_3	-0.029358049	b_3	0.235608589
a_4	-3.976320625	b_4	-0.00550418
a_5	0.040289721	b_5	0.002570736

a_6	0.429537911	b_6	-0.003416047
a_7	0.086420031	b_7	0.120813626
a_8	-1.771913844	b_8	0.007168094
a_9	-0.004921159	b_9	0.000421948
a_{10}	0.005952919	b_{10}	0.038056529
a_{11}	-1.274823713	b_{11}	0.000846899
a_{12}	-0.007804927	b_{12}	-0.155828198
a_{13}	0.212538737	b_{13}	0.00083605
a_{14}	-0.000689277	b_{14}	-0.000231149

Table 6-The error rate values using the 4th order polynomial equations (7a & 7b)

	rate error at X easting (m)	rate error at Y northing (m)
Maximum	0.154	0.286
Minimum	0.000	0.002
	0.025	0.0498

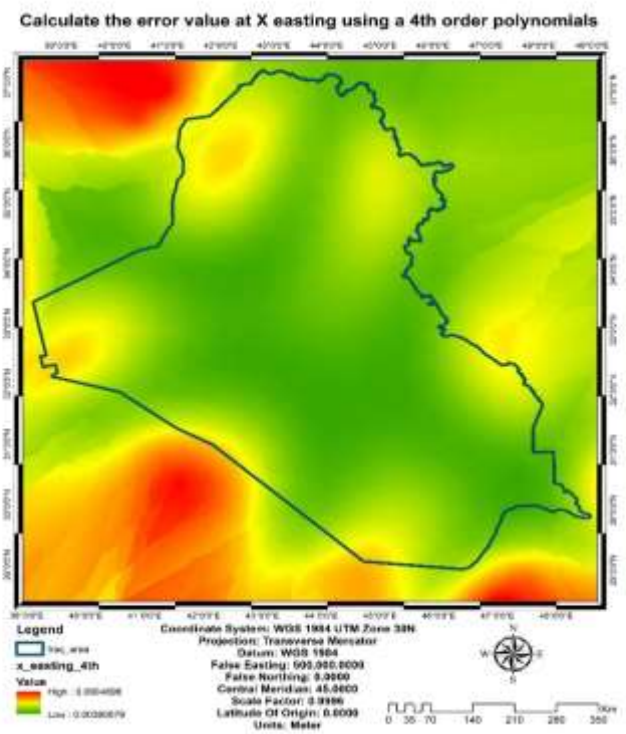

Figure 7-present the distribution of error values at X easting using a 4th order polyno

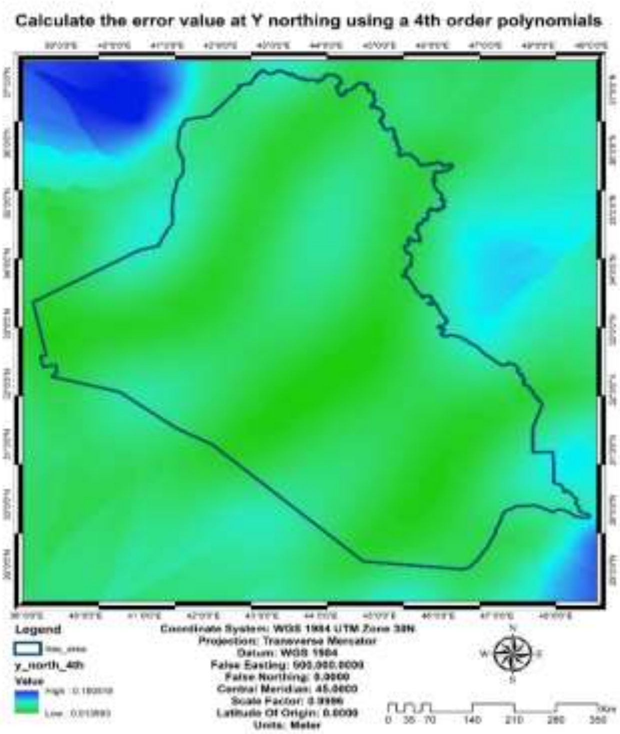

Figure 8- present the distribution of error values at Y northing using a 4th order polynomials

Average

4.2.2 For Backward Representation

This step represents the calculation of the rate of error for the first test at Longitude and Latitude using the 2ed, 3rd, and 4th order polynomials equations depending on the normalization conversion process.

3. Tables- (7 and 8) represent the values of coefficients and rate error, respectively, at longitude and latitude using the 2nd order polynomial equations (5c and 5d). Also, we could draw the distributions of error as shown in Figures-(10 and 11).

