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Gradient of Magnetic Field in Pulsar Star

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Abstract

The magnetic field of pulsar (**B**), is considered to be magnetic dipole field.The pulsar is a neutron star. The pulsar magnetic field is about (10^8 T), with high curvature and radius of (10^4m). The work is interested on the geometry of the magnetic field within the light cylinder($R_L=3\times10^8$m).The model of pulsar, which we adopted, is that of (Goldriech & Julian), the dipole axis is in the same axis of the star rotation it is an aligned rotator.

Within this consideration we studied the types of gradient with changing position angle (λ) .The first type, -B-Gradient in (r) direction ($\nabla_r B$), and the second type,-B-Gradient in (λ) direction ($\nabla_\lambda B$).The results indicated that the assumed magnetic field configuration enable us to estimate, why the two types of Grad.-B. increase towards the limits of the light cylinder.Therefore we noted the magnetic field inside the light cylinder is very strong with high curvature for star surface.

Also, we concluded, after the boundary of (R_L), the magnetic field lines are open and the charges will be accelerate with high velocity and flow out the light cylinder.

الخلاصة

النجم النابض هو نجم نيوتروني نابض دوار ذو مجال ثنائي قطب مغناطيسي يتميز بشدته بحدود (10^8 T) وانحنائه العالي بسبب صغر حجم النجم(10^4 m). ركزنا في هذا البحث على دراسة هندسة المجال المغناطيسي داخل منطقة اسطوانة الضوء (R_L) ضمن الاطار الذي افترضه (Coldriech&Jolian)، حيث يقع محور الدوران على المحور المغناطيسي للنجم.

ووفقا لهذه للافتراضات درسنا انوع انحدار المجال المغناطيسي مع تغير زاوية الموقع (λ). النوع الاول انحدار المجال باتجاه(r) ($\vec{\nabla}_r B$) والنوع الثاني انحدار المجال باتجاه(λ) ($\nabla_\lambda B$).

من خلال بحثنا هذا استنتجنا ان انحدار المجال المغناطيسي بنوعيه يتزايد باتجاه حدود اسطوانة الضوء حيث ان قوة المجال المغناطيسي عند الاقطاب وصغر حجم النابض والانحناء العالي لسطح النجم هي من الاسباب الرئيسية لهذه الزيادة باتجاه حدود اسطوانة الضوء. كما لاحظنا ايضا انه خطوط المجال المغناطيسي بعد حدود منطقة اسطوانة الضوء تصبح مفتوحة ولذلك فان الشحنات تتعجل بسرعة عالية (بحدود سرعة الضوء) هاربة بعيدا عن حدود الاسطوانة الضوئية.

1. Introduction

Pulsar is a celestial radio source emitting short bursts of radio emission. These objects had been discoverd by Hewish and Bell,in the end of 1967. Pulsars have astrong magnetic field, (B) which is about 108 tesla [1].

As in the ordinary case in cosmically plane we will consider the dipole field model.In cosmically physics the electromagnetic phenomena like, aurora, magnetic storms, which accrue in certain regions around the geomagnetic poles, also there are a number of electromagnetic phenomena at the sun, as sun spots, solar flares, these processes are very importance. The first stellar magnetic field was observed by H.W. Bacock in 1947,[1].

The determination of surface magnetic field strengths of pulsar estimated by measuring the effect of cyclotron scattering on the wave forms and spectra of pulsing x-ray stars, [2,3].For Her x-1 the magnetic field B is 4×10^{8}T if the hard x-ray spectrum is due to cyclotron scattering at $\leq$ 42 keV, and B~6×10^{8}T when the hard x-ray is due to cyclotron emission at 58 keV, [4].

For 440115+63 the magnetic field B~2×10^{8}T if the hard x-ray is due to cyclotron scattering at ~25 keV, [4]. however most pulsar magnetic field lies within a decade of 10^{8} T. The magnetic field at the surface of the neutron star may be related to the period of the star (P) and its derivative$(\dot{P})$, [5], As:

$$B = k\sqrt{P\dot{P}}$$

Where k is constant related to the radius and moment of inertia of the star.

