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Fixed-Point Theorems for Jaggi Type Contraction in Partial Metric Spaces

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Abstract
In this paper, the partial metric space is adopted as the background of the work. As known, a partial metric space is a general case of a metric space where self-distances are not always zero. A generalization of the $(\beta\text{-}\ \psi)$ -Jaggi type contractive mapping is presented by adding some factors while retaining the control functions. The existence and uniqueness of a fixed point for this type of mappings are studied and discussed. It is very useful in extending the current findings of the corresponding literature.

Keywords: Admissible functions, (c)-Comparison function, Fixed point, Jaggi-type contractions, Partial metric space.

مبرهنات النقطة الصامدة لتطبيق انكماشي من نوع جاجي في الفضاءات المترية الجزئية

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الخلاصة
خلال هذا البحث, تم اعتماد الفضاء المتري الجزئي كخلفية للعمل. من المعروف ان الفضاء المتري الجزئي هو حالة عامة للفضاء المتري حيث لا تكون المسافات الذاتية صفرًا دائمًا. يتم تقديم تعميم التطبيق الانكماشي من نوع $(\beta-\ \psi)$-جاجي عن طريق إضافة بعض العوامل مع الاحتفاظ بدوال التحكم. تمت دراسة ومناقشة وجود و وحدانية نقطة الصامدة لهذا النوع من التطبيقات. إنه مفيد جدًا في توسيع النتائج الحالية للأدبيات المقابلة.

1. Introduction

Over the past decades, many authors have generalized the concept of the metric space such as quasi metric space by Maurice Frechet [1-2], b-metric space by Bakhtin [3-4], G-metric space by Mustafa and Sims [5-6], partial metric space by Matthews [7-9], Wangwe and Kumar [10] formed varied results on fixed-point for F-Hardy-Roger's multi-valued maps in partial metric spaces with ordering, etc. For spaces with special installations, see [11-12], where were discussed the proximity properties in fuzzy normed spaces and modular spaces

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respectively. A partial metric space (shortly, PMS) provides a powerful and flexible framework for studying spaces with self-referential distances, making them valuable in theoretical computer science and fixed-point analysis. Vijayabaskerreddy and Srinivas [13] proved a common fixed-point theorem for two pair of self-maps using the weakly compatible mappings through F-contraction. In [14], the authors proposed the notion of double controlled partial metric type spaces and studied the existence of fixed points for Kannan type contractions.

The work throughout this search is depended on the PMS to present some results about the existence fixed points. Banach's contraction assignment principle is the base of many extended fixed-point theories [15-16]. It has wide-ranging applications in many branches of mathematics and other science [17-19] in this field.

In this paper, we will deal with a type of contractive mapping called Jaggi contractive which was shown by Jaggi [20] in 1977 to showing the existence of fixed points. Karapınar and Fulga [21] combined of a Jaggi type contraction and interpolative type contraction in the metric spaces to present a hybrid type contraction and investigate the existence and uniqueness of it. Also, showed an application these results by solving fractional differential equations. Pankaj and Kumar [22] proved the existence and uniqueness weak (ψ-φ)-Jaggi type contraction. The concept of Jaggi-Wardowski-type contraction is introduced by Shagari and et al., [23] in G-metric space to give results of fixed points and then applied it to solve Fredholm-type integral equation under new conditions.

2. Preliminaries

In the following, some basic notions are recalled in PMS.

Definition 2.1: [24] Let G be a non-empty set and $\rho: G \times G \rightarrow \mathfrak{R}^+$ be a function where $\mathfrak{R}^+ = [0, \infty)$ such that for all $g, s, h \in G$,

1. $\rho(g,g) \leq \rho(g,s)$;
2. if $0 \leq \rho(g,g) = \rho(g,s) = \rho(s,s)$ then, $g = s$;
3. $\rho(g,s) = \rho(s,g)$;
4. $\rho(g,h) + \rho(s,s) \leq \rho(g,s) + \rho(s,h)$.

Then the pair (G, ρ) is called a partial metric space.

Remark 2.2: [25-26]

1. If $\rho(g,s) = 0$, then from Definition (2.1, 1- 2), we have $g = s$.
2. Any metric space is a partial metric space, but converse not always true, as the following, if $G = \mathfrak{R}^+$ and $\rho(g,s) = max\{g,s\}$. Then (G,ρ) is a partial metric space and $\rho(g,g) \neq 0$. For all $g \in G \setminus \{0\}$, but it is not metric space.
3. A partial metric ρ on G generates a T_0 topology τ_ρ on G with a base of the family of open ρ-balls is $B_\rho(g,\epsilon) = \{s \in G: \rho(g,s) < \rho(g,g) + \epsilon\}$ for all $g \in G$, and $\epsilon > 0$.
4. Hausdorff status may not be achieved in PMS.

To clarify the Remark 2.2, the following examples are presented:

Example 2.3: Let $G = \mathfrak{R}^+$ and $\rho: G \times G \rightarrow \mathfrak{R}^+$ be $\rho(g,s) = |g-s| + max\{g^2, s^2\}$, $B_\rho(0,1) = \{s \in G: \rho(0,s) < 1\}$, such that $\{g \in G: |g| + g^2 < 1\}$, then $(G,\rho)$is PMS. Now, to describe the ball in this status, there are two cases: if

i. $g \geq 0, \Longrightarrow |g| = g$. Then $0 < g < \frac{-1+\sqrt{5}}{2}$ for all $g \in G$.

ii. $g < 0, \Longrightarrow |g| = -g$. Then $\frac{1-\sqrt{5}}{2} < g < 0$. And then $B_\rho(0,1)$ is an open interval $\left(\frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$.

Example 2.4: Consider a set $\mathcal{A}$ enhanced with element ô such that ô $\notin \mathcal{A}$. Define a partial order on $G = \mathcal{A} \cup \{ ô \}$ such that ô $< s, \forall s \in \mathcal{A}$ and no other order relations exist. Define the function $\rho: G \times G \to \{0,1\}$ as $\rho(h,s) = 0$, if $h = s$ and $h \neq$ ô, and $\rho(h,s) = 1$ if $h \neq s$. Then $\rho(h,h) = 0, \forall\, h \neq ô$, $\rho(ô, ô) = 1$ and $\rho(h, ô) = \rho(ô, h) = 1, \forall\, h \neq ô$.

Definition 2.5: [27, 28] Let $(G,\ \rho)$ be a PMS. Then a partial metric ρ induces non-Hausdorff topology because any open set containing h will be also intersect with any open set containing ô, preventing the separation of a Hausdorff type.

1. A sequence $\{g_n\}$ in (G,ρ) converges to $g \in G$ if $\rho(g,g) = \lim_{n\to\infty} \rho(g, g_n)$;
2. A sequence $\{g_n\}$ in (G,ρ) is called a Cauchy sequence if $\rho(g,g) = \lim_{n,m\to\infty} \rho(g_m, g_n)$ exists (and is finite);
3. (G,ρ) is said to be complete if every Cauchy sequence $\{g_n\}$ in G converges, with respect to $\Im_\rho$, to a point $g \in G$ such that $\rho(g,g) = \lim_{n,m\to\infty} \rho(g_m, g_n)$;

Definition 2.6: [26] A subset $\mathcal{A}$ of G is

1. Closed $\mathcal{A}' \subset \mathcal{A}$ where ($\mathcal{A}'$ is the set of all accumulation points of G), if a sequence $\{g_n\}$ in $\mathcal{A}$ converges to some $g \in G$, then $g \in \mathcal{A}$.
2. Bounded if there exists $g_0 \in G$ and $M > 0$ such that for all $a \in \mathcal{A}$, we have $a \in \beta_\rho(g_0, M)$, that is, $\rho(g_0, a) < \rho(a,\ a) + M$.

