Iraqi Journal of Science, 2020, Vol. 61, No. 6, pp: 1433-1439
DOI: 10.24996/ijs.2020.61.6.22

ISSN: 0067-2904

Bayesian Estimation for the Parameters and Reliability Function of Basic Gompertz Distribution under Squared Log Error Loss Function

Manahel Kh. Awad*, Huda A. Rasheed
Department of Mathematics, College of Science, Mustansiriyah University, Baghdad, Iraq

Received: 29/7/2019 Accepted: 22/10/2019

Abstract
In this paper, some estimators for the unknown shape parameters and reliability function of Basic Gompertz distribution were obtained, such as Maximum likelihood estimator and some Bayesian estimators under Squared log error loss function by using Gamma and Jefferys priors. Monte-Carlo simulation was conducted to compare the performance of all estimates of the shape parameter and Reliability function, based on mean squared errors (MSE) and integrated mean squared errors (IMSE's), respectively. Finally, the discussion is provided to illustrate the results that are summarized in tables.

Keywords: Basic Gompertz distribution, Maximum likelihood estimator, Bayes estimator, Squared log error loss function, Reliability function, Mean squared errors.

التقدير البيزي لمعلمة ودالة المعولية لتوزيع Basic Gompertz Distribution تحت دالة الخسارة اللوغاريتمية التربيعية

مناهل خالد عواد*، هدى عبدالله رشيد
قسم الرياضيات، كلية العلوم، الجامعه المستنصرية، بغداد، العراق

الخلاصة
في هذا البحث، تم الحصول على بعض المقدرات لمعلمة الشكل غير المعروفة ودالة المعولية لتوزيع Basic Gompertz، مثل مقدّر الإمكان الأعظم وبعض المقدرات البيزية تحت دالة الخسارة اللوغاريتمية التربيعية باستخدام دوال اسبقية كاما وجيفري. تم إجراء محاكاة مونت كارلو لمقارنة أداء جميع تقديرات معلمة الشكل ودالة المعولية ، استنادًا إلى متوسط مربعات الخطأ (MSE) ومتوسط مربعات الخطأ التكاملي (IMSE)، على التوالي. أخيراً قدمت مناقشة لتوضيح النتائج التي تم تلخيصها في جداول.

1. Introduction

The Gompertz distribution plays an important role in modeling survival times, human mortality and actuarial data. It was formulated by Gompertz (1825) to fit mortality tables[1].
The probability density function of the Gompertz distribution is defined as follows [2]:

$$f(t\,;\lambda) = \lambda \exp\left[ct + \frac{\lambda}{c}(1 - e^{ct})\right] \;\; ; \quad t \geq 0 \quad c, \lambda > 0$$

where c is the scale parameter and λ is the shape parameter of the Gompertz distribution.
In this paper, we'll assume that c=1, which is a special case of Gompertz distribution known as Basic Gompertz distribution with the following probability density function [3]

$$f(t\,;\lambda) = \lambda \exp\left[t + \lambda(1 - e^{t})\right] \;\; ; \quad t \geq 0 \quad \lambda > 0 \qquad (1)$$

*Email: hudamath@uomustansiriyah.edu.iq

The corresponding cumulative distribution function F(t) and reliability or survival function R(t) of Basic Gompertz distribution are given by

$$F(t) = 1 - \exp[\lambda(1 - e^{t})] \quad ; \quad t \geq \qquad (2)$$

$$R(t) = \overline{F(t)} = \exp[\lambda(1 - e^{t})] \quad ; \quad t \geq 0$$

2. Maximum likelihood Estimator of the Shape Parameter (λ)

Assume that $\underline{t} = t_1, t_2, \ldots, t_n$ are a random sample of size n from the Basic Gompertz distribution defined by eq.(1), then the likelihood function for the sample observation will be as follows [4]

$$L(\mathbf{t_1}, \mathbf{t_2}, \ldots, \mathbf{t_n}; \lambda) = \prod_{i=1}^{n} f(t_i; \lambda)$$

$$= \lambda^{n} \exp[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n} (1 - e^{t_i})] \qquad (3)$$

By letting $\frac{\partial}{\partial\lambda} \ln L(\mathbf{t_i}; \lambda) = 0$, the MLE of λ becomes

$$\hat{\lambda}_{ML} = \frac{-n}{T} \qquad (4)$$

where $T = \sum_{i=1}^{n}(1 - e^{t_i})$

Based on the invariant property of the MLE, the MLE for R(t) will be as follows

$$\hat{R}(t)_{ML} = \exp[\hat{\lambda}_{ML}(1 - e^{t})]$$

3. Bayesian Estimation

We provide Bayesian estimation method for estimating λ and R(t) of Basic Gompertz distribution, including informative and non-informative priors.

