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# Analysis of the Distribution of Globular Clusters - Specific Frequency in Different Galaxy Types.

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**Abstract**

Globular clusters (GC) are important objects for tracing a galaxy's early evolution. The origin of the scaling parameters of the globular clusters as a function of galaxy mass, i.e., specific frequency, ($S_N$), specific mass ($S_M$), and specific number ($T$) of globular clusters is investigated to understand the origin of the relations between these parameters and the V-band magnitude ($M_V$) of their host galaxies. A wide sample of data from literature, covering different galaxy masses, types, and environments, including a study of the number of globular clusters ($N_{GC}$) has been used as the basis for our analysis. The sample covers the entire range of galaxy ($M_V = -11$ to $-25$ mag), where this large dataset confirms that, irrespective of the type of galaxy, the specific frequencies of GCs increase around a galaxy magnitude, of $M_V \simeq -18$ mag. Additionally, it is shown by our findings that the specific mass of GCs tends to increase significantly in galaxies with $M_V \simeq -21$ mag, while the specific number rises considerably in lower-luminosity galaxies with $M_V \simeq -17$ mag. Thus, highlighting the variations of GC formation efficiency, across different galaxy types.

**Keywords:** Globular clusters, number of GCs, specific frequency, specific masses, specific numbers.

# تحليل توزيع العناقيد الكروية – التردد النوعي في أنواع المجرات المختلفة

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**الخلاصة**

تعد العناقيد الكروية (GC) من المكونات الأساسية لتتبع التطور المبكر للمجرات. في هذه الدراسة، نحقق في أصل معلمات القياس للعناقيد الكروية بوصفها دالة لكتلة المجرة، بما في ذلك التردد النوعي($S_N$) ، والكتلة النوعية($S_M$) ، والعدد النوعي ($T$) للعناقيد الكروية، وذلك لفهم العلاقة بين هذه المعلمات والقدر المطلق في نطاقV – ($M_V$) للمجرات المضيفة لها. يعتمد تحليلنا على عينة واسعة من البيانات المستمدة من الأدبيات، والتي تشمل مجرات ذات كتل وأنواع وبيئات مختلفة، بالإضافة إلى دراسة عدد العناقيد الكروية($N_{GC}$). تغطي العينة نطاقًا واسعًا من كتل المجرات $M_V = -11$ إلى $-25$ mag، حيث تؤكد هذه البيانات الموسعة أن التردد النوعي للعناقيد الكروية يزداد عند قدر مجري $M_V \simeq -18$ mag بغض النظر عن نوع المجرة. علاوة على ذلك، تشير نتائجنا إلى أن الكتلة النوعية للعناقيد الكروية تزداد بشكل ملحوظ في المجرات ذات القدر $M_V \simeq$

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$M_V \simeq -17$ ،−21 mag بينما يرتفع العدد النوعي بشكل كبير في المجرات منخفضة اللمعان ذات القدر
mag، مما يسلط الضوء على الاختلافات في كفاءة تكوين العناقيد الكروية عبر الأنواع المختلفة من المجرات.

## 1. Introduction

One of the oldest systems formed at the beginning of the universe was star clusters. specifically, globular clusters (GCs) which, are found within various morphological type. They emerged during the main stages of star formation within galaxies, making them valuable sources of information and critical insights for the initial conditions which led to the formation of the host galaxies [1][2]. Globular clusters, are dense systems of low-metallicity stars they are gravitationally bound. Most of these stars are formed simultaneously from gas with similar chemical composition. Their total mass ranges, from ($10^4$ to $10^6$) $M_\odot$, and they are characterized by a relatively high central stellar density, reaching up to $10^6 M_\odot pc^{-3}$. The main region of these globular clusters is galactic halo [3] [4]. For a long time, globular clusters have play a central role in the study of astrophysics due to their ancient origins, as well as the critical insights they offer regarding the formation and, evolution of galaxies in the early epochs. The history of star formation can be reconstructed across different types of galaxies by analyzing their properties. These stellar systems provide a unique window, into the early conditions that shaped galaxies as we observe them today.

With their integral properties such as mass age and metallicity. along with their features within host galaxies. The total numbers, as well as dynamical distributions are essential for the initial physical conditions related to their development and formation, so transforming GC. Thus, we identify the universal features of GCs systems as interesting tools for investigating the main episodes regarding galactic star formation and as observational limits to separate several models of galaxy development [5] [6] [7]. It is clear nowadays that the main features of GCs are special markers to explain several astrophysical processes [8] [9].

Emphasizing the overall properties of GCs, one of the most fundamental parameters regarding a GC system is regarded to be the specific frequency related to Globular clusters ($S_N$) in a galaxy. The $S_N$ is considered a measure related to GCS richness in elliptical galaxies [10]. Since that time, $S_N$ has been used extensively in many investigations on various morphological environments and types (from galaxies in dense clusters to those in loose groupings and the field) [11]. Especially in the knowledge of their origin and formation processes, the specific frequency of GCs is a fundamental factor influencing the masses as well as the luminosities of stellar populations.

The $S_N$ serves as a crucial indicator of the efficiency, of GC relative to the field stars, which provides essential insights into the star formation processes as well as galaxy evolution. Utilizing merger tree data derived from the Lectea II simulation, M. Brockamp et al. [12] investigate GC evolution and formation in a Milky Way analog galaxy. They primarily focused on the formation efficiency during several cosmic times. Notably, their findings revealed that factors such as metallicity distributions. Galactocentric distances, and the formation efficiency of GCs, which, vary as functions of redshift and halo mass, are highly sensitive to the environmental, conditions during their formation within galaxy precursors or halos. The various models of the origin of galaxies and their subsystems can be effectively described using the specific frequency. $S_N$ values exhibit significant variation across different galaxy types, making them a valuable diagnostic tool for distinguishing between morphological classifications and environmental conditions. High $S_N$ are typically associated with early-type and dwarf ellipticals, whereas lower values are more commonly observed in late-type spiral galaxies [13] [14]. By utilizing deep observations, from the Hubble Space Telescope and wide-field based investigations, it was found that the specific values of

globular clusters differ significantly across galaxies, particularly between, low-mass dwarf galaxies and the brightest ellipticals [15].
Most galaxies have specific frequency values between 0.5 and 20, so the number of GCs scales roughly, but not exactly, with galaxy luminosity. Spiral galaxies have minimal variation in their $S_N$ values, often falling within the range of 0.5 to 2 [16] [17]. For luminous elliptical galaxies, $S_N$ is about 2 to 10 and tends to increase with luminosity. This suggests that these galaxies may have undergone more intense periods of star formation, leading to a larger population of globular clusters per unit stellar mass [13] [18]. In contrast, for early-type dwarf elliptical (dE) galaxies whose GC systems are predominantly metal-poor, their average $S_{N,}$ value increases with the decrease of galaxy luminosity, from only a few to a few tens [15] [19]- [21]. Consequently, the specific frequency serves as a useful tracer for comprehending the history of star formation as well as the consequent development of GC systems in various types of galaxies.

Referring to the total mass of the GC system relative to the total mass, of the host galaxy, the specific mass ($S_M$) can be defined as a physically relevant quantity and a fundamental factor in the knowledge of galaxy dynamics. It offers an understanding of the fraction of a galaxy contained in GCs [15]. Moreover, the luminosity of the galaxy determines the mean of GCs; thus, more galaxies usually host more massive GCs. The relation between galaxy luminosity as well as, the specific mass of their globular clusters was previously investigated in detail by [22]. Their analysis shows that high luminous galaxies usually have higher $S_M$ in their GCs. This implies that the features of their host galaxies greatly the formation conditions of GCs. Strader et al. (2011) found that the features of their host galaxies, especially their luminosity. influence the specific mass of GCs. This study highlights that more luminous galaxies create environments favorable for the development of more massive GCs.

The specific number ($T$) is defined, as the $N_{GC}$ normalized through, the stellar mass ($M_{G\star}$) related to the galaxy [23][24]. This quantity the relationship between globular clusters and the characteristics of the galaxy, aiding in the understanding of their distribution and impact within the cosmic context. The specific number helps to understand how efficiently a galaxy converts its stellar into clusters. Like $S_{M,}$ $T$ varies depending on the type of galaxy, and its evolutionary history. It is beneficial in comparing galaxies, of different masses and morphologies, as it highlights trends in globular cluster formation efficiency across diverse environments, and galaxy types. Mistani et al. (2016) examined how particular, morphological types galaxies affect a specific GCs. Their discovery of a higher specific number of GCs in elliptical galaxies relative to irregular, and spiral galaxies points to a link between galaxy formation processes as well as GC formation [25].

In this study, we analyze the total number of globular clusters ($N_{GC}$) within each galaxy. thereby providing a direct metric of the richness of the galaxy's GCS. The outline of this paper is as, follows: Section 2 offers a brief introduction to the galaxy samples used in this study, along with a discussion and description of the data sources and methodology employed to extract the key parameters each type of galaxy. Section 3 details the parameters of the clusters' system. Section 4 presents the discussion and results. Finally, Section 5 concludes with our findings.

## 2. The Data Sample

The methods applied and datasets used to investigate the distribution across various galaxy types are presented in this section. This study depends on thorough data gathered from many sources that, offer specific knowledge. On galaxy populations in different environments, such as more isolated field galaxies, as well as dense galaxy clusters. Those datasets help to compute fundamental GC parameters like specific mass, specific frequency, and specific number for several kinds of galaxies.

Our methodology begins with consolidating data on the total globular clusters in each galaxy, along with their stellar masses and luminosities. From this foundation, we proceed the calculate the relevant parameters for clusters. Subsequently, these calculated values are plotted and analyzed to trends across galaxy types, offering insights into the formation and evolution of globular systems. The primary datasets, encompass observations and analyses of both early- and late-type galaxies, providing a well-rounded view of cluster characteristics in varied galactic environments. This approach allows for a robust comparative analysis. Enhancing the understanding of globular systems in relation to galaxy morphology and environment.

### 2.1 Early-type galaxies in the ACS Virgo Cluster Survey

A main observational program aiming, at imaging 100 early-type galaxies in, Virgo with the use of the Hubble Camera, for the (ACS) aboard the Hubble Space Telescope (HST) [26]. In the Virgo Cluster, one of the most and closest galaxy clusters available for thorough study. This survey is essential for early-type galaxy study. For research on GC systems, it thus becomes one of the fundamental databases.

Our sample comprises 100 early-type galaxies from the ACS Virgo Cluster Survey, using data compiled from Peng et al. (2008) catalog (hereafter Eric08). The Virgo Cluster is well-studied and provides an ideal setting for in-depth comparisons of specific frequency, specific mass, and specific number values across galaxies with varying luminosities and masses.

### 2.2 Ultra-diffuse Galaxies (UDGs) in the Coma Cluster Treasury Survey

The Coma Cluster Treasury Survey is a program that images the Coma Cluster core, a massive galaxy cluster containing over 1000 galaxies, making it one of the largest known cosmic structures. Using the HST, the survey captures images in two filters. The imaging depth is sufficient for the detection of, globular clusters at the distance of the Coma Cluster, down to the mean value of the GC luminosity function [27].

We have conducted an in-depth study of globular clusters within ultra- galaxies (UDGs), which are peculiar galaxies characterized by their low luminosities, resembling classical dwarf galaxies, but with sizes up to about 5 times larger than expected for their mass. From this survey, we selected 43 ultra-diffuse galaxies for detailed analysis [28] (hereafter Lim18). The primary motivation for choosing sample lies in the unique nature of UGs and their notably high specific frequencies, of globular clusters, which pose significant challenges to traditional galaxy formation models.

### 2.3 Early-type Satellites in the Exploration of Local Volume Satellites Survey

The Exploration of Local Volume Satellite (ELVES) Survey at detecting and characterizing low mass dwarf satellite galaxies surrounding nearby, massive hosts within the Local Volume. This relies on ground-based observations to gather information about these galaxies. Ours comprises 177 dwarf galaxies, significantly enhancing the statistical basis, for studying the properties of clusters associated with dwarfs in environments. This provides essential comparison to samples obtained nearby clusters. We utilize dwarf galaxy catalogs, from the ELVES Survey, as referenced [29] (hereafter Scott22).

### 2.4 Irregular Galaxies in the Local Group and Isolated Late-type Dwarf Galaxies

The Local Group is a gravitationally collection of over 50 galaxies, including major members as the Milky Way. Andromeda (M31), and the Large and, Small Magellanic Clouds (LMC and SMC). This group contains, a broad variety of types, from massive spirals to smaller irregular and dwarf galaxies [30]. Its proximity provides, a necessary laboratory for research on galaxy evolution, formation, and interactions, hence illuminating large-scale structure related to the universe. Not quite of the Local Group (LG), late-type isolated dwarf galaxies, particularly dwarf spiral galaxies, exist low-density environments. Those galaxies might be found in more distant and usually form alone instead of being gravitationally connected to the larger LG galaxies. There are 35 galaxies total in sample, six irregular galaxies from, LG and, 29 isolated late-type dwarf galaxies. Forbes et al. (2018) [hereinafter DAF 18] [31] provided the data for work.

### 2.5 Ultra-diffuse Galaxies in Coma Cluster and Entire Clusters of Galaxies

In this study, we incorporate data from [32] [hereafter WEH17]. Which focuses on globular clusters in ultra- galaxies, and the broader context, of galaxy clusters. One of the most massive and largest galaxy clusters known. Coma Cluster, boasts important observations of UGs included in this dataset. Understanding the GC inside UDGs depends on Harris's catalog. Especially because of their very high specific frequencies that distinguish them from other galaxy types. This unique characteristic makes UDGs important objects, for improving theoretical models regarding galaxy evolution and formation. Using Harris's dataset, we have investigated the link between the $N_{GC}$ and several galaxy parameters, including stellar mass and luminosity. Those results, are essential in understanding the function of GCs in more general as well as evolutionary contexts of galaxies. The thorough nature of datasets, covering a broad range of morphologies, types, and, environments. Offers a perfect basis for a comparison of GC systems.

## 3. Globular Cluster System Parameters

### 3.1 The Specific Frequency of GC

The specific frequency of clusters is defined as the number of globular clusters per unit luminosity, of a galaxy, specifically for a galaxy with absolute magnitude of $M_V$= -15:

$$S_N = N_{GC} \times 10^{\,0.4(M_V+15)} \quad (1)$$

Originally, proposed by [10], the concept regarding specific frequency is still fundamental for theories galaxy formation, especially those examining the evolution of giant elliptical galaxies by means of disk mergers. This parameter serves as an indicator of the relative efficiencies in the formation of star clusters versus field stars. Offering insight into resilience of clusters that have survived, the various disruptive processes over cosmic time.

$S_N$ calculating lies in its potential to reveal whether globular cluster formation is proportional to the overall star formation activity across galaxies of varying types and masses. In our study, we computed the specific frequency for our sample galaxies using data from three major sources [15], [28], [29]. This study seeks to clarify the link between star formation as well as GC formation efficiency, thereby enhancing theories in galaxy development and formation by means of this analysis.

### 3.2 Specific Mass GC system

The specific mass of a globular cluster, system is defined as the ratio of the total stellar mass of the globular cluster, system, $M_{GCS}$, to the stellar mass of the host galaxy, $M_{G\star}$, expressed as [15]:

$$S_M = 100 \times \frac{\mathrm{M}_{GCS}}{\mathrm{MG}\star} \quad (2)$$

$M_{G\star}$ is the stellar mass of the host galaxy; $M_{GCS}$ is the total stellar mass in GCs. This offers an understanding, of the GC formation's relative efficiency of galaxy stellar mass. It might provide clearer knowledge regarding the relation between GCs, as well as the total mass of their host galaxy [15]. With the use of the total stellar mass values for the GCs obtained directly from three main references [10], [15], [31]. In our research, we computed the specific mass for every galaxy in our sample.

### 3.3 The Specific Number of GC (*T*)

The specific number globular clusters are defined as the $N_{GCs}$ per $10^9$ $M_\Theta$ of the host galaxy's stellar mass, which is calculated as follows [24]:

$$T = 10^9 M_\odot \times \frac{N_{GC}}{M_{G*}} \quad (3)$$

This method provides a consistent means to measure GC system richness relative to galaxy mass by allowing direct comparison regarding GC populations across galaxies with various mass-to-light ratios. We ascertained the specific number of GC depending on stellar masses was directly adopted from the original studies [15] [31] [32].

### 3.4 Total Number of the Globular Clusters ($N_{GC}$)

One of the most obvious characteristics of a galaxy is its $N_{GCs}$, which are closely related to its stellar mass and provide important understanding of its evolutionary history. From a few clusters in dwarf galaxies to tens of thousands in massive ellipticals, the distribution related to $N_{GC}$ differs greatly among the several morphological forms of galaxies. Crucially, galaxy environment and morphology affect the non-constant ratio of $N_{GC}$ to stellar luminosity. To ensure robust cross-referencing as well as data consistency, $N_{GC}$ values for this work have been systematically acquired from a comprehensive set of surveys and catalogs [15], [28], [29], [31], [32]. We investigated the relation between $N_{GC}$ and other galaxy characteristics, such as stellar mass, absolute magnitude, and GC total stellar mass. We analyze the correlation between the richness of GC systems and galaxy mass and luminosity, illuminating underlying formation processes and evolutionary pathways.

## 4. Results and Discussion

### 4.1 $S_N$ Values

The specific frequency, which measures the $N_{GCs}$ relative to galaxy luminosity, was calculated for our sample according to Equation (1). Figure 1 shows the galaxy correlation, between $S_N$ and absolute magnitude [15], [28], [29].

The Eric08 dataset includes 100 early galaxies from the Advanced Camera for Surveys Virgo Survey, primarily consisting, of giant elliptical galaxies. These galaxies, spanning a broad range of absolute magnitudes from $M_V$∼−15 to −23, exhibit relatively low $S_N$ values, primarily 0.1 and 12.5. This suggests a moderate efficiency in GC formation relative their stellar mass. Notably, the giant cD galaxy M87, positioned at the cluster's center, has an exceptionally high $S_N$ of 12.6 ± 0.8, consistent with its massive GC population. Which may be attributed to its central location, and dominant role to the Virgo Cluster [33], [34].

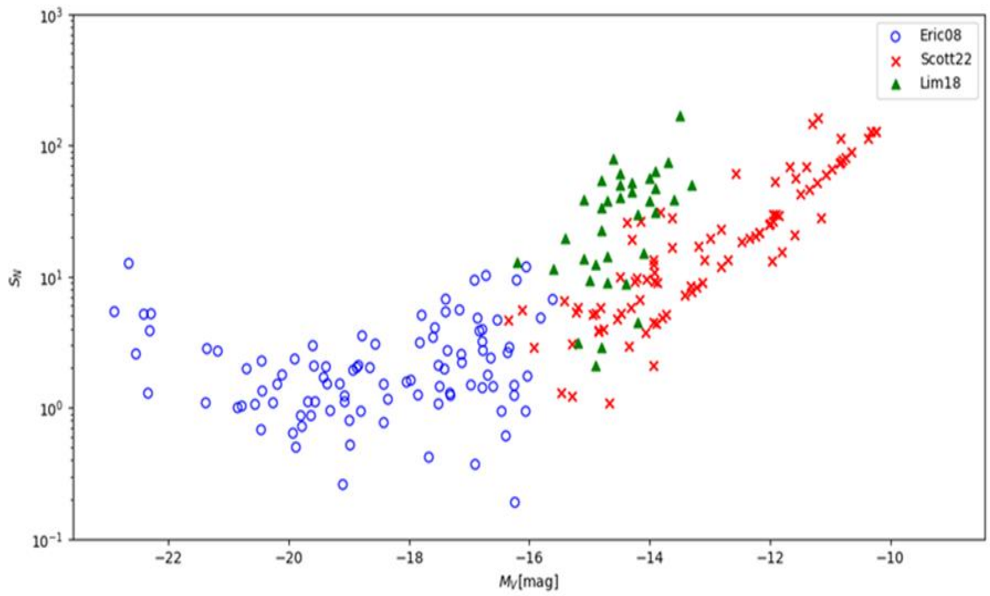


**Figure 1:** Globular cluster specific frequency, $S_N$, vs. absolute galaxy magnitude, $M_V$. The visualization of $S_N$ as a function of $M_V$ across different galaxy samples. Blue circles represent early-type galaxies within Virgo Cluster (Eric08) [15]. Red crosses depict early dwarf galaxies in low-density, environments, (Scott22) [29]. Green triangles correspond, to ultra-diffuse galaxies (UGs) in the Comam Cluster (Lim18) [28].

Early-type galaxies (Lenticular and Fainter Galaxies) with, intermediate luminosity ($-22< M_V <-18$). Which include lenticular (S0) types, typically show $S_N$ values around 1.5, reflecting limited GC formation. Fainter galaxies ($M_V >-18$) exhibit a broader $S_N$ range, with some showing minimal GC presence. While others exhibit unexpectedly high $S_N$, comparable, to luminous galaxies such as M87. The relatively flat $S_N$ distribution among these Virgo galaxies suggests, a stable GC formation process, likely to the galaxy type and cluster environment [34].

The UDGs from Lim18, display significantly elevated $S_N$ values, ranging approximately 0.2 to 348.3, with $M_V$ spanning from -12 to -16. Despite their low luminosities, UGs are remarkably efficient at hosting GCs. Suggesting an atypical GC process. This high GC abundance in UDGs may reflect their unique formation and evolutionary pathways, which, appear distinct more typical early-type galaxies [35].

Early-type Dwarf Galaxies in Low-Density Environments, Scott22 data, comprises 177 early-type, dwarf satellite galaxies within the Local Volume, with magnitudes ranging from $M_V \sim -10$ to $-16$. These galaxies exhibit a notable trend, of increasing, $S_N$ with the decrease of luminosity. For the faintest galaxies ($M_V \sim -10.2$), $S_N$ exceeds 127, indicating an exceptionally high efficiency of GC relative to stellar mass. This trend implies early-type dwarfs in sparse environments can support a disproportionately large $N_{GCs}$, potentially due to isolated evolutionary histories, and absence of disruptive interactions [36].

Across all datasets, consistent pattern emerges. $S_N$ tends to increase as $M_V$ becomes fainter, especially evident in Scott22 and Lim18 samples. This inverse correlation, between luminosity and specific frequency underscores increased GC formation efficiency in lower-luminosity systems. Particularly in dwarf and ultra-diffuse galaxies. Such findings, suggest environmental factors and intrinsic galaxy properties play crucial roles driving GC populations in these systems [37]. In contrast, the relatively stable $S_N$ values in the Eric08 sample of Virgo galaxies highlight the more predictable GC formation behavior densely clustered, high-mass galaxies.

**4.2 $S_M$ Values**

The specific mass representing ratio, of total globular cluster mass to stellar mass of the host galaxy which determined for our sample using Equation (2). Figure 2 illustrates, the relationship between $S_M$ and, absolute magnitude [15], [31], [32].

The Virgo Cluster Early-type Galaxies [15]. Which include early-type galaxies, primarily massive elliptical galaxies ($1.4\times10^8$ $-5.3\times10^{11}$) $M_\odot$. These galaxies exhibit $S_M$ values generally ranging 0.05 to 4, with a relatively flat distribution of $S_M$ across this luminosity range. This indicates a steady proportion of GC to stellar mass, suggesting that, in massive galaxies, GC formation efficiency is fairly consistent regardless of slight variations in luminosity. However, at the brightest end ($M_V <-22$), $S_M$ values show a subtle decrease. Which could imply that stellar mass grows rapidly than GC mass in the most luminous elliptical, likely due to favoring the accumulation of stellar mass, over GCs in these high-density environments.

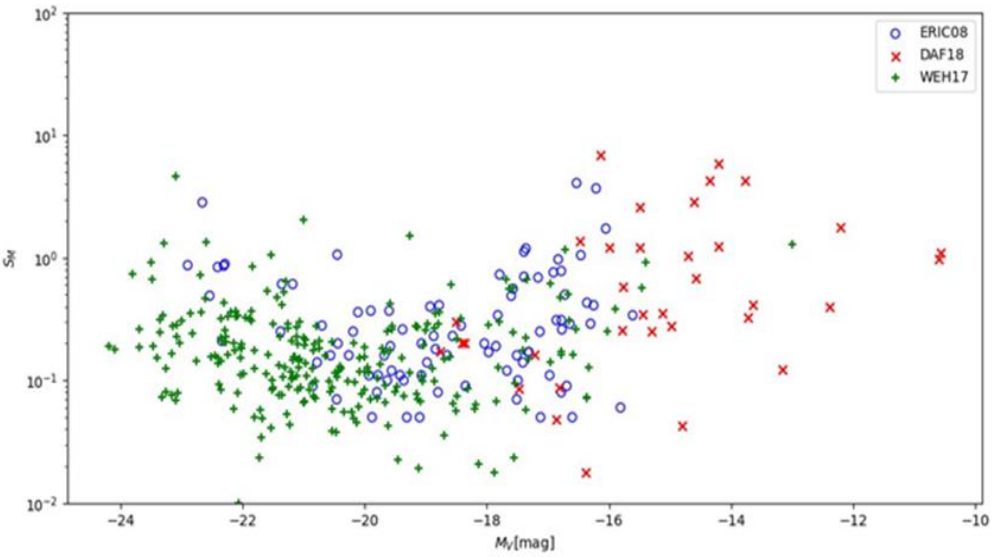


**Figure 2:** Globular cluster specific mass, $S_M$, vs. V-Band absolute galaxy magnitude. $M_V$, showcasing data across various galaxy morphologies. Masses, and environments. The symbols in the plot indicate data sources as follows: green plus signs for ultra-diffuse galaxies from WEH17[32]. Red crosses for early- and late-type dwarf galaxies from, DAF18[31], and blue circles for early-type galaxies within the Virgo from Eric08[15].

The Dwarf Galaxies in Low-density Environments (DAF18), represented by red crosses, include 35 early- and late- dwarf galaxies with magnitudes predominantly brighter than $M_V =-16$. These galaxies display a strikingly pattern from the Eric08 data, with $S_M$ values that increase significantly as luminosity decreases. For the faintest dwarf galaxies, $S_M$ values exceed 6, indicating a disproportionately high GC mass relative to stellar mass. This elevated $S_M$ in low-luminosity. Low-density environments suggest, that dwarf galaxies retain a larger fraction of GC mass relative to their total stellar mass, possibly due to isolated evolution with fewer interactions and mergers. Which might otherwise strip GCs, or disrupt their formation [38] [39].

The Ultra-Diffuse Galaxies in the Coma from the WEH17 dataset, denoted by green plus signs. Cover a broad absolute magnitude range, from $M_V \approx -11$ to -24 with specific mass values ranging from approximately 0.005 to 4. These UDGs generally show lower $S_M$ values compared to dwarf galaxies. Which eflecting their diffuse nature and lower overall mass. The relatively low $S_M$ in UDGs may be due to their lower density. Which could limit their ability to retain GCs. Nonetheless, the presence non-negligible $S_M$ values in UDGs supports the hypothesis that these galaxies. Despite their diffuse structure and faint luminosities, can host a substantial GC population relative to stellar mass. This aligns with theories suggesting, that UDGs may have undergone unique formation processes. Which either promote GC retention, or facilitate GC formation even in low-density stellar environments [31].

Generally, $S_M$ increases as galaxy decreases, particularly noticeable among dwarf galaxies and UDGs. This inverse relationship between $M_V$ and $S_M$ suggests that smaller, fainter galaxies are more efficient at retaining GCs relative to their stellar mass than more massive galaxies. In contrast, the flatter $S_M$ distribution in more luminous early-type galaxies, such as those in the Virgo Cluster (Eric08), indicates a relatively constant GC formation efficiency within a well-defined luminosity range. This dichotomy, in $S_M$ behavior reflects the influence of galaxy mass, morphology. Environmental factors on GC populations, highlighting how dwarf and UDG systems retain GCs more efficiently than their massive counterparts' high-density environments, where stellar mass accumulation may dominate GC formation.

## 4.3 *T* Values

In the Virgo Cluster sample, which comprises early galaxies with magnitudes ranging from $M_V \sim -15$ to −23, *T* values predominantly fall 0.7 and 133.8 [15]. This relatively low and stable range of *T* suggests, a uniform GC formation efficiency in galaxies. For more massive, giant elliptical galaxies, the distribution of *T* remains relatively flat. Indicating that the $N_{GCs}$ are roughly proportional to the stellar mass. This consistency reflects, the very stable environments and evolutionary histories of early-type galaxies in dense cluster settings, such as Virgo.

Particularly in the fainter luminosity range ($M_V > -16$), Local Group dwarf galaxies, which span magnitudes from $M_V \approx -11$ to −19, show noticeably higher and more variable *T* values. Although such galaxies often have low stellar masses, they might have unusually high $N_{GCs}$, which would provide *T* values often exceeding 1000. This higher *T* implies that such dwarfs, residing in less dense environments, might preserve more of their GC systems and undergo evolutionary processes unique from those of Virgo Cluster galaxies, maybe including isolated evolution with minimum tidal disruption.

Within a broader luminosity range ($M_V \approx -12$ to −24), the ultra-diffuse galaxies in the Coma Cluster show even more uniform distribution in *T* values. Various UDGs, especially those with $M_V > -18$, have remarkably high *T* values, which could be ascribed to significant GC populations even in their diffuse, low-luminosity stellar profiles. This implies that, maybe in response to their extended profiles and possible formation in isolated or low-density environments before being accreted into the Coma Cluster, UDGs are able to produce or retain disproportionately large GC systems relative to their stellar masses.

Across all datasets, a notable trend emerges: as galaxy luminosity decreases (moving rightward along the $M_V$ axis), *T* values tend to increase. This inverse correlation suggests that lower-luminosity, low-mass galaxies, such as Local Group dwarfs and Coma Cluster UDGs, are unusually efficient in forming or retaining GCs relative to their stellar masses.

This behavior most certainly reflects their different formation as well as evolutionary pathways. Whereas smaller, isolated galaxies could form or retain larger GC populations, unimpeded through the stripping as well as tidal forces common in dense environments, massive, early-type galaxies in clusters like Virgo seem to experience regulated evolution with stable GC formation efficiency.

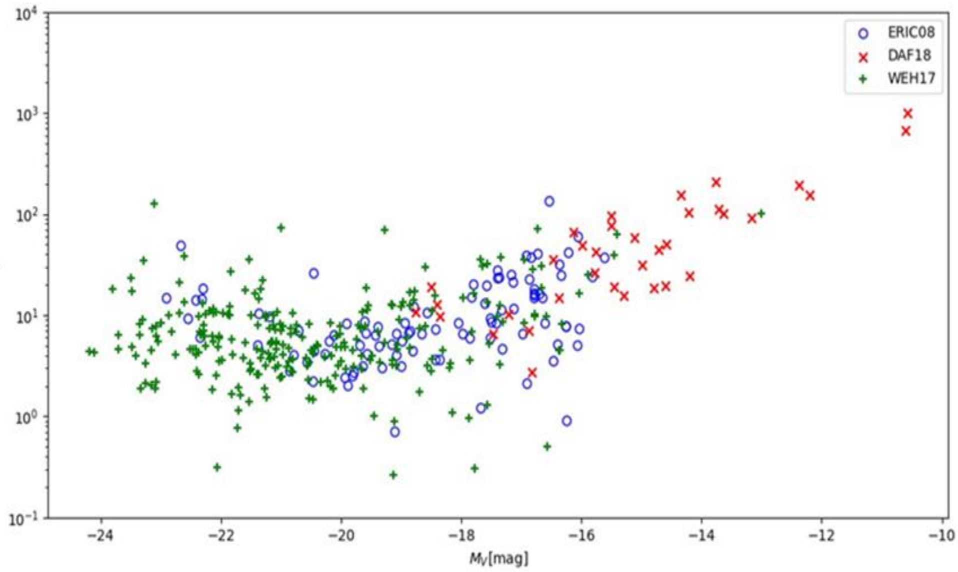


**Figure 3:** The specific number of globular clusters (*T*) vs. absolute magnitude of the galaxy $M_V$ of various galaxy samples, for the same sample as in Figure 2.

The detected variations in *T* values highlight how environment and evolutionary history affect GC systems of different kinds throughout galaxies. Dwarf as well as ultra-diffuse galaxies, for example, might undergo unique processes like tidal stripping or external accretion, so improving or preserving their GC systems. Often, these galaxies form in isolation or low-density regions prior to joining larger clusters. Early-type galaxies in dense cluster environments show more uniform GC formation efficiency, perhaps because of their stable formation processes and more frequent interactions. By highlighting the complex interaction among galaxy mass. Environment, luminosity, and GC formation efficiency, this study clarifies the governing GC retention and assembly. The significant difference in, *T* among low-mass galaxies suggests the possibility of different evolutionary histories in which, isolated environments and minimal tidal disruption help to retain large GC populations. Knowing such dynamics helps one to better appreciate, how the environment shapes the GC systems of galaxies over the luminosity spectrum.

### 4.4 $N_{GC}$ Values

We present a complete regarding GC distributions over a wide spectrum of galaxy environments and types. Figure 4 illustrates the relationship between $N_{GC}$ and the absolute magnitude $M_V$ of galaxies, in various environments. A clear inverse correlation is observed, with more luminous galaxies hosting larger GC populations. Two fitted lines with, distinct slopes represent this negative trend, capturing the behavior in different MV ranges. For bright galaxies ($M_V$ between -23 and -16), the relation follows $y=0.0e^{-1.64x}$, while for faint galaxies ($M_V$ between -16 and -10), it is fitted as $y=0.01e^{-0.47x}$. This trend suggests, that the development and complexity of a galaxy's GC system correlate strongly with luminosity. Particularly indicating that more massive galaxies, with higher luminosities have more extensive and structured GC populations. In the magnitude range of $M_V$ =−11 to -16, however, there is considerable scatter in $N_{GC}$, indicating that dwarf galaxies generally have fewer GCs than giant galaxies. This suggests that dwarf galaxies lack the mass, and potential depth to retain or develop as many GCs, an observation that aligns with idea of galaxy formation and

evolution in low-mass systems. The analysis also, reveals an enhanced fraction of dwarf galaxies with rich GC systems within the Virgo compared to lower-density environments. Specifically, the fraction of Virgo dwarfs with $N_{GC} > 10$ is approximately twice that observed in the Local Volume sample, emphasizing environmental effects, on GC richness in dwarf galaxies.
Figure 5 shows, the relation between $N_{GC}$ and the stellar mass of the host galaxy, for comparative studies across galaxies with diverse morphologies, and star formation histories. This relation can be influenced young stellar populations. The stellar mass of the galaxy provides a more stable tracer of the galaxy's total mass. A strong positive correlation between $N_{GC}$ and $M_{star}$ (Figure 5). Showing that as stellar mass increases, so does the richness of the GC system. For galaxies with $M_{star} < 10^8 M_{\odot}$, the $N_{GCs}$ remain relatively low, consistent with the limited mass available dwarf galaxies to support extensive GC populations. Conversely, galaxies with $M_{star} > 10^9 M_{\odot}$ exhibit a rapid increase in $N_{GC}$, underlining the higher complexity of GCs systems in more massive galaxies. This correlation confirms that stellar mass is a fundamental parameter that is linked to the GC population, supporting the hypothesis that more massive galaxies have not only the gravitational potential but also the enriched histories needed to form and retain larger GC populations.

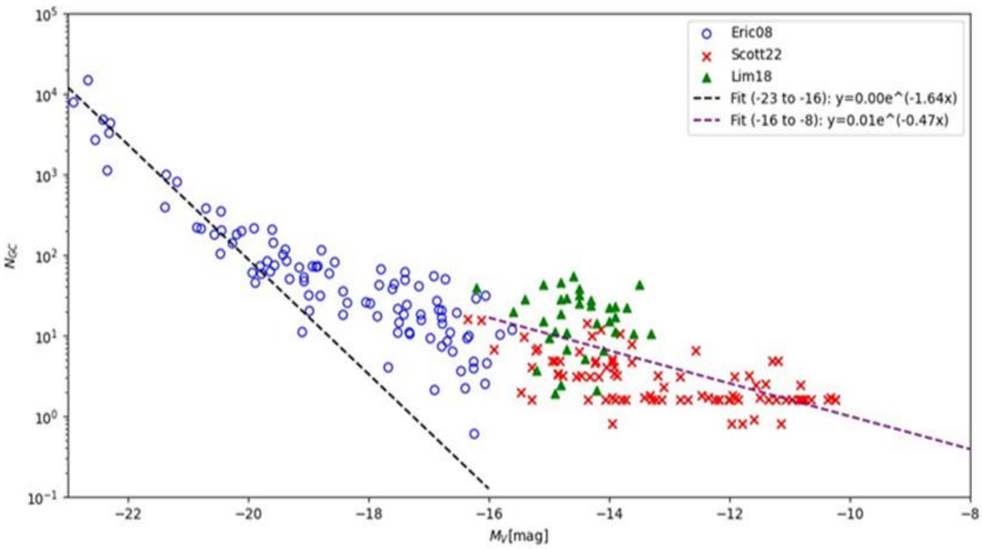


**Figure 4:** The relationship between $N_{GC}$ and the absolute magnitude $M_V$ of galaxies in various environments. The blue circles are early-type galaxies in the Virgo Cluster (Eric08) [15]. Galaxies in the Coma Cluster are green triangles (Lim18) [28] and early-type dwarfs in low-density environments are red crosses (Scott22) [29].

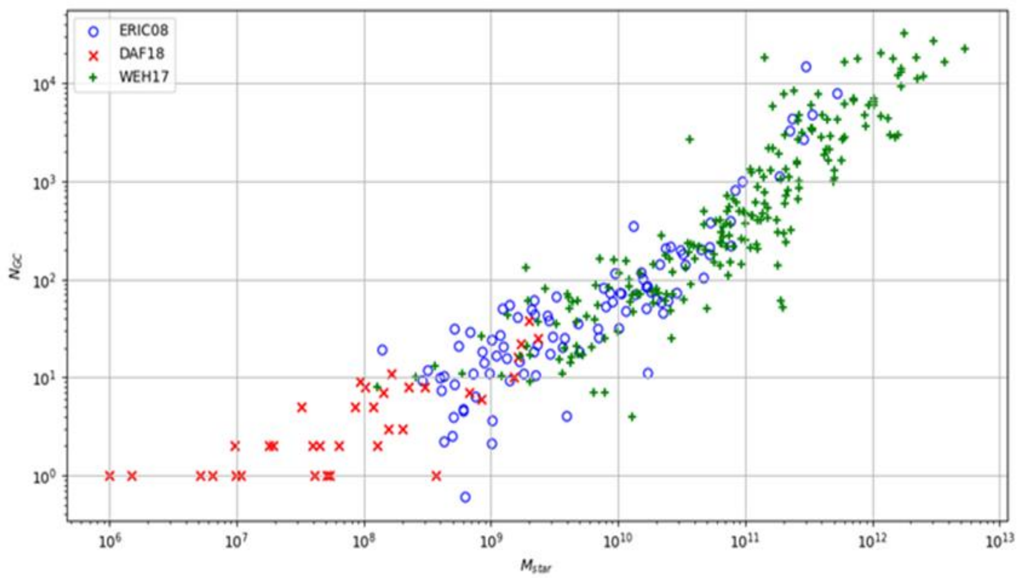


**Figure 5:** The $N_{GCs}$ vs. Stellar mass of the galaxy[Mʘ], the green plus signs indicate data from [WEH17] [32], while big red multiplication signs to data from [DAF18] [31], and the circles indicate data from [Eric08] [15].

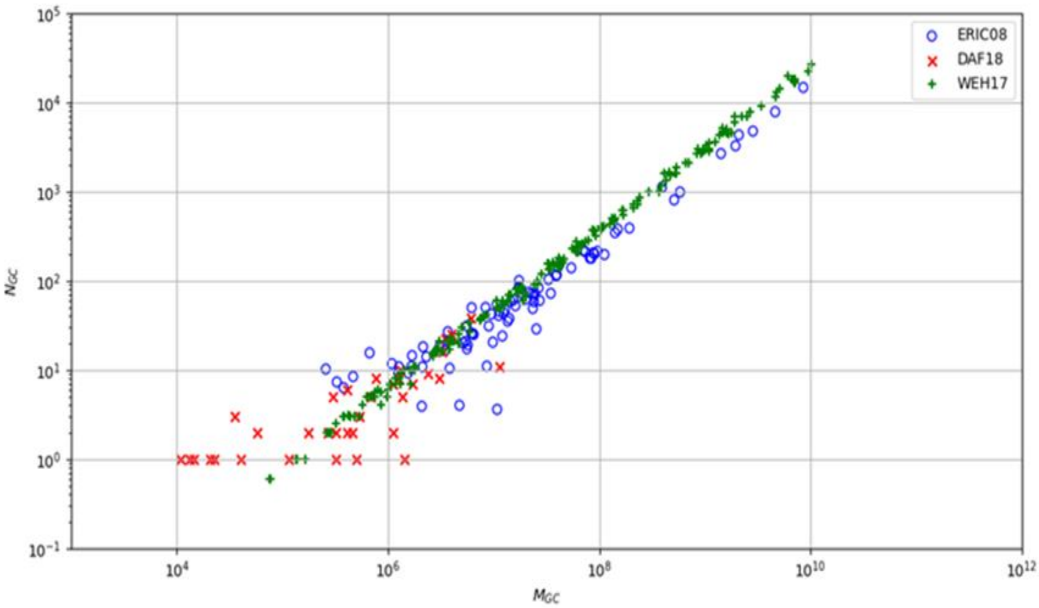


**Figure 6:** The correlation between the $N_{GCs}$ and their total stellar mass ($M_{GC}$) across various galaxy types, for the same sample as in Figure 5.

Figure 6 shows the correlation between the $N_{GCs}$ and the total stellar mass of globular clusters. This analysis reveals that globular clusters are not merely supplementary stellar populations but integral components of a galaxy's stellar mass structure.

Consistent across a range of galaxy types from massive to dwarf, the observed linear relationship between the $N_{GCs}$ as well as their cumulative stellar mass proposes a basic process in galaxy formation or evolution regulating the accumulation regarding GCs relative to galaxy mass. This proportionality most certainly represents universal processes governed, by merger histories and accretion events, that control the relation between $M_{GC}$ and $N_{GC}$, therefore offering an understanding of galactic evolution and formation. The stability related to correlation, independent of galaxy size. Suggests possible universal dynamics affected by elements, such as gravitational potential, galactic mass, or environmental variables, which might influence GC formation, and retention efficiency. These results are in agreement with previous studies [40], [41], [42], which confirm the theory that GCs are fundamental to the stellar population of galaxies and significantly influence galactic evolution. Our findings provide useful constraints for models of galaxy formation. Galaxies with more GCs show a correspondingly higher cumulative stellar mass in such clusters, highlighting the crucial part GCs play in understanding the stellar mass composition as well as assembly processes of galaxies.

## 5. Conclusions

- The key scaling parameters, including specific frequency, specific mass, specific number, and the normalized total $N_{GCs}$ relative to galaxy luminosity and stellar mass, exhibit a clear functional dependence on galaxy luminosity, with both low- and high-luminosity galaxies showing elevated scaling values and a pronounced minimum near $M_V \approx -20$ mag.
- Low-luminosity systems, particularly ultra-diffuse galaxies and early-type dwarfs in low-density environments, have markedly higher globular cluster formation efficiencies relative to their stellar masses, as indicated by elevated $S_N$ values.
- High-luminosity elliptical galaxies tend to show lower GC formation efficiency, likely due to the influence of dense environments and evolutionary processes that govern these massive systems.
- Specific mass and specific number $T$ parameters suggest that dwarf galaxies, especially those in isolation, are adept at retaining globular clusters, possibly due to fewer disruptive interactions.
- The distinctive trends in dwarf and ultra-diffuse galaxies provide valuable insights into the unique evolutionary pathways these systems may undergo, including isolated evolution and minimal tidal stripping. This provides a crucial basis for further exploration into the underlying mechanisms of globular cluster formation and galaxy evolution.

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