Definition 2.7: [29] Let $(G,\ \rho)$ be a PMS, a self-mapping Γ on G is called to be continuous, if for each sequence $\{g_n\}$ in G converges to $u \in G$, that is

$$\rho(u,u) = \lim_{n\to\infty} \rho(g_n, u) = \lim_{n\to\infty} \rho(g_n, g_{n+k}).$$

Remark 2.8: [30] The above definition implies to:

1. $\rho(\Gamma u, \Gamma u) = \lim_{n\to\infty} \rho(\Gamma g_n, \Gamma u) = \lim_{n\to\infty} \rho(\Gamma g_n, \Gamma g_{n+k})$.
   Notice that, the equality above can be expressed as
   $\lim_{n\to\infty} \rho(\Gamma g_n, \Gamma u) = \lim_{n\to\infty} \rho(g_{n+1}, \Gamma u) = \lim_{n\to\infty} \rho(g_{n+1}, g_{n+k+1}) = \rho(u,u)$.
2. The non-Hausdorffian of G implies to non-uniqueness of the limit of a convergent sequence. To explain this, follow the below example

Example 2.9: Consider a PMS G = [0, 1] with, $\rho(g,s) = max\,\{g,s\}$, the sequence $\left\{\frac{1}{n}\right\}_{n \in \mathbb{N}}$
Then, (G,ρ). Note $\rho(0,\ 0) = \lim_{n\to\infty} \rho\left(0,\ \frac{1}{n}\right) = 0$ and $\rho(1,\ 1) = \lim_{n\to\infty} \rho\left(1,\ \frac{1}{n}\right) = 1$.

Lemma 2.10: [31]

1. Let $\{g_n\}$ and $\{s_n\}$ be two sequences in a PMS G such that
$\rho(g,g) = \lim_{n\to\infty} \rho(g_n, g) = \lim_{n\to\infty} \rho(g_n, g_n)$ and $\rho(s,s) = \lim_{n\to\infty} \rho(s_n, s) = \lim_{n\to\infty} \rho(s_n, s_n)$,
Then, $\rho(g,s) = \lim_{n\to\infty} \rho(g_n, s_n)$.
In particular, $\rho(g,h) = \lim_{n\to\infty} \rho(g_n, h)$, for every $h \in G$.

2. For g, s in G

 i. If $\rho(g,s) = 0$, then $g = s$,
 ii. If $g \neq s$, then $\rho(g,s) > 0$.

As, in [32], investigated reform the following two definitions in the PMS:

Definition 2.11: Let a mapping $\Gamma: G \to G$ and $\beta: G \times G \to [0,\infty)$ be a function. Then Γ is called β-orbital admissible if

$$\beta(g,\Gamma g) \geq 1 \Rightarrow \beta(\Gamma g,\Gamma^2 g) \geq 1.$$

Definition 2.12: Let a mapping $\Gamma: G \to G$ and $\beta: G \times G \to [0,\infty)$, be a function. Then Γ is called β-admissible if

$\beta(g,s) \geq 1 \Rightarrow \beta(\Gamma g,\Gamma s) \geq 1$, satisfied for all $g,s \in G$.

Obviously, every β-admissible is β-orbital admissible by the following example:

Example 2.13: Consider (G, ρ), $G = \{0,2,3,4\}$, $\Gamma: G \to G$ and $\rho: G \times G \to \mathfrak{R}$, $\rho(g,s) = |g-s|$ for all $g,s \in G$, $\beta: G \times G \to [0,\infty)$, when $\beta(g,s) = 0$ if $(g,s) \in \{(2,4),(4,2)\}$ and $\beta(g,s) = 1$ if $\{(g,s) \notin \{(2,4),(4,2)\}$ then Γ is β-orbital admissible, but not β-admissible function, where $\Gamma_0 = \Gamma_2 = 4, \Gamma_4 = \Gamma_5 = 5$.

Definition 2.14: The function $\psi: [0,\infty) \to [0,\infty)$ is called a (c)-comparison if it is non-decreasing and satisfies

there exists $k_0 \in \mathbb{N}$ and $a \in (0,1)$ and $\sum_{k=1}^{\infty} v_k < \infty$ such that $v_k \geq 0$ and
$\psi^{k+1}(t) \leq a\psi^k(t) + v_k$, for $k \geq k_0$ and any $t \geq 0$.

Lemma 2.15: Let $\Psi =: \{\psi: \psi$ is (c)-comparison function$\}$. Then ψ satisfies the following:

1. $(\psi^n(t))_{n \in \mathbb{N}}$ converges to 0 as $n \to \infty$ for all $t \in \mathfrak{R}^+$.
2. $\psi(t) < t$, for all $t \in \mathfrak{R}^+$.
3. ψ is continuous at 0.
4. The series $\sum_{k=1}^{\infty} \psi^k(t)$ converges for all $t \in \mathfrak{R}^+$.

Samet [28] suggested a contraction type self-mapping to unify several previous results in the literature by control functions.

The rational expression for contraction had been introduced for the first time by Jaggi [19], as the following definition shown:

Definition 2.16: Let (G,ρ) be a metric space, and $\Gamma: (G,\rho) \to (G,\rho)$ be mapping. Γ is called a ψ Jaggi Type Contraction denoted if, there exist $\psi \in \Psi$ such that

$$\rho\big(\Gamma(g),\Gamma(s)\big) \leq \psi\left(\varrho_1 \frac{\rho(g,\Gamma g).\rho(s,\Gamma s)}{\rho(g,s)} + \varrho_2 \rho(g,s)\right) \quad (1)$$

for all $g,s \in G$, such that $\varrho_1 + \varrho_2 < 1$.

3. Main Results

In the following is a new version of Jaggi contraction type via control functions in PMS.

Definition 3.1: Let Γ be a self-mapping on a PMS (G,ρ) and β, ψ be real functions as in Definitions 2.11 and 2.14, respectively then Γ is called (β- ψ) generalized Jaggi contraction if

$$\beta(g,\Gamma s)\rho\big(\Gamma(g),\Gamma(s)\big) \leq \psi\big(\gamma^q\Gamma(g,s)\big).$$

Where $\gamma^q\Gamma(g,s) =$

$$\sqrt[q]{\varrho_1\left(\frac{\rho(g,\Gamma g).\rho(s,\Gamma s)}{\rho(g,s)}\right)^q + \varrho_2\big(\rho(g,s)\big)^q + \varrho_3\left[max\left\{\frac{\rho(g,\Gamma g)+\rho(s,\Gamma s)}{2}, \atop \rho(g,s),\rho(s,\Gamma g)\right\}\right]^q} \quad (2)$$

for all $g,s \in G$, $q > 0$ and $\varrho_i \geq 0, i = 1,2,3$ such that $\varrho_1 + \varrho_2 + \varrho_3 = 1$.

Theorem 3.2: Let $\Gamma: (G,\rho) \to (G,\rho)$ be a continuous mapping on a complete PMS (G,ρ), if Γ is (β- ψ) generalized Jaggi contraction. Then Γ has a fixed point $g \in G$. Moreover, for any $g_0 \in G$, the sequence $\{\Gamma^n g_0\}$ converges to g.