3.1 Posterior Density Functions Using Gamma Distribution

In this subsection, we assumed that λ is distributed Gamma as a prior distribution with density [5].

$$g_1(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \quad ; \quad \lambda > 0, \quad \alpha, \beta > 0 \qquad (5)$$

In general, the posterior probability density function of unknown parameter λ with prior g(λ) can be expressed as

$$\pi(\lambda|\underline{t}) = \frac{L(\mathbf{t_1}, \mathbf{t_2}, \ldots, \mathbf{t_n}; \lambda)\, g(\lambda)}{\int_{\forall\lambda} L(\mathbf{t_1}, \mathbf{t_2}, \ldots, \mathbf{t_n}; \lambda)\, g(\lambda) d\lambda} \qquad (6)$$

Now, combining eq. (3) with eq. (5) in eq. (6) yields:

$$\pi_1(\lambda|\underline{t}) = \frac{\lambda^{n+\alpha-1} e^{-\lambda[\beta - \sum_{i=1}^{n}(1 - e^{t_i})]}}{\int_0^{\infty} \lambda^{n+\alpha-1} e^{-\lambda[\beta - \sum_{i=1}^{n}(1 - e^{t_i})]} d\lambda}$$

After simplification, we get

$$\pi_1(\lambda|\underline{t}) = \frac{(\beta - T)^{n+\alpha} \lambda^{n+\alpha-1} e^{-\lambda(\beta - T)}}{\Gamma(n + \alpha)}$$

Notice that, the posterior p.d.f. of the parameter λ is obviously Gamma distribution, i.e.

$\lambda|\underline{t} \sim \text{Gamma}(n + \alpha, \beta - T)$; with: $E(\lambda|\underline{t}) = \frac{n+\alpha}{\beta-T}$, $Var(\lambda|\underline{t}) = \frac{n+\alpha}{(\beta-T)^2}$

3.2 Posterior Density Functions Using Jeffreys Prior

Assume that λ has a non-informative prior density defined, using Jeffrey's prior information $g_2(\lambda)$, as follows [6]

$$g_2(\lambda) \propto \sqrt{I(\lambda)}$$

where $I(\lambda)$ is Fisher information which is defined as

$I(\lambda) = -nE\left(\frac{\partial^2 \ln f}{\partial \lambda^2}\right)$

Hence,

$$g_2(\lambda) = k\sqrt{-nE\left(\frac{\partial^2 \ln f(t;\lambda)}{\partial \lambda^2}\right)} \quad , \qquad (7)$$

where k is a constant.

By taking the natural logarithm for p.d.f of Basic Gompertz distribution and taking the second partial derivative with respect to λ, we get

$$E\left(\frac{\partial^2 \ln f(t;\lambda)}{\partial \lambda^2}\right) = -\frac{1}{\lambda^2}$$

After substitution into eq. (7), we have

$$g_2(\lambda) = \frac{k}{\lambda}\sqrt{n} \qquad , \quad \lambda > 0$$

After substituting in eq.(6), the posterior density function based on Jeffreys prior can be written as

$$\pi_2(\lambda|\underline{t}) = \frac{\lambda^{n-1} e^{-\lambda\sum_{i=1}^{n}(e^{t_i}-1)}}{\int_0^\infty \lambda^{n-1} e^{-\lambda\sum_{i=1}^{n}(e^{t_i}-1)} d\lambda}$$

$$= \frac{P^n \lambda^{n-1} e^{-\lambda P}}{\Gamma(n)}$$

where $P = \sum_{i=1}^{n}(e^{t_i} - 1)$ = -T

The posterior density $\pi_2(\lambda|\underline{t})$is recognized as the density of the Gamma distribution, i.e.

