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On Partially Ordered Ps-Dislocated Metric Spaces

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Abstract:
Dislocated metrics play an important role in logical programming. Moreover, they are pivotal in fields such as topology and electronic engineering. These spaces are significant because they have the property of modifying self-distance property. This feature made many authors undertake extensive investigations of this metric, where they examined the fixed points of maps that exist in this space that meet specific requirements and studied the characteristics that set it apart. Additionally, they looked into whether R-contractions have the best proximity in the framework of dislocated metric spaces, and specific requirements be established to ensure that a best proximity point for these contractions is unique. Researchers worked on generalizing the concept of dislocated metric space to include b-dislocated metric space, S-dislocated metric space, and other forms of metric structures. This paper introduces the concept of Ps-dislocated metric spaces and the concepts of Ps-proximal contraction map and generalized Ps-proximity contraction mapping. Also, we study the existence of a best proximity point of non-self-map via generalized α_{Ps}-proximity contraction R map and given the condition to be unique. After that, we used these results to derive the theorems' best proximity point on Ps-dislocated metric spaces endowed with a partial order, and we gave an example that cleared these concepts in this space.

Keywords: Partially Ordered Set, Best Proximity Point, Simulation Function, Approximately Compact

حول الفضاءات المترية المخلوعة – Ps المرتبة جزئيا

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الخلاصه:
تلعب المقاييس المخلوعة دورا مهما في البرمجة المنطقية. علاوة على ذلك، فهي محورية في مجالات مثل التبولوجيا والهندسة الإلكترونية. تأتي أهمية هذه المساحة كونها تحتوي على خاصية تعديل قيمة المسافة الذاتية. هذه الميزة جعلت العديد من المؤلفين يجرون تحقيقات مكثفة في هذا المقياس، حيث قاموا بفحص النقاط الثابتة للدوال الموجودة في هذا الفضاء والتي تلبي متطلبات محددة، ودرسوا الخصائص التي تميزه. بالإضافة إلى ذلك، نظروا فيما إذا كانت الدوال الانكماشية نوع R تتمتع بأفضل قرب في إطار الفضاءات المترية المخلوعة، وقاموا بوضع متطلبات محددة لضمان أن أفضل نقطة قرب لهذه الانقباضات تكون وحيدة.

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كذلك عمل الباحثون على تعميم مفهوم الفضاء المتري المخلوع ليشمل الفضاء المتري المخلوع b، والفضاء المتري المخلوع S، وأنواع أخرى من البناءات المترية.
تقدم هذه الورقة مفهوم الفضاء المتري المخلوع Ps ونوع من دوال القرب الانكماشيةPs وايضا نوع من دوال القرب الانكماشية المعممة Ps. أيضا، قمنا بدراسة وجود أفضل نقطة قرب للدوال الغير ذاتية باستعمال دوال القرب نوع R الانكماشية المعممةα_{Ps} وإعطاء الشروط اللازمة لتكون نقطة القرب وحيده. بعد ذلك، استخدمنا هذه النتائج لاشتقاق نظريات أفضل نقطة قرب للفضاء المتري المخلوع Ps والمرتب ترتيبا جزئيا، وقدمنا مثالا يوضح هذه المفاهيم في الفضاء المتري المخلوع Ps .

1-Introduction and Preliminaries

In light of the importance of metric spaces in mathematical analysis, researchers have undertaken extensive studies of them. They have developed new types of metric spaces. One of these spaces is dislocated metric space, and the most distinguishing feature is that self-distance does not always equal zero. This hallmark of this space made it, since Seda and Hitzler [1] introduced it in 2000, very important in topology, electronics engineering, and many other important fields [2]. This has motivated many researchers to explore the subject in depth [3-5].

Another space is the S-metric space. This space was introduced in 2012 [6] by Shaban Sedghi, who gave it some properties of a multivalued map. Many researchers have been interested in this space [7-13]. Also, a b-metric space was proposed by Bakhtin [14] and refined by Czerwik [15], and others have worked around it, see [16-18]. For example, in 2019, Parvaneh and Hosseini [19] introduced the notion of p-metric space, an extension of the b-metric space. Another concept that plays a very important role in the mathematical analysis is the concept of a fixed point, which began in 1922 [20] when Banach advertised his famous theorem about the contraction principle. Since then, many authors have worked on the expansion of this theorem; see [21-32]. The work of researchers has led to expanding the concept of the fixed point, and they have obtained a new concept, which is a best proximity point. In this work, we generalize the results in [33] to a new space called the dislocated metric space and examine some findings that have been investigated in other spaces, see [34-45].

Now, we give the following concepts

Definition 1.1: Let $\mathrm{D} \neq \emptyset$. A map $d_{\mathrm{Ps}}: \mathrm{D} \times \mathrm{D} \times \mathrm{D} \rightarrow [0, \infty)$ is called Ps $-$dislocated (dis) metric if $\exists\, \Omega: [0, \infty) \rightarrow [0, \infty)$ continuous such that $\Omega(\mathrm{s}) \leq \mathrm{s}, \forall \mathrm{s} \in [0, \infty)$ and $\Omega(0) = 0$, $\forall\, \mathrm{u}, \mathrm{v}, \mathrm{w} \in \mathrm{D}$, we have

1)If $d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{v}, \mathrm{w}) = 0$ then $\mathrm{u} = \mathrm{v} = \mathrm{w}$,

2)$d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{v}) = d_{\mathrm{Ps}}(\mathrm{v}, \mathrm{v}, \mathrm{u})$,

3)$d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{v}, \mathrm{w}) \leq \Omega(d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{e}) + d_{\mathrm{Ps}}(\mathrm{v}, \mathrm{v}, \mathrm{e}) + d_{\mathrm{Ps}}(\mathrm{w}, \mathrm{w}, \mathrm{e}))$, $\mathrm{e} \in \mathrm{D}$.

Then $(\mathrm{D}, d_{\mathrm{Ps}})$ is Ps $-$dis metric space.

Definition 1.2: A map $\mathrm{L}: (\mathrm{D}, d_{\mathrm{Ps}}) \rightarrow (\mathrm{D}, d_{\mathrm{Ps}})$ is Ps-continuous if $\forall \langle \mathrm{u}_n \rangle$ convergence to $\mathrm{u} \in \mathrm{D}$, implies that $\lim_{n\rightarrow\infty} d_{\mathrm{Ps}}(\mathrm{Lu}_n, \mathrm{Lu}_n, \mathrm{Lu}) = d_{\mathrm{Ps}}(\mathrm{Lu}, \mathrm{Lu}, \mathrm{Lu})$.

Definition 1.3: Let $\emptyset \neq \mathrm{U}$, and S be subsets of Ps-dis metric space $(\mathrm{D}, d_{\mathrm{Ps}})$ and $\mathrm{L}: \mathrm{U} \rightarrow \mathrm{S}$ be a map. An element $\mathrm{u} \in \mathrm{U}$ is called a best proximity (prox) point of L if

$d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{Lu}) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S})$. And:

$d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}) = \inf\{d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, s): \mathrm{u} \in \mathrm{U}, s \in \mathrm{S}\}$.

$\mathrm{u} \in \mathrm{U}, d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{S}) = \inf\{d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, s): s \in \mathrm{S}\}$.

$\mathrm{U}_\circ = \{\mathrm{u} \in \mathrm{U}: d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, s) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S})$, for some $s \in \mathrm{S}\}$.

$\mathrm{S}_\circ = \{s \in \mathrm{S}: d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, s) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S})$, for some $\mathrm{u} \in \mathrm{U}\}$.

Definition 1.4: [33] A map $Z: [0,\infty) \times [0,\infty) \to \mathbb{R}$ is called a simulation if:
1- $Z(0,0) = 0$.
2- $Z(\mathrm{v}, \mathrm{h}) < \mathrm{h} - \mathrm{v}$ that for all $\mathrm{v}, \mathrm{h} > 0$.
3-If $\langle \mathrm{v}_n \rangle$, $\langle \mathrm{h}_n \rangle$ are sequences in $(0,\infty)$ such that
$\lim_{n\to\infty} \mathrm{v}_n = \lim_{n\to\infty} \mathrm{h}_n > 0 \Longrightarrow \lim_{n\to\infty} sup\ Z(\mathrm{v}_n, \mathrm{h}_n) < 0$.

Definition 1.5: [34] Let $\mathrm{U} \subseteq \mathbb{R}$. A function $\mathrm{R}: \mathrm{U} \times \mathrm{U} \to \mathbb{R}$ is an R-function if it satisfies
1-If $\langle \mathrm{u}_n \rangle \subset (0,\infty) \cap \mathrm{U}$ such that $\mathrm{R}(\mathrm{u}_{n+1}, \mathrm{u}_n) > 0\ \ \forall\, n \in N$, then $\langle \mathrm{u}_n \rangle \to 0$.
2-If $\langle \mathrm{u}_n \rangle$, $\langle \mathrm{v}_n \rangle \subset (0,\infty) \cap \mathrm{U}$ are two sequences converging to the same limit.
$b \geq 0$ and $\mathrm{b} < \mathrm{u}_n$ and $\mathrm{R}(\mathrm{u}_n, \mathrm{v}_n) > 0, \forall\ n \in N$, then $b = 0$.
The set of all R-function R denoted by $\mathrm{R}_{\boldsymbol{c}}$.
A function $\mathrm{R} \in \mathrm{R}_{\boldsymbol{c}}$ is called a strong R-function if:
3-If $\langle \mathrm{u}_n \rangle, \langle \mathrm{v}_n \rangle \subset (0,\infty) \cap \mathrm{U}$ such that $\langle \mathrm{v}_n \rangle \to 0$ and $\mathrm{R}(\mathrm{u}_n, \mathrm{v}_n) > 0\ \forall\ n \in N$ then $\langle \mathrm{u}_n \rangle \to 0$.
Let $\mathrm{R}_{\boldsymbol{c}}^{s}$ denote the set of all strong R-function R.

Definition 1.6: Let $\alpha_{\mathrm{Ps}}: \mathrm{U} \times \mathrm{U} \times \mathrm{U} \to [0,\infty)$. A map $\mathrm{L}: \mathrm{U} \to \mathrm{S}$ is called α_{Ps}-prox admissible if $\forall\ \mathrm{u}_1, \mathrm{u}_2, \mathrm{w}_1, \mathrm{w}_2 \in \mathrm{U}$:

$$\begin{cases} \alpha_{\mathrm{Ps}}(\mathrm{u}_1, \mathrm{u}_1, \mathrm{u}_2) \geq 1; \\ d_{\mathrm{Ps}}(\mathrm{w}_1, \mathrm{w}_1, \mathrm{Lu}_1) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}) \\ d_{\mathrm{Ps}}(\mathrm{w}_2, \mathrm{w}_2, \mathrm{Lu}_2) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}). \end{cases} \Longrightarrow \alpha_{\mathrm{Ps}}(\mathrm{w}_1, \mathrm{w}_1, \mathrm{w}_2) \geq 1;$$

It is called a triangular α_{Ps}-admissible (trianα_{Ps}-admis) map if
$\mathrm{u}, \mathrm{s}, \mathrm{v} \in \mathrm{U},\ \alpha_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{s}) \geq 1$ and , $\alpha_{\mathrm{Ps}}(\mathrm{s}, \mathrm{s}, \mathrm{v}) \geq 1 \Longrightarrow \alpha_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{v}) \geq 1$.

Definition 1.7: Let $\alpha_{\mathrm{Ps}}: \mathrm{U} \times \mathrm{U} \times \mathrm{U} \to [0,\infty)$. A map $\mathrm{L}: \mathrm{U} \to \mathrm{S}$ is called α_{Ps}-prox R-contraction (cont) if $\forall\ \mathrm{u}, \mathrm{v}, \mathrm{w}_1, \mathrm{w}_2 \in \mathrm{U}$:

$$\begin{cases} \alpha_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{v}) \geq 1; \\ d_{\mathrm{Ps}}(\mathrm{w}_1, \mathrm{w}_1, \mathrm{Lu}) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}) \\ d_{\mathrm{Ps}}(\mathrm{w}_2, \mathrm{w}_2, \mathrm{Lv}) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}). \end{cases} \Longrightarrow \mathrm{R}\big(d_{\mathrm{Ps}}(\mathrm{w}_1, \mathrm{w}_1, \mathrm{w}_2), d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{v})\big) > 0;$$

Definition 1.8: A map $\mathrm{L}: \mathrm{U} \to \mathrm{S}$ is called generalized α_{Ps}-prox con R-cont if $\forall\ \mathrm{u}, \mathrm{v}, \mathrm{w}_1, \mathrm{w}_2 \in \mathrm{U}$, where $\alpha_{\mathrm{Ps}}: \mathrm{U} \times \mathrm{U} \times \mathrm{U} \to [0,\infty)$ we have

$$\begin{cases} \alpha_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{v}) \geq 1; \\ d_{\mathrm{Ps}}(\mathrm{w}_1, \mathrm{w}_1, \mathrm{Lu}) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}) \\ d_{\mathrm{Ps}}(\mathrm{w}_2, \mathrm{w}_2, \mathrm{Lv}) = d_{\mathrm{Ps}}(\mathrm{U}, \mathrm{U}, \mathrm{S}). \end{cases} \Longrightarrow \mathrm{R}\big(d_{\mathrm{Ps}}(\mathrm{w}_1, \mathrm{w}_1, \mathrm{w}_2), \mathrm{M}(\mathrm{u}, \mathrm{u}, \mathrm{v})\big) > 0;$$

Where $\mathrm{R} \in \mathrm{R}_{\boldsymbol{c}}$ and $\mathrm{M}(\mathrm{u}, \mathrm{u}, \mathrm{v}) = \max\{d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{w}_1), d_{\mathrm{Ps}}(\mathrm{v}, \mathrm{v}, \mathrm{w}_2), d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{v})\}$.

Definition 1.9: The set S is said to be approximately compact if $\forall\ \langle \mathrm{u}_n \rangle$ of C has a convergent subsequence, where C denotes all sequence $\langle \mathrm{u}_n \rangle$ of S satisfying $d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{u}_n) = d_{\mathrm{Ps}}(\mathrm{u}, \mathrm{u}, \mathrm{S})$.

Definition 1.10: We say that $(\mathrm{D}, d_{\mathrm{Ps}}, \leq)$ is regular if for every nondecreasing sequence $\langle \mathrm{u}_n \rangle \subset \mathrm{D}$ such that $\mathrm{u}_n \to \mathrm{u} \in \mathrm{D}$, as $n \to \infty$, there exists a subsequence $\langle \mathrm{u}_{n_k} \rangle$ of $\langle \mathrm{u}_n \rangle$ such that $\mathrm{u}_{n_k} \leq \mathrm{u}$ for all k.

2-Main Results

will remember the following theorem and prove new theorems based on the order property.

Theorem 2.1: Let $(\mathrm{D}, d_{\mathrm{Ps}})$ be a Ps-complete Ps-dis metric space and let U and S be Ps - closed in D such that S is approximately compact with respect to U and $\mathrm{L}: \mathrm{U} \to \mathrm{S}$ be a trian generalized α_{Ps}-prox cont admis R, with $\mathrm{L}(\mathrm{U}_\circ) \subseteq \mathrm{S}_\circ$ such that:

1- $R(p,q) < q - p \ \forall\, p, q > 0$.
2- $\exists u_\circ, u_1 \in U_\circ$ such that $d_{Ps}(u_1, u_1, Lu_\circ) = d_{Ps}(U,U,S), \alpha_{Ps}(u_\circ, u_\circ, u_1) \geq 1$.
3- L is Ps-continuous or, if $\langle u_n \rangle$ is a sequence in U such that $\alpha_{Ps}(u_n, u_n, u_{n+1}) \geq 1 \ \forall\, n \in N$, $u_n \to u$, when $n \to \infty$ for some $u \in U$. Then $\alpha_{Ps}(u_n, u_n, u) \geq 1 \forall\ n \in N$. And, L has the best prox point $f \in U$ that $d_{Ps}(f, f, L f) = d_{Ps}(U, U, S)$ also,
4- $d_{Ps}(f, f, f) = 0 \ \forall\, (u, u, r) \in U_\circ \times U_\circ \times U_\circ \exists\, h \in U_\circ$ s.t $\alpha_{Ps}(u, u, h) \geq 1, \alpha_{Ps}(r, r, h) \geq 1$, then L has a unique best prox point.

Proof: Let $u_\circ, u_1 \in U_\circ$, $d_{Ps}(u_1, u_1, Lu_\circ) = d_{Ps}(U, U, S)$ with , $\alpha_{Ps}(u_\circ, u_\circ, u_1) \geq 1$.
We construct $\langle u_n \rangle$ such that
$d_{Ps}(u_{n+1}, u_{n+1}, Lu_n) = d_{Ps}(U, U, S)$ with $\alpha_{Ps}(u_n, u_n, u_{n+1}) \geq 1$, if $u_{n+1} = u_n \Longrightarrow$ L has the best prox point. If not, then $u_{n+1} \neq u_n \forall\, n \in N_\circ$. (1)
By $\alpha_{Ps}(u_n, u_n, u_{n+1}) \geq 1$
$d_{Ps}(u_n, u_n, Lu_{n-1}) = d_{Ps}(u_{n+1}, u_{n+1}, Lu_n) = d_{Ps}(U, U, S)$

$$R(d_{Ps}(u_n, u_n, u_{n+1}), M(u_{n-1}, u_{n-1}, u_n)) > 0. \tag{2}$$

When

$$M(u_{n-1}, u_{n-1}, u_n) = max\{d_{Ps}(u_{n-1}, u_{n-1}, u_n), d_{Ps}(u_n, u_n, u_{n+1})\}. \tag{3}$$

If for some $n_\circ \in N$ we have:
If $max\{d_{Ps}(u_{n_\circ-1}, u_{n_\circ-1}, u_{n_\circ}), d_{Ps}(u_{n_\circ}, u_{n_\circ}, u_{n_\circ+1})\} = d_{Ps}(u_{n_\circ}, u_{n_\circ}, u_{n_\circ+1})$.
We get $R\,(d_{Ps}(u_{n_\circ}, u_{n_\circ}, u_{n_\circ+1}), d_{Ps}(u_{n_\circ}, u_{n_\circ}, u_{n_\circ+1})) > 0$. (4)
Then $max\{d_{Ps}(u_{n-1}, u_{n-1}, u_n), d_{Ps}(u_n, u_n, u_{n+1})\} = d_{Ps}(u_{n-1}, u_{n-1}, u_n) \quad \forall n \in N$
$R(d_{Ps}(u_n, u_n, u_{n+1}), d_{Ps}(u_{n-1}, u_{n-1}, u_n)) > 0 \quad \forall n$

$$\text{So,} \lim_{n\to\infty} d_{Ps}(u_n, u_n, u_{n+1}) = 0. \tag{5}$$

Thus, $\forall \xi > 0$ and subsequence$\langle u_{n_k} \rangle$ and $\langle u_{m_k} \rangle$ let $m_k > n_k > k$ with
$d_{Ps}(u_{n_k}, u_{n_k}, u_{m_k-1}) < \xi \leq d_{Ps}(u_{n_k}, u_{n_k}, u_{m_k}) \ \forall k \in N$.
Then, triangle inequality implies
$\langle u_k \rangle = \langle d_{Ps}(u_{n_k}, u_{n_k}, u_{m_k}) \rangle \to \xi$ and $\langle v_k \rangle = \langle d_{Ps}(u_{n_k-1}, u_{n_k-1}, u_{m_k-1}) \rangle \to \xi$.
Also, from trian α_{Ps}-admis we get $\alpha_{Ps}(u_n, u_n, u_m) \geq 1, \forall n \geq m$. So,
$R(u_k, v_k) = R(d_{Ps}(u_{n_k}, u_{n_k}, u_{m_k}), d_{Ps}(u_{n_k-1}, u_{n_k-1}, u_{m_k-1})) > 0$
$\forall\, k \in N$, since $\xi < u_k \Longrightarrow \xi = 0$, which contradicts. Then $\langle u_n \rangle \subset U$ is a Ps-Cauchy sequence. There exists $c, f \in U$ such that by (5) we get

$$d_{Ps}(c, c, f) = \lim_{n\to\infty} d_{Ps}(u_n, u_n, f) = \lim_{\substack{n\to\infty \\ m\to\infty}} d_{Ps}(u_n, u_n, u_m) = 0. \tag{6}$$

If L is Ps-continuous we have

$$d_{Ps}(L\,c, L\,c, Lf) = \lim_{n\to\infty} d_{Ps}(Lu_n, Lu_n, Lf) = \lim_{\substack{n\to\infty \\ m\to\infty}} d_{Ps}(Lu_n, Lu_n, Lu_m) = 0$$

$d_{Ps}(U, U, S) = \lim_{n\to\infty} d_{Ps}(u_{n+1}, u_{n+1}, Lu_n) = d_{Ps}(c, c, Lf)$. That is, $f \in U$ is a best prox point of L. Next,
$d_{Ps}(c, c, S) \leq d_{Ps}(c, c, Lu_n)$
$\leq \Omega\,(d_{Ps}(c, c, u_{n+1}) + d_{Ps}(c, c, u_{n+1}) + d_{Ps}(Lu_n, Lu_n, u_{n+1}))$
$= 2d_{Ps}(c, c, u_{n+1}) + d_{Ps}(U, U, S)$
$\leq 2d_{Ps}(c, c, u_{n+1}) + d_{Ps}(c, c, S)$.
When $n \to \infty$ then, $\lim_{n\to\infty} d_{Ps}(c, c, u_{n+1}) = 0$. And
$d_{Ps}(c, c, S) \leq \lim_{n\to\infty} d_{Ps}(c, c, Lu_n) \leq d_{Ps}(c, c, S) \Longrightarrow \lim_{n\to\infty} d_{Ps}(c, c, Lu_n) = d_{Ps}(c, c, S)$.
Therefore, $\langle Lu_n \rangle$ has a subsequence $\langle Lu_{n_i} \rangle$ such that
$\lim_{n\to\infty} Lu_{n_i} = h$ for some $h \in S$. Then
$d_{Ps}(c, c, h) = \lim_{n\to\infty} d_{Ps}(u_{n_i}, u_{n_i}, Lu_{n_i}) = d_{Ps}(U, U, S)$.We obtain $d_{Ps}(w, w, Lf) = d_{Ps}(U, U, S)$,

$w \in U$.

Now,

$$\begin{cases} \alpha_{Ps}(u_n, u_n, f) \geq 1; \\ d_{Ps}(u_{n+1}, u_{n+1}, Lu_n) = d_{Ps}(U, U, S) \Longrightarrow R\,(d_{Ps}(u_{n+1}, u_{n+1}, w), M(u_n, u_n, f)) > 0; \\ d_{Ps}(w, w, Lf) = d_{Ps}(U, U, S). \end{cases}$$

$$M(u_n, u_n, f) > d_{Ps}(u_{n+1}, u_{n+1}, w).$$

As $n \to \infty \Longrightarrow d_{Ps}(f, f, w) < M(f, f, f) < max\{d_{Ps}(f, f, f), d_{Ps}(f, f, w)\}$.

As $n \to \infty$,we have $f = w$ .Then, L has the best prox point f. Finally, to prove the uniqueness. Let $f_1 \neq f_2 \in U$ are two best prox points of L. So,

$$d_{Ps}(f_1, f_1, Lf_1) = d_{Ps}(f_2, f_2, Lf_2) = d_{Ps}(U, U, S).$$

If $d_{Ps}(f_1, f_1, f_2) \neq 0$, then we have two cases:

1- If $\alpha_{Ps}(f_1, f_1, f_2) \geq 1 \Longrightarrow R(d_{Ps}(f_1, f_1, f_2), d_{Ps}(f_1, f_1, f_2)) > 0$. That is a contradiction.

2- Or $\alpha_{Ps}(f_1, f_1, f_2) < 1 \Longrightarrow \exists \langle c_n \rangle \in U_\circ$ such that

$\alpha_{Ps}(f_1, f_1, c_n) \geq 1;$

$\alpha_{Ps}(f_2, f_2, c_n) \geq 1.$

Since $L(U_\circ) \subseteq S_\circ \to Lc_n \subset S_\circ$, then by Definition 1.7:

$d_{Ps}(f_1, f_1, Lf_1) = d_{Ps}(c_{n+1}, c_{n+1}, Lc_n) = d_{Ps}(U, U, S) \ \forall n \in N_\circ$.

By R-cont $\to R(d_{Ps}(f_1, f_1, c_{n+1}), d_{Ps}(f_1, f_1, c_n)) > 0$, by Definition 1.5:

$\lim_{n\to\infty} d_{Ps}(f_1, f_1, c_n) = 0$. Similar, $\lim_{n\to\infty} d_{Ps}(f_2, f_2, c_n) = 0$.

Then $d_{Ps}(f_1, f_1, f_2) \leq \Omega(d_{Ps}(f_1, f_1, c_n) + d_{Ps}(f_1, f_1, c_n) + d_{Ps}(f_2, f_2, c_n))$.

When $n \to \infty$ we get $d_{Ps}(f_1, f_1, f_2) \leq \Omega(0) = 0 \Longrightarrow d_{Ps}(f_1, f_1, f_2) = 0$.

Then L has a unique best proximity point.■

Definition 2.2: Let $(D, d_{Ps}, \leq)$ be a Ps-dis ordered metric space. A map $L: U \to S$ is prox ordered $\Longleftrightarrow \forall u_1, u_2, r_1, r_2 \in U$ we have

$$\begin{cases} r_1 \leq r_2; \\ d_{Ps}(u_1, u_1, Lr_1) = d_{Ps}(U, U, S) \quad \Longrightarrow u_1 \leq u_2; \\ d_{Ps}(u_2, u_2, Lr_2) = d_{Ps}(U, U, S). \end{cases}$$

Theorem 2.3: Let U be Ps-closed, S approximate compact respect to U lies in $(D, d_{Ps}, \leq)$, and L be a prox ordered with $L(U_\circ) \subseteq S_\circ$. If

1-$R(p, q) < q - p \ \forall\, p, q > 0$.

2- $\exists\, u_\circ, u_1 \in U_\circ$ s.t $d_{Ps}(u_1, u_1, Lu_\circ) = d_{Ps}(U, U, S)$.

3- $\forall\, u, r, w_1, w_2 \in U$,

$$\begin{cases} u \leq r, u \neq r; \\ d_{Ps}(w_1, w_1, Lu) = d_{Ps}(U, U, S) \quad \Longrightarrow R(d_{Ps}(w_1, w_1, w_2), M(u, u, r)) > 0; \\ d_{Ps}(w_2, w_2, Lr) = d_{Ps}(U, U, S), \end{cases}$$

where $M(u, u, r) = max\{d_{Ps}(u, u, w_1), d_{Ps}(r, r, w_2), d_{Ps}(u, u, r)\}$.

4-L be Ps-continuous, or$(D, d_{Ps}, \leq)$ is regular. Then L has the best prox point $f \in U$, that $d_{Ps}(f, f, Lf) = d_{Ps}(U, U, S)$.

5-If $d_{Ps}(f, f, f) = 0 \ \forall (u, u, r) \in U_\circ \times U_\circ \times U_\circ, \exists\ h. \in U_\circ$ s.t $u \leq h$, and $r \leq h$.

Then L has a unique best prox point.

Proof: Define $\alpha_{Ps}: U \times U \times U \to [0, \infty)$ as $\alpha_{Ps}(u, u, r) = \begin{cases} 1 & u \leq r; \\ 0 & \text{otherwise.} \end{cases}$

$$\begin{cases} \alpha_{Ps}(u, u, r) \geq 1; \\ d_{Ps}(w_1, w_1, Lu) = d_{Ps}(U, U, S); \\ d_{Ps}(w_2, w_2, Lr) = d_{Ps}(U, U, S). \end{cases}$$

$$\begin{cases} u \le r; \\ d_{Ps}(w_1, w_1, Lu) = d_{Ps}(U, U, S); \\ d_{Ps}(w_2, w_2, Lr) = d_{Ps}(U, U, S). \end{cases}$$

So, $w_1 \le w_2$ and $\alpha_{Ps}(w_1, w_1, w_2) \ge 1$. Then we derive L is generalized α_{Ps}-prox-cont R.Then, all conditions above Theorem 2.1 hold. So, L has a unique best prox point.■

Example 2.4: Let $D = \{(0,2), (2,0), (0,-2), (-2,0)\}$.
Define $\le$ on D , by
$(u_1, v_1, r_1) \le (u_2, v_2, r_2) \Leftrightarrow u_1 \le u_2$, $v_1 \le v_2$ and $r_1 \le r_2$
$d_{Ps}: D \times D \times D \longrightarrow [0, \infty)$

$$d_{Ps}((u_1, v_1), (u_1, v_1), (u_2, v_2)) = \begin{cases} 0 & (u_1, v_1) = (u_2, v_2) \in \{(0,2), (2,0)\}; \\ \frac{1}{2} & u_1 + u_2 = v_1 + v_2 \ ; \\ 2 & \text{Otherwise}. \end{cases}$$

The $(D, d_{Ps}, \le)$ is a complete dis ordered metric space.
Let $U = \{(0,2), (0,-2)\}$, $S = \{(2,0), (-2,0)\}$. Then $d_{Ps}(U, U, S) = \frac{1}{2}$ and $U = U_\circ$, $S = S_\circ$.
Let $L: U \longrightarrow S$, such that $L(u, v) = (v, u)$. Also, $L(U_\circ) \subseteq S_\circ$.Then

$$\begin{cases} u \le v, u \ne v; \\ d_{Ps}(w_1, w_1, Lu) = d_{Ps}(U, U, S) = \frac{1}{2} \\ d_{Ps}(w_2, w_2, Lv) = d_{Ps}(U, U, S) = \frac{1}{2}; \end{cases} \Longrightarrow R(d_{Ps}(w_1, w_1, w_2), M(u, u, v)) > 0;$$

$$\begin{cases} (0,-2) < (0,2); \\ d_{Ps}(w_1, w_1, L(0,-2)) = d_{Ps}(U, U, S) = \frac{1}{2}; \\ d_{Ps}(w_2, w_2, L(0,2)) = d_{Ps}(U, U, S) = \frac{1}{2}; \end{cases}$$

$$\begin{cases} (0,-2) < (0,2) \\ d_{Ps}((0,-2), (0,-2), (-2,0)) = d_{Ps}(U, U, S) = \frac{1}{2}; \\ d_{Ps}((0,2), (0,2), (2,0)) = d_{Ps}(U, U, S) = \frac{1}{2}. \end{cases}$$

$$M(u, u, v) = \max\{d_{Ps}((0,-2), (0,-2), (0,-2)), d_{Ps}((0,2), (0,2), (0,2)), d_{Ps}((0,-2), (0,-2), (0,2))\}$$
$$= \max\left\{2, 0, \frac{1}{2}\right\} = 2.$$

Then $w_1 = (0,-2)$, $w_2 = (0,2)$ and $d_{Ps}(w_1, w_1, w_2) = \frac{1}{2}$ and $w_1 < w_2$
$R: [0, \infty) \times [0, \infty) \to \mathbb{R}$. By

$$R(p, q) = \begin{cases} \frac{1}{2}q - p \ , & p < q; \\ -1 & q = p = 0; \\ 0 & \text{o.w} \end{cases}$$

$$R(d_{Ps}(w_1, w_1, w_2), M(u, u, v))$$
$$R\left(\frac{1}{2}, 2\right) = \frac{1}{2}q - p = \frac{1}{2}(2) - \frac{1}{2} = \frac{1}{2} > 0.$$

Thus $(0,-2), (0,2)$ are best proximity points of L.

Definition 2.5: [35] Let $L: D \to D$ be a map and $\alpha_{Ps}: D \times D \times D \to [0, \infty)$ be a function. We say L is an α_{Ps}-admis map if $u, r \in D$, $\alpha_{Ps}(u, u, r) \ge 1 \Longrightarrow \alpha_{Ps}(Lu, Lu, Lr) \ge 1$.

Also, L is called α_{Ps}-orbital admi map if u , r ∈ D
$\alpha_{Ps}(u, u, Lu) \geq 1 \Rightarrow \alpha_{Ps}(Lu, Lu, L^2u) \geq 1$, when $L^2 = LoL$.
Moreover, L is called trian α_{Ps}-orbital admis if L is α_{Ps}-orbital admis and $\alpha_{Ps}(u, u, r) \geq 1, \alpha_{Ps}(r, r, Lr) \geq 1 \Rightarrow \alpha_{Ps}(u, u, Lr) \geq 1$.
Theorem 2.6: Let (D, d_{Ps}) be a Ps-complete. If $L: D \to D$ is non-decreasing and trianr α_{Ps}-orbital admi such that:
1- $R(p,q) < q - p \; \forall \, p, q > 0$;
2- $\exists u_\circ \in D$ s.t $\alpha_{Ps}(u_\circ, u_\circ, Lu_\circ) \geq 1$;
3- $\forall u, r \in D$ such that $\alpha_{Ps}(u, u, r) \geq 1$ we have $R(d_{Ps}(Lu, Lu, Lr), M(u, u, r)) > 0$,
where $M(u, u, r) = \max\{d_{Ps}(u, u, r), d_{Ps}(u, u, Lu), d_{Ps}(r, r, Lr)\}$.
4- L is Ps-continuous. Then L has a fixed point
5-When f is a fixed point, then for a $(f_1, f_1, f_2) \in D \times D \times D$, $\exists h \in D$ such that $\alpha_{Ps}(f_1, f_1, h) \geq 1$and
$\alpha_{Ps}(f_2, f_2, h) \geq 1$.Then L has a unique fixed point.
Proof: Let $u_\circ, u_1 \in D \Rightarrow \alpha_{Ps}(u_\circ, u_\circ, Lu_\circ) \geq 1, \alpha_{Ps}(u_\circ, u_\circ, u_1) \geq 1$.
Since $L: D \to D$ Then $L(D) \subseteq D$.
By the assumption (3), we get $d_{Ps}(Lu_\circ, Lu_\circ, Lu_1) = d_{Ps}(D, D, D) = 0$.
$$d_{Ps}(Lu_1, Lu_1, Lu_1) = d_{Ps}(D, D, D) = 0.$$
$R(d_{Ps}(Lu_\circ, Lu_\circ, Lu_1), M(u_\circ, u_\circ, u_1)) > 0$.
Then L is generalized α_{Ps}-prox R-cont. All conditions of Theorem 2.1 hold; therefore, L has a unique best prox point $c \in D$, such that $d_{Ps}(c, c, Lc) = d_{Ps}(D, D, D) = 0 \Rightarrow c = L\,c$.
Then L has a unique fixed point. ■
Theorem 2.7: Let $(D, d_{Ps}, \leq)$ be a partially ordered Ps-complete. If
1-$R(p,q) < q - p \; \forall \, p, q > 0$;
2- $\exists \, u_\circ \in D$, s.t $u_\circ \leq Lu_\circ$;
3- $\forall u, r \in D$, we have $u \leq r, u \neq r \Rightarrow R(d_{Ps}(Lu, Lu, Lr), M(u, u, r)) > 0$,
where $M(u, u, r) = max\{d_{Ps}(u, u, r), d_{Ps}(u, u, Lu), d_{Ps}(r, r, Lr)\}$.
4-L is Ps-continuous or $(D, d_{Ps}, \leq)$ is regular.
Then L has a fixed point.
5-If f is a fixed point, then for a $(f_1, f_1, f_2) \in D \times D \times D \;\; \exists h \in D$ s.t $f_1 \leq h, f_2 \leq h$.
Then L has a unique fixed point.
Proof: Let $u_\circ \in D$, , the assumption (2) $u_\circ \leq Lu_\circ$, if $u_\circ = Lu_\circ$. Then $u_\circ$ is a fixed point. Let $u_\circ \neq Lu_\circ$ (we suppose $Lu_\circ = u_1$) we get:
$u_\circ \leq u_1, u_\circ \neq u_1$ and
$d_{Ps}(Lu_\circ, Lu_\circ, Lu_\circ) = d_{Ps}(D, D, D) = 0$.
$d_{Ps}(Lu_1, Lu_1, Lu_1) = d_{Ps}(D, D, D) = 0$.
Since $R(d_{Ps}(Lu_\circ, Lu_\circ, Lu_1), M(u_\circ, u_\circ, u_1)) > 0$.
Then L is a prox order-preserving respect to the Ps-dislo metric. All conditions of the above theorem hold; therefore, L has a unique best prox point $c \in D$. ■

3- Conclusions

In this paper, a Ps-dis metric space, Ps-proximal contraction map, and the generalized Ps-prox cont map are introduced. Also, the existence and uniqueness of a best approximation point for the non-self-map in this space are studied. Then, we used these results to obtain theorems of a best approximation point on Ps-dis metric spaces with partial ordering. Also, an example to clarify these concepts and show that if the condition of uniqueness is not satisfied, we will get a best proximity point, but it does not necessarily have to be unique.

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