Proof: Since Γ is β-orbital admissible, then there exists g_0 in G, such that $\beta(g_0,\Gamma g_0) = \beta(g_0,g_1) \geq 1$, so $\beta(\Gamma g_0,\Gamma^2 g_0) = \beta(g_1,g_2) \geq 1$. And so on $\beta(g_{n+1},g_{n+2}) = \beta(\Gamma g_n,\Gamma g_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, where $g_1 = \Gamma g_0$, and $g_2 = \Gamma g_1 = \Gamma^2 g_0 \ldots\ldots, g_n = \Gamma g_{n-1} = \Gamma^{n-1} g_0$.

By assuming that $\rho(g_n, g_{n+1}) > 0$ for all $n \in \mathbb{N}$, if it is not true that there exists $k_0 \in \mathbb{N}$ such that $\rho\left(g_{k_0}, g_{k_0+1}\right) = 0$, Therefore, $g_{k_0} = g_{k_0+1} = \Gamma g_{k_0}$ then it is the proof is completed, since g_{k_0} forms a fixed point. Now, when q ≥ 0, by letting $\gamma^{q}\Gamma(g, s)$ for $g = g_n$ and $s = g_{n+1}$, so

$$\beta(g, \Gamma s)\rho\big(\Gamma(g), \Gamma(s)\big) \leq \psi\big(\gamma^{q}\Gamma(g, s)\big).$$

Becomes

$$\rho(g_{n+1}, g_{n+2}) = \rho\big(\Gamma(g_n), \Gamma(g_{n+1})\big) \leq \beta(g_n, \Gamma g_{n+1})(\rho\big(\Gamma(g_n), \Gamma(g_{n+1})\big) \leq \psi\big(\gamma^{q}\Gamma(g_n, g_{n+1}\,)\big) \tag{3}$$

where

$$\gamma^{q}\Gamma(g_n, g_{n+1}\,) = \sqrt[q]{\varrho_1\left(\frac{\rho(g_n,\Gamma g_n)\cdot\rho(g_{n+1},\Gamma g_{n+1})}{\rho(g_n,g_{n+1})}\right)^{q} + \varrho_2\big(\rho(g_n, g_{n+1})\big)^{q} + \varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(g_n,\Gamma g_n)+\rho(g_{n+1},\Gamma g_{n+1})}{2}, \\ \rho(g_n, g_{n+1}), \rho(g_{n+1}, \Gamma g_n)\end{matrix}\right\}\right]^{q}}\,. \tag{4}$$

Therefore,

$$\rho(g_{n+1}, g_{n+2}) = \rho\big(\Gamma(g_n), \Gamma(g_{n+1})\big) \leq \beta(g_n, \Gamma g_{n+1})(\rho\big(\Gamma(g_n), \Gamma(g_{n+1})\big)$$

$$\leq \psi\left(\sqrt[q]{\varrho_1\left(\frac{\rho(g_n, \Gamma g_n)\cdot\rho(g_{n+1}, \Gamma g_{n+1})}{\rho(g_n, g_{n+1})}\right)^{q} + \varrho_2\big(\rho(g_n, g_{n+1})\big)^{q} + \varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(g_n, \Gamma g_n) + \rho(g_{n+1}, \Gamma g_{n+1})}{2}, \\ \rho(g_n, g_{n+1}), \rho(g_{n+1}, \Gamma g_n)\end{matrix}\right\}\right]^{q}}\right) \tag{5}$$

$$\leq \psi\left(\sqrt[q]{\varrho_1\left(\frac{\rho(g_n,g_{n+1})\cdot\rho(g_{n+1},g_{n+2})}{\rho(g_n,g_{n+1})}\right)^{q} + \varrho_2\big(\rho(g_n, g_{n+1})\big)^{q} + \varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(g_n,g_{n+1})+\rho(g_{n+1},g_{n+2})}{2}, \\ \rho(g_n, g_{n+1}), \rho(g_{n+1}, g_{n+1})\end{matrix}\right\}\right]^{q}}\right) \tag{6}$$

$$\leq \psi\left(\sqrt[q]{\varrho_1\big(\rho(g_{n+1}, g_{n+2})\big)^{q} + \varrho_2\big(\rho(g_n, g_{n+1})\big)^{q} + \varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(g_n,g_{n+1})+\rho(g_{n+1},g_{n+2})}{2}, \\ \rho(g_n, g_{n+1}), \rho(g_{n+1}, g_{n+1})\end{matrix}\right\}\right]^{q}}\right). \tag{7}$$

Suppose that, if
$\rho(g_{n+1}, g_{n+2}) > \rho(g_n, g_{n+1})$, putting in (7). Then

$$\rho(g_{n+1}, g_{n+2}) \leq \psi\sqrt[q]{\varrho_1\big(\rho(g_{n+1}, g_{n+2})\big)^{q} + \varrho_2\big(\rho(g_{n+1}, g_{n+2})\big)^{q} + \varrho_3\left[\max\left\{\begin{matrix}\rho(g_{n+1}, g_{n+2}), \\ \rho(g_{n+1}, g_{n+2}), \rho(g_{n+1}, g_{n+1})\end{matrix}\right\}\right]^{q}} \tag{8}$$

$$\leq \psi\sqrt[q]{\varrho_1\big(\rho(g_{n+1}, g_{n+2})\big)^{q} + \varrho_2\big(\rho(g_{n+1}, g_{n+2})\big)^{q} + \varrho_3[\max\{\rho(g_{n+1}, g_{n+2})\}]^{q}} \tag{9}$$

$$\leq \psi\sqrt[q]{\big(\rho(g_{n+1}, g_{n+2})\big)^{q}\,(\varrho_1 + \varrho_2 + \varrho_3\,)} \tag{10}$$

$$= \psi\sqrt[q]{\big(\rho(g_{n+1}, g_{n+2})\big)^{q}} \tag{11}$$

$< \rho(g_{n+1}, g_{n+2})$ by $\varrho_1 + \varrho_2 + \varrho_3 = 1$, and $\psi(t) < t$. That is a contradiction.

assume that if
$\rho(g_{n+1}, g_{n+2}) \le \rho(g_n, g_{n+1})$, putting in (2). Then

$$\rho(g_{n+1}, g_{n+2}) \le \psi \sqrt[q]{\begin{matrix} \varrho_1\big(\rho(g_n, g_{n+1})\big)^q + \varrho_2\big(\rho(g_n, g_{n+1})\big)^q \\ +\varrho_3 \left[\max\left\{\begin{matrix} \frac{\rho(g_n,g_{n+1})+\rho(g_n,g_{n+1})}{2}, \\ \rho(g_n, g_{n+1}), \rho(g_{n+1}, g_{n+1}) \end{matrix}\right\}\right]^q \end{matrix}} \quad (12)$$

$$\rho(g_{n+1}, g_{n+2}) \le \psi \sqrt[q]{\begin{matrix} \varrho_1\big(\rho(g_n, g_{n+1})\big)^q + \varrho_2\big(\rho(g_n, g_{n+1})\big)^q \\ +\varrho_3 \left[\max\left\{\begin{matrix} \rho(g_n, g_{n+1}), \\ \rho(g_n, g_{n+1}), \rho(g_{n+1}, g_{n+1}) \end{matrix}\right\}\right]^q \end{matrix}} \quad (13)$$

getting,

$$\le \psi \sqrt[q]{\varrho_1\big(\rho(g_n, g_{n+1})\big)^q + \varrho_2(\rho(g_n, g_{n+1}))^q + \varrho_3[\max\{\rho(g_n, g_{n+1})\}]^q} \quad (14)$$

$$\le \psi \sqrt[q]{\big(\rho(g_n, g_{n+1})\big)^q (\varrho_1 + \varrho_2 + \varrho_3)} \quad (15)$$

