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Computer Simulations of Astronomical Objects as seen by the Next Generation Optical Space Telescopes

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Abstract

Computer simulations are carried out to quantify the quality of astronomical objects as seen by the Next Generation Space Telescope (NGST). These simulations are studied in terms of the modulation transfer function, MTF, of a reference star, and the power spectrum of a binary star. The results of these simulations are compared with the simulated results for the same astronomical objects as seen via Hubble Space Telescope (HST), and a 4 m Ground-Based Optical Telescope (GBT) in the absence and presence of atmospheric turbulence.
This study is also extended to examine the percentage frequency recovery of these telescopes from a binary star of separation is just within the full extent of Hubble's psf in the absence and presence of atmospheric turbulence.

الخلاصة

ثم محاكاة نوعية صور الاجسام الفلكية كما ترى بواسطة الجيل الجديد من التلسكوبات البصرية الفضائية. المحاكاة تمت من خلال دراسة دالة تضمين الانتقال لمصدر نقطي وكذالك طيف القدرة لنجم ثنائي. نتائج هذه المحاكاة قورنت معالنتائج لنفس الاجسام الفلكية ولكن مأخوذه بواسطة هابل تلسكوب وتلسكوب بصري ارضي قطره 4 متر. الدراسة تضمنت كذلك تحديد كمية الترددات المسترجعة من نجم ثنائي (في حدود قدرة تحديد تلسكوب هابل) صور بواسطة التلسكوبات اعلاه.

Introduction

The NGST is a major element of NASA's original program. It is planned to deploy a telescope with an 8 m diameter aperture and sensitivity approximately equals 1000 times greater than any currently existing telescope [1].
The NGST is designed for observations in the far visible to the mid infrared part of the spectrum. This wavelengths coverage is different from that of the HST which covers the range from the ultraviolet to the near infrared. The NGST will be able to detect the likely presence of planetary systems around nearby stars from their infrared radiation. Details of the design of the NGST are found in [2], [3] and [4]. It has been suggested that the NGST could be built and lunched by about 2009.
The origin of galaxies is one of the major projects in observational cosmology [5]. Some simulated observations have already done with NGST. Burgarella et.al [6] have studied the detectability of very high redshift galaxies and tried to characterize Wide Deep-Field observations. The expected Mid-IR molecular hydrogen line emission from the first objects in the universe with NGST has been studied [7]. The optimal instrumentation to achieve high resolution observation with NGST is discussed [8]. The operation and advantages of using an imaging Fourier transform spectrometer (IFTS) for the NGST is demonstrated [9].

After this brief review of the works that have been carried out with the NGST, it seems that the literatures are of lack of the quantitative studies of astronomical objects as seen via NGST, GBT and HST. Therefore, it is now of interest to do some image simulations in order to quantify the recovered information by these telescopes.

According to the Rayleigh criterion, the theoretical resolutions of NGST, GBT, and HST are 0.0125", 0.025", and 0.044" respectively using a wavelength of 400 nm. These results demonstrate the resolution in the absence of any seeing conditions.

The random spatial and temporal variations of refraction of the Earth's atmosphere is severely limits the resolution of GBT. Star image of the order of 1" is typically observed. The atmospheric MTF is an important parameter, particularly in the selection of a site for astronomical observatory or for the specification of the quality of an astronomical telescope. The limitations imposed by atmospheric turbulence on GBT requires knowledge of the MTF of the atmosphere. This is generally carried out by observing point sources such as stars. A detailed discussion of the atmospheric transfer function and other relevant properties of the turbulence could be found in [10-15], particularly in the comprehensive review given by Roddier [12,15].

The objective of this paper is to implement statistical approach in order to quantify the astronomical objects as seen via:

1. NGST
2. GBT (in the absence and presence of atmospheric turbulence).
3. HST

Theory

In this section, we present a brief theory of the image formation model, i.e., the image that recorded by an optical telescope in the presence of atmospheric turbulence. First, we shall neglect the turbulent atmosphere and just consider the optical telescope. The fundamental equation to be used for the formation of an image by an ideal optical system is given by:

$$i(x,y)=\iint_{-\infty}^{\infty} o(x',y')\,psf(x-x',y-y')\,dx'dy' \quad (1)$$

$$i(x,y)=o(x,y)\otimes psf(x,y) \quad (2)$$

Equations (1) and (2) are equivalent and representing a convolution equation. Where $i(x,y)$ is the observed image intensity, $o(x,y)$ is the object intensity, $psf(x,y)$ represents the image blurring function caused by the telescope and $\otimes$ denotes convolution operator. The Fourier transform of (2) is given by:

$$I(u,v)=O(u,v).\,T(u,v) \quad (3)$$

where $I(u,v)$ and $O(u,v)$ are, respectively, the complex Fourier transforms of the image intensity $i(x,y)$, and the object intensity $o(x,y)$; $T(u,v)$ which represents the Fourier transform of the *psf*, is an important function known as the optical transfer function (OTF). The modulation or amplitude of the complex function $T(u,v)$ is called MTF. In general, the resolution of an imaging system is limited only by the luck of large optical elements that are free from inherent distortions.

Now consider an extremely distant quasimonochromatic point source located on the optical axis of a simple imaging system. In the absence of atmospheric turbulence, this source would generate a plane wave normally incident on the lens. In the presence of the atmosphere, the plane wave incident on the inhomogeneous medium propagates into the medium, and ultimately a perturbed wave falls on the lens. The field distribution incident on the lens can be expressed as,

$$U(\eta,\gamma)=e^{i\phi(\eta,\gamma)} \quad (4)$$

where $\phi(\eta,\gamma)$ is the random phase of the incident wavefront and the variables (η,γ) represent distances in the pupil function. The instantaneous *psf* of the entire telescope atmosphere system is given by:

$$psf(x,y)=\left|FT\left[H(\eta,\gamma)U(\eta,\gamma)\right]\right|^2 \quad (5)$$

where $H(\eta,\gamma)$ represents the pupil function and FT denotes Fourier Transform operator. The corresponding OTF is the Fourier Transform of the *psf*, thus

$$T(u,v)=FT[psf(x,y)] \quad (6)$$

Equation (6) can also be written in terms of the pupil function and the field distribution incident on the lens as,

$$T(u,v)=\iint_{-\infty}^{\infty} H(\eta,\gamma)H^*(\eta-\eta',\gamma-\gamma').U(\eta,\gamma)U^*(\eta-\eta',\gamma-\gamma')\,d\eta' d\gamma' \quad (7)$$

where * denotes complex conjugate. The variables η and γ are related to the Fourier space variables u and ν by $\eta = \lambda f u$, $\gamma = \lambda f \upsilon$, where λ is the wavelength and f is the focal length.

Results and Discussions

The pupil function $H^{(\eta,\gamma)}$ of NGST, GBT and HST are taken to be a 2- dimensional circular function of diameters (2R) 200, 100, and 60 pixels respectively via the following equation,

$$H(\eta,\gamma) = \begin{cases} 1 & \text{if} \quad \sqrt{\eta^2 + \gamma^2} < R \\ 0 & \text{elsewhere} \end{cases} \qquad (8)$$

Each pixel is taken to be 4 cm.

If we consider the object to be imaged is an extremely distant quasimonochromatic point source located on the axis of an optical telescope. In the absence of atmospheric turbulence, this source would generates a plane wave as mentioned before and so that $\phi(\eta,\gamma)$ in eq. (4) becomes zero and consequently $U(\eta,\gamma) = 1$

The *psfs* of the NGST, GBT and HST are computed following eq.(5) and the corresponding MTFs are also computed via eq.(6) or eq.(7). The results are normalised to one at maximum and Fig.(1) display the central lines through the MTFs.

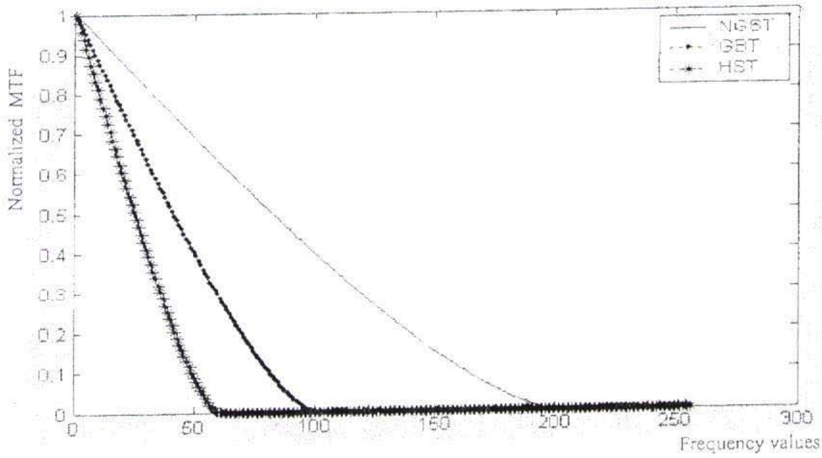

Fig.(1): *MTF*s of NGST, GBT and HST.