$(\lambda|t_1, \dots, t_n) \sim \text{Gamma}\ (n, P)$, with $E(\lambda|t_1, \dots, t_n) = \frac{n}{p}$; $ver(\lambda|t_1, \dots, t_n) = \frac{n}{p^2}$

3.3 Bayes Estimation under Squared Log Error Loss Function

This loss function was used by Brown in 1968 and takes the following formula [7]

$$L(\hat{\lambda}, \lambda) = (\ln\hat{\lambda} - \ln\lambda)^2 = \left(\ln\frac{\hat{\lambda}}{\lambda}\right)^2$$

This is coordinated with $\lim L(\hat{\lambda}, \lambda) \to \infty$ as $\hat{\lambda} \to 0$ or ∞.

Any equivalent loss function considers the estimation error and the convenience quality, however, the unequal loss function simply gets the estimation error. This loss function is convex toward $\frac{\hat{\lambda}}{\lambda} \leq e$ and otherwise is concave, yet its risk function has a unique minimum with $\hat{\lambda}$ [8].

According to the above mentioned loss function, we drive the corresponding Bayes estimators for λ using Risk function $R(\hat{\lambda} - \lambda)$, which minimizes the posterior risk,

$$R(\hat{\lambda}, \lambda) = E\left[L(\hat{\lambda}, \lambda)\right] = \int_0^\infty (\ln\hat{\lambda} - \ln\lambda)^2 \pi(\lambda|t_1 \dots\dots\dots..t_n) d\lambda$$

$$= (\ln\hat{\lambda})^2 - 2(\ln\hat{\lambda})\, E(\ln\lambda\,|\underline{t}) + E((\ln\lambda)^2|\underline{t})$$

Taking the partial derivative for $R(\hat{\lambda}, \lambda)$ with respect to $\hat{\lambda}$ and setting it equal to zero, gives

$\ln\hat{\lambda} = E(\ln\lambda|\underline{t})$

Hence,

$$\hat{\lambda} = \text{Exp}\left(E(\ln\lambda|\underline{t})\right) \qquad (8)$$

3.3.1 Bayes Estimation under Squared Log Error Loss Function with Gamma Prior

Bayes estimator relative to Squared log error loss function based on Gamma prior can be derived as follows

$$E(\ln\ \lambda|\underline{t}) = \int_0^\infty \ln\lambda \frac{(\beta - T)^{n+\alpha}}{\Gamma(n+\alpha)} \lambda^{n+\alpha-1} e^{-\lambda(\beta-T)} d\lambda \qquad (9)$$

By using the transformation technique by assuming that,

$y = \lambda(\beta - T)$, which implies that,

$$\lambda = \frac{y}{(\beta - T)} \Rightarrow \ d\lambda = \frac{dy}{(\beta - T)}$$

Substituting into eq. (9) gives

$$E(\ln\lambda|\underline{t}) = (\beta - T)^{n+\alpha} \int_0^\infty \frac{1}{\Gamma(n+\alpha)} \ln\left(\frac{y}{\beta - T}\right)\left(\frac{y}{\beta - T}\right)^{n+\alpha-1} e^{-y} \frac{dy}{\beta - T}$$

$$= \int_0^\infty \frac{\ln y \, y^{n+\alpha-1} e^{-y}}{\Gamma(n+\alpha)} dy - \frac{\ln(\beta - T)}{\Gamma(n+\alpha)} \int_0^\infty y^{n+\alpha-1} e^{-y} dy \quad (10)$$

Recall that $\Gamma(n+\alpha)$ is a Gamma function which is defined as

$$\Gamma(n+\alpha) = \int_0^\infty y^{n+\alpha-1} e^{-y} dy \quad (11)$$

And we can say that

$$\int_0^\infty \frac{\ln y \, y^{n+\alpha-1} e^{-y}}{\Gamma(n+\alpha)} dy = \frac{d}{d(n+\alpha)} \ln\Gamma(n+\alpha) = \psi(n+\alpha) \quad (12)$$

Such that, $\psi(n) = \frac{\Gamma'(n)}{\Gamma(n)}$, where $\psi(n)$ is a Digamma function.