$< \rho(g_{n+1}, g_{n+2})$ by $\varrho_1 + \varrho_2 + \varrho_3 = 1$, and $\psi(t) < t$. That is a contradiction.
Continuing in this way. To get

$$\rho(g_{n+1}, g_n) \le \psi^n \rho(g_0, g_1) \text{ for any } n \ge 1. \quad (16)$$

Let z ∈ $\mathbb{N}_0$, and by the Definition 2.1, getting

$$\begin{aligned} \rho(g_n, g_{n+z}) &\le \rho(g_n, g_{n+1}) + \cdots + \rho(g_{n+z-1}, g_{n+k}) - \sum_{j=1}^{k-1} \big(\rho(g_{n+j}, g_{n+j})\big) \\ &\le \sum_{j=n}^{n+z-1} \psi^j\big(\rho(g_1, g_0)\big) \\ &\le \sum_{j=n}^{\infty} \psi^j\big(\rho(g_1, g_0)\big) \to 0, \text{ for } n \to \infty, \end{aligned} \quad (17)$$

together with properties of (c)-comparison function obtaining $\lim_{n\to\infty} \rho(g_n, g_{n+z}) = 0$.
Hence $\{g_n\}$ is a Cauchy sequence (G, ρ) and the completeness of G implies that $\exists\, v \in G$; $\lim_{n\to\infty} \rho(g_n, v) = 0 = \lim_{n\to\infty} \rho(g_n, g_{n+z}) = \rho(v, v)$.

By continuity mapping of Γ by Remark 2.8, getting
$\lim_{n\to\infty} \rho(g_{n+1}, \Gamma v) = \lim_{n\to\infty} \rho(\Gamma g_n, \Gamma v) = 0$, so v is a fixed point of Γ, that is $v = \Gamma v$.
Since (G, ρ) is complete, then there exists $g \in G$ such that $\lim_{n\to\infty} \rho(g_n, g) = 0$.
And since Γ is a continuous mapping, so conclude that
$\rho(g, \Gamma g) = \lim_{n\to\infty} \rho(g_n, \Gamma g_n) = \lim_{n\to\infty} \rho(g_n, g_{n+1}) = 0$.
We find that g is a fixed point of a mapping.

The following example satisfied Theorem 3.2.

Example 3.3: Consider $G = [0,2]$, $\rho(g, s) = \begin{cases} \max\{g, s\}, & if\ g \ne s \\ 0 & , if\ g = s \end{cases}$ which is a PMS [29, 33].

Define $\Gamma: G \to G$ as $\Gamma(g) = \begin{Bmatrix} 0, if\ g \in \left[0, \frac{1}{2}\right) \\ \frac{1}{4}, if\ g \in \left[\frac{1}{2}, 1\right] \\ \frac{1}{2}, if\ g \in \left[1, \frac{3}{2}\right] \\ \frac{3}{4}, if\ g \in (\frac{3}{2}, 2] \end{Bmatrix}$ is not continuous mapping on G.

Therefore, $\Gamma^2(g) = \begin{Bmatrix} 0, if\ g \in [0, \frac{1}{2}] \\ \frac{1}{4}, if\ g \in (\frac{1}{2}, 2] \end{Bmatrix}$ is not continuous mapping on G,

therefore, $\Gamma^3(g) = 0$ is continuous mapping on G, then just claim that Γ satisfies of the Theorem 3.2 by putting in Equation 2 and assume that $\beta(g, \Gamma s) = 1$and $\psi(t) = t, q = 2$, $\varrho_1 = \varrho_2 = \frac{1}{4}, \varrho_3 = \frac{1}{2}$.

To prove the existence (unique) of a fixed point of a mapping Γ under assumption of continuity and by (β- ψ) generalized Jaggi contraction.

Theorem 3.4: Let Γ: $(G, \rho) \to (G, \rho)$ be a (β- ψ) generalized Jaggi contraction, and (G, ρ) be a complete PMS. If for some $M \in \mathbb{N}$, $M$is integer $M > 1$, Γ^M is continuous. Then Γ has a unique fixed point.

Proof: By similar way of proof Theorem 3.2, there exists the sequence $\{g_n\}$, $g_n = \Gamma g_{n-1}$ for all $n \in \mathbb{N}$ such that $\rho(g_n, g) = 0$ is convergent to some $g \in G$, a sequence $\{g_n\}$ has a subsequence $\{g_{n(k)}\}$ where $n(k) = k.M$, $k \in \mathbb{N}$ and $M$is integer $M > 1$.
Therefore, let Γ^0 is identity mapping in G, thus getting $g_{n(k)} = \Gamma^M g_{n(k)-M}$ then
$\rho(g, \Gamma^M g) = \lim_{n\to\infty} \rho(g, \Gamma^M g_{n(k)-M}) = \lim_{n\to\infty} \rho(g_{n(k)}, g) = \rho(g, g) = 0$.
Therefore, g is a fixed point of Γ^M. Assume that $\Gamma g \neq g$, getting
$\Gamma^{M-p-1}g \neq \Gamma^{M-p}g$ for any $p = 0, 1, \dots, k-1$. Used by (β- ψ) generalized Jaggi contraction, by putting in (2) $g = \Gamma^{M-p-1}g$ and $s = \Gamma^{M-p}g$, the expression
$\gamma^q\Gamma(\Gamma^{M-p-1}g, \Gamma^{M-p}g)$

$$= \sqrt[q]{\begin{matrix} \varrho_1\left(\frac{\rho(\Gamma^{M-p-1}g, \Gamma(\Gamma^{M-p-1}g)).\rho(\Gamma^{M-p}g, \Gamma(\Gamma^{M-p}g)}{\rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g)}\right)^q \\ +\varrho_2\left(\rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(\Gamma^{M-p-1}g, \Gamma\Gamma^{M-p-1}g) + \rho(\Gamma^{M-p}g, \Gamma\Gamma^{M-p}g)}{2}, \\ \rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g), \rho(\Gamma^{M-p}g, \Gamma(\Gamma^{M-p-1}g))\end{matrix}\right\}\right]^q \end{matrix}}. \tag{18}$$

Then

$$= \sqrt[q]{\begin{matrix} \varrho_1\left(\frac{\rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g).\rho(\Gamma^{M-p}g, \Gamma^{M-p+1}g)}{\rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g)}\right)^q + \varrho_2\left(\rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g) + \rho(\Gamma^{M-p}g, \Gamma^{M-p+1}g)}{2}, \\ \rho(\Gamma^{M-p-1}g, \Gamma^{M-p}g), \rho(\Gamma^{M-p}g, \Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}}$$

$$= \sqrt[q]{\begin{matrix} \varrho_1\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q + \varrho_2\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)+\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)}{2}, \\ \rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g),\rho(\Gamma^{M-p}g,\Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}} \quad (19)$$

getting,

$$\rho\left(\Gamma(\Gamma^{M-p-1}g),\Gamma(\Gamma^{M-p}g)\right) \le \beta(\Gamma^{M-p-1}g,\Gamma^{M-p+1}g)(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)$$

$$\le \psi\left(\sqrt[q]{\begin{matrix} \varrho_1\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q + \varrho_2\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)+\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)}{2}, \\ \rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g),\rho(\Gamma^{M-p}g,\Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}}\right). \quad (20)$$

Suppose that, if

$\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g) > \rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)$, putting in (20). Then,

$$\rho\left(\Gamma(\Gamma^{M-p-1}g),\Gamma(\Gamma^{M-p}g)\right) \le \beta(\Gamma^{M-p-1}g,\Gamma^{M-p+1}g)(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g) \le \psi\left(\gamma^q\Gamma(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right) =$$

$$\psi\left(\sqrt[q]{\begin{matrix} \varrho_1\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q + \varrho_2\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)+\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)}{2}, \\ \rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g),\rho(\Gamma^{M-p}g,\Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}}\right)$$

$$\le \psi\left(\sqrt[q]{\begin{matrix} \varrho_1\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q + \varrho_2\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g), \\ \rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g),\rho(\Gamma^{M-p}g,\Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}}\right)$$

$$\le \psi\sqrt[q]{\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q(\varrho_1+\varrho_2+\varrho_3)},$$

$$= \psi\sqrt[q]{\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right)^q}, \quad (21)$$

$$= \psi\left(\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)\right), \quad (22)$$

$< \rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g)$by $\varrho_1+\varrho_2+\varrho_3 = 1$, and $\psi(t) < t$. That is a contradiction.