Table 7-Values of coefficients using the 2nd order polynomial equations (5c & 5d)

	Coefficients		Coefficients
c_0	-0.005277382	d_0	-0.000558245
c_1	0.093634296	d_1	0.001283094
c_2	-0.010933052	d_2	0.996970094
c_3	0.000192554	d_3	-0.001977919
c_4	0.035602483	d_4	0.001464073
c_5	-0.002324351	d_5	-0.000721924

Table 8-The error rate values using the 2nd order polynomial equations (5c & 5d)

	rate error at Longitude (sec)	rate error at Latitude (sec)
Maximum	37.16646	21.56515
Minimum	0.006929	0.02133
Average	8.603299	4.401512

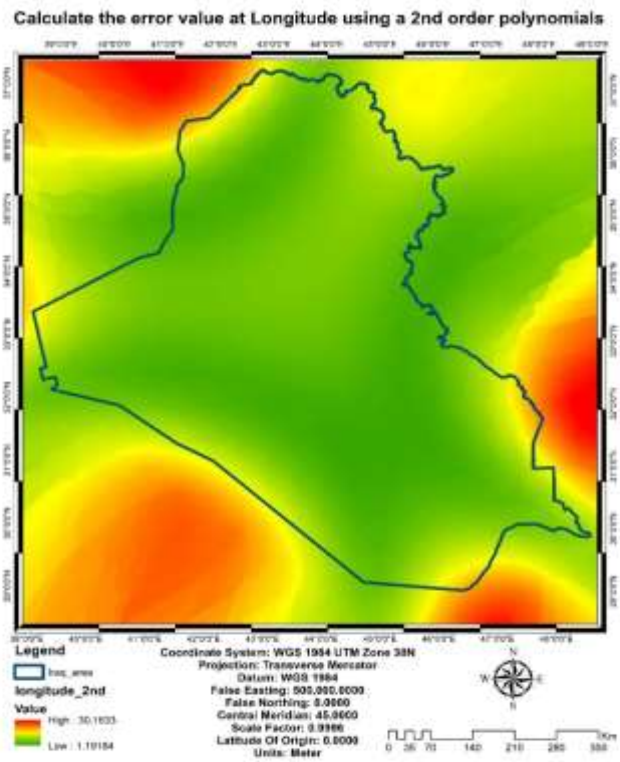

Figure 9-present the distribution of error values at Longitude using a 2nd order polynomials

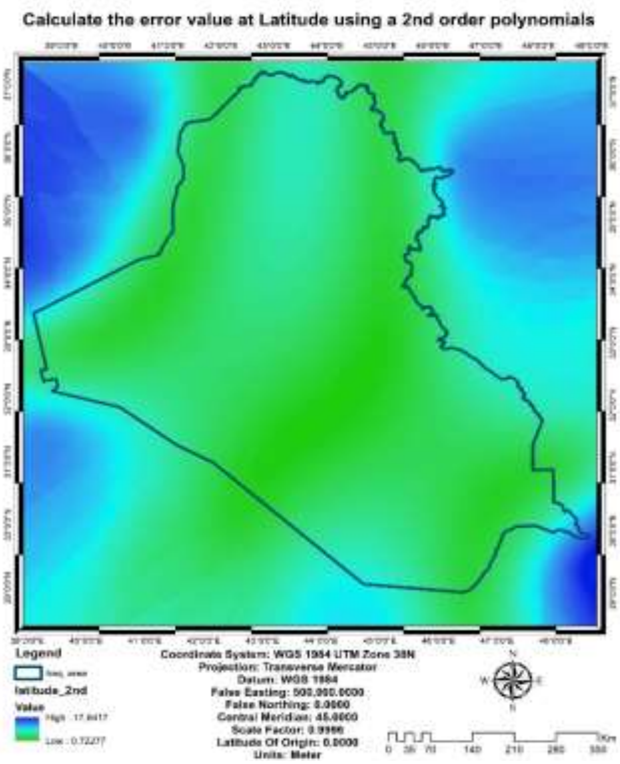

Figure 10-present the distribution of error values at Latitude using a 2nd order polynomials

4. Tables-(9 and 10) represent the values of coefficients and rate error, respectively, at longitude and latitude using the 3rd order polynomial equations (6c and 6d). Also, we could draw the distributions of error as shown in Figures-(12 and 13).