Substituting eq. (11) and eq. (12) into eq. (10) gives

$E(\ln\lambda|\underline{t}) = \psi(n+\alpha) - \ln(\beta - T)$

From eq. (9)and eq. (8), we have

$\hat{\lambda} = \text{Exp}(\psi(n+\alpha) - \ln(\beta - T))$

Thus, Bayesian estimation for the shape parameter of Basic Gompartz distribution under Squared log error loss function with Gamma prior is

$$\hat{\lambda}_{BG} = \text{Exp}(\psi(n+\alpha) - \ln(\beta - T)) \quad (13)$$

Now, according to eq.(8), the Bayesian estimation for R(t) under Squared log error loss function with Gamma prior can be obtained as follows

$$\hat{R}(t) = \text{Exp}\left(E(\ln\lambda|\underline{t})\right) \quad (14)$$

$$E(\ln R(t)|\underline{t}) = \int_0^\infty \lambda(1 - e^t) \frac{(\beta - T)^{n+\alpha}}{\Gamma(n+\alpha)} \lambda^{n+\alpha-1} e^{-\lambda(\beta - T)} d\lambda$$

$$= \int_0^\infty \frac{(\beta - T)^{n+\alpha}}{\Gamma(n+\alpha)} \lambda^{n+\alpha} e^{-\lambda(\beta - T)} d\lambda - e^t \int_0^\infty \frac{(\beta - T)^{n+\alpha}}{\Gamma(n+\alpha)} \lambda^{n+\alpha} e^{-\lambda(\beta - T)} d\lambda$$

$$E(\ln R(t)|\underline{t}) = \frac{n+\alpha}{\beta - T} \ (1 - e^t) \quad (15)$$

Combining equations (14) and (15) gives

$$\hat{R}(t)_{BG} = Exp\left(\frac{n+\alpha}{\beta - T} \ (1 - e^t)\right)$$

Where, $\hat{R}(t)_{BG}$ represents Bayesian estimation for R(t) under Squared log error loss function with Gamma prior.

3.3.2 Bayes Estimation under Squared Log Error Loss Function with Jefferys Prior

Similarly, we can obtain the Bayes estimator for the shape parameter λ under Jefferys prior by using eq. (8) as follows

$$E(\ln\lambda|\underline{t}) = \int_0^\infty \ln\lambda \frac{P^n}{\Gamma(n)} \lambda^{n-1} e^{-\lambda P} d\lambda \quad (16)$$

By letting $y = \lambda P$ which implies that,

$\lambda = \frac{y}{P}$ and $d\lambda = \frac{dy}{P}$

After substituting into eq.(16), we get

$$E\left(ln\lambda\middle|\underline{t}\right) = \int_0^\infty ln\left(\frac{y}{P}\right)\frac{P^n}{\Gamma(n)}\left(\frac{y}{P}\right)^{n-1} e^{-y}\frac{dy}{P}$$

$$= \int_0^\infty \frac{ln\, y y^{n-1} e^{-y}}{\Gamma(n)} dy - \frac{ln\, P}{\Gamma(n)}\int_0^\infty y^{n-1} e^{-y} dy \quad (17)$$

$$E\left(ln\lambda\middle|\underline{t}\right) = \psi(n) - ln\, P \quad (18)$$

So, from eq. (8) and eq.(18), we get the Bayes estimator of parameter λ under Squared log error loss function, based on Jefferys prior, as the following form

$$\hat{\lambda}_{BJ} = Exp(\psi(n) - ln\, P)$$

Similarly, we can find the corresponding estimator for R(t) by using eq. (14) where

$$E\left(lnR(t)\middle|\underline{t}\right) = \int_0^\infty lnR(t)\pi_2\left(\lambda\middle|\underline{t}\right)d\lambda \quad (19)$$

$$\mathrm{E}\left(\ln\mathrm{R(t)}\middle|\underline{\mathrm{t}}\right) = \frac{\mathrm{P^n}}{\Gamma(\mathrm{n})}\int_0^\infty \lambda(1-\mathrm{e^t})\lambda^{\mathrm{n}-1}\mathrm{e}^{-\lambda\mathrm{P}}\mathrm{d}\,\lambda$$

$$= \frac{\mathrm{n}}{\mathrm{P}}\int_0^\infty \frac{\mathrm{P^{n+1}}}{\Gamma(\mathrm{n}+1)}\lambda^{\mathrm{n}}\mathrm{e}^{-\lambda\mathrm{P}}\mathrm{d}\lambda - \mathrm{e^t}\frac{\mathrm{n}}{\mathrm{P}}\int_0^\infty \frac{\mathrm{P^{n+1}}}{\Gamma(\mathrm{n}+1)}\lambda^{\mathrm{n}}\mathrm{e}^{-\lambda\mathrm{P}}\mathrm{d}\lambda$$

$$\mathrm{E}\left(\ln\mathrm{R(t)}\middle|\underline{\mathrm{t}}\right) = \frac{\mathrm{n}}{\mathrm{P}}\,(1-\,\mathrm{e^t}) \quad (20)$$

