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Modeling of Fuzzy Soft Paranormal-Type Operator Associated with a Fuzzy Soft Hilbert Domain

Salim Dawood Mohsen*, Mohammed Jassim Mohammed

Department of Mathematics, College of Education, Mustansiriyah University, Baghdad, Iraq

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Abstract

This article presents new findings in the realm of operator theory (OT) related to fuzzy soft set theory (FST). A new version of the fuzzy soft paranormal operator in a fuzzy soft Hilbert space, called fuzzy soft M-paranormal operator, is formulated in a more comprehensive and general form. The most significant characteristic of the proposed operator is described as allied to other its leading traits. In addition, the requisite conditions to acquire various interesting merits of this novel operator are highlighted.

Keywords: Soft set; Fuzzy soft; Fuzzy soft Hilbert space, Fuzzy soft paranormal operator, Fuzzy soft M −paranormal operator.

نمذجة نوع من المؤثر فوق السوي الضبابي الناعم المرتبط مع مجال هلبرت الضبابي الناعم

سالم داود محسن * ، محمد جاسم محمد

قسم الرياضيات ، كلية التربية ، الجامعة المستنصرية ، بغداد ، العراق

الخلاصه:

تقدم هذه المقالة نتائج جديدة في مجال نظرية المؤثرات وتحديدا في النظرية الضبابية الناعمة، يتم تقديم مصطلح مؤثر فوق السوي الضبابي الناعم في فضاء هلبرت الضبابي الناعم يسمى المؤثر فوق السوي-M الضبابي الناعم وهي صيغة اكثر شمولا وعمومية. يتم وصف الصفة الاكثر أهمية للمؤثر المقترح مع صفاته المهمة الأخرى، بالإضافة الى ذلك يتم تسليط الضوء على الشروط الضرورية لتقديم خواص مثيرة للاهتمام مختلفة لهذا المؤثر الجديد.

1. Introduction

There is a lot of ambiguity and uncertainty in the world, so it has a set of complex problems, in numerous domains such as economics, engineering, and the environment. In other words, the modern approach has been applied in various fields, including data analysis and decision-making, demonstrating its versatility. In 1965, Zadeh [1] provided the fuzzy set theory (FS-theory), which is as a more general extension of set theory. This considerably work provided a formal definition of fuzzy sets, providing a flexible mathematical framework for dealing with uncertainty and imprecision in various fields of study. Fuzzy sets have become the cornerstone of advances in artificial intelligence, decision-making, and control systems. Its ability to model ambiguity and partial membership has made it an essential tool

*Email: dr_salim2015@yahoo.com

for solving real-world problems where classical binary logic fails, [2-5]. On the other hand, Molodtsov [6] in 1999 developed this discipline by introducing a new generalization of Set Theory known as Soft Set Theory (SS-theory) to deal with uncertainty in a more comprehensive way. Following the pioneering work of Molodtsov, Maji et al. [7] in 2002 employed SS-theory for modeling problems and decision-making rely on the mathematical frame of rough sets. Thence, Maji et al. [8] in 2003 performed exacting and widely studies on the SS-theory. During the past two decades, the SS-theory has received great attention from numerous researchers who have introduced new topics, and these topics are considered extended, such as soft metric space [9], soft normed space [10], soft inner product spaces [11], and soft seminorm [12]. In this connection, researchers have investigated many soft operators for their importance in scientific fields. For instance, Daraby et al. [13] in 2018 offered several attributes of Felbin-sort fuzzy inner product spaces (FFIP-spaces) alongside fuzzy bounded linear operators (FBL-operators) acting on FFIP-spaces with several operator norms. Moreover, they considered the principle of fuzzy orthonormality and constructed a version of a Bessel's inequality based on Felbin's norm. Later, Yazar et al. [14] in 2019 imposed new theories and attributes of soft Hilbert space (SH-space). Other studies have appeared that are interested in this theme, see [15-25]. Continuing this line of investigation, in 2001, Maji et al. [26] invented a more general theory, namely the Fuzzy Soft Set Theory (FS-ST), which merges the soft set principle and fuzzy soft principle to introduce more accurate and comprehensive outcomes. The Fuzzy soft point (FS-point) and the fuzzy soft normed space (FSN-space) were constructed to expand the scope of investigations. Recently, Faried et al. [27] in 2020 offered fuzzy soft inner product space (FSIN-space) and fuzzy soft Hilbert space (FSH-space). In [28], Faried et al. in 2020 also introduced the Fuzzy Soft Operator Theory (FS-OT) by studying certain fuzzy soft linear operators on FS-H, such as the fuzzy soft right operator alongside the left shift operator and discussing a variety of related merits of Fuzzy Soft Spectral Theory (FSS-ST). In addition, several scholars proposed various applications of FSS-theory [29-30], which made it possible to introduce new ideas for fuzzy operators [31-35]. This note introduces a more general version of the fuzzy soft paranormal operator in a fuzzy soft Hilbert space, called fuzzy soft M-paranormal operator, to deduce a significant characteristic relies on inequality principle. This characteristic includes the fuzzy soft norm (FS-Norm). To find the required approach by considering a variety of algebraic and analytical merits. The advantages are discussed theoretically. This focuses on the studying on the conditions requisite to gain diverse merits from this novel operator.