Also, suppose if

$\rho(\Gamma^{M-p-1}g,\Gamma^{M-p}g) \le \rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)$, putting in (22). Then,

getting,

$$\rho\left(\Gamma(\Gamma^{M-p-1}g),\Gamma(\Gamma^{M-p}g)\right) \le \beta(\Gamma^{M-p-1}g,\Gamma^{M-p+1}g)(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)$$

$$\le \psi\left(\sqrt[q]{\begin{matrix} \varrho_1\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q + \varrho_2\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\frac{\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)+\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)}{2}, \\ \rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g),\rho(\Gamma^{M-p}g,\Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}}\right)$$

$$\le \psi\left(\sqrt[q]{\begin{matrix} \varrho_1\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q + \varrho_2\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q \\ +\varrho_3\left[\max\left\{\begin{matrix}\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g), \\ \rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g),\rho(\Gamma^{M-p}g,\Gamma^{M-p}g)\end{matrix}\right\}\right]^q \end{matrix}}\right)$$

$$\le \psi\sqrt[q]{\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q(\varrho_1+\varrho_2+\varrho_3)}, \quad (23)$$

$$= \psi\sqrt[q]{\left(\rho(\Gamma^{M-p}g,\Gamma^{M-p+1}g)\right)^q},$$

$= \psi\big(\rho(\Gamma^{M-p}g, \Gamma^{M-p+1}g)\big),$ (24)
$< \rho(\Gamma^{M-p}g, \Gamma^{M-p+1}g)$ by $\varrho_1 + \varrho_2 + \varrho_3 = 1$, and $\psi(t) < t$. That is a contradiction.
Thus Γ has a fixed point say g.
Corollary 3.5: The continuous mapping Γ on a complete PMS G has a fixed point g^*, if

$$\rho\big(\Gamma(g), \Gamma(s)\big) \leq \delta\left(\sqrt{\begin{array}{c}\varrho_1\left(\frac{\rho(g,\Gamma g).\rho(s,\Gamma s)}{\rho(g,s)}\right)^2 + \varrho_1\big(\rho(g,s)\big)^2 + \\ \varrho_1\left[\max\left\{\frac{\rho(g,\Gamma g)+\rho(s,\Gamma s)}{2}, \rho(g,s), \rho(s,\Gamma g)\right\}\right]^2\end{array}}\right), \quad (25)$$

for any $g, s \in G$ and $g \neq s, \delta \in (0,1)$.
Proof: Putting $\beta(g, \Gamma s) = 1$ and $\psi(t) = t, q = 2, \varrho_1 = \varrho_2 = \frac{1}{4}, \varrho_3 = \frac{1}{2}$ in Theorem 3.2, and in Equation (2).
Now, the following definition leads us to new results in PMS without condition of continuity.

Definition 3.6 [32]: Suppose that G is a PMS, and β is admissible function. Then G is said to be regular if for each iterative sequence $\{g_n\}$ in G provides that $\beta(g_n, g_{n+1}) \geq 1$ for all n and $g_n \to g \in G$, as $n \to \infty$, then there exists a subsequence $\{g_{n(k)}\}$ of $\{g_n\}$ such that $\beta\big(g_{n(k)}, g\big) \geq 1$ for all k.

Theorem 3.7: Let (G, ρ) be a complete PMS and $\Gamma: G \to G$ be $(\beta$- $\psi)$ generalized Jaggi contraction. Suppose that

i. Γ is β −orbital admissible;
ii. There exists $g_0 \in G$ such that $\beta(g_0, \Gamma g_0) \geq 1$;
iii. Γ is regular.

Then, there exists $u \in G$ such that $\Gamma u = u$ and $\rho(u, u) = 0$.
Proof: By condition (iii), there exists a subsequences $\{g_{n(k)}\}$ of $\{g_n\}$ such that $\beta\big(g_{n(k)}, u\big) \geq 1$ for all k, get that

$$\rho\big(g_{n(k)+1}, \Gamma u\big) = \rho\big(\Gamma g_{n(k)}, \Gamma u\big) \leq \beta\big(g_{n(k)}, u\big)\rho\big(\Gamma g_{n(k)}, \Gamma u\big)$$
$$\leq \psi\left(\gamma^q\Gamma\big(g_{n(k)}, u\big)\right) < \gamma^q\Gamma\big(g_{n(k)}, u\big).$$

Where

$$\gamma^q\Gamma\big(g_{n(k)}, u\big) = \sqrt[q]{\begin{array}{c}\varrho_1\left(\frac{\rho\big(g_{n(k)}, \Gamma g_{n(k)}\big).\rho(u,\Gamma u)}{\rho\big(g_{n(k)}, u\big)}\right)^q + \varrho_2\left(\rho\big(g_{n(k)}, u\big)\right)^q \\ +\varrho_3\left[\max\left\{\begin{array}{c}\frac{\rho\big(g_{n(k)}, \Gamma g_{n(k)}\big) + \rho(u,\Gamma u)}{2} \\ , \rho\big(g_{n(k)}, u\big), \rho\big(u, \Gamma g_{n(k)}\big)\end{array}\right\}\right]^q\end{array}}. \quad (26)$$

Therefore,
$\rho(u, \Gamma u) \leq \psi\sqrt[q]{\big(\rho(u,\Gamma u)\big)^q(\varrho_1 + \varrho_2 + \varrho_3)} = \psi\big(\rho(u, \Gamma u)\big) < \rho(u, \Gamma u)$. By $\varrho_1 + \varrho_2 + \varrho_3 = 1$ and $\psi(t) < t$. That is a contradiction.
Thus, we have $\rho(u, \Gamma u) = 0$. Therefore, $u = \Gamma u$.
Corollary 3.8: Let (G, ρ) be a complete PMS and $\beta: G \times G \to [0, \infty)$ and $\psi \in \Psi$. If $\Gamma: G \to G$ such that

$$\beta(g, s)\rho(\Gamma g, \Gamma s) \leq \psi(\max(\frac{\rho(g,\Gamma g) + \rho(s,\Gamma s)}{2}, \rho(g, s), \rho(s, \Gamma g))) \quad (27)$$

for all $g, s \in G$. Assume also that

i. Γ is β −orbital admissible;
ii. There exists $g_0 \in G$ such that $\beta(g_0, \Gamma g_0) \geq 1$;

iii. either Γ is continuous or regular.
Then there exists $u \in G$ such that $\Gamma u = u$ and $\rho(u,u) = 0$.