Substitute eq. (20) into eq. (14) gives

$$\hat{\mathrm{R}}(t)_{BJ} = \exp\left(\frac{\mathrm{n}(1-\mathrm{e^t})}{\mathrm{P}}\right)$$

Recall that P = -T, thus, $\hat{\mathrm{R}}(t)_{BJ}$ is equivalent to MLE for $\hat{\mathrm{R}}(t)_{\mathrm{ML}}$

4. Simulation Study

In this section, a Monte Carlo simulation was performed to compare the performance of the different estimators of the unknown shape parameter λ and Reliability function R(t) for Basic Gompertz distribution. The process was repeated 5000(L=5000) times with different sample sizes (n = 15, 50, and100).

The default values of the shape parameter λ and two values of the Gamma prior parameters were chosen to be less than and greater than one, as λ= 0.5, 3; α=0.8, 3; β=0.5,3.

All estimators for λ that were derived in the previous section are evaluated based on their mean squared errors (MSE's), where,

$$\mathrm{MSE}\left(\hat{\lambda}\right) = \frac{\sum_{\mathrm{i=0}}^{\mathrm{L}}\left(\hat{\lambda}_{\mathrm{i}}-\lambda\right)^2}{\mathrm{L}} \quad ; \quad \mathrm{i} = 1,2,3,\ldots,\mathrm{L}$$

The integrated mean squared error (IMSE) was employed to compare the performance of the Bayesian estimators for R(t). IMSE is an important global measure and more accurate than MSE, which is defined as the distance between the estimated value and actual value of reliability function given by equation, where,

$$\mathrm{IMSE}(\hat{\mathrm{R}}(\mathrm{t})) = \frac{1}{\mathrm{L}}\sum_{\mathrm{i=1}}^{\mathrm{L}}\left[\frac{1}{n_t}\sum_{\mathrm{j=1}}^{n_t}(\hat{\mathrm{R}}_i(t_j) - \mathrm{R}(t_j))\right]^2$$

$$= \frac{1}{n_t}\sum_{\mathrm{j=1}}^{n_t}\mathrm{MSE}(\hat{\mathrm{R}}_i(t_j))$$

where *i*=1,2,…,*L*,n_t the random limits of t_i.

In this paper, we chose t=0.1, 0.2, 0.3, 0.4, 0.5.

The expected values (EXP) and mean squared errors (MSEs) of all estimates of the shape parameter of Basic Gompertz distribution λ were tabulated in Tables-(1-2) for all sample sizes. The integrated mean squared error (IMSE) values for the estimates of R(t) were tabulated in Tables- (3-4).

Table 1- Expected values (EXP) and MSEs of the different estimates of the shape parameter λ of Basic Gompertz distribution when λ =0.5

n	Estimate / Criteria	MLE	Jeffreys prior	Gamma prior			
				α=0.8		α= 3	
				β=0.5	β= 3	β=0.5	β=3
15	EXP	0.5353497	0.5169348	0.5351083	0.4893851	0.6118625	0.5595807
	MSE	0.0235674	0.0210955	0.0225615	**0.0146673**	0.0403994	0.0225792
50	EXP	0.5102127	0.5026106	0.5080449	0.4952185	0.5301548	0.5167699
	MSE	0.0056782	0.0054159	0.0055315	**0.0049467**	0.0068623	0.0056429
100	EXP	0.5053571	0.4978777	0.5005705	0.4942800	0.5114384	0.5050112
	MSE	0.0026505	0.0025493	0.0025594	**0.0024645**	0.0028022	0.0025636

Table 2-Expected values (EXP) and MSEs of the different estimates of the shape parameter λ of Basic Gompertz distribution when λ = 3

n	Estimate / Criteria	MLE	Jeffreys prior	Gamma prior			
				α=0.8		α= 3	
				β=0.5	β= 3	β=0.5	β= 3
15	EXP	3.2120990	3.1016020	2.9363160	1.9585270	3.3574840	2.2394540
	MSE	0.8484274	0.7594381	**0.5280226**	1.1832750	0.8128504	0.7073729
50	EXP	3.0612750	3.0156660	2.9713070	2.5815020	3.1006220	2.6938410
	MSE	0.2044161	0.1949720	**0.1780807**	0.2751655	0.2031455	0.2026478
100	EXP	3.0321400	2.9872730	2.9656800	2.7578700	3.0300660	2.8177440
	MSE	0.0954172	0.0917731	**0.0887214**	0.1239551	0.0922897	0.1014135

