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Approximate Solutions of Nonlinear Smoking Habit Model

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Abstract:

The work in this paper focuses on solving numerically and analytically a nonlinear social epidemic model that represents an initial value problem of ordinary differential equations. A recent smoking habit model from Spain is applied and studied here. The accuracy and convergence of the numerical and approximation results are investigated for various methods; for example, Adomian decomposition, variation iteration, Finite difference and Runge-Kutta. The discussion of the present results has been tabulated and graphed. Finally, the comparison between the analytic and numerical solutions from the period 2006-2009 has been obtained by absolute and difference measure error.

Keywords: Ordinary differential equations, Adomian decomposition method, Variation iteration method, Finite difference method, Runge-Kutta method, social epidemiology.

الحلول التقريبية للنموذج اللاخطي لعادة التدخين

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الخلاصة

الهدف من هذه الدراسة هو حل النموذج الوبائي الغير خطي عدديا وتحليليا والذي هو من مسائل القيم الابتدائية للمعادلات التفاضلية الاعتيادية. نموذج عادة التدخين في اسبانيا هو تطبيق لدراستنا الحالية. تم استخدام طريقتان تحليليتان هما طريقة ادمين و التغيرات والفروقات النسبية ورائكا كوتا لحل نموذج عادة التدخين. تم جدولة مناقشة النتائج الحالية ورسمها بيانيا. اخيرا، تم المقارنة بين الحلول التحليلية والعددية للطرق المستخدمة من سنة 2006 الى 2009 التي تم الحصول عليها بواسطة الخطأ المطلق ومقياس الفرق.

1. Introduction

Social epidemiological models are studied to analyze epidemic stages and infectious diseases. The advantage of the current study is to know if the the social habits under study is epidemically extending or dwindling in the next years. Many researchers analyzed the social habit models. For example, Guerrero, Santonja and Villanueva examined the Spanish smoke-free legislation of 2006 [1]. In 2011, Sánchez *et al.* predicted the cocaine consumption in Spain [2]. Mohammed, Noor, Siri and Ibrahim created in 2018 a non-conventional hybrid numerical approach to solve the multi-dimensional random sampling for cocaine abuse in Spain [3].

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In 2010, the economic cost of alcohol consumption was studied in Spain by Santonja *et al.* [4]. The mathematical modeling of the social obesity which is epidemic in the region of Valencia in Spain was achieved in 2010 by Santonja *et al.* [5]. Mohammed, Noor, Ibrahim and Siri numerically solved the weight reduction model due to health campaigns in Spain in 2015 using several types of Runge-Kutta method [6]. In the purely hyperbolic case, an adequate definition of the numerical viscosity required by the 'WENO' scheme when capillary effects exist was provided in 2013 [7].

There are some methods used in this paper; the first one is the Adomian Decomposition Method (ADM) which is considered to be a reliable method and needs less computation to solve many linear and nonlinear different problems, such as the ordinary differential equations, partial differential equations and integral equations [8, 9]. ADM was applied on epidemic models and on the fuzzy fractional order differential algebraic equations [10, 11, 12] and it has wide applications in life. The second method is the Variational Iteration method (VIM) which was established by Ji-Huan in 1997 and is regarded as one of the reliable repetitive methods that give approximate solutions to the differential equations [13,14]. This method is widely used in scientific and engineering applications to solve linear, nonlinear, homogeneous and inhomogeneous equations, as in the autonomous ordinary differential systems in 1999 [13] and the differential equations of fractional order in 2000 [15]. Moreover, VIM is a modification of the general Lagrange multiplier method into an iteration method that is called the correction functional method [16]. The difference between ADM and VIM is that VIM does not require specific treatments for the nonlinear problems [9]. The third method is the Finite Difference Method (FDM) which is one of the approximate methods that is used to solve the different types of differential equations. Mohammed, Ibrahim, Siri, & Noor created the new method in 2019, that mixed between the Mean Monte Carlo simulation process and the finite difference numerical iteration method to sample randomly from a nonlinear epidemic model [17]. Finally, the iteration method is the Runge-Kutta for the 4th order which is a numerical technique that is used to solve the first and higher order ordinary differential equations. This method is used for high accuracy with the order $O(h^4)$ to decrease the errors [18].

The above methods are used to solve the social epidemic model under the current study. The importance of using these methods is to provide an available approximate solution to solve a nonlinear system that may have no exact solution. Moreover, these methods are reliable to give an accurate approximate solution for nonlinear systems that have multiple variables. This study is organized as follows: in Section 2, the mathematical model of smoking habit is described; Section 3 derived the analytic methods ADM and VIM; Section 4 applies the numerical methods FDM and RK4 to solve the nonlinear system of the smoking habit model used in Spain. In Section 5, the results of the presented methods are discussed tabularly and graphically. Finally, Section 6 is devoted to the conclusion of the research.

2. Mathematical Model

The current model has been used successfully to predict the evolution of the smoking habit in Spain after the Spanish smoke-free law in 2006 was applied [1]. The population consists of four types of individuals, a, b, c and d , representing non-smokers, normal smokers, excessive smokers and ex-smokers, respectively. These groups are functions of time. The governing equations for the smoking habit is given by the first order non-linear ordinary differential equations:

$$a'(t) = \mu(1 - a(t)) - \beta a(t)(b(t) + c(t)), \quad (1)$$

$$b'(t) = \beta a(t)(b(t) + c(t)) + \rho d(t) + \alpha c(t) - (\gamma + \lambda + \mu)b(t), \quad (2)$$

$$c'(t) = \gamma b(t) - (\alpha + \delta + \mu)c(t), \quad (3)$$

$$d'(t) = \lambda b(t) + \delta c(t) - (\rho + \mu)d(t). \quad (4)$$

Tables-(1 and 2) represent variables a, b, c, d and parameters $\mu, \beta, \rho, \alpha, \gamma, \delta$, respectively. Equations 1-4 have to be solved subject to the initial conditions:

$$\begin{aligned} a(0) &= 0.5045, & b(0) &= 0.2059, \\ c(0) &= 0.1559, & d(0) &= 0.1337, \end{aligned} \quad (5)$$

with the predicted parameters that are given as:

$$\begin{aligned} \mu &= 0.01, & \beta &= 0.0381, & \rho &= 0.0425, & \alpha &= 0.1244, \\ \gamma &= 0.11750, & \lambda &= 0.0498 & \text{and} & & \delta &= 0.0498. \end{aligned} \quad (6)$$

Table 1-Variables of the smoking habit model

$a(t)$	The social class of people who never smoke from the total population.
$b(t)$	The social class of people who smoke less than 20 cigarettes per day.
$c(t)$	The social class who smoke more than 20 cigarettes per day.
$d(t)$	The social class of ex-smokers.

Table 2- Parameters of smoking habit model

μ	Rate of births in Spain.
β	The transmission of smoke infection because of social pressure to adopt the smoking habit.
ρ	The rate of returns to smoking.
α	The rate of smokers who are excessively and who are becoming a normal smoker by reducing the number of cigarettes per day.
γ	The rate of smokers who are normal and who are becoming excessive smokers by increasing the number of cigarettes per day.
λ	The rate of normal smokers who stop smoking.
δ	The rate of excessive smokers who stop smoking.

3. Analytic Methods for Solving the Smoking Habit Model

3.1 Adomian Decomposition Method (ADM)

The nonlinear system of equations 1-4 of the smoking habit model can be solved by the Adomian decomposition method with the given initial conditions 5 and 6.

Let l be an operator that is given by $l = \frac{d}{dt}$ and the inverse of this operation is $l^{-1} = \int_0^t (\cdot) dt$. Then by applying l^{-1} for both sides of equations 1-4:

$$a(t) - a(0) = l^{-1} (\mu(1 - a(t)) - \beta a(t)(b(t) + c(t))), \quad a_0(t) = 0.5045.$$

$$b(t) - b(0) = l^{-1} (\beta a(t)(b(t) + c(t)) + \rho d(t) + \alpha c(t) - (\gamma + \lambda + \mu)b(t)), \quad b_0(t) = 0.2059.$$

$$c(t) - c(0) = l^{-1} (\gamma b(t) - (\alpha + \delta + \mu)c(t)), \quad c_0(t) = 0.1559,$$

$$d(t) - d(0) = l^{-1} (\lambda b(t) + \delta c(t) - (\rho + \mu)d(t)), \quad d_0(t) = 0.1337.$$

The above equations can be generated with k iterations, $k \geq 0$.

$$a_{k+1}(t) = l^{-1} (\mu(1 - a_k(t)) - \beta A_k(t) - \beta B_k(t)), \tag{7}$$

$$b_{k+1}(t) = l^{-1} (\beta A_k(t) + \beta B_k(t) + \rho d_k(t) + \alpha c_k(t) - (\gamma + \lambda + \mu)b_k(t)) \tag{8}$$

$$c_{k+1}(t) = l^{-1} (\gamma b_k(t) - (\alpha + \delta + \mu)c_k(t)), \tag{9}$$

$$d_{k+1}(t) = l^{-1} (\lambda b_k(t) + \delta c_k(t) - (\rho + \mu)d_k(t)), \text{ for all } k \geq 0. \tag{10}$$

The general forms of the non-linear terms $A_k(t)$ and $B_k(t)$ have to be:

$$A_k(t) = (\sum_{n=0}^2 a_n(t))(\sum_{n=0}^2 b_n(t)) \tag{11}$$

$$B_k(t) = (\sum_{n=0}^2 a_n(t))(\sum_{n=0}^2 c_n(t)), \quad k = 0,1,2. \tag{12}$$

The solution at $k=0$ has been determined by allowing

$$A_0(t) = a_0(t)b_0(t) \text{ and } B_0(t) = a_0(t)c_0(t), \text{ and Substituting in (5)-(8)}$$

$$a_1(t) = -0.00199932t, \quad b_1(t) = -0.00447553t,$$

$$c_1(t) = -0.00452353t, \quad d_1(t) = 0.01099838t.$$

For $k=1$,

$$A_1(t) = a_0(t)b_1(t) + b_0(t) + a_1(t), \tag{and}$$

$$B_1(t) = a_0(t)c_1(t) + c_0(t)a_1(t).$$

Substituting in (5)-(8)

$$a_2(t) = 0.01t + 0.00011026t^2, \quad b_2(t) = 0.00029629t^2, \\ c_2(t) = 0.00015368t^2, \quad d_2(t) = -0.000512785t^2.$$

Finally, for $k=2$,

$$A_2(t) = a_0(t)b_2(t) + a_1(t)b_1(t) + b_0(t)a_2(t) \\ B_2(t) = a_0(t)c_2(t) + a_1(t)c_1(t) + c_0(t)a_2(t)$$

Substituting in (5)-(8)

$$a_3(t) = 0.01t - 0.00011892t^2 - 0.00000399t^3, \\ b_3(t) = 0.00006892t^2 - 0.00001618t^3, \\ c_3(t) = 0.00000216t^3, \\ d_3(t) = 0.0498t + 0.00011028t^3.$$

The Adomian decomposition method for functions $a(t), b(t), c(t)$ and $d(t)$ is applied as follows:

$$a(t) = \sum_{k=0}^{\infty} a_k(t) = a_0(t) + a_1(t) + a_2(t) + a_3(t) + \dots \\ a(t) = 0.5045 - 0.00199932t - 0.00000866t^2 - 0.00000399t^3 + \dots \tag{13}$$

$$b(t) = \sum_{k=0}^{\infty} b_k(t) = b_0(t) + b_1(t) + b_2(t) + b_3(t) + \dots \\ b(t) = 0.2059 - 0.00447553t + 0.00036521t^2 - 0.00001618t^3 + \dots \tag{14}$$

$$c(t) = \sum_{k=0}^{\infty} c_k(t) = c_0(t) + c_1(t) + c_2(t) + c_3(t) + \dots \\ c(t) = 0.1559 - 0.00452353t + 0.00015368t^2 + 0.00000216t^3 + \dots \tag{15}$$

$$d(t) = \sum_{k=0}^{\infty} d_k(t) = d_0(t) + d_1(t) + d_2(t) + d_3(t) + \dots \\ d(t) = 0.1337 + 0.01099838t - 0.000512785t^2 + 0.00011028t^3 + \dots \tag{16}$$

3.2 Variation Iteration Method (VIM)

The nonlinear system of the smoking habit model 1-4 can be solved by the VIM with the given initial conditions 5 and 6. The correction functional for the system of equations 1-4 becomes:

$$a_{k+1}(t) = a_k(t) + \int_0^t \lambda \left(a'_k(t) - \left(\mu(1 - a_k(t)) - \beta a_k(t)(b_k(t) + c_k(t)) \right) \right), \quad k \geq 0, \tag{17}$$

$$b_{k+1}(t) = b_k(t) + \int_0^t \lambda \left(b'_k(t) - \left(\beta a_k(t)(b_k(t) + c_k(t)) + \rho d_k(t) + \alpha c_k(t) - (\gamma + \lambda + \mu)b_k(t) \right) \right), \quad k \geq 0, \tag{18}$$

$$c_{k+1}(t) = c_k + \int_0^t \lambda \left(c'_k(t) - (\gamma b_k(t) - (\alpha + \delta + \mu)c_k(t)) \right), \quad k \geq 0, \tag{19}$$

$$d_{k+1}(t) = d_k(t) + \int_0^t \lambda \left(d'_k(t) - (\lambda b_k(t) + \delta c_k(t) - (\rho + \mu)d_k(t)) \right), \quad k \geq 0. \tag{20}$$

where λ is a general Lagrange multiplier. By choosing $\lambda = -1$ and putting in 17-20 with initial condition 5 and 6, i.e $k=0$:

$$a_1(t) = 0.5045 - 0.00199932t \\ b_1(t) = 0.2059 - 0.00447554t \\ c_1(t) = 0.1559 - 0.00452354t \\ d_1(t) = 0.1337 + 0.01099838t$$

And at $k=1$:

By the same way, when $k = 1$, equations 17-19 will become the following:

$$a_2(t) = 0.5045 - 0.00199932t + 0.00011026t^2 - 2.2849871610^{-7}t^3, \\ b_2(t) = 0.2059 - 0.00447554t + 0.00024884t^2 + 2.2849871610^{-7}t^3, \\ c_2(t) = 0.1559 - 0.00452353t + 0.00015368t^2,$$

$$d_2(t) = 0.1337 + 0.01099838t - 0.00051279t^2.$$

For $k=2$:

$$a_3(t) = 0.5045 - 0.00199932t + 0.00011026t^2 - 0.00000527t^3 + 1.99202672 * 10^{-8}t^4 - 5.52507601 * 10^{-10}t^5 + 4.65135788 * 10^{-13}t^6 + 8.52541875 * 10^{-16}t^7, \tag{21}$$

$$b_3(t) = 0.2059 - 0.00447554t + 0.00024884t^2 - 0.00001228t^3 - 2.69344475 * 10^{-8}t^4 + 3.50390346 * 10^{-10}t^5 - 4.2405435 * 10^{-13}t^6 - 2.84180625 * 10^{-16}, \tag{22}$$

$$c_3(t) = 0.1559 - 0.00452354t + 0.00015368t^2 + 3.10383361 * 10^{-7}t^3 + 6.71214979 * 10^{-9}t^4, \tag{23}$$

$$d_3(t) = 0.1337 + 0.01099838t - 0.00051279t^2 + 0.00001566t^3 + 2.84480901 * 10^{-9}t^4. \tag{24}$$

The process continues in order to get a better approximation:

$$a(t) = \lim_{k \rightarrow \infty} a_k(t), b(t) = \lim_{k \rightarrow \infty} b_k(t), c(t) = \lim_{k \rightarrow \infty} c_k(t) \text{ and } d(t) = \lim_{k \rightarrow \infty} d_k(t).$$

4. Numerical Methods for Solving the Smoking Habit Model

4.1 Finite Difference Method (FDM)

The nonlinear system 1-4 of the smoking habit model can be solved using the finite difference method with the initial conditions 5 and 6 and step size $h = \{1, 0.5, 0.25\}$ such that $h = \frac{\text{Upper limit} - \text{Lower limit}}{m}$, and $m=16$ which refers to a number of years (2006-2022). In order to find $a_1(t), b_1(t), c_1(t)$ and $d_1(t)$, using Backward Finite Difference (BFDM) as follows:

$$a_1(t) = a_0(t) + h(\mu(1 - a_0(t)) - \beta a_0(t)(b_0(t) + c_0(t))), \tag{25}$$

$$b_1(t) = b(t) + h(\beta a_0(t)(b_0(t) + c_0(t)) + \rho d_0(t) + \alpha c_0(t) - (\gamma + \lambda + \mu)), \tag{26}$$

$$c_1(t) = c_0(t) + h(\gamma b_0(t) - (\alpha + \delta + \mu)c_0(t)), \tag{27}$$

$$d_1(t) = d_0(t) + h(\lambda b_0(t) + \delta c_0(t) - (\rho + \mu)d_0(t)). \tag{28}$$

After substituting (5) and (6) in equations (25)-(28), we have: $a_1(t) = 0.50250068$, $b_1(t) = 0.20142446$, $c_1(t) = 0.15137647$ and $d_1(t) = 0.14469839$.

Furthermore, using the Central Finite Difference method (CFDM) to find the other terms:

$$a_{i+1}(t) = a_{i-1}(t) + 2h(\mu(1 - a_i(t)) - \beta a_i(t)(b_i(t) + c_i(t))), \tag{29}$$

$$b_{i+1}(t) = b_{i-1}(t) + 2h(\beta a_i(t)(b_i(t) + c_i(t)) + \rho d_i(t) + \alpha c_i(t) - (\gamma + \lambda + \mu)), \tag{30}$$

$$c_{i+1}(t) = c_{i-1}(t) + 2h(\gamma b_i(t) - (\alpha + \delta + \mu)c_i(t)), \tag{31}$$

$$d_{i+1}(t) = d_{i-1}(t) + 2h(\lambda b_i(t) + \delta c_i(t) - (\rho + \mu)d_i(t)), \quad i=1,2,\dots,m \tag{32}$$

4.2 Runge-Kutta of 4th Order (RK4) Method

RK4 is one of the most accurate iteration numerical methods. The nonlinear system (1)- (4) of the smoking habit model can be solved by RK4 with the initial condition (5) and the predicted parameters (6), $i = 1, 2, \dots, m$.

$$a_{i+1} = f(t_i, a_i, b_i, c_i, d_i),$$

$$= a_i + \frac{1}{6}(ka_1 + 2ka_2 + 2ka_3 + ka_4)h,$$

$$b_{i+1} = f(t_i, a_i, b_i, c_i, d_i),$$

$$= b_i + \frac{1}{6}(kb_1 + 2kb_2 + 2kb_3 + kb_4)h,$$

$$c_{i+1} = f(t_i, a_i, b_i, c_i, d_i),$$

$$= c_i + \frac{1}{6}(kc_1 + 2kc_2 + 2kc_3 + kc_4)h,$$

$$d_{i+1} = f(t_i, a_i, b_i, c_i, d_i),$$

$$= d_i + \frac{1}{6}(kd_1 + 2kd_2 + 2kd_3 + kd_4)h.$$

where:

$$ka_1 = f_1(t_i, a_i, b_i, c_i, d_i),$$

$$kb_1 = f_1(t_i, a_i, b_i, c_i, d_i),$$

$$kc_1 = f_1(t_i, a_i, b_i, c_i, d_i),$$

$$kd_1 = f_1(t_i, a_i, b_i, c_i, d_i).$$

$$ka_1 = -0.0016, kb_1 = -0.003, kc_1 = -0.0039, kd_1 = 0.0091.$$

$$ka_2 = f_2\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right),$$

$$kb_2 = f_2\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right),$$

$$kc_2 = f_2\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right),$$

$$kd_2 = f_2\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right).$$

$$ka_2 = -0.0015, kb_2 = -0.0034, kc_2 = -0.0038, kd_2 = 0.0087.$$

$$ka_3 = f_3\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$kb_3 = f_3\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$kc_3 = f_3\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$kd_3 = f_3\left(t_i \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right).$$

$$ka_3 = -0.0015, kb_3 = -0.