Corollary 3.9: Let $(G,\ \rho)$ be a complete partial metric space and $\beta: G\ \times G \to\ [0,\infty)$ and $\psi\ \in\ \Psi$. If $\Gamma: G\ \to G$ such that $\beta(g,s)\rho(\Gamma g,\Gamma s\ ) \le\ \psi(\max\ (\rho(g,s),\rho(g,\Gamma s),\rho(s,\Gamma g)))$ for all $g,s \in G$, assume also that

i. Γ is β −orbital admissible;
ii. There exists $g_0 \in G$ such that $\beta(g_0,\Gamma g_0\ ) \ge\ 1$;
iii. either Γ is continuous or regular.

Then there exists $u \in G$ such that $\Gamma u = u$ and $\rho(u,u) = 0$.

Abdeljawad [34] introduced an interesting results about generalized weakly contraction mapping. In this work, the existence of fixed points is studied for generalized weakly contraction mappings in PMS.

Definition 3.10: Let (G,ρ) be a PMS, $\psi\ \in\ \Psi$ and φ : [0,∞) → [0,∞) is a strictly increasing function with $\varphi(0) = 0$. A mapping Γ: $(G,\rho) \to (G,\rho)$ is called $(\beta - \psi - \varphi)$ Jaggi weakly contraction if

$$\beta(g,\Gamma s)\psi(\rho(\Gamma(g),\Gamma(s))) \le\ \psi\big(\gamma^{q}\Gamma(g,s)\big) - \varphi\big(\mu^{q}\Gamma(g,s)\big)$$

where, $\gamma^{q}\Gamma(g,s)$

$$= \sqrt[q]{\varrho_1\left(\frac{\rho(g,\Gamma g).\rho(s,\Gamma s)}{\rho(g,s)}\right)^{q} + \varrho_2\big(\rho(g,s)\big)^{q} + \varrho_3[\max\{\rho(g,s),\rho(s,\Gamma s)\}]^{q}}$$

and $\mu^{q}\Gamma(g,s)$

$$= \sqrt[q]{\begin{array}{c}\varrho_1\left(\dfrac{\rho(g,\Gamma g).\rho(s,\Gamma s)}{\rho(g,s)}\right)^{q} + \varrho_2\big(\rho(g,s)\big)^{q} \\ +\varrho_3\left[\max\left\{\dfrac{1}{2}[\rho(g,s)+\rho(s,\Gamma s)],\rho(g,s),\rho(s,\Gamma s),\rho(s,\Gamma s)\right\}\right]^{q}\end{array}} \tag{28}$$

for all $g,s \in G$, $q \ge 0$ and $\varrho_i\ \ge 0, i = 1, 2,3$ such that $\varrho_1 + \varrho_2 + \varrho_3 = 1$.

Theorem 3.11: Let Γ: $(G,\rho) \to (G,\rho)$ be a continuous mapping, and (G,ρ) is complete PMS of $(\beta - \psi - \varphi)$ Jaggi weakly contraction, such that there exists $g_0 \in G$. Then, Γ has a unique fixed point.

Proof: Use of assumption β-orbital admissible, then there exists g in G, assume that $g = g_0$, such that $\beta(g_0,\Gamma g_0) =\ \beta(g_0,g_1) \ge 1$, then $\beta(\Gamma g_0,\Gamma^2 g_0) =\ \beta(g_1,g_2) \ge 1$ and obtain $\beta(g_{n+1},g_{n+2}) =\ \beta(\Gamma g_n,\Gamma g_{n+1}) \ge 1$ for all $n \in\ \mathbb{N}$, by assume that $\rho(g_n,g_{n+1}) > 0$ for all $n \in\ \mathbb{N}, g_1 =\ \Gamma g_0$, and $g_2 =\ \Gamma g_1 = \Gamma^2 g_0 \ldots\ldots, g_n = \Gamma g_{n-1}\ =\ \Gamma^{n-1} g_0$.
On the contrary, if the inequality above does not hold, that is, if there exists $k_0 \in \mathbb{N}$ such that $\rho\big(g_{k_0},g_{k_0+1}\big) = 0$. Therefore, $g_{k_0} = g_{k_0+1} =\ \Gamma g_{k_0}$. Since g_{k_0} forms a fixed point, it terminates the proof. Then, when $q \ge 0$, by letting $\gamma^{q}\Gamma(g,s)$ for $g =\ g_n$ and $s = g_{n+1}$, the expression

$$\beta(g,\Gamma s)\psi(\rho(\Gamma(g),\Gamma(s))) \le\ \psi\big(\gamma^{q}\Gamma(g,s)\big) - \varphi\big(\mu^{q}\Gamma(g,s)\big).$$

putting in Equation (28) then getting,

$$\rho(g_{n+1},g_{n+2})=\rho(\Gamma(g_n),\Gamma(g_{n+1}))\leq\beta(g_n,\Gamma g_{n+1})(\rho(\Gamma(g_n),\Gamma(g_{n+1}))$$
$$\leq\psi\left(\sqrt[q]{\begin{array}{c}\varrho_1\left(\frac{\rho(g_n,\Gamma g_n).\rho(g_{n+1},\Gamma g_{n+1})}{\rho(g_n,g_{n+1})}\right)^q+\varrho_2(\rho(g_n,g_{n+1}))^q\\+\varrho_3[\max\{\rho(g_n,g_{n+1}),\rho(g_{n+1},\Gamma g_{n+1})\}]^q\end{array}}\right)$$
$$-\varphi\left(\sqrt[q]{\begin{array}{c}\varrho_1\left(\frac{\rho(g_n,\Gamma g_n).\rho(g_{n+1},\Gamma g_{n+1})}{\rho(g_n,g_{n+1})}\right)^q+\varrho_2(\rho(g_n,g_{n+1}))^q\\+\varrho_3\left[\max\left\{\begin{array}{c}\frac{1}{2}[\rho(g_n,g_{n+1})+\rho(g_{n+1},\Gamma g_{n+1})],\\\rho(g_n,g_{n+1}),\rho(g_{n+1},\Gamma g_{n+1}),\rho(g_{n+1},\Gamma g_{n+1})\end{array}\right\}\right]^q\end{array}}\right)\quad(29)$$
$$\leq\psi\left(\sqrt[q]{\begin{array}{c}\varrho_1\left(\frac{\rho(g_n,g_{n+1}).\rho(g_{n+1},g_{n+2})}{\rho(g_n,g_{n+1})}\right)^q+\varrho_2(\rho(g_n,g_{n+1}))^q\\+\varrho_3[\max\{\rho(g_n,g_{n+1}),\rho(g_{n+1},g_{n+2})\}]^q\end{array}}\right)$$
$$-\varphi\left(\sqrt[q]{\begin{array}{c}\varrho_1\left(\frac{\rho(g_n,g_{n+1}).\rho(g_{n+1},g_{n+2})}{\rho(g_n,g_{n+1})}\right)^q+\varrho_2(\rho(g_n,g_{n+1}))^q\\+\varrho_3\left[\max\left\{\begin{array}{c}\frac{1}{2}[\rho(g_n,g_{n+1})+\rho(g_{n+1},g_{n+2})],\\\rho(g_n,g_{n+1}),\rho(g_{n+1},g_{n+2}),\rho(g_{n+1},g_{n+2})\end{array}\right\}\right]^q\end{array}}\right).$$