The magnetic field now seem to be general characteristic of a certain category of pulsars stars. The magnetic fields for single rotating neutron star (N.S) would have grow inversely with the radius R: $B \propto \frac{1}{R^{3}}$

The pulsar has small area where its radius is about 10^{4}m, then the magnetic field may be very strong (10^{8} T). Fig(1-a,b,c) shows the distribution of the pulsar with [4]:

a- Pulsar period for 328 pulsars

b- Period first derivative for 256 pulsars.

c- Magnetic field strength at assuming a dipolar magnetic field, star radius 10^{4} and moment of inertia 10^{38} kg m^{2}, for 256 pulsars [5].

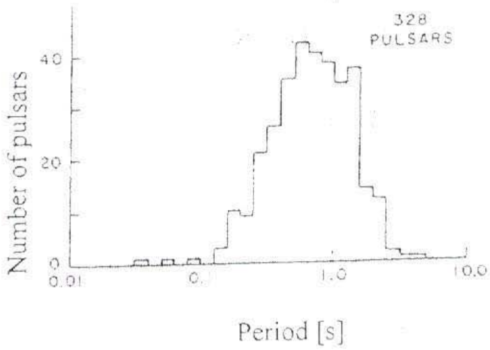

Fig (1-a) The observed distribution of pulsar periods

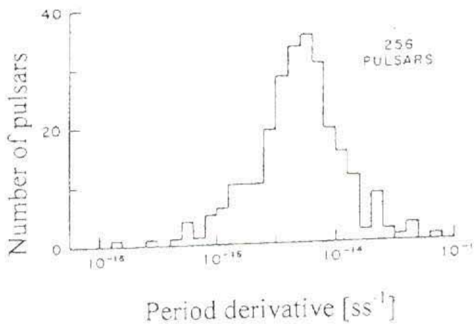

Fig (1-b) The observed distribution of period derivative

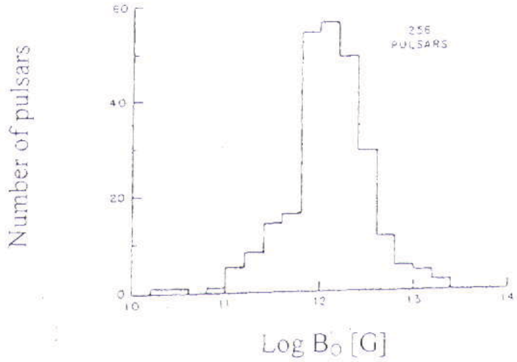

Fig (1-c) Distribution of the field strength at the surface of the pulsar star

2. Equation of dipole magnetic field pulsar star

In cosmic bodies the magnetic field for these stars considered to be as a magnetic dipole field, [1]. Gold in 1967, was the first suggest that the source of the pulsating signal could have its origins in the rapidly rotating magnetic dipole field associated with the star [6]. This suggestion was followed by many studies, which considered the magnetic field as a dipole field. Therefore we will consider a magnetic dipole field model to study the magnetic field for pulsar star.

The magnetic dipole is generated by a closed current loop of dipole moment m (m = iA). Where A is the area of the loop. Solution of Biot-Savart Low may be the way to determine the field **B** at a given point (p). In astrophysics there are many properties of solutions according to the wanted accuracy, [1].

However, in this work we will consider the cases of:

1- spherical coordinates
2- for small dipole (point).
3- first approximation .

The magnetic field (B) equation is given by Biot-Savart low:

$$\vec{B} = \frac{\mu_O}{4\pi} \int \frac{\vec{J} \times \vec{r}}{r^3} dv \qquad (1)$$

Where (B) is the magnetic field induction produced a current density J at point P. $\vec{r}$ the vector that directed from the point of integration (source point of $\vec{J}$) toward the field point P.dv is the volume element that content the source $\vec{J}$.

In comical bodies the magnetic field generally derives from currents induced by rotation in such a body. Let (r,θ,ϕ) be the spherical coordinates of the system centered in the dipole and having its axis parallel to the magnetic moment (a) where, a= μ_O m/4π and the latitude λ is given by (see fig.2):

$$\lambda = \frac{1}{2}\pi - \theta \qquad (2)$$

Then, the magnetic field components are:

$$B_r = B_p \ \mathrm{Sin}\lambda$$
$$B_\lambda = -\frac{1}{2} B_p \ \mathrm{Cos}\lambda \qquad (3)$$

The magnetic field in point(p) is given by:

$$B_p = 2a / r^3 \qquad (4)$$

According to alven notation [1], By= B_r and B_x=Bλ. So Br and Bλ don't refer to radial and angular components. Bϕ =0 constant. (no component in (B direction) as shown in Fig.(3), [1]

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Or the magnetic field is given by the relation:

$$B = \frac{a}{r^3}[1 + 3\sin^2 \lambda]^{\frac{1}{2}} \qquad (5)$$

Equation (5), represent the first approximation of the magnetic dipole field. The dipole moment of this field is a ≅ BR^3, where R the pulsar radius [7]. The magnetic line of force has the form:

$$r = r_e \cos^2\lambda \qquad (6)$$

Where (re) is the distance from origin in the equatorial plane ($\lambda = 0$), as shown in Fig. (3).
The dipole magnetic moment for pulsars was estimated to be $2x10^{18} - 2x10^{21}$ Tm^3 [4] with the typical value being ~ $10^{20}Tm^3$.The magnetic field is calculated by using Eq. (5) and (6), with the variation of both r_e andλ.

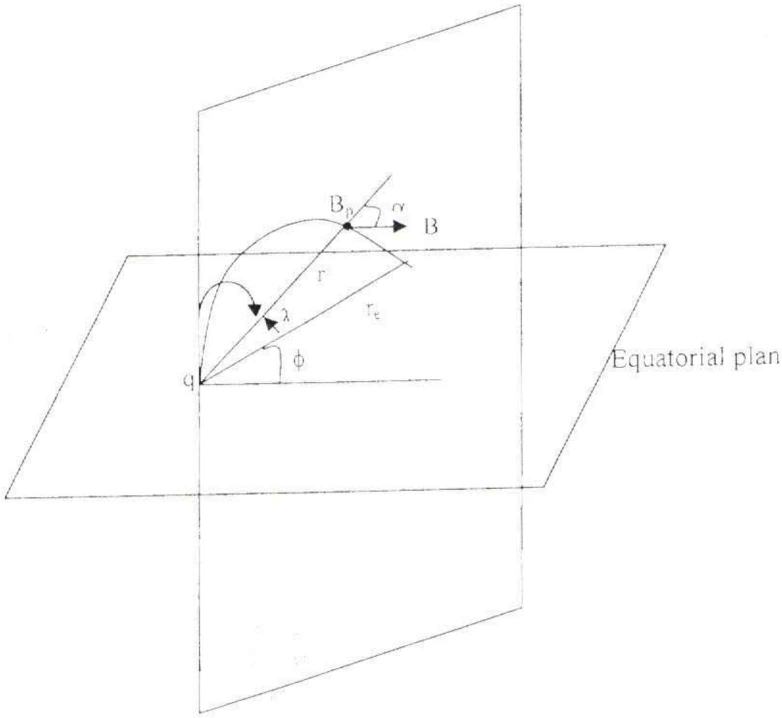


Fig.(2) magnetic line of force from a dipole q

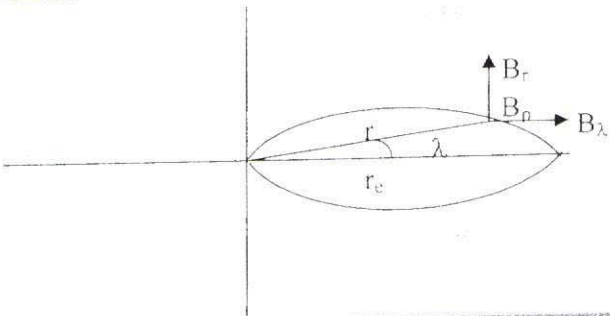


Fig.(3) illustrates the components of magnetic field in spherical coordinate

3. Gradient of the magnetic field

There are two types of gradient with respect to the magnetic field direction:

1- $\vec{\nabla} B \perp \vec{B}$

2- $\vec{\nabla} B \,//\, \vec{B}$

For the alignment magnetic field there are two component (Eq.3). B varies in bath $\vec{r}$ and λ direction. So there are B-gradient in r and λ direction.

3.1 B-Gradient in (r) direction ($\vec{\nabla}_r B$)

According to the equation of magnetic dipole field, where B is given by eq.(5)
And differentiating this with respect to r, or:

$$\frac{\partial B}{\partial r} = \nabla_r B$$

We can get:

$$\nabla_r B \approx \frac{-3a}{r^4}(1+3\sin^2\lambda)^{1/2} \quad (7)$$

Where r is defined by Eq. (6), a is the magnetic moment for pulsar star

3.2 B-Gradient in (λ) direction ($\nabla_\lambda B$)

The second type ($\nabla_\lambda B$) called, B-gradient in (λ) direction. Differentiating Eq.(5) with respect to λ:

$$\frac{\partial B}{\partial \lambda} = \nabla_\lambda B$$, we obtain

$$\nabla_\lambda B = \frac{6a}{r_e^3}\frac{\sin\lambda}{\cos\lambda}(1+3\sin^2\lambda)^{1/2} + \frac{3a\sin\lambda}{r_e^3\cos^5\lambda}\times(1+3\sin^2\lambda)^{-1/2} \quad (8)$$

This equation represent the gradient of magnetic field in λ direction.

4. Calculation and results

As illustrated in fig.(4) the single field line, the line is characterized by r_e. In this work we are restricted by $r_e \approx 2\times10^4$m which is approximately near the surface. The upper boundary is the radius of light cylinder, (R_L). This radius depends on the angular velocity (ω) of the pulsar;

$R_L = c/\omega$

Where c is the light velocity. In case, of choosing 1 sec^{-1} as angular velocity of pulsar, one gets that R_L is of order 3×10^8m.

This radius is the boundary of the special relativity, and can not exceed this value, after this value as we illustrated the magnetic field lines are open, the charge will be accelerated when it have the light velocity and flow out the light cylinder, [6,8,9].

4.1 First approximation equation

By using Eq.(1), the components of magnetic dipole field are computed. The method involved the following steps:

1 –The chosen values for r_e are: $r_e = 2\times10^4$ m, 2×10^6m, 2×10^8m.
2 – Calculation of (r) by Eq. (6) for $0 \le \lambda \le 90$.
3 – Calculation of B_p for any position by using the Eq. (4)
4 – Calculation of the components, B_r, B_λ.
5 – calculation the total B by using the Equation, (1) or (5).

These relation provided the basis for estimating the surface field strength from the varies position angle (λ) and r_e values.

The relations between B and position angle (λ) for different values of r_e are shown in Fig(5).

These curves illustrate that magnetic field has maximum values at poles, while the minimum values at equatorial plane (λ=0).

4.2 B-Gradient in ($\bar{r}$), ($\nabla_r B$) with changing position angle (λ)

For different values of r_e, and by using computer program we calculated the values of $\nabla_r B$ by Eq.(7)

Fig.(6), illustrates that, $\nabla_r B$ is changing universally with varying in position angle (λ) illustrates that, when r_e has maximum value (r_e = 3 $\times10^8$ m) radius of light cylinder, $\nabla_r B$ would be increasing and having maximum value at poles (λ=90). While towards the center of star, $\nabla_r B$ may be decreasing

4-3 B-Gradient in (λ), with changing position angle (λ)

The magnetic field strength, for pulsar star has maximum values at poles (λ=0) that is belong to high curvature for pulsar surface and small size for this star (radius ~10^4 m).$\nabla_\lambda B$ is calculated by using eq.(8), the results is shown in Fig(7). the gradient increase with position angle. There is a raped change after $\lambda=75^\circ$ that is owing to the high curvature near the poles.

The gradient is changed with the field line, so as the r_e increases the gradient decrease.

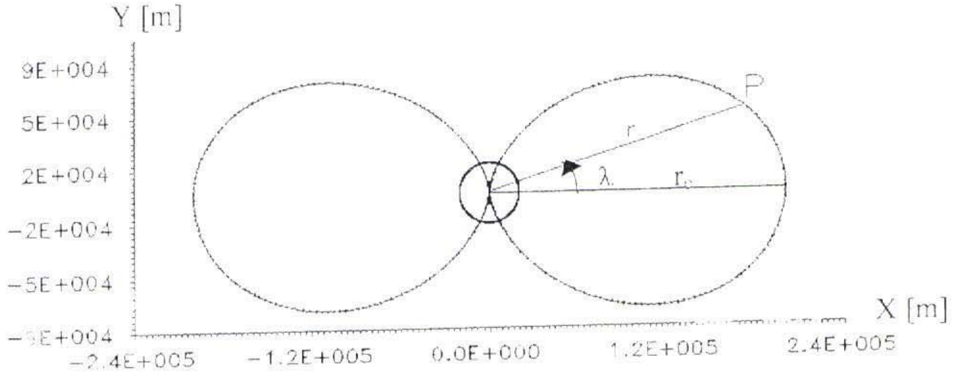


Fig. (4), The shape of single magnetic field line of magnetic dipole

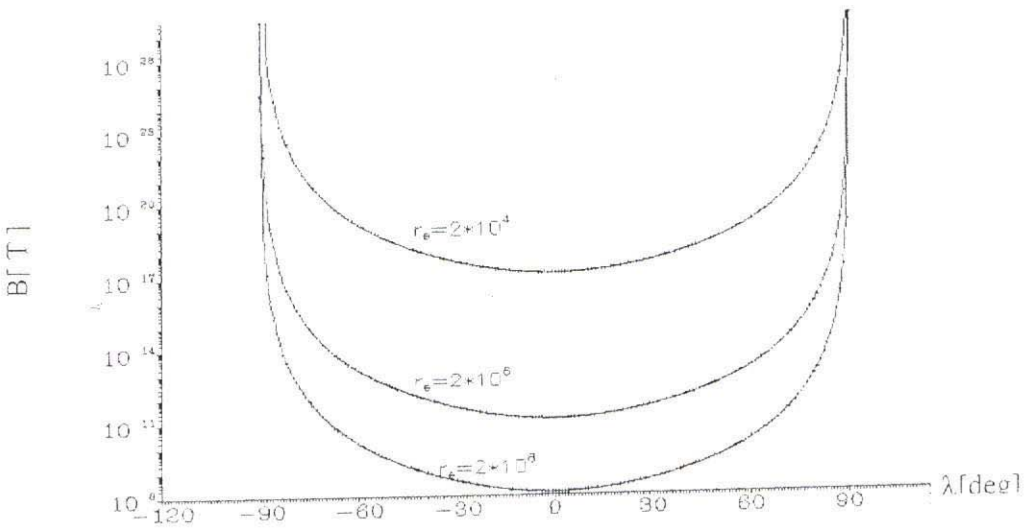
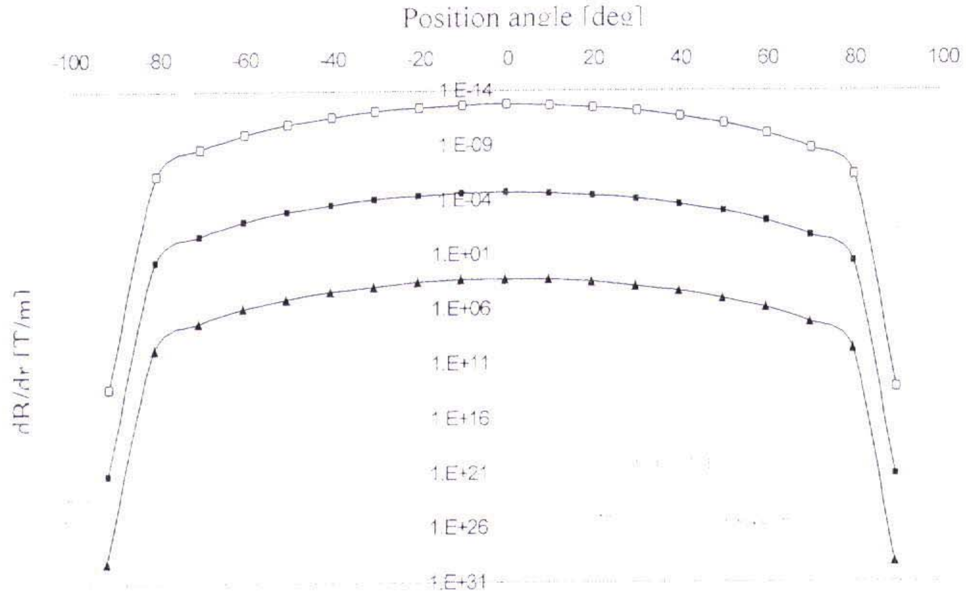


Fig. (5), The Relation between total magnetic field (B) and Position Angle (λ) for different r_e



Fig(6) The Relation between the gradient (dB/dr) and Position Angle (λ) when r_e have different values

$r_e = 2*10^8$ m □ □
$r_e = 2*10^6$ m ■ ■
$r_e = 2*10^4$ m ▲ ▲

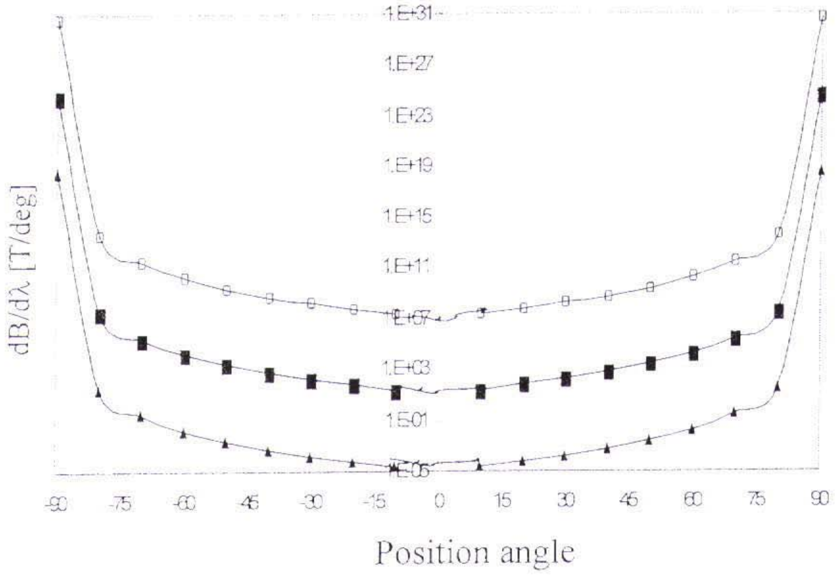


Fig. 7 The Relation between Gradient B (dB/dλ) and Angle (λ):

$r_e = 2*10^4$ □ □
$r_e = 2*10^6$ ■ ■
$r_e = 2*10^8$ ▲ ▲

5. Conclusions

From the previous results and discussion we can state the following remarks:

1- The magnetic field inside the light cylinder very strong and the particles would be very connected with the magnetic field, thus the field pressure is very large near the pulsar surface.

2- The grad.(B) increase towards the limits of light cylinder, where the magnetic field decrease, therefore the particles will accelerated with high energy and large velocity to escape beyond the light cylinder.

6. References

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