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Study the Effect of Isospin Dependence and Short-Range Correlations through Inclusive Nuclear Structure for ^{28}Si and ^{32}S Nuclei

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Abstract

Pair densities and the correlation functions that go along with them are crucial resources for successfully integrating many-body correlations into nuclear theories. The explicit impacts of spin and isospin dependence on nuclear charge density distributions are examined in this work. The folding model is used to compute the density of charge distribution, involving each short-range correlation (SRC) and tensor correlation and uses an effective two-body densities formula that accounts for spin and isospin interactions. Calculations were made for the ^{28}Si and ^{32}S nuclei elastic electron scattering form factors $F(q)$ and the root of the mean square charged radii $\langle r^2 \rangle^{(1/2)}$. Additionally, the inelastic longitudinal electron scattering form factors associated with the isoscalar transition, $T = 0$ of the $(0^+ \rightarrow 2^+)$ and $(0^+ \rightarrow 4^+)$ were calculated, including core polarization effects using the Tassie model. The results exhibit good agreement with experimental data, demonstrating the accuracy and robustness of the adopted theoretical framework.

Keyword: Short Range Correlation (SRC), Charge Density Distribution (CDD), Core Polarization Effects, Tassie-Model and Transition Densities .

دراسة تأثير الارتباطات قصيرة المدى والاعتماد الأيزوسبيني لدوال الارتباط النووي لنوى السليكون ^{28}Si و الكبريت ^{32}S

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الخلاصة

تمثل كثافات الأزواج ودوال الارتباط المرتبطة بها أدوات أساسية لدمج تأثيرات الأجسام المتعددة في النظريات النووية. تبحث هذه الدراسة التأثيرات المباشرة للاعتماد على spin و isospin في توزيعات كثافة الشحنة النووية. باستخدام folding model تم حساب توزيع كثافة الشحنة مع مراعاة كل من الارتباطات قصيرة المدى (SRC) وارتباطات tensor، من خلال صيغة فعالة لكثافة الجسمين تأخذ في الاعتبار تفاعلات spin و isospin. شملت الحسابات عوامل شكل التشتت الإلكتروني المرن $F(q)$ ونصف قطر جذر

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متوسط الشحنة التربيعي $\langle r^2 \rangle^{1/2}$ لنواتي Si^{28} و S^{32} ، بالإضافة إلى عوامل التشكل للاستطارة الطولية الغير مرنة للانتقالات النووية المتساوية القياس ($T=0$) من $(0^+ \rightarrow 2^+)$ و $(0^+ \rightarrow 4^+)$ ، مع تضمين تأثيرات استقطاب النواة باستخدام نموذج Tassie. أظهرت النتائج توافقاً جيداً مع البيانات التجريبية، مما يؤكد دقة ومثانة الإطار النظري المعتمد.

Introduction

One of nature's most intricate systems is the atomic nucleus. Understanding the brief interparticle portion of the nuclear wave function is one of the primary difficulties characterizing nuclei. The difficulty arises from the nucleus's high density and complex nucleon–nucleon interaction. The latter causes all the relevant scales of the system (nucleon size, average distance, and interaction range) to be comparable, making effective theoretical descriptions very demanding. The binding nucleon, in addition to free neutron structure functions [1,2], neutrino-nucleus interactions, neutrino oscillation experimentation, neutron-star framework, nuclear symmetry energy, and more, all depend on a thorough knowledge regarding these Short-Range Correlations (SRCs). The field of nuclear physics makes extensive use of correlation functions regarding previous studies [3-6]. A nucleus is a dense, quantum mechanical, many-body system having complex interactions amongst its nucleons that interact intensely. The inclusive approach here integrates all nucleon interactions—from hard-core SRCs and tensor forces to core polarization (Tassie model) to compute ground-state charge densities $\rho_{ch}(r)$, elastic form factors $F(q)$, and inelastic transitions $(0^+ \rightarrow 2^+)$ and $(0^+ \rightarrow 4^+)$. By folding these correlations into an effective two-body density operator and validating against experimental RMS radii and form factors, the work achieves two key aims: first, quantifying how SRCs and isospin asymmetry redistribute charge. Second, resolving discrepancies in high- q scattering data through configuration mixing (OXBASH) and collective enhancements (Tassie). This framework, grounded in universal SRC behavior [7, 8] and benchmarked against electron scattering [9, 10], advances a unified description of nuclear structure, bridging microscopic interactions with macroscopic observables for applications in neutrino physics and astrophysics. Since, the core potential is not fundamental, correlations must be present. An early study that represented nuclear correlation functions was employed to show how correlation functions affect fields of study in a wide range of calculations, including the strong and weak interactions. Correlation functions are essential components in more contemporary instances, such as parity violations in nuclei [11], shadows in deep inelastic scattering [12], nuclear transparency in quasielastic scattering [13], and computations of neutrinoless double beta decay [14]. Although correlation functions are widely used, less attention has been paid to their spin and isospin dependency. Both spin and isospin dependence in the nucleon-nucleon interactions become significant over a short range, resulting in occurrences like the substantial predilection for proton–neutron short-range correlated couples [7]. Calculations determining the spin and isospin decomposition corresponding to the two-body density defines the correlation function using a formal model of nuclear contacts. Our formalism is predicated on a scale of distinction that is intrinsic to nuclei's long- and short-range structures [7]. Although the underlying short-range behavior is a universal characteristic of the two-body nuclear interaction, the aggregate effect of long-range interactions at small distances may be captured by nucleus-specific coefficients known as "contacts". Investigating the impact of SRC and isospin dependency on the nuclear correlation functions for ^{28}Si and ^{30}S is the goal of this investigation. In this work, a created framework incorporating comprehensive two-body correlation functions, such as SRC and Tensor Correlations (TCs), was used to calculate charge, elastic, and inelastic form factors. In order to ensure compatibility with observed root mean square charge radii, distributions of two-body charge densities for the ground states (2BCDDs) were generated utilizing harmonic