Then

$$\leq\psi\left(\sqrt[q]{\begin{array}{c}\varrho_1(\rho(g_{n+1},g_{n+2}))^q+\varrho_2(\rho(g_n,g_{n+1}))^q\\+\varrho_3[\max\{\rho(g_n,g_{n+1}),\rho(g_{n+1},g_{n+2})\}]^q\end{array}}\right)$$
$$-\varphi\left(\sqrt[q]{\begin{array}{c}\varrho_1(\rho(g_{n+1},g_{n+2}))^q+\varrho_2(\rho(g_n,g_{n+1}))^q\\+\varrho_3\left[\max\left\{\begin{array}{c}\frac{1}{2}[\rho(g_n,g_{n+1})+\rho(g_{n+1},g_{n+2})],\\\rho(g_n,g_{n+1}),\rho(g_{n+1},g_{n+2}),\rho(g_{n+1},g_{n+2})\end{array}\right\}\right]^q\end{array}}\right).\quad(30)$$

Suppose that, if
$\rho(g_{n+1},g_{n+2})>\rho(g_n,g_{n+1})$, putting in Equation (30) then getting,

$$\rho(g_{n+1},g_{n+2})\leq\psi\left(\sqrt[q]{\begin{array}{c}\varrho_1(\rho(g_{n+1},g_{n+2}))^q+\varrho_2(\rho(g_{n+1},g_{n+2}))^q\\+\varrho_3\left[\max\left\{\frac{\rho(g_{n+1},g_{n+2})+\rho(g_{n+1},g_{n+2})}{2}\right\}\right]^q\end{array}}\right)$$
$$-\varphi\left(\sqrt[q]{\begin{array}{c}\varrho_1(\rho(g_{n+1},g_{n+2}))^q+\varrho_2(\rho(g_{n+1},g_{n+2}))^q\\+\varrho_3\left[\max\left\{\begin{array}{c}\frac{1}{2}[\rho(g_{n+1},g_{n+2})+\rho(g_{n+1},g_{n+2})]\\,\rho(g_{n+1},g_{n+2}),\rho(g_{n+1},g_{n+2}),\rho(g_{n+1},g_{n+2})\end{array}\right\}\right]^q\end{array}}\right)\quad(31)$$
$$\leq\psi\left(\sqrt[q]{(\rho(g_{n+1},g_{n+2}))^q(\varrho_1+\varrho_2+\varrho_3)}\right)-\varphi\left(\sqrt[q]{(\rho(g_{n+1},g_{n+2}))^q(\varrho_1+\varrho_2+\varrho_3)}\right)$$
$$\leq\psi\left(\sqrt[q]{(\rho(g_{n+1},g_{n+2}))^q}\right)-\varphi\left(\sqrt[q]{(\rho(g_{n+1},g_{n+2}))^q}\right)$$

$\leq (\rho(g_{n+1}, g_{n+2})) - (\rho(g_{n+1}, g_{n+2}))$, by $\varrho_1 + \varrho_2 + \varrho_3 = 1$, and $\psi(t) < t$. That is a contradiction.
Moreover, assume that if $\rho(g_{n+1}, g_{n+2}) \leq \rho(g_n, g_{n+1})$, that is clear, a contradiction.
Continuing in this way. To get $\rho(g_{n+1}, g_n) \leq \psi^n \rho(g_0, g_1)$ for any $n \geq 1$.
Let $z \in \mathbb{N}_0$, and by the Definition 2.1, getting

$$\rho(g_n, g_{n+z}) \leq \rho(g_n, g_{n+1}) + \cdots + \rho(g_{n+z-1}, g_{n+k}) - \sum_{j=1}^{k-1} \left(\rho(g_{n+j}, g_{n+j})\right)$$
$$\leq \sum_{j=n}^{n+z-1} \psi^j(\rho(g_1, g_0))$$
$$\leq \sum_{j=n}^{\infty} \psi^j(\rho(g_1, g_0)) \to 0, \text{ for } n \to \infty,$$

together with properties of (c)-comparison function obtaining $\lim_{n\to\infty} \rho(g_n, g_{n+z}) = 0$.
Hence $\{g_n\}$ is a Cauchy sequence (G, ρ) and the completeness of G implies that $\exists\, v \in G$ such that $\lim_{n\to\infty} \rho(g_n, v) = 0 = \lim_{n\to\infty} \rho(g_n, g_{n+z}) = \rho(v, v)$. The continuity mapping of Γ and Remark 2.8, getting
$\lim_{n\to\infty} \rho(g_{n+1}, \Gamma v) = \lim_{n\to\infty} \rho(\Gamma g_n, \Gamma v) = 0$, so, v is a fixed point of Γ, that is $v = \Gamma v$.
Since (G, ρ) is complete, then there exists $g \in G$ such that $\lim_{n\to\infty} \rho(g_n, g) = 0$.
Since Γ is a continuous mapping, we conclude that $\rho(g, \Gamma g) = \lim_{n\to\infty} \rho(g_n, \Gamma g_n) = \lim_{n\to\infty} \rho(g_n, g_{n+1}) = 0$. Thus g is fixed point of Γ.

4.Conclusions

PMS have important applications in computer science, especially in domain theory, fixed-point computations and semantics of programming languages [35-36]. This is due to its ability to handle self-referential structures and incomplete information makes them well-suited for computational models [37]. The PMS provides a flexible and powerful environment for studying spaces with self-referencing distances, making them valuable in theoretical computer science. A new formula for the contractive condition was presented through Definition 3.1 and used this condition to find the uniqueness and existences of fixed point in partial metric space and the application was done in Example 3.3, as well as using another link between the Definition 3.6 and the contraction condition, by [34] development and introduce Definition 3.10 to find the fixed point. Finally, it is not bad to mention some useful results for future work, such as, Nuray [38] who studied the statistical convergence in partial metric spaces. It may be a catalyst for new findings in this area. Also, for future work one can study demonstrates the applicability of theorems through applications to equations as in [39-40].

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References

[1] S. S. Chauhan, P. Garg, and K. Thakur, "Study of Metric Space and Its Variants" *Journal of Mathematics*, vol. 2022, pp. 1-22, 2022.

[2] C. Yuan, and L. Yang, "An Efficient Multi-Metric Learning Method by Partitioning the Metric Space" *Neuro computing Elsevier*, vol. 529, no. 7, pp. 56-79, 2023.

[3] M. A. Mamud, and K. K. Tola, ,"Fixed Point Results for Generalized (α, ψ)-Contraction Mapping in Rectangular *b*-Metric Spaces"*Abstract and Applied Analysis,*vol. 2022, no.1, pp. 1-12, 2022.

[4] Sh.S. Albundi, "Iterated Function System in ϑ-Metric Spaces" *Bulletin of Parana´s*

Mathematical Society, vol. 4, no. 3s, pp.1-10, 2022.

[5] R. A. Thirumalai, and S. Thalapathiraj, "A Novel Approach For Some Fixed Point Results in Complete G-Metric Space By Using Asymptotically Regular Mapping in Digital Image Compression And Decompression Applications" *International Journal of Applied Mathematics*, vol. 37, no. 3, pp. 311–332, 2024.

[6] S.Q. Latif, and S.S. Abed, "Types of Fixed Points of Set-Valued Contraction Mappings for Comparable Elements" *Iraqi Journal of Science*, special issue, pp. 190–195, 2020.

[7] O. Valero, "On Banach Fixed Point Theorems for Partial Metric Spaces" *Applied General Topology*, vol. 6, no.2, pp. 229-240, 2005.