Table 9-Values of coefficients using the 3rd order polynomial equations (6c & 6d)

	Coefficients		Coefficients
c_0	-0.005244783	d_0	-0.00053587
c_1	0.093622695	d_1	0.001232322
c_2	-0.010410525	d_2	0.997474459
c_3	9.98601E-05	d_3	-0.002055162
c_4	0.034756186	d_4	0.001557791
c_5	-0.008846928	d_5	-0.002855796
c_6	-9.6549E-05	d_6	-7.26E-06
c_7	0.00024294	d_7	-0.002572616
c_8	0.02920124	d_8	0.000388714
c_9	-0.000340033	d_9	8.01967E-05

Table 10-The error rate values using the 3rd order polynomial equations (6c & 6d)

	rate error at Longitude (sec)	rate error at Latitude (sec)
Maximum	1.57353	0.609209
Minimum	0.001294	0.000471
Average	0.334704	0.124751

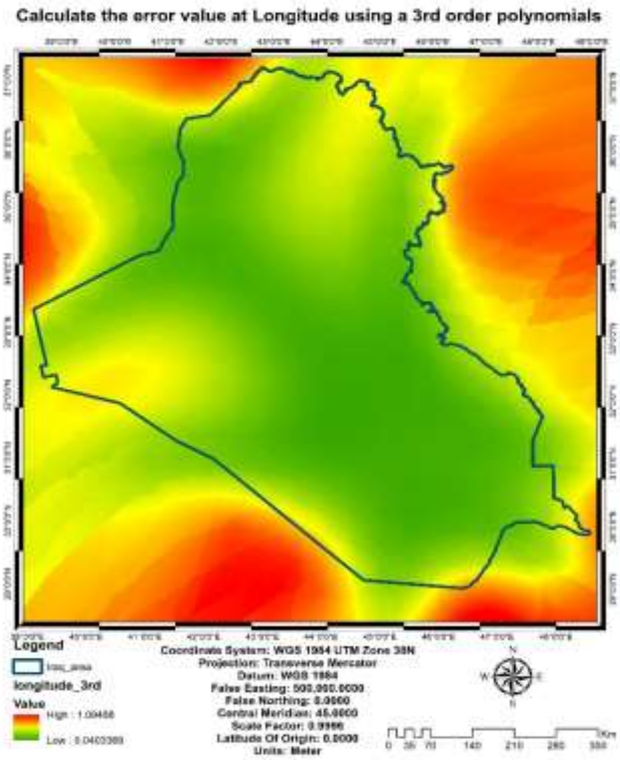

Figure 11-present the distribution of error values at Longitude using a 3rd order polynomials

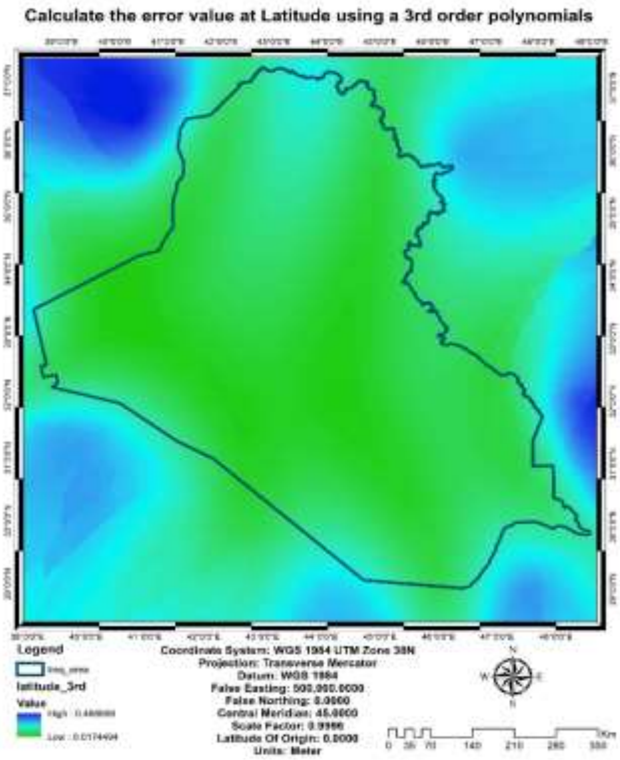

Figure 12-present the distribution of error values at Latitude using a 3rd order polynomials

6. Tables-(11 and 12) represent the values of coefficients and rate error, respectively, at the longitude and latitude using the 4th order polynomial equations (7c and 7d). Also, we could draw the distributions of error as shown in Figures-(14 and 15).

Table 11-Values of coefficients using the 4th order polynomial equations (7c & 7d)

	Coefficients		Coefficients
c_0	-0.0052448	d_0	-0.000535607
c_1	0.093626764	d_1	0.001230629
c_2	-0.010379846	d_2	0.997480429
c_3	9.43143E-05	d_3	-0.002058185
c_4	0.034719336	d_4	0.001541364
c_5	-0.008476756	d_5	-0.002716049
c_6	-0.000104558	d_6	-4.9682E-06
c_7	0.000145654	d_7	-0.002572951
c_8	0.028262383	d_8	0.000585766
c_9	-0.004602165	d_9	-0.000539866
c_{10}	-6.20839E-07	d_{10}	3.54477E-06
c_{11}	-0.000147522	d_{11}	-1.34117E-05
c_{12}	0.000218654	d_{12}	-0.000989218
c_{13}	0.015117426	d_{13}	0.000108676
c_{14}	0.000652888	d_{14}	0.000217122