2. Concepts

This section outlines the crucial principles relevant to the study of fuzzy soft M-paranormal operator (M-FSPO) in the fuzzy soft Hilbert space (FS-HS).

Definition 2.1: [28] Assume that U is the universal set, the fuzzy set $\tilde{A}$ in U is a set of ordered pairs; $\tilde{A} = \{(x, \mu_{\tilde{A}}(x): x \in U\}$, where $\mu_{\tilde{A}}: U \to [0,1]$ is said the membership function.

Definition 2.2: [8] Assume that U is the universal set, and D is a set of parameters, $P(U)$ the power set of U, and $\tilde{A} \subseteq D$. Let H be a function from $\tilde{A}$ to $P(U)$, $H_{\tilde{A}} = \{H(e) \in, P(U): e \in \tilde{A}\}$. $H_{\tilde{A}}$ or the pair is $(H, \tilde{A})$ said soft set on U with respect to $\tilde{A}$.

Definition 2.3: [33] The soft set $H_{\tilde{A}}$ is said fuzzy soft set on U, when H is mapping $\tilde{A} \to I^U$, I^U be the power set of all fuzzy subsets of U, symbolize by FSS.

Definition 2.4: [10] The fuzzy soft point (FS-Point) on U, it is a special case of FSS. In brief $\tilde{x}_{\mu H(e)}$, if $e \in \tilde{A}$ and $x \in U$, $\mu_{H(e)}(x) = \{\alpha, if\ x = x_o \in U$ and $e = e_0 \in \tilde{A}$, if $x \in U - \{x_0\}$ or, $e \in \tilde{A} - \{e_0\}$, where $\alpha \in (0,1]$.