This figure illustrates clearly the diffraction limited cut-off frequency for each described telescope. The total frequency components of each frame of *MTF* after normalization is computed and given below. The normalization is done according to the total frequency components of NGST.

MTF of NGST = 1
MTF of GBT = 0.2497
MTF of HST = 0.0898

Now, in the presence of atmospheric turbulence, we have to make some assumptions about the form of the probability density function of the complex amplitude of the perturbed wavefront from the point source (reference star).

A description of the nature of the wavefront perturbation introduced by the atmosphere is provided by Kolmogorov model developed by Tatarski [16]. This model assumes that the wavefront perturbations are due to the variations in the refractive index of the atmosphere. The phase fluctuations in Tatarski's model are usually assumed to have Gaussian random distribution with the following second order structure function,

$$D_\phi(\rho) = \langle \left| \phi(r) - \phi(\rho - r) \right|^2 \rangle \qquad (9)$$

Where $D_\phi(\rho)$ is the atmospherically induced variance between the phase at two parts of the wavefront separated by a distance ρ in the aperture plane , and <...> represents the enseble average.

A number of authors [17, 18, 19, 20] have suggested alternative to the Gaussian random model. It has been shown that no significant experimental evidence has been put forward which strongly favours any suggested model [21, 22]. The Gaussian random model is still the most widely used and is the principle model of atmospheric seeing [23, 24].

The degree of turbulence in the atmosphere can be presented by the parameter r_o . This parameter is Fried's coherence parameter (i.e., seeing correlation length [10]), which plays an important rule in optical telescope imaging.

To generate the *psf* of the GBT in the presence of the atmosphere, it is necessarily to define two functions. The first is $H(\eta,\gamma)$, which is taken to be a circular function with D=2R=400 cm. The second is $U(\eta,\gamma)$, which is taken to be a complex Gaussian random process. $U(\eta,\gamma)$ is convolved with a normal Gaussian function of standard deviation $\sigma = r_o/2$. The result is a complex wavefront for the seeing r_o. The amplitude of this complex wavefront at the exit pupil are shown in Fig.(2a & b). To generate *psfs* of GBT at seeing conditions 0.5" and 1", r_o must be taken 40, and 20 cm respectively. $H(\eta,\gamma)$ and $U(\eta,\gamma)$ are used in eq.(5), and the corresponding result represent short exposure *psfs (see, fig.2c &d)*. It should be pointed out here that the scales of x and y-axis in this figure represent the extraction of the central parts from the actual array of size x=256 and y=256 pixels.

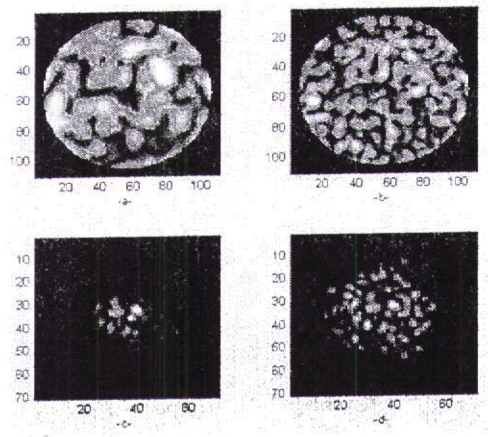

Fig.(2):
a- Amplitude of the complex wavefront at 0.5".
b- Amplitude of the complex wavefront at 1".
c- Short exposure *psf* at 0.5".
d- Short exposure *psf* at 1".

Two-hundreds short exposure *psfs* at seeing 0.5" and 1" are generated in this way and averaged out. The results are then normalized to one at maximum. A smoothed normal Gaussian functions are then fitted. These new Gaussian functions represent the long exposure *psfs* at seeing 0.5" and 1" as shown in Fig.(3).

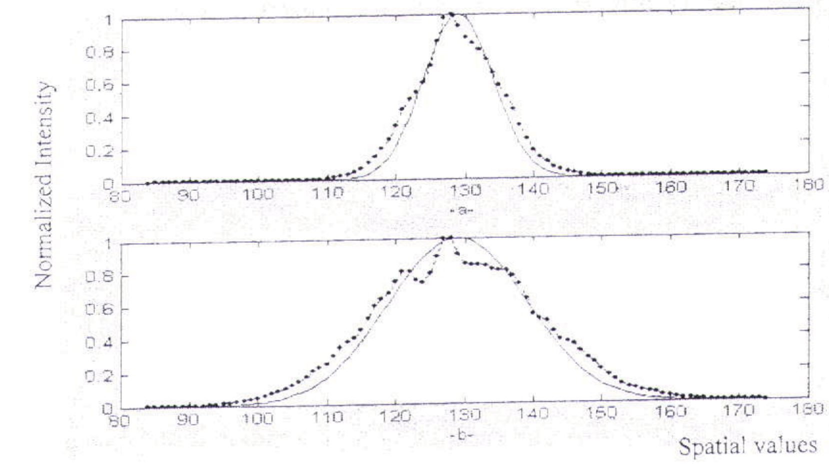

Fig.(3): **Central lines through the results of averaging 200 short-exposure *psfs* (dotted line) and normal Gaussian psf (solid line).**
a- 0.5". b- 1".

The corresponding short and long exposure *MTFs* at different seeing conditions are then computed and normalised to 1 at maximum. The results are shown in Fig.(4) and Fig.(5) respectively.

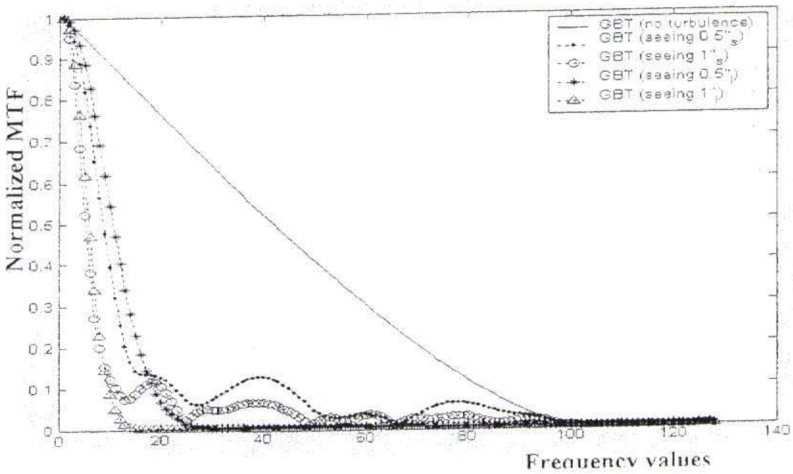

Fig.(4): Central lines through *MTF*s when one short exposure *psf* is computed.

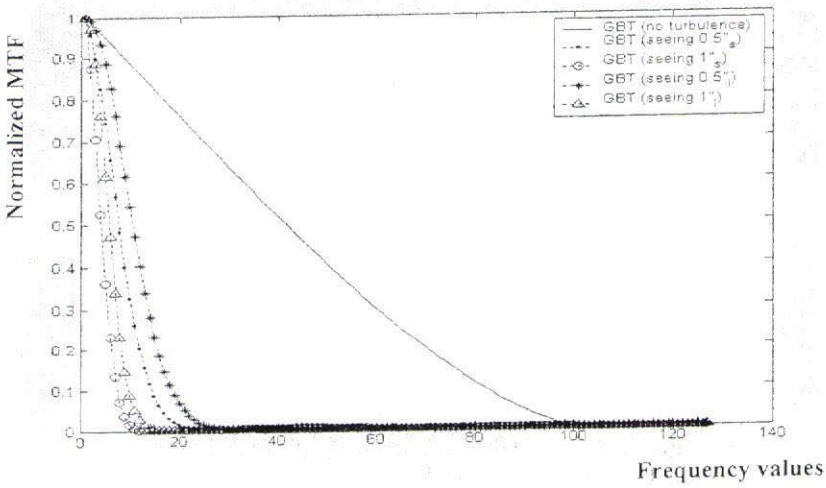

Fig.(5): Central lines through *MTF*s when 200 short-exposure *psf*s are computed and averaged out.

The total frequency components of each frame of *MTF* after normalization is computed and given below. The normalisation is done according to the total frequency components of *MTF* of GBT in the absence of atmospheric turbulence.

MTF of GBT (no turbulence)=1.
MTF of GBT at seeing 0.5" (using one short exposure *psf*) = 0.2056.
MTF of GBT at seeing 1" (using one short exposure *psf*) = 0.1233.
MTF of GBT at seeing 0.5" after averaging 200 short exposure *psf*s = 0.0464.
MTF of GBT at seeing 1" after averaging 200 short exposure *psf*s = 0.0133.
MTF of GBT at seeing 0.5" after fitting a smoothed Gaussian function =0.0532.
MTF of GBT at seeing 1" after fitting a smoothed Gaussian function =0.0156.
Up to now, our study was concentrated on the *psf* and *MTF* of the described telescopes.
It is now important to extent this study to include a binary star. A binary star of equal magnitude and of a separation is just within the full extent of a Hubble *psf* is simulated (see fig.6). This is considered to be a reference binary star. This binary star is convolved with the *psf*s of NGST, GBT, HST, GBT/atmosphere at seeing 0.5", and 1". The results of these simulations are shown in Fig.(7).

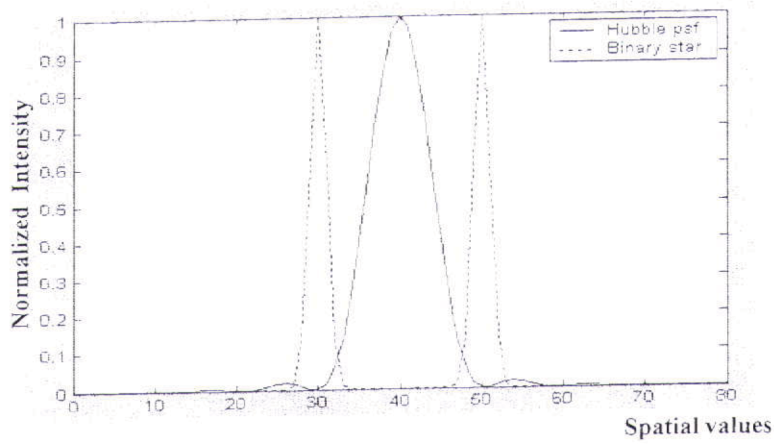

Fig.(6): Central lines through a binary star and Hubble *psf*.

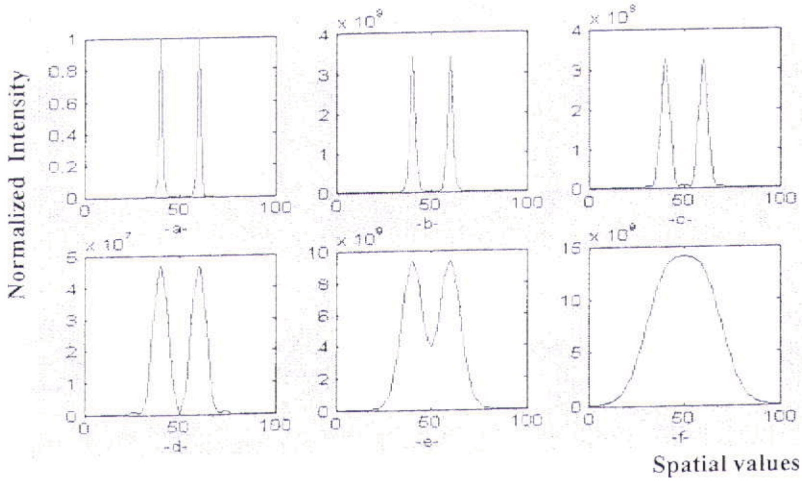

Fig.(7): Central lines through a binary star
a-Original. b-Via NGST. c-Via GBT. d-Via HST.
e-Via GBT at 0.5". f-Via GBT at 1".

The power spectrum, $P(u,v)$, of each image of a binary star, $b(x,y)$, that described in Fig.(7) is computed by the following equation and the results are shown in Fig.(8).

$$P(u,v)=|FT[b(x,y)]|^2 \qquad (9)$$

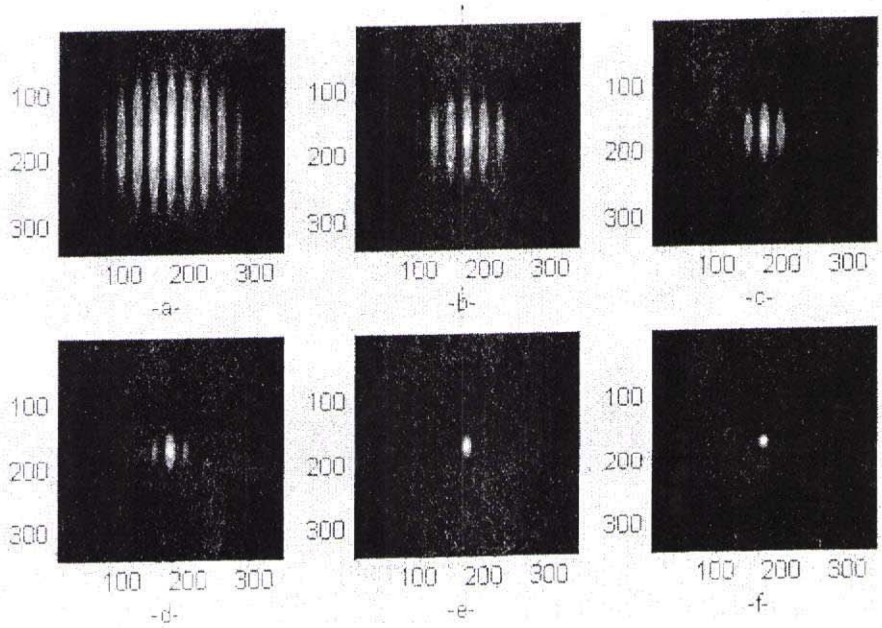

Fig.(8): Power spectrum of an image of a binary star:
a-Original. b-Via NGST. c-Via GBT. d-Via HST.
e-Via GBT at 0.5''. f-Via GBT at 1''.

The normalized central lines of Fig.(8) are shown in Fig.(9).

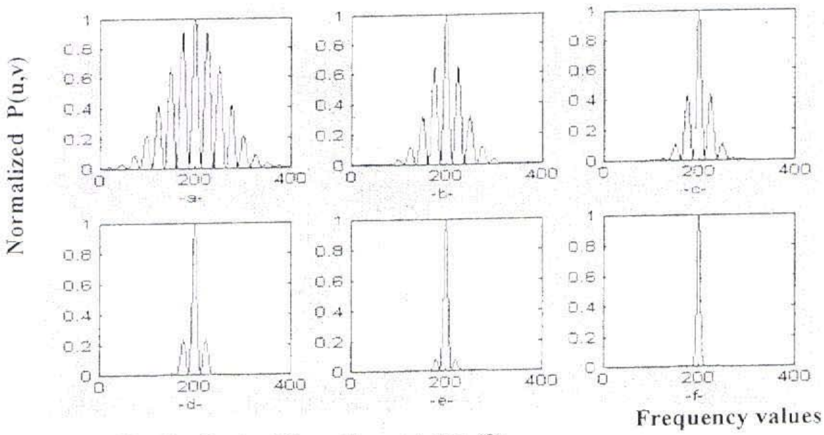

Fig.(9): Central lines through Fig.(8).
a-Original. b-Via NGST. c-Via GBT. d-Via HST.
e-Via GBT at 0.5''. f-Via GBT at 1''.

It is now so crucial to quantify the revealed frequency components of Fig.(8). The total frequency components of each frame of a power spectrum is computed. The normalization is done according to the total frequency components of the power spectrum of original binary star.

Total frequency components of original binary star=1.

Total frequency components of a binary star as seen via NGST = 0.5585

Total frequency components of a binary star as seen via GBT= 0.3483

Total frequency components of a binary star as seen via HST = 0.2284

Total frequency components of a binary star as seen via GBT(seeing 0.5'') = 0.1688

Total frequency components of a binary star as seen via GBT(seeing 1") = 0.1262

Conclusions

Diffraction theory and Fourier techniques have been applied to model the effects of telescope apertures and atmospheric turbulence on image quality.

The conclusions to be drawn from these simulations are as follows:

1- In the absence of atmospheric turbulence, the total frequency components of *MTF* of a reference star via NGST is 4 times greater than that of GBT and nearly 11 times than that of HST.

2- In the presence of atmospheric turbulence, the total frequency components of the *MTF* of the same reference star via GBT (no turbulence) is 18.79 greater than that of the same telescope operating at seeing 0.5" and 64 times at seeing 1".

3- The NGST is recovered 56% of the total frequency components of a binary star of a separation is just within the diameter of Hubble *psf*, while GBT and HST recovered 35% and 23% respectively. In the presence of atmospheric turbulence and at seeing 0.5" and 1", GBT recovered about 17% and 13% respectively.

References

1. Coulter, D.R. and David, N. "*Technology for the Next Generation Space Telescope*". SPIE, 4013,**2000**.
2. Smith, E. P., et al. "*The Next Generation Space Telescope Design Reference Mission*". SPIE, 3356,**1998**.
3. Mather, J. C., et al. "*Scientific Mercies for the Next Generation Space Telescope*". SPIE, 3356,**1998**.
4. Bernard, D.S., et. Al. "*NASA's Next Generation Space Telescope Visiting a Time When Galaxies were Young*". Astro-ph /9808053, **1998**.
5. Dressler A. et. Al. **1996** "*Hubble Space Telescope and Beyond*". AURA Reports. Washington D.C.
6. Burgarella et. Al. "*Simulated NGST Observations*". Astro-ph /9807169v2, 17 Jul. **1998**.
7. Benedetta Ciardi and Andrea Ferrar. "*Detection the First Objects in the mid-IR with NGST*". Astro-ph/ 0005461 v2. 13 Dec. **2000**.
8. Bensammar, S. and Melchior, A. L. "*Pixel and Micro-Lensing with NGST*". Paper presented at the NGST Science and Technology Exposition (Hyannis, USA). 13-16 September **1999**, Published by PASP.
9. Graham, J. R. "*An Imaging Fourier Transform Spectrometer for the Next Generation Space Telescope*". Paper presented at the NGST Science and Technology Exposition (Hyannis, USA). 13-16 September **1999**, Published by PASP.
10. Fried, D. L. "*Limiting resolution looking down through the atmosphere*". J.O.S.A., 56(**1966**)1380-1384.
11. Roddier, F., Ricort, G. and Roddier, C. "*Defocusing effects in astronomical speckle interferometery*". Opt. Commun., 24(**1978**) 281-284.
12. Roddier, F. "*The effects of atmospheric turbulence in optical astronomy*". Prog. In Opt., 19(**1981**)283-368.
13. Dainty, J. C. and Scaddan, R. J. "*Measurements of the atmospheric transfer function at Mauna Kea-Hawai*". Mon. Not. R. Astr. Soc., 170(**1975**)519-532.
14. Mohammed, A. T. and Burge, R.E. "*Quantitative limitations imposed by atmospheric turbulence and the quantum nature of light on image quality*". J. Phys. D.: Appl. Phys., 21(**1988**) 1078-1084.
15. Roddier, F. "*Adaptive Optics in Astronomy*" Cambridg University Press, First Published **1999**.
16. Tatarski, V. I. "*Wave propagation in a turbulent medium*". Dover press, New York, **1967**, pp.93-163.
17. Kim, Y. and Jaggard, D. L. "*Band Limited Fractal Model of Atmospheric Refractivity Fluctuation*". Optical Society of America. 5(**1988**), pp.475-480.
18. Siggia, E. D. "*Model of Intermittency in three-dimensional turbulence*". Physical Review A, 17(**1978**),pp.1166-1176.
19. Frisch , U. Sulem, P., & Nelkin, M. "*A simple dynamical model of intermittent fully developed turbulence*. Journal of Fluid Mechanics. 87(**1978**)pp. 719-736.
20. Mandelbrot, B. B. "*Intermittent turbulence in self-similar cascades:divergence of high moments and dimension of the caririer*". Journal of Fluid Mechanics. 62(**1974**)pp.331-358.
21. Racine, R. "*Temporal Fluctuations of Atmospheric Seeing*". Publications of the Astronomical Society of the Pacific.108(**1996**)372.

22. Vernin, J. & Munoz-Tunon, C. "*The temporal behaviour of seeing*".New Astronomy Review.42(**1998**)pp.451-454.
23. Robert Nigel Tubbs. '*Lucky exposures: Diffraction Limited Astronomical Imaging Through the Atmosphere*". Ph.D thesis. St. Johns College, Cambridge University, Sep. **2003**.
24. Mohammed, A. T., Rashid, N. M. and Sadik, A.R. Optics and Laser in Engineering. "*Computer simulations of astronomical objects as seen by ground-based optical telescopes*". 12(**1990**)233-243.