oscillator wave functions and these correlations. In addition to demonstrating the substantial influence of SRC and isospin effects on nuclear structural parameters, these computations seek to give a more precise interpretation of the scattered electron data.

Theory

In order to represent the charge effect and nucleon interactions in a nucleus, the two-body structure of the nucleon density operator is considered. This form takes into consideration the inner structure of the nucleus as well as nucleon-nucleon interactions. Using the following two-body density operator, the charge distribution—which is affected by electromagnetic interactions, it can be shown:

$$\hat{\rho}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{(A-1)} \sum_{i \neq j} \left\{ \delta \left[\sqrt{2} \vec{r} - \vec{R}_{ij} - \vec{r}_{ij} \right] + \delta \left[\sqrt{2} \vec{r} - \vec{R}_{ij} + \vec{r}_{ij} \right] \right\} \quad (1)$$

Through folding the two-body correlations functions using an operator from Equation (1), it could be possible to develop an effective two-body density of charges operators (such as usage with uncorrelated wave functions), $\tilde{T}_{ij}$ [15].

$$\hat{\rho}_{eff}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)} \sum_{i \neq j} \tilde{T}_{ij} \left\{ \delta \left[\sqrt{2} \vec{r} - \vec{R}_{ij} - \vec{r}_{ij} \right] + \delta \left[\sqrt{2} \vec{r} - \vec{R}_{ij} + \vec{r}_{ij} \right] \right\} \tilde{T}_{ij} \quad (2)$$

the form of $\tilde{T}_{ij}$ is provided by [16]:

$$\tilde{T}_{ij} = T(r_{ij}) \nabla_1 + T(r_{ij}) \{1 + \beta(A) S_{ij}\} \nabla_2 \quad (3)$$

It is evident that two different kinds of correlations were included in Equation (3). The first term, $T(r_{ij})$, stands for brief-range correlations between two bodies, while ∇_1 works as a projection operator onto the space of functions of two-body waves. With close ranges when particles are forced away by the repulsive core, these short-range correlations lower the two-body wave function. They are essential components of particle-pair separation. As the gap increases and the interactions get weaker, $T(r_{ij})$, progressively becomes closer to unity. A simple model form of two-body SRC is given as [8]:

$$T(r_{ij}) = g_{NN}(r) C_{NN}^S + \kappa(1 - g_{NN}(r)) \quad (4)$$

In order to account for behavior up to $(r \leq 0.9)$ fm, a model was provided that combines the correlated density from the nuclear contact formalism together with a longer-ranged part when correlations should be less relevant. Nucleon correlations were modelled using a two-component approach: First, short-range term $(r \leq 0.9 \text{ fm})$: dominated by SRCs, described by $g_{NN}(r) \cdot C_{NN}^S(r)$, where $g_{NN}(r)=1$ (constant) and $C_{NN}^S(r)$ is the contact coefficient for pp/pn/nn pairs (spin $s=0,1$). The contact coefficient $C_{NN}^S(r)$ is fitted in Equation (5) using nuclear radius (b) and mass optimization, critical for quantifying correlated nucleon pairs. This hybrid model ensures accurate SRC description while smoothly merging with long-range nuclear structure.

$$C_{PP}^0 = bW^2 / A \quad (5)$$

Here, the b-size harmonic oscillator parameter was applied, and W is the mass optimization parameter in the contact formalism. Equation (6) was used to predict W, which is determined by fitting nuclear mass using the Coulomb (0.711 MeV) and asymmetry (23.7 MeV) terms [8]:

$$W \approx \frac{A}{2} \left[1 - \frac{a_c A^{2/3}}{4a_A} \right] \approx \frac{A}{2} [1 - 0.0075 A^{2/3}] \quad (6)$$

At distances $(r > 0.9)$, the two-body correlations function $g_{NN}(r)$ gradually transitions to account for the uncorrelated density's behaviour. The definition of $g_{NN}(r)$ in the suggested framework is as follows [8]:

$$g_{NN}(r) = \begin{cases} 1 & (r \leq 0.9 \text{ fm}) \\ \frac{1}{k} (k - 1 + e^{(0.9 \text{ fm} - r)/a}) & (r > 0.9 \text{ fm}) \end{cases} \quad (7)$$