Table 12-The error rate values using the 4th order polynomial equations (7c & 7d)

	rate error at Longitude (sec)	rate error at Latitude (sec)
Maximum	0.094228	0.038256
Minimum	0.000124	2.36E-05
Average	0.018406	0.005913

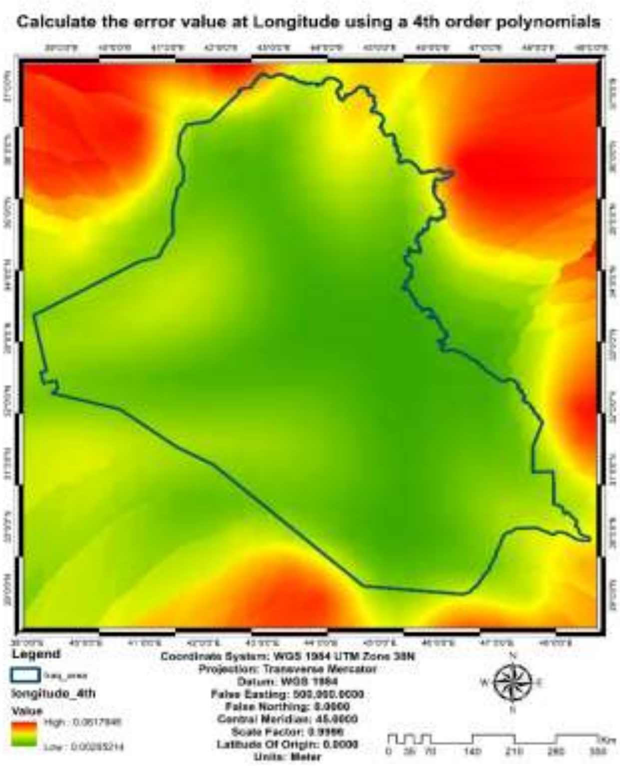

Figure 13-present the distribution of error values at Longitude using a 4th order polynomials

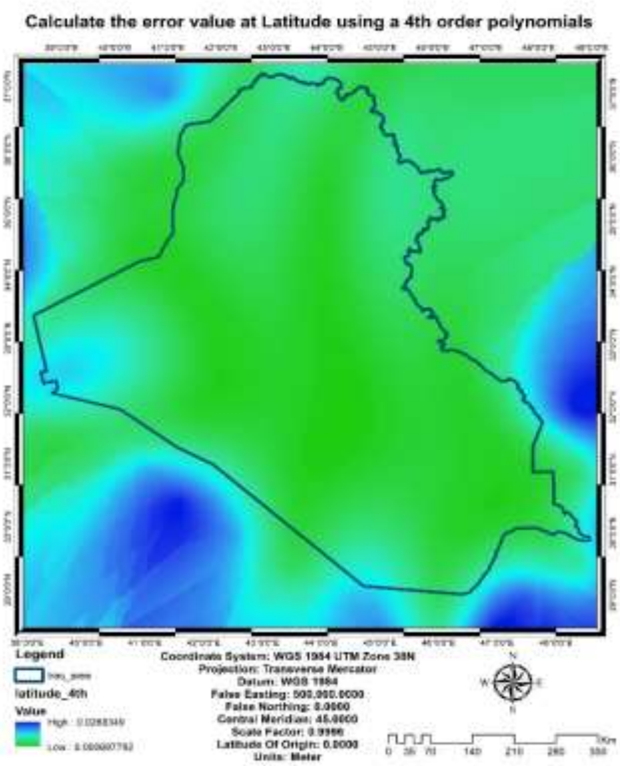

Figure 14-present the distribution of error values at Latitude using a 4th order polynomials

5. Conclusions

When the first test was used, the attained results held a very high error rate, leading to the failure of the conversion process. But, when the second test of the normalization method was used, the rate of error was gradually reduced to the lowest ratio, using the fourth-order polynomials equation. This resulted in the success of the conversion process and produced results that were near to idealism, as described in the following:

Using the second-order polynomial equation, the result of the error average values was approximately 120 meters at X east and about 50 meters at Y north. The average error value decreased when the third-order polynomial equation was used, where the average error values were less than 4 meters at the X east and about 5 meters at the Y north. The final conversion process was successful, causing a very large drop of average error values when the fourth order polynomial equations were used, where the average error values were less than 3 cm at X east and about 5 cm at Y north. These results are very good and we can depend on this model for the conversion process of X east and Y north to longitude and latitude for the Iraqi territory, due to the high precision of this model.

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