Definition 2.5: [27] Let $\widetilde{U}$ be an FS-vector, and a function $\widetilde{\|.\|}: \widetilde{U} \to \tilde{R}^{+}(\tilde{A})$ is said FS-Norm on $\widetilde{U}$, if $\widetilde{\|.\|}$ satisfies the next conditions:
(a) $\|\widetilde{\tilde{x}_{\mu_{H(e)}}}\| \geq \tilde{0}\ \forall \tilde{x}_{\mu_{H(e)}} \in \widetilde{U}$, and $\|\widetilde{\tilde{x}_{\mu_{H(e)}}}\| = \tilde{0} \leftrightarrow \tilde{x}_{\mu_{H(e)}} = \tilde{\theta}$.
(b) $\|\widetilde{\tilde{r} \cdot \tilde{x}_{\mu_{H(e)}}}\| = |\tilde{r}|\|\widetilde{\tilde{x}_{\mu_{H(e)}}}\|, \forall\, \tilde{x}_{\mu_{H(e)}} \in \widetilde{U}, \forall\, \tilde{r} \in \tilde{R}^{+}(\tilde{A})$.
(c) $\|\widetilde{\tilde{x}_{\mu_{1H(e_1)}} + \tilde{y}_{\mu_{2H(e_2)}}}\| \leq \|\widetilde{\tilde{x}_{\mu_{1H(e_1)}}}\| + \|\widetilde{\tilde{y}_{\mu_{2H(e_2)}}}\|, \forall\ \tilde{x}_{\mu_{1H(e_1)}}, \tilde{y}_{\mu_{2H(e_2)}} \in \widetilde{U}$.
An FS-vector $\widetilde{U}$ with FS-Norm $\widetilde{\|.\|}$ is called an FS-Normed space (FSN-Space).
Definition 2.6: [27] Let $\widetilde{H}$ be an FS-inner product space. It is said to be a fuzzy soft Hilbert space if $\widetilde{H}$ is complete space and symbolize by FSH-space.
Definition 2.7: [31] Assume that $\widetilde{H}$ is an FSH-space and $\widetilde{Ŧ}: \widetilde{H} \to \widetilde{H}$ is an FS-operator, thus $\widetilde{Ŧ}$ is said to be a fuzzy soft linear operator (FSL-operator) if:
a) $\widetilde{Ŧ}(\widetilde{\tilde{x}_{\mu 1H(e1)} + \tilde{y}_{\mu 2H(e2)}}) = \widetilde{Ŧ}(\widetilde{\tilde{x}_{\mu 1H(e1)}}) + \widetilde{Ŧ}(\widetilde{\tilde{y}_{\mu 2H(e2)}})$, for all $\tilde{x}_{\mu 1H(e1)}, \tilde{y}_{\mu 2H(e2)} \in \widetilde{H}$.
b) $\widetilde{Ŧ}(\widetilde{\lambda \tilde{x}_{\mu 1H(e1)}}) = \lambda \widetilde{Ŧ}(\widetilde{\tilde{x}_{\mu 1H(e1)}})$, for all $\tilde{x}_{\mu 1H(e1)} \in \widetilde{H}$ and $\lambda \in \tilde{R}^{+}(\tilde{A})$.
Definition 2.8: [31] Assume that $\widetilde{H}$ is an FSH-space and $\widetilde{Ŧ}$ belongs to $\widetilde{Ƀ}(\widetilde{H})$, then FS-adjoint operator $\widetilde{Ŧ}^{*}$is defined as:
$\langle \widetilde{\widetilde{Ŧ}\tilde{x}_{\mu 1H(e1)}, \tilde{y}_{\mu 2H(e2)}} \rangle \cong \langle \widetilde{\tilde{x}_{\mu 1H(e1)}, \widetilde{Ŧ}^{*}\tilde{y}_{\mu 2H(e2)}} \rangle\ \forall\, \tilde{x}_{\mu 1H(e1)}, \tilde{y}_{\mu 2H(e2)}\ \tilde{\in}\ \widetilde{H}$.
Where $\widetilde{Ƀ}(\widetilde{H})$ is the set of all bounded linear operator on $\widetilde{H}$.
Definition 2.9: [32] Assume that $\widetilde{H}$ is an FSH-space and $\widetilde{Ŧ}$ belongs to $\widetilde{Ƀ}(\widetilde{H})$, then $\widetilde{Ŧ}$ is called a fuzzy soft normal operator (FS-Normal) if $\widetilde{Ŧ}\widetilde{Ŧ}^{*} \cong \widetilde{Ŧ}^{*}\widetilde{Ŧ}$.
Definition 2.10: [31] Let $\widetilde{Ŧ}$ be an FS-operator of $\widetilde{H}$. It is said to be an FS-self adjoint operator if $\widetilde{Ŧ} \cong \widetilde{Ŧ}^{*}$.
Definition 2.11: [21] Assume that $\widetilde{H}$ is an FSH-space and $\widetilde{Ŧ}$ belongs to $\widetilde{Ƀ}(\widetilde{H})$, then $\widetilde{Ŧ}$ is said an FS- isometry operator if it achieves
$\langle \widetilde{\widetilde{Ŧ}\, \tilde{x}_{\mu 1H(e1)}, \widetilde{Ŧ}\, \tilde{y}_{\mu 2H(e2)}} \rangle \cong \langle \widetilde{\tilde{x}_{\mu 1H(e1)}, \tilde{y}_{\mu 2H(e2)}} \rangle\ \forall\, \tilde{x}_{\mu 1H(e1)}, \tilde{y}_{\mu 2H(e2)}\ \tilde{\in}\ \widetilde{H}$,
where $\widetilde{H}$ is Hilbert space
Definition 2.12: [34] An FS-operator Fuzzy $\tilde{I}$, is called a soft identity operator $\tilde{I} : \widetilde{H} \to \widetilde{H}$ if $\tilde{I}(\tilde{x}_{\mu 1H(e1)}) = \tilde{x}_{\mu 1H(e1)}, \forall \tilde{x}_{\mu 1H(e1)}\ \tilde{\in}\ \widetilde{H}$.
Definition 2.13: [36] Assume that $\widetilde{H}$ is an FSH-space and $\widetilde{Ŧ}$ belongs to $\widetilde{Ƀ}(\widetilde{H})$, then $\widetilde{Ŧ}$ is said to be an FS-unitary operator if it achieves $\widetilde{Ŧ}\widetilde{Ŧ}^{*} = \tilde{I} = \widetilde{Ŧ}^{*}\widetilde{Ŧ}$.
Definition 2.14: [37] Let $\widetilde{H}$ be an FSH-space and $\widetilde{Ŧ}$ belongs to $\widetilde{Ƀ}(\widetilde{H})$, then $\widetilde{Ŧ}$ is an FS-paranormal operator if it achieves: $\|\widetilde{Ŧ^{2}\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\widetilde{Ŧ}\tilde{x}_{\mu H(e)}}\|^{2} \forall\, \tilde{x}_{\mu H(e)}\ \tilde{\in}\ \widetilde{H}$, so: $\|\widetilde{\widetilde{Ŧ}^{2}\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\widetilde{Ŧ}\tilde{x}_{\mu H(e)}}\|^{2}$ for all $\tilde{x}_{\mu H(e)}$ belongs in $\widetilde{H}$.
We know that $\widetilde{H}$ is an FS-Hilbert space with $\widetilde{Ŧ}\ \tilde{\in}\ \widetilde{Ƀ}(\widetilde{H})$, hence $\widetilde{Ŧ}$ is said to be an FS-paranormal operator and symbolizes by (FSPO).
Theorem 2.15: [37] Assume that $\widetilde{Ŧ}_1$ and $\widetilde{Ŧ}_2$that belong in $\widetilde{Ƀ}(\widetilde{H})$ are self-adjoint operators and FSPO then $\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2$ is also an FSPO.
Proposition 2.16: [36] If $\widetilde{Ŧ}_1$and $\widetilde{Ŧ}_2$ are two FSPO that belong in $\widetilde{Ƀ}(\widetilde{H})$ and they are self-adjoint operators then $\widetilde{Ŧ}_1.\widetilde{Ŧ}_2$ is also an FSPO.

3. Main features

This section presents significant outcomes and discoveries relating to the algebraic and analytic merits of the proposed fuzzy soft M-paranormal operator (M-FSPO) in FSH-space.
Definition 3.1: Let $\widetilde{H}$ be an FSH-space with $\widetilde{Ŧ}$ belongs in $\widetilde{Ƀ}(\widetilde{H})$, then $\widetilde{Ŧ}$ is said to be M-fuzzy soft paranormal operator if it achieves the condition, $\|\widetilde{\widetilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\widetilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}}\|^{2}$, and m ∈ N and in brief M-FSPO.
Theorem 3.2: Consider that $\widetilde{Ŧ}_1$ and $\widetilde{Ŧ}_2$ belong in $\widetilde{Ƀ}(\widetilde{H})$ are FS-self adjoint operators then

$\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2$ is M-FSPO.

Proof: For $\tilde{x}_{\mu H(e)} \tilde{\in} \widetilde{H}$.

Let $\|(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}\|^2 \cong \langle(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, (\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}\rangle$

$\cong \langle((\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1})^*(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1^* + \widetilde{Ŧ}_2^*)^{m+1}(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{2m+2}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{2(m+1)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$.

$\|(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}\|^2 \lesssim \|(\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2)^{2(m+1)}\tilde{x}_{\mu H(e)}\|\|\tilde{x}_{\mu H(e)}\|$, therefore $\widetilde{Ŧ}_1 + \widetilde{Ŧ}_2$ M-FSSPO.

Theorem 3.3: Consider that $\widetilde{Ŧ}_1$ and $\widetilde{Ŧ}_2$ belong in $\widetilde{B}(\widetilde{H})$ are FS-self- adjoint operators then $\widetilde{Ŧ}_1.\widetilde{Ŧ}_2$ is M-FSPO.

Proof: For $\tilde{x}_{\mu H(e)} \tilde{\in} \widetilde{H}$.

Let $\|(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}\|^2 \cong \langle(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, (\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}\rangle$

$\cong \langle((\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1})^*(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1^*.\widetilde{Ŧ}_1^*)^{m+1}(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{2m+2}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{2(m+1)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$.

$\|(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{m+1}\tilde{x}_{\mu H(e)}\|^2 \lesssim \|(\widetilde{Ŧ}_1.\widetilde{Ŧ}_2)^{2(m+1)}\tilde{x}_{\mu H(e)}\|\|\tilde{x}_{\mu H(e)}\|$, therefore $\widetilde{Ŧ}_1.\widetilde{Ŧ}_2$ M-FSPO.

Corollary 3.4: If $\widetilde{Ŧ}$ is M-FSPO and self -adjoint operator, where $\widetilde{Ŧ}$ belongs to $\widetilde{B}(\widetilde{H})$ then $\widetilde{Ŧ}^n$ is M-FSPO.

Proof: For all $\tilde{x}_{\mu H(e)} \tilde{\in} \widetilde{H}$ with we have $\widetilde{Ŧ}$ is an FSPO, so $\|\widetilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}\|^2 \lesssim \|\widetilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}\|$, thus must show $\left\|(\widetilde{Ŧ}^n)^{2(m+1)}\tilde{x}_{\mu H(e)}\right\| \|\tilde{x}_{\mu H(e)}\| \gtrsim \left\|(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}\right\|^2$.

Let $\|(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}\|^2 \cong \langle(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}, (\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}\rangle$

$\cong \langle((\widetilde{Ŧ}^n)^{m+1})^*(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle((\widetilde{Ŧ}^*)^n)^{m+1}(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}^n)^{m+1}(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$

$\cong \langle(\widetilde{Ŧ}^n)^{2(m+1)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$.

$\|(\widetilde{Ŧ}^n)^{m+1}\tilde{x}_{\mu H(e)}\|^2 \lesssim \left\|(\widetilde{Ŧ}^n)^{2(m+1)}\tilde{x}_{\mu H(e)}\right\| \|\tilde{x}_{\mu H(e)}\|$, therefore $\widetilde{Ŧ}^n$ is M-FSPO.

Theorem 3.5: Every FSPO is a 0-FSSPO**.**

Proof: For $\tilde{x}_{\mu H(e)} \tilde{\in} \widetilde{H}$, since $\widetilde{Ŧ} \tilde{\in} \widetilde{B}(\widetilde{H})$ is an FSPO, we have

$$\|\widetilde{Ŧ}\tilde{x}_{\mu H(e)}\|^2 \lesssim \|\widetilde{Ŧ}^2\tilde{x}_{\mu H(e)}\|\|\tilde{x}_{\mu H(e)}\|$$

$$\|\widetilde{Ŧ}^{(0+1)}\tilde{x}_{\mu H(e)}\|^2 \lesssim \|\widetilde{Ŧ}^{2(0+1)}\tilde{x}_{\mu H(e)}\|\|\tilde{x}_{\mu H(e)}\|$$

$\|\widetilde{Ŧ}^1\tilde{x}_{\mu H(e)}\|^2 \lesssim \|\widetilde{Ŧ}^{2.1}\tilde{x}_{\mu H(e)}\|\|\tilde{x}_{\mu H(e)}\|$, so $\widetilde{Ŧ}$ is a 0-FSPO.

Next, submitted the equivalent definition of M-FSPO by method in the following theorem.

Theorem 3.6: Assume that $\widetilde{Ŧ}$ belongs in $\widetilde{B}(\widetilde{H})$, then $\widetilde{Ŧ}$ is M-FSPO if and only if $\widetilde{Ŧ}^{*2(m+1)}\widetilde{Ŧ}^{2(m+1)} - 2\lambda\widetilde{Ŧ}^{*(m+1)}\widetilde{Ŧ}^{m+1} + \lambda^2 \gtrsim 0$ for all non-negative λ.

Proof: For all $\tilde{x}_{\mu H(e)} \tilde{\in} \tilde{H}$.
Suppose that $\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)} - 2\lambda\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{m+1} + \lambda^2 I \gtrapprox 0$, to prove $\tilde{Ŧ}$ is M-FSPO.
$\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)} - 2\lambda\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{m+1} + \lambda^2 \gtrapprox 0$

$$\langle(\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)} - 2\widetilde{\lambda\tilde{Ŧ}^{*(m+1)}}\tilde{T}^{m+1} + \lambda^2)\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle \gtrapprox 0$$

$$\langle\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle - 2\lambda\langle\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle + \lambda^2\langle\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle \gtrapprox 0$$

$$\langle\tilde{Ŧ}^{2(m+1)}\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle - 2\lambda\langle\tilde{Ŧ}^{(m+1)}\tilde{Ŧ}^{(m+1)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle + \lambda^2\langle\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle \gtrapprox 0$$

$$\|\widetilde{\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}}\|^2 - 2\lambda\|\widetilde{\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}}\|^2 + \lambda^2\|\widetilde{\tilde{x}_{\mu H(e)}}\|^2 \gtrapprox 0$$

$$4\|\widetilde{\tilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}}\|^4 - 4\|\widetilde{\tilde{x}_{\mu H(e)}}\|^2\|\widetilde{\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}}\|^2 \lessapprox 0$$

$$\|\widetilde{\tilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}}\|^4 \lessapprox \|\widetilde{\tilde{x}_{\mu H(e)}}\|^2\|\widetilde{\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}}\|^2$$

$\|\widetilde{\tilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}}\|^2 \lessapprox \|\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{Ŧ}^{2(m+1)}\tilde{x}_{\mu H(e)}}\|$ hence, $\tilde{Ŧ}$ is M-FSPO.
Conversely, let $\tilde{Ŧ}$ be M-FSPO to prove $\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)} - 2\lambda\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{m+1} + \lambda^2 I \gtrapprox 0$.
The expression
$\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)} - 2\lambda\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{m+1} + \lambda^2 I$, can be rewritten in a form specifically:
$x^2 - 2xy + y^2 = (x-y)^2 \geq 0$, Such that $x = \tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{(m+1)}, y = \lambda$, I is identity operator.
Thus: $(\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{(m+1)} - \lambda I)^2 \geq 0$.
By Definition 3.1, M-FSSPO it satisfies the positive condition.
$\langle\tilde{Ŧ}^{*(m+1)}\widetilde{\tilde{Ŧ}^{(m+1)}}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle \gtrapprox 0$, $\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{(m+1)}$ positive operator and λI also positive since $\lambda > 0$ by assumption.
$\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{(m+1)} - \lambda I$. It also remains positive, as does its square $(\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{(m+1)} - \lambda I)^2$. The sequence of any bounded operator is always positive $(\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{(m+1)} - \lambda I)^2 \geq 0$.
We get $\tilde{Ŧ}^{*2(m+1)}\tilde{Ŧ}^{2(m+1)} - 2\lambda\tilde{Ŧ}^{*(m+1)}\tilde{Ŧ}^{m+1} + \lambda^2 I \gtrapprox 0$.

Theorem 3.7: Assume that Ŧ belongs in $\tilde{B}(\tilde{H})$, if $\tilde{T}$ is M-FSPO then $\tilde{Ŧ}^*$ is M-FSPO.

Proof: We have $\tilde{Ŧ}$ is M-FSPO, so that $\|\tilde{Ŧ}^{2(m+1)}\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\tilde{Ŧ}^{m+1}\tilde{x}_{\mu H(e)}}\|^2$
Let $\|\widetilde{(\tilde{Ŧ}^*)^{m+1}\tilde{x}_{\mu H(e)}}\|^2 \cong \langle(Ŧ^*)^{m+1}\tilde{x}_{\mu H(e)}, (\tilde{Ŧ}^*)^{m+1}\tilde{x}_{\mu H(e)}\rangle$

$$\cong \langle((\tilde{Ŧ}^*)^{m+1})^* (\tilde{Ŧ}^*)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$
$$\cong \langle((\tilde{Ŧ}^*)^*)^{m+1} (\tilde{Ŧ}^*)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$
$$\cong \langle(\tilde{Ŧ}^*)^{m+1} (\tilde{Ŧ}^*)^{m+1}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$
$$\cong \langle(\tilde{Ŧ}^*)^{2m+2}\, \tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$
$$\cong \langle(\tilde{Ŧ}^*)^{2(m+1)}\, \tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$

$\|\widetilde{(\tilde{Ŧ}^*)^{m+1}\tilde{x}_{\mu H(e)}}\|^2 \lessapprox \|(\tilde{Ŧ}^*)^{2(m+1)}\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$, therefore $\tilde{Ŧ}^*$ is M-FSPO.

Theorem 3.8: Assume $\tilde{Ŧ}$ belongs in $\tilde{B}(\tilde{H})$ and is an FSPO, then $\tilde{Ŧ}$ is M-FSPO.

Proof: For every $\tilde{x}_{\mu H(e)} \tilde{\in} \tilde{H}$.

We have $\tilde{Ŧ}$ is an FSPO $\left\|\tilde{Ŧ}^2\widetilde{\tilde{x}_{\mu H(e)}}\right\|\left\|\widetilde{\tilde{x}_{\mu H(e)}}\right\| \gtrapprox \left\|\tilde{Ŧ}\widetilde{\tilde{x}_{\mu H(e)}}\right\|^2$.
Through the mathematical induction,
Case1: m=1 it is right since $\tilde{Ŧ}$ is an FSPO
Assume $\|\tilde{Ŧ}^{2.(1+1)}\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \cong \|\tilde{Ŧ}^4\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$
$\|\tilde{Ŧ}^{2.(1+1)}\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\tilde{Ŧ}^{1+1}\tilde{x}_{\mu H(e)}}\|^2$
$\|\tilde{Ŧ}^{2(2)}\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\tilde{Ŧ}^2\tilde{x}_{\mu H(e)}}\|^2$
This means $\|\tilde{Ŧ}^{2(1+1)}\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\tilde{Ŧ}^{1+1}\tilde{x}_{\mu H(e)}}\|^2$
Case2: m=k is right, thus
$\|\tilde{Ŧ}^{2.(k+1)}\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\tilde{Ŧ}^{k+1}\tilde{x}_{\mu H(e)}}\|^2$
To prove m=k+1 is right

Assume $\|\widetilde{\mathrm{T}^{(k+1)+1}\tilde{x}_{\mu H(e)}}\|^2 \cong \langle \widetilde{\mathrm{T}}^{(k+1)+1}\tilde{x}_{\mu H(e)}, \widetilde{\mathrm{T}}^{(k+1)+1}\tilde{x}_{\mu H(e)}\rangle$

$$\cong \langle \widetilde{\mathrm{T}}^{(k+2)}\tilde{x}_{\mu H(e)}, \widetilde{\mathrm{T}}^{(k+2)}\tilde{x}_{\mu H(e)}\rangle$$

$$\cong \langle \widetilde{\mathrm{T}}^{2}\widetilde{T}^{k}\tilde{x}_{\mu H(e)}, \mathrm{T}^{2}\widetilde{\mathrm{T}}^{k}\tilde{x}_{\mu H(e)}\rangle$$

$$\cong \|\widetilde{\mathrm{T}}^{2}\widetilde{\mathrm{T}^{k}\tilde{x}_{\mu H(e)}}\|^2$$

$$\lessapprox \|\widetilde{\mathrm{T}}^{2}\|^2\|\widetilde{\mathrm{T}^{k}\tilde{x}_{\mu H(e)}}\|^2$$

$$\lessapprox \|\widetilde{\mathrm{T}}^{2}\|^2\|\widetilde{\mathrm{T}^{2k}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$$\lessapprox \|\widetilde{\mathrm{T}^{4}}\|\|\widetilde{\mathrm{T}^{2k}\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$$\lessapprox \|\widetilde{\mathrm{T}}^{4}\widetilde{\mathrm{T}^{2k}\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$$\lessapprox \|\widetilde{\mathrm{T}}^{2k}\widetilde{\mathrm{T}^{4}\tilde{x}_{\mu H(e)}}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$$\lessapprox \|\widetilde{\mathrm{T}}^{2\widetilde{(k+2)}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$$\lessapprox \|\widetilde{\mathrm{T}}^{2\widetilde{((k+1)+1)}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$\|\widetilde{\mathrm{T}}^{\widetilde{(k+1)+1}}\tilde{x}_{\mu H(e)}\|^2 \lessapprox \|\widetilde{\mathrm{T}}^{2\widetilde{((k+1)+1)}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$ thus, as a result $\widetilde{\mathrm{T}}$ is M-FSPO.

Theorem 3.9: If $\widetilde{\mathrm{T}}$ belongs in $\widetilde{\mathrm{B}}(\widetilde{H})$ is invertible M-FSPO, then $\widetilde{\mathrm{T}}^{-1}$ is M-FSPO.

Proof: We have $\widetilde{\mathrm{T}}$ is M-FSPO $\|\widetilde{\mathrm{T}}^{2(m+1)}\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\mathrm{T}}^{m+1}\widetilde{\tilde{x}_{\mu H(e)}}\|^2$

we can replace $\tilde{x}_{\mu H(e)}$ by $(\widetilde{\mathrm{T}}^{-1})^{m+1}\tilde{x}_{\mu H(e)}$ then

$$\|\widetilde{\mathrm{T}}^{m+1}\widetilde{(\widetilde{\mathrm{T}}^{-1})^{(m+1)}}\tilde{x}_{\mu H(e)}\|^2 \lessapprox \|\mathrm{T}^{2(m+1)}(\widetilde{\mathrm{T}}^{-1})^{\widetilde{m+1}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$$\|\mathrm{T}^{m+1}(\widetilde{\widetilde{\mathrm{T}}^{m+1}})^{-1}\tilde{x}_{\mu H(e)}\|^2 \lessapprox \|\widetilde{\tilde{x}_{\mu H(e)}}\|\|\widetilde{\mathrm{T}}^{2(m+1)}(\widetilde{\mathrm{T}}^{-1})^{\widetilde{(m+1)}}\tilde{x}_{\mu H(e)}\|$$

$$\|(\widetilde{\mathrm{T}}^{-1})^{\widetilde{m+1}}\tilde{x}_{\mu H(e)}\|^2 \lessapprox \|(\widetilde{\mathrm{T}}^{-1})^{2\widetilde{(m+1)}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$\|(\widetilde{\mathrm{T}}^{-1})^{2\widetilde{(m+1)}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|(\widetilde{\mathrm{T}}^{-1})^{\widetilde{(m+1)}}\tilde{x}_{\mu H(e)}\|^2$ as a result, $\widetilde{\mathrm{T}}^{-1}$ is M-FSPO, $m \in N$.

Theorem 3.10: Assume $\widetilde{\mathrm{T}}$ belongs in $\widetilde{\mathrm{B}}(\widetilde{H})$ is an FSPO, and $\widetilde{\mathrm{T}}$ is unitary equivalent to $\tilde{P}$ Thereafter, $\widetilde{\mathrm{T}}$ is M-FSPO.

Proof: For $\widetilde{\mathrm{T}}$ unitary equivalent to $\tilde{P}$, we have $\widetilde{\mathrm{T}} \cong \widetilde{U}\tilde{P}\widetilde{U}^*$ then $\widetilde{\mathrm{T}}^{2(m+1)} \cong \widetilde{U}\tilde{P}^{2(m+1)}\widetilde{U}^*$

$$\|\widetilde{\mathrm{T}}^{2(m+1)}\widetilde{\tilde{x}_{\mu H(e)}}\| \cong \|\widetilde{U}\tilde{P}^{2(m+1)}\widetilde{U}^*\widetilde{\tilde{x}_{\mu H(e)}}\|$$

Let $\|\widetilde{\mathrm{T}}^{m+1}\widetilde{\tilde{x}_{\mu H(e)}}\|^2 \cong \|\widetilde{(\widetilde{U}\tilde{P}\widetilde{U}^*)^{m+1}}\,\tilde{x}_{\mu H(e)}\|^2$

$$\langle \widetilde{\mathrm{T}}^{m+1}\widetilde{\tilde{x}_{\mu H(e)}, \widetilde{\mathrm{T}}^{m+1}}\tilde{x}_{\mu H(e)}\rangle \cong \langle (\widetilde{U}\tilde{P}\widetilde{U}^*)^{m+1}\widetilde{\tilde{x}_{\mu H(e)}, (\widetilde{U}}\tilde{P}\widetilde{U}^*)^{m+1}\,\tilde{x}_{\mu H(e)}\rangle$$

$$\cong \langle (\widetilde{U}\tilde{P}^{m+1}\widetilde{U}^*)\widetilde{\tilde{x}_{\mu H(e)}, (\widetilde{U}}\tilde{P}^{m+1}\widetilde{U}^*)\tilde{x}_{\mu H(e)}\rangle$$

$$\cong \langle (\widetilde{U}\tilde{P}^{m+1}\widetilde{U}^*)^*(\widetilde{U}\widetilde{\tilde{P}^{m+1}}\widetilde{U}^*)\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$

$$\cong \langle (\widetilde{U}\tilde{P}^{m+1}\widetilde{U}^*)(\widetilde{U}\widetilde{\tilde{P}^{m+1}}\widetilde{U}^*)\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$

$$\cong \langle (\widetilde{U}\tilde{P}^{2m+2}\widetilde{\widetilde{U}^*)\tilde{x}_{\mu H(e)}}, \tilde{x}_{\mu H(e)}\rangle$$

$$\cong \langle (\widetilde{U}\tilde{P}^{2(m+1)}\widetilde{\widetilde{U}^*)}\tilde{x}_{\mu H(e)}, \tilde{x}_{\mu H(e)}\rangle$$

$$\lessapprox \|\widetilde{U}\tilde{P}^{2\widetilde{(m+1)}\widetilde{U}}{}^*\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$$

$\|\widetilde{\mathrm{T}}^{m+1}\widetilde{\tilde{x}_{\mu H(e)}}\|^2 \lessapprox \|\widetilde{U}\tilde{P}^{2\widetilde{(m+1)}\widetilde{U}}{}^*\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\|$. So, $\|\widetilde{\mathrm{T}}^{2\widetilde{(m+1)}}\tilde{x}_{\mu H(e)}\|\|\widetilde{\tilde{x}_{\mu H(e)}}\| \gtrapprox \|\widetilde{\mathrm{T}}^{\widetilde{m+1}}\tilde{x}_{\mu H(e)}\|^2$ as a result $\widetilde{\mathrm{T}}$ is M-FSPO $m \in N$.

4. Conclusions

In this investigation, an *M*-FSPO is constructed in FSH-space, acting on the class of paranormal type FSL-operators. Additionally, various interesting algebraic and analysis of M-FSPO are examined; as a result, this *M*-FSPO determined in terms of the FS-norm. Under some conditions, the proposed operator offers other merits. Our study is posed in the domain of the FSH-space, specifically fuzzy soft theory (FS-T). For future work, the proposed operator can be employed to develop distinct classes of FS-operators. Furthermore, *M*-FSPO can be employed to gain solutions of differential equations.

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