The model that describes the two-body density and correlation functions in nuclear systems heavily relies on the parameters κ and α . Both of these parameters play crucial roles in modeling nuclear correlations. The parameter κ is a scaling factor that controls the relative weight of the uncorrelated two-body density at large distances, while the parameter α is a length scale that determines how smoothly the transition occurs between short-range (correlated) and long-range (uncorrelated) behavior, guaranteeing that $g_{NN}(r)$ reaches $(\kappa-1)/\kappa$ as $r \rightarrow \infty$. The isospin and nucleus type can affect each κ and α , indicating variations in the degree of interactions between p-p, n-n, and p-n pairings. In a long-range, correlation behavior becomes isospin sensitive as a result of such reliance. The second term in Equation (3) displays two-body tensor correlations, which have a larger range and are created due to the nucleon-nucleon force's strong tensor component. In this case, V_2 is an operator for translation over only states 3S_1 and 3D_1 . S_{ij} , the standard tensor operator, was explained by Mahmood and Flaiyh [17] and is produced by taking a second-rank operators in structure space and internally spin structure then multiplying it with a scalar:

$$S_{ij} = \frac{3}{r_{ij}^2} (\vec{\sigma}_i \cdot \vec{r}_{ij}) \cdot (\vec{\sigma}_j \cdot \vec{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j \quad (8)$$

For tensor correlation strength, shown by the parameter $\beta(A)$ in Equation (3), is not a zero just on both of the 3S_1 and 3D_1 channels. In Equation (2), the expected value of the efficient operator for two-body charge density yields the ground state distribution of two-body charge densities $\rho_{ch}(r)$, which may be written as:

$$\langle \Psi | \hat{\rho}_{eff}^{(2)}(\vec{r}) | \Psi \rangle = \sum_{ij} \langle i j | \hat{\rho}_{eff}^{(2)}(\vec{r}) [| ij \rangle - | ji \rangle] \quad (9)$$

where $|ij\rangle$ is the two-particle wave functions.

The definition for (the radius of the nuclear mean square charge $\langle r^2 \rangle$) is equal to [18]:

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho_{ch}(r) r^4 dr \quad (10)$$

With spin-zero nuclei, the two-body charge density distributions in the ground state $\rho_{ch}(r)$ of Equation (10), is associated with the elastic electron scattering form factor. The two-body charge density distributions in the ground state $\rho_{ch}(r)$ of Equation (10), which is associated with the elastic electron scattering structure for zero spin nuclei. Given that the Plane Wave Born Approximation (PWBA) treats incoming and scattering electron waves as plane waves. Where $\rho_{ch}(r)$ is spherical-symmetric and real, the form factor is only the Fourier transformed value of $\rho_{ch}(r)$, deduced by [9,10]:

$$F(q) = \frac{4\pi}{Z} \int_0^\infty \rho_o(r) j_0(qr) r^2 dr \quad (11)$$

Here q represents the transfer of momentum from the outgoing electron towards the target's nucleus and $(j_0(qr) = \sin(qr)/qr)$ is a spherical Bessel function of zeroth degree. It is possible to describe Equation (11) as:

$$F(q) = \frac{4\pi}{qZ} \int_0^\infty \rho_o(r) \sin(qr) r dr \quad (12)$$

In the case of inelastic electron scattering, longitudinal or Coulomb scattering is an effect of the electron's interaction via the nucleus's charge distribution, with the multipole operators' matrix components $\hat{T}_J^L(q)$, the C4 and C2 longitudinal shape factors relate to the charge density distributions (CDD) [19].

$$|F_J^L(q)|^2 = \frac{4\pi}{Z^2 (2J_i+1)} | \langle f | \hat{T}_J^L(q) | i \rangle |^2 |F_{cm}(q)|^2 |F_{fs}(q)|^2 \quad (13)$$

When using the shell model wave function, the centre of mass correction eliminates the false state caused by the centre of mass's mobility, $F_{cm}(q)$, where Z is the nucleus' atomic number and is given by [20]:

$$F_{cm}(q) = e^{q^2 \cdot b^2 / 4A} \quad (14)$$

Where A represents the nuclear number for mass. Assumed to be the same for protons and neutrons. The independent nucleon form factor is represented by the function $F_{fs}(q)$, having the form [21]:

$$F_{fs}(q) = e^{-0.43 \cdot q^2 / 4} \quad (15)$$

The definition of the longitudinal operator is [9]:

$$\hat{T}_{Jt_z}^L(q) = \int dr j_J(qr) Y_J(\Omega) \rho(r, t_z) \quad (16)$$

The spherical harmonic wave structure is represented by $Y_J(\Omega)$, whereas the charge densities operators are represented by $\rho(r, t_z)$, and $j_J(qr)$ is the spherical Bessel function. The reduced matrix components between the system's the initial and ending multiple particle states can be expressed into spin as well as isospin space using OBDM components times the longitudinal operator's single particle matrices elements, involving configuration mixing [22], i.e.

$$\langle f || \hat{T}_{JT}^L || i \rangle = \sum_{a,b} OBDM^{JT}(i, f, J, a, b) \langle b || \hat{T}_{JT}^L || a \rangle \quad (17)$$

There are two components to the longitudinal operator's many particles reduced matrix elements: the core polarization matrix component with the model field [23]:

$$\langle f || \hat{T}_J^L(\tau_z, q) || i \rangle = \left\langle f || \hat{T}_J^{ms}(\tau_z, q) || i \right\rangle + \left\langle f || \hat{T}_J^{cor}(\tau_z, q) || i \right\rangle \quad (18)$$

In Equation (18), the model space matrix element takes the following form [22]:

$$\left\langle f || \hat{T}_J^{ms}(\tau_z, q) || i \right\rangle = e_i \int_0^\infty dr r^2 j_J(qr) \rho_{J, \tau_z}^{ms}(i, f, r) \quad (19)$$

where ρ_{J, τ_z}^{ms} is the transition density of charges for the model space, and is supplied by [22]:

$$\rho_{J, \tau_z}^{ms}(i, f, r) = \sum_{jj'(ms)} OBDM(i, f, J, j, j', \tau_z) \langle j || Y_J || j' \rangle R_{nl}(r) R_{nl}(r) \quad (20)$$

The element of the core polarization matrix is provided by [22]:

$$\left\langle f || \hat{T}_J^{cor}(\tau_z, q) || i \right\rangle = e_i \int_0^\infty dr r^2 j_J(qr) \rho_J^{cor}(i, f, r) \quad (21)$$

The transition density of core polarization, denoted as ρ_J^{cor} , is contingent upon the core polarisation model. The core polarization transition density used to characterize all the modes within nuclei is supplemented by the transition density of the model space to account for the core polarization effects. The overall density of transitions becomes:

$$\rho_{J, \tau_z}(i, f, r) = \rho_{J, \tau_z}^{ms}(i, f, r) + \rho_J^{cor}(i, f, r) \quad (22)$$

Based on the combined modes of nuclei, the (Tassie form) establishes the core polarizations transition density [24]:

$$\rho_{Jt_z}^{core}(i, f, r) = N \frac{1}{2} (1 + \tau_z) r^{J-1} \frac{d\rho_o(i, f, r)}{dr} \quad (23)$$

which, when represented by Equation (2) and includes a proportionality constant N, reflects the base state two-body distribution of charge density.

For this model, the Coulomb form factor is:

$$F_J^L(q) \sqrt{\frac{4\pi}{2J_i+1}} \frac{1}{Z} \left\{ \int_0^\infty r^2 j_J(qr) \rho_J^{ms}(i, f, r) dr + N \int_0^\infty dr r^2 j_J(qr) r^{J-1} \frac{d\rho_o(i, f, r)}{dr} \right\} F_{cm}(q) F_{fs}(q) \quad (24)$$

The radial integral $\int_0^\infty dr r^{J+1} j_J(qr) \frac{d\rho_o(i, f, r)}{dr}$ can be written as:

$$-\int_0^\infty \frac{d}{dr} \{ r^{J+1} j_J(qr) \rho_o(i, f, r) \} dr - \int_0^\infty dr (J+1) r^J j_J(qr) \rho_o(i, f, r) - \int_0^\infty dr r^{J+1} \frac{d}{dr} j_J(qr) \rho_o(i, f, r) \quad (25)$$

Since the first term adds nothing, both the second and third ones can be combined as follows:

$$-q \int_0^\infty dr r^{J+1} \rho_o(i, f, r) \left[\frac{d}{d(qr)} + \frac{J+1}{qr} \right] j_J(qr) \quad (26)$$

According to the spherical Bessel function's recursion relation:

$$\left[\frac{d}{d(qr)} + \frac{J+1}{qr} \right] j_J(qr) = j_{J-1}(qr) \quad (27)$$

$$\therefore \int_0^\infty dr r^{J+1} j_J(qr) \frac{d\rho_o(i, f, r)}{dr} = -q \int_0^\infty dr r^{J+1} j_{J-1}(qr) \rho_o(i, f, r) \quad (28)$$

As a result, Equation (24) form factor gets the form:

$$F_J^L(q) = \left(\frac{4\pi}{2J_i+1} \right)^{1/2} \frac{1}{Z} \left\{ \int_0^\infty r^2 j_J(qr) \rho_{Jt_z}^{ms} dr - Nq \int_0^\infty dr r^{J+1} \rho_o(i, f, r) j_{J-1}(qr) \right\} F_{cm}(q). F_{fs}(q) \quad (29)$$

The proportionality constant N can be determined from the form factor evaluated at $q=k$ (photon point) = $Ex/\hbar c$ (Ex is the excitation energy) i.e., substituting $q=k$ in Equation (29), we obtain:

$$N = \frac{\int_0^\infty dr r^2 j_J(kr) \rho_{Jt_z}^{ms}(i, f, r) - \sqrt{\frac{(2J_i+1) B(CJ)}{(2J+1)!!}} k^J}{k \int_0^\infty dr r^{J+1} \rho_o(i, f, r) j_{J-1}(kr)} \quad (30)$$

where the decreased likelihood of a transition is represented by B(CJ).

Results and discussion

For the ^{28}Si and ^{32}S nuclei, calculations were performed for the ground-state two-body charge density distributions (2BCDD's), $\rho_{ch}(r)$, the root-mean-square (RMS) charge radii $\langle r^2 \rangle^{1/2}$, and both elastic and inelastic electron scattering form factors, $F(q)$'s. Higher states $1d_{3/2}$ and $2p_{3/2}$ were included in these computations, and their occupancy probabilities were handled as free parameters. For all nuclei under consideration, the repulsive core of nucleon-nucleon interactions is uniformly represented by a uniform hard-core radius of ($r=0.9$) fm. In order to appropriately account for spin-isospin-dependent tensor forces and short-range correlations (SRCs), the estimated $\langle r^2 \rangle^{(\frac{1}{2})}$ values were fitted to experimental data to determine the tensor correlation strengths $\beta(A)$. Tables 1 and 2 contain all the parameters needed to calculate $\rho_{ch}(r)$, $\langle r \rangle^{(\frac{1}{2})}$ as well as $F(q)$'s, including size parameter(b), the strength of tensor correlations $\beta(A)$, correlation transition $k(A)$, exponential decay length $\alpha(A)$, as well as the states' occupation probability η .

Table 1: The ground state 2BCDD properties.

Nuclei	b	$\beta(A)$	k(A)	$\alpha(A)$	$\eta_{1s\frac{1}{2}}$	$\eta_{1p\frac{3}{2}}$	$\eta_{1p\frac{1}{2}}$	$\eta_{1d\frac{5}{2}}$	$\eta_{2s\frac{1}{2}}$	$\eta_{1d\frac{3}{2}}$	$\eta_{2p\frac{3}{2}}$
^{28}Si	1.96	0.081	7.5	0.55	1	1	1	0.708	0	0.0625	0.375
^{32}S	1.95	0.075	8.0	0.75	1	1	1	1	0.25	0.125	0.25

Table 2: Root-Mean-Square (rms) Charge Radii

Nuclei	$\left(\frac{1}{2}\right)$ $\langle r^2 \rangle_{\text{without}}$	$\left(\frac{1}{2}\right)$ $\langle r^2 \rangle_{\text{with corr.}}$	$\langle r^2 \rangle_{\text{exp.}}^{1/2}$ [19]
^{28}Si	2.97	3.084	3.12
^{32}S	3.096	3.205	3.26

The distribution of two-body charge densities $\rho_{\text{ch}}(r)$ is depicted in Figure (1) for ^{28}Si [Fig.1(a)] and ^{32}S [Fig.1(b)] nuclei, using various representations: the solid curve incorporates both tensor (TCs) and short-range correlations (SRCs), the dashed curve shows calculations without correlations, and the discrete points or dotted line indicates experimental data [24,25]. In the absence of correlations, the dashed curve shows the simplistic assumption of independent nucleons and differs greatly from actual data, especially at small radii (r). By redistributing protons from the center area to the surface, adding SRCs and TCs, as the solid curve illustrates, lowers the central peak and marginally increases the tail. By bringing the theoretical conclusions into line with the actual data, this modification highlights how important spin and isospin-dependent nucleon-nucleon interactions are to properly modeling nuclear charge densities.

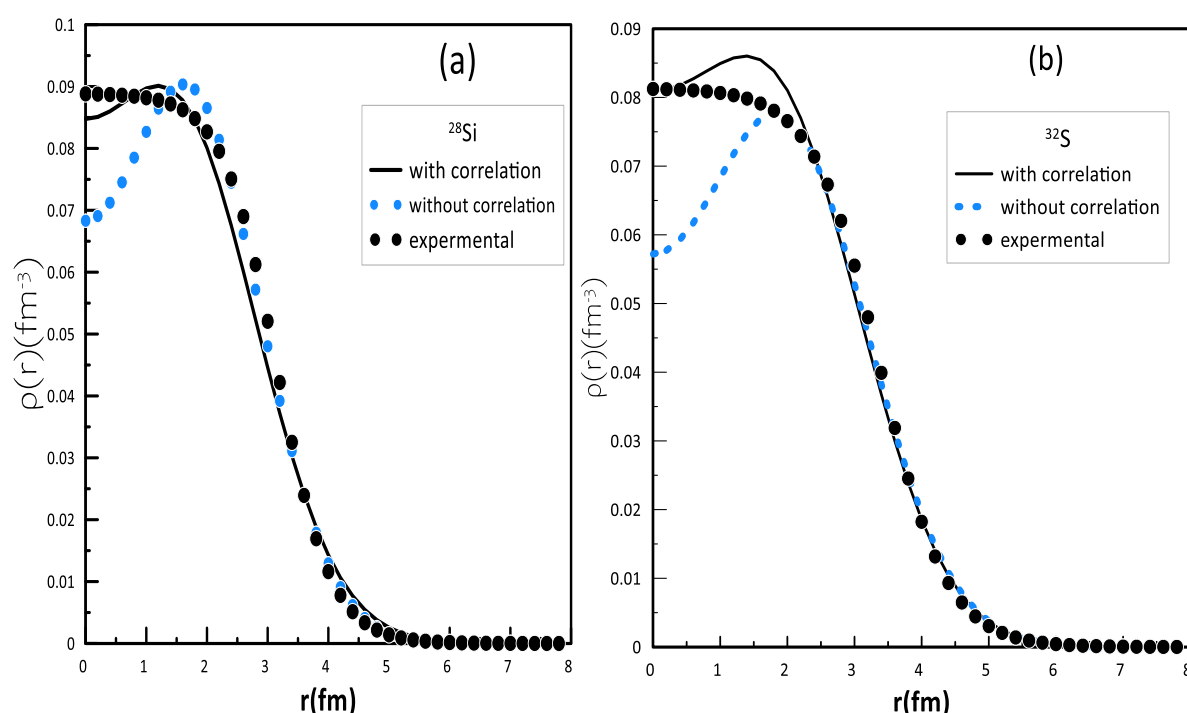


Figure 1: 2BCDD's reliance on $r(\text{fm})$ for the ^{28}Si (Fig. a) and ^{32}S (Fig. b) nuclei. The experimental data of Ref. [24,25] are represented by the dotted symbols.

The elastic electron scattering form factor $F(q)$ for ^{28}Si and ^{32}S nuclei exhibited an oscillating pattern due to interference between scattered electron waves, reflecting the spatial distribution of nucleons within the nucleus. Equation (12) was calculated by applying the distributions of the ground state two-body charge density of Equation (9). Figure (2) for ^{28}Si [Fig.2(a)] and ^{32}S [Fig.2(b)] nuclei, compares the computed $F(q)$ s with data from experiments for the (2s-1d) shell nuclei under study. The dotted symbols represent experimental data [26], while the calculated $F(q)$'s both with and without the influence of

SRC and TC are shown by the dashed and solid distributions, in each case. The black plot calculations provide a good description of all three of the diffraction minima for the (^{28}Si and ^{32}S) nuclei that are shown in the experimental data. It is evident that the data at the area for transfer of momentum $q > 1.2 \text{ fm}^{-1}$ is underestimated based on the computed $F(q)$ s shown by the dash and black plots.

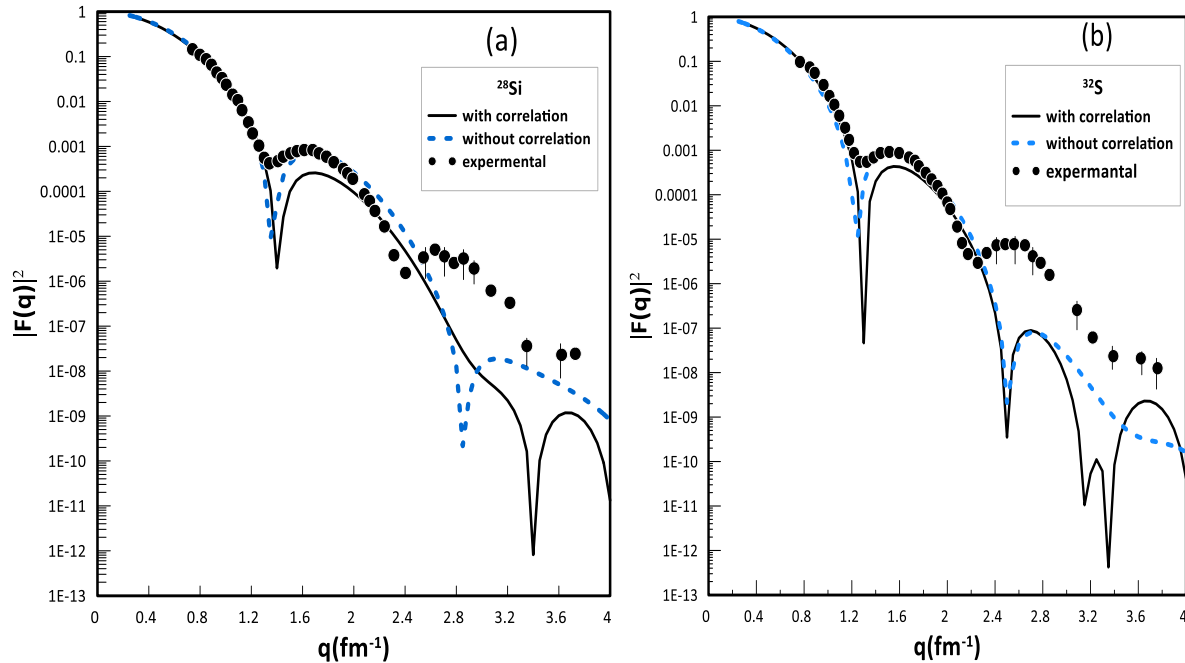


Figure 2: The $F(q)$ s for both ^{28}Si (Fig. a) and ^{32}S (Fig. b) nuclei are dependent on $q \text{ (fm}^{-1}\text{)}$. Ref. [26] findings from experiments are represented by the dotted symbols.

The ^{28}Si and ^{32}S nuclei for transition $J_i^\pi T_i \text{ to } J_f^\pi T_f = (0^+0 \rightarrow 2^+0)$ longitudinal inelastic electron form factors are shown in Figure (3) for ^{28}Si [Fig.3(a)] and ^{32}S [Fig.3(b)] nuclei. Equation (22) provides the model space transition density, and the OXBASH code is used to determine the (OBDM) components of the aforementioned nuclei [27]. Excitation energies $E_x = 1.78 \text{ MeV}$ [26] and experimental value of $B(\text{C}2) = 415 \text{ e}^2 \cdot \text{fm}^4$ [23], for ^{28}Si and $E_x = 2.237 \text{ MeV}$ [28] and experimental value of $B(\text{C}2) = 235 \text{ e}^2 \cdot \text{fm}^4$ [23] for ^{32}S relate to the C2 transition. The core polarizations via Tassie model in conjunction with the distribution of ground-state two-body charge densities (2BCDD) computed from Equation (9). The blue curves reflect the core polarizations contribution, which introduces collective nuclear modes; the model space contribution is shown by red curves, which take configuration mixing into account but do not include collective effects; whereas, the black curves illustrate the overall contribution, integrating model space and core polarizations effects. The success of incorporating core polarizations is demonstrated by the outcomes of the experiment; they are displayed as solid circles that closely match the black curves. The C2 form factors peaking in the first and second are enhanced by core polarizations effects, which also make the computed values quite similar to the experimental data. There is a small displacement of the diffraction minimum positions compared to the sd-shell model (red lines). The core polarizations findings are moved towards lower values of q for greater values of q , which makes the theoretical results quite similar to the experimental data.

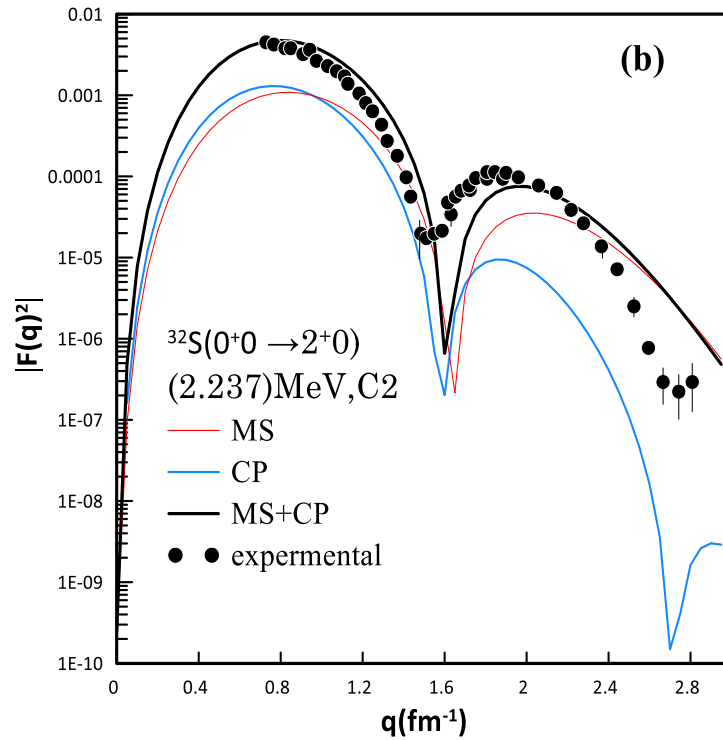


Figure 3: The ^{28}Si (Fig. a) with ^{32}S (Fig. b) nuclei's inelastic longitudinal C2 form factors.

The C4 form factor of ^{28}Si nuclei is shown in Figure (4), which is depicted in relation to the momentum transfer q for the transitions $(J_i^\pi T_i \rightarrow J_f^\pi T_f) = (0^+0 \rightarrow 4^+0)$ with an observed excitation energies of 4.617 (MeV) and experimental $B(C4)$ are 27500 (e^2fm^4) [29]. As seen in Figure (4), the form factors significantly outperform the model space outcomes. When considering the core polarizations effects, the computed form factors for the momentum transfer area, $q > 1.6 \text{ fm}^{-1}$, are somewhat moved towards lower values of q . This brings the computed findings of the distributions of solids extremely near to the results of the experiment.

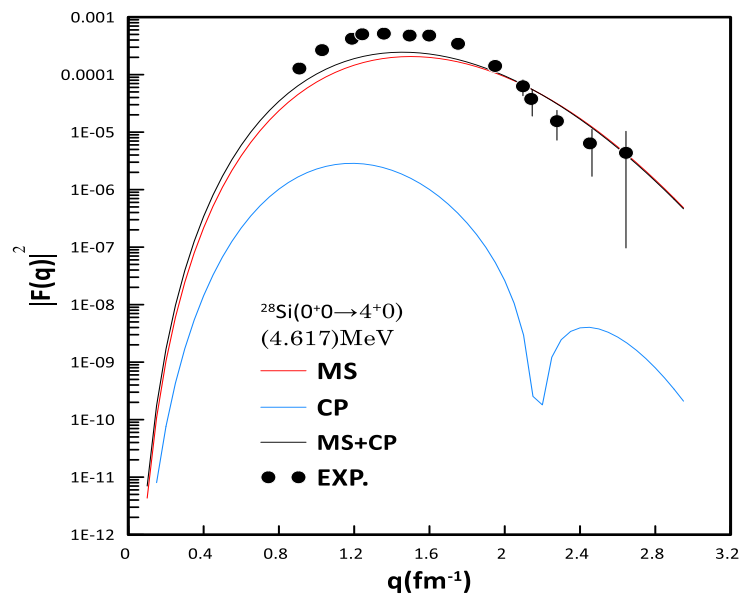


Figure 4 : The ^{28}Si nuclei's inelastic longitudinal C4 form factors

For ^{32}S , no C4 electron scattering data have been available for analysis. Figure (5) illustrates the inelastic longitudinal coulomb C2 and C4 form factors for ^{32}S nuclei,

respectively. The longitudinal C2 and C4 form factors that have been computed are depicted in relation to the momentum transfer q . For the transition ($0^+ \rightarrow 2^+$) and ($0^+ \rightarrow 4^+$) for ^{32}S with observed excitation energies of 4.282 MeV and 4.46 MeV [28], respectively. The experimental value of $B(\text{C2})$ and $B(\text{C4})$ of the above nucleus are 68.8, 50700 (e^2fm^4) [28]. The blue dash plot symbolizes the influence of the C2, and black solid plot symbolizes C4, which includes both contributions of the model space and core polarization effect, which is adjusted for the effect of two bodies. The red solid plot symbolizes the overall investigated data, calculated by combining the C2 and C4 for ^{32}S . The black “dotted symbols” is the observational data [28]. These plots demonstrate that the model space is incapable of providing an accurate explanation for the entire area of momentum transfer with the observation data. However, the results for the longitudinal coulomb C4 form factors become logical and consistent with the observation data all through the entire area of momentum transfer q , as evidenced by the black curves in these plots.

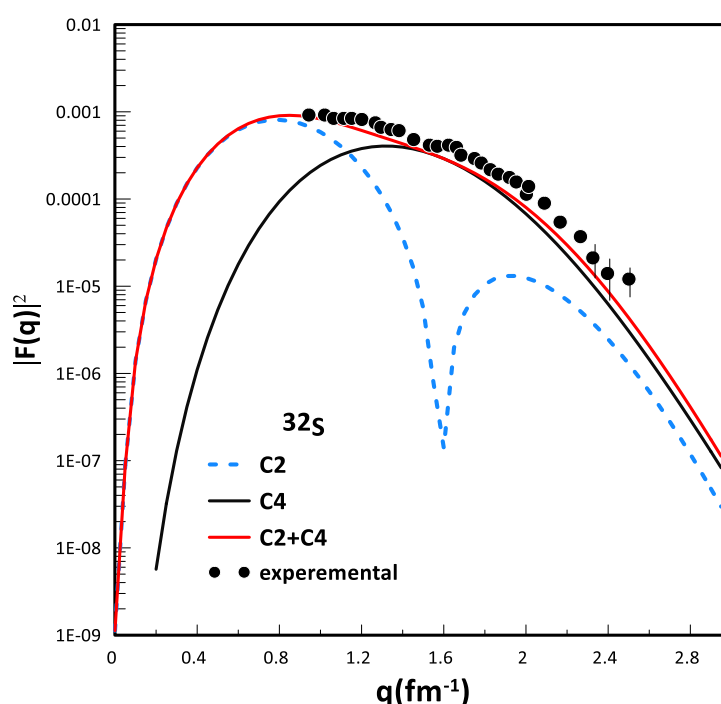


Figure 5: The ^{32}S nuclei's inelastic longitudinal (C2+C4) form factors

Conclusions

The introduction of spin- and isospin-dependent interactions results in accurate predictions for the root-mean-square (RMS) charge radii of the ^{28}Si and ^{32}S nuclei, as well as for their elastic electron scattering form factors. In particular, the application of the Tassie model for treating core polarization effects dramatically enhances the precision of inelastic form factor calculations for ground-state to excited-state transitions, especially for $0^+ \rightarrow 2^+$ and $0^+ \rightarrow 4^+$. The results demonstrate the strength of the theoretical formalism in reproducing experimental data with very high accuracy, leading to improved physical insight into nuclear structure and a deeper understanding of the role that correlations play in determining nuclear properties. In summary, this work emphasizes the importance of precision in nuclear charge density distributions and electron scattering form factors when accounting for spin and isospin dependence, especially in relation to short-range correlations (SRCs). Through the use of an effective two-body density version of the folding model, the resulting charge density distributions accurately reflect these correlations.

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