Table 3- IMSEs of the different estimates for reliability function R(t) of Basic Gompertz distribution when λ =0.5

n	MLE	Jeffreys prior	Gamma prior			
			α=0.8		α= 3	
			β=0.5	β= 3	β=0.5	β= 3
15	0.0014595	0.0014595	0.0012712	0.0011462	0.0021770	**0.0007793**
50	0.0003847	0.0003847	0.0003709	0.0003508	0.0004564	**0.0003111**
100	0.0001869	0.0001869	0.0001837	0.0001780	0.0002053	**0.0001678**

Table 4-IMSEs of the different estimates for reliability function R(t) of Basic Gompertz distribution when λ =3

n	MLE	Jeffreys prior	Gamma prior			
			α=0.8		α= 3	
			β=0.5	β= 3	β=0.5	β= 3
15	0.0069471	0.0069471	0.0080101	0.0797167	**0.0055115**	0.0629822
50	0.0021114	0.0021114	0.0022277	0.0175773	**0.0019502**	0.0145029
100	0.0010669	0.0010669	0.0010965	0.0059061	**0.0010238**	0.0050060

4. Results, Discussion and Analysis

The discussion of the results obtained from applying the simulation study can be summarized as follows:

1. When the shape parameter λ=0.5,

- The best estimator for λ is Bayes estimator under Squared log error loss function based on Gamma prior, with α=0.8 and β=3 for all sample sizes (see Table-1).
- From Table-3, it is clear that the best estimator for R(t) is Bayes estimator under Squared log error loss function with Gamma prior, when α = 3 and β = 3 for all sample sizes.

2. When the shape parameter λ=3,

- From Table-2, notice that the performance of Bayes estimator under Squared log error loss function based on Gamma prior, is the best with α=0.8 and β=0.5 for all sample sizes .
- Table-4 shows that the best estimate for R(t) is Bayes estimator under the Squared log error loss function based on Gamma prior with α=3 and β=0.5 for all sample sizes.

3. In general, MSEs and IMSEs are increasing with the increase of the shape parameter value.
4. From Tables-(3, 4), it is observed that IMSE values for Maximum likelihood and Bayes estimates under the Squared log error loss function with Jefferys prior are the same.

ACKNOWLEDGMENT

The authors would like to thank the Department of Mathematics, College of Science, Mustansiriyah University, for supporting this work.

References

1. Gompertz, B. **1852**. On the nature of the function expressive of the law of human mortality and on a new mode of determining the value of Life Contingencies. *Philosophical Transactions of the Royal Society of London*. **115** (Part 1-2).

2. Moala, F. A. and Dey, S. **2018**. Objective and subjective prior distributions for the Gompertz distribution. *Annals of the Brazilian Academy of Sciences*, **90**(3): 2643-2661.

3. Rasheed, H. A., Hasanain, W. S. and Al-Obedy, N. J. **2018**. Comparing Some Estimators of the Parameter of Basic Gompertz distribution. Special issue of the first international scientific conference of Iraqi Al-Khawarizmi Society. *Journal of Iraqi Al-Khwarizmi Society (JIKhS)*, **2**: 178-187.

Soliman, A. A. and Al Sobhi, M. M. **2015**. Bayesian MCMC Inference for the Gompertz Distribution Based on Progressive First-Failure Censoring Data. The 2nd ISM International statistical Conference (ISM-II), AIP Conf. Proc. (1643): (125-134).

4. Abu-Zinadah_H. H. **2014**. Bayesian Estimation on the Exponentiated Gompertz Distribution under Type II Censoring. *International Journal of Contemporary Mathematical Sciences*, **9**(11): 497 - 505.

5. Ismail, A. A. **2010**. Bayes estimation of Gompertz distribution parameters and acceleration factor under partially accelerated life tests with type-I censoring. *Journal of Statistical Computation and Simulation*, **80**(11): 1253-1264.

6. Dey S. **2010**. Bayesian estimation of the shape parameter of the generalised exponential distribution under different loss functions. *Pakistan Journal of Statistics and Operation Research*, **VI**(2): 163-174.

7. Brown, L. **1968**. Inadmissibility of the usual estimators of scale parameters in problems with unknown location and scale parameters, *The Annals of Mathematical Statistics*, **39**: 29–48.