[8] D. Gopal, and S. Jain, *Fixed Point Theory in Partial Metric Spaces*,1st Edition, Chapman and Hall/CRC, New York, pp. 16, 2021.

[9] E. Güner and H. Aygün, "A New Approach to Fuzzy Partial Metric Spaces" *Hacettepe Journal of Mathematics & Statistics*, vol. 01, no.12, pp. 1563 -1576, 2022.

[10] L. Wangwe and S. Kumar, "Fixed Point Theorems for Multi-valued α−F-contractions in Partial metric spaces with an Application" *Results in Nonlinear Analysis*, vol. 4, no. 3, pp. 130-148, 2021.

[11] S.I. Raghad, and A. A. Buthainah, "Best Proximity Point Theorem for $\tilde{\alpha}-\tilde{\psi}$ −Contractive Type Mapping in Fuzzy Normed Space", *Baghdad Science Journal,* vol. 20, no. 5, pp. 1722-1037, 2023.

[12] N.J. Mohammed and S.S. Abed, "On Invariant Approximations in Modular Spaces" *Iraqi Journal of Science*, vol. 62, no. 9, pp. 3097-3101, 2021.

[13] B. Vijayabaskerreddy and V. Srinivas, "Matthews Partial Metric Space Using F-Contraction" *Communications on Applied Nonlinear Analysis*, vol. 31, no. 5s, pp. 449-459, 2024.

[14] H. Ahmad, and M. Younis, "Mehmet Emir Köksal Double Controlled Partial Metric Type Spaces and Convergence Results" *Journal of Mathematics*, vol. 11, no. 1, pp. 1-11, 2021.

[15] H.M. Abu-Donia, H.A. Ati and E. Safaa, "Fixed Point Theorems in Partial 2-Metric Spaces", *Wseas Transactions on Mathematics*, vol.21, pp. 153-159, 2022.

[16] M. Noorwali and S.S.Yeşilkaya, "New Fixed Point Theorems for Admissible Hybrid Maps", *Journal of Function Spaces*, vol. 2022, no.1 , pp.1-8, 2022.

[17] A. J. Gnanaprakasam, G. Mani, R. Ramaswamy, O. Ola, A. A. Abdelnaby, K. H. Khan, and S. Radenović, "Application of Fixed-Point Results to Integral Equation through F-Khan Contraction" *Symmetry*, vol. 15, no. 3, pp.1-12, 2023.

[18] K. Shah, S. Sarwar, M. Khan, A. N. Mlaiki and M.F. Azmi, "Solving Integral Equations via Fixed Point Results Involving Rational-Type Inequalities"*Axioms*, vol. 15, no. 3, pp.1-12, 2023.

[19] B. S. Choudhury and P. Chakraborty, "Strong Fixed Points of Φ-couplings and Generation of Fractals" *Chaos Solitons & Fractals,* vol. 163, 2022.

[20] D.S. Jaggi, "Some Unique Fixed Point Theorems" *Indian Journal of Pure and Applied Mathematics*, vol. 8, pp. 223–230, 1977.

[21] E. Karapinar and A. Fulga, "A Hybrid Contraction that Involves Jaggi Type" *Symmetry*, vol. 11, no. 5, pp. 1-9, 2019.

[22] Pankaj and M. Kumar, "A Fixed Point Theorem for Weak (ψ- φ)-Jaggi Type Contraction" *Boletim da Sociedade Paranaense de Matemática*, vol. 1–5, no. 3s, 2024.

[23] M. S. Shagari, R. O. Ogbumba and S. Yahaya, "A New Approach to Jaggi-Wardowski-Type Fixed Point Theorems" *Probl.Anal.Issues Anal*, vol. 13 (31), no. 1, 2024, pp. 50–70, 2024.

[24] S. Anwar, M. Nazam, H. Al Sulami, A. Hussain, Kh. Javed and M. Arshad, "Existence Fixed-Point Theorems in the Partial Metric Spaces and an Application to the Boundary Value Problem", *AIMS Mathematics*, vol. 7, no. 5, pp. 8188-8205, 2022.

[25] G.S. Saluja "Some Common Fixed Point Theorems on Partial Metric Spaces Satisfying Implicit Relation" *Math Moravica*, vol. 24, no. 1, pp. 29-43, 2020.

[26] D. Bugajewski, P. Mackowiak and R. Wang, "On Compactness and Fixed Point Theorems in Partial Metric Spaces", *Fixed Point Theory*, vol.23, no.1, pp. 163-178, 2022.

[27] E. O. Austine and D. Igbokwe, "New Faster Four Step Iterative Algorithm for Suzuki Generalized Nonexpansive Mappings with an Application" *Advances in the Theory of Nonlinear Analysis and its Application*, vol. 5, no. 3, pp. 482–506, 2021.

[28] S. S. Yesilkaya, "On Interpolative Hardy-Rogers Contractive of Suzuki Type Mappings" *Topological Algebra and its Applications*, vol. 9, no. 1, pp. 13–19, 2021.

[29] B. Samet, C. Vetro, and P.Vetro, "Fixed Point Theorem for α - ψ Contractive Type Mappings" *Nonlinear Anal,* vol. 75, pp. 2154–2165, 2012.

[30] H. Aydi, M. Abbas, and C. Vetro, "Partial Hausdorff metric and Nadler's fixed point Theorem on Partial Metric Spaces" *Topology and Its Applications,* vol. 159, pp. 3234–3242, 2012.

[31] E. Karapınar, "A Note on Common Fixed-Point Theorems in Partial Metric Spaces," *Miskolc Mathematical* Notes, vol. 12, no. 2, pp.185–191, 2011.

[32] O. Popescu, " Some New Fixed-Point Theorems for α-Geraghty Contraction Type Maps in Metric Spaces*" Fixed Point Theory and Applications,* vol. 190, pp. 1-12, 2014.

[33] E. Karapınar1,"On Jaggi Type Contraction Mappings" *UPB Scientific Bulletin, Series A: Applied Mathematics and Physics*, vol. 80, no. 4, pp. 1223-7027, 2018.

[34] T. Abdeljawad, "Fixed Points for Generalized Weakly Contractive Mappings in Partial Metric Spaces" *Mathematical and Computer Modelling,* vol. 54, pp. 2923-2927, 2011.

[35] H. Jabar, M.S.Abd, and S.F. Behadili, " Unveiling Patterns of Nomophobia Using Data Mining Techniques" *Iraqi Journal of Science*, vol. 65, no.8, pp. 4623–4632, 2024.

[36] S. Sameer and S. A. Salam, " Cart based Approach for Discovering Emerging Patterns in Iraqi Biochemical Dataset" *Iraqi Journal of Science,* vol. 63, no. 1, pp. 353-362, 2022.

[37] O.O. Otafudu and O. Valero, "On Information Orders on Metric Spaces*"*, *Information*, vol.12, no. 427, pp. 1-13, 2021.

[38] F. Nuray, "Statistical convergence in partial metric spaces", *Korean Journal of Mathematics*, vol. 30, no. 1, pp. 155-160, 2022.

[39] C. Tunc, F.S. Alshammari, F.T. Akyildiz, "Existence and Uniqueness of Solutions of Hammerstein-Type Functional Integral Equations", *Symmetry*, vol. 15, no. 2205, pp. 1-15, 2023.

[40] U. Ishtiaq, M. Din, Y. Rohen, Kh. Alnowibet and L. Popap, "Certain Fixed-Point Results for (e, ψ, Φ)-Enriched Weak Contractions via Theoretic Order with Applications", *Axioms*, vol. 14, no. 135, pp. 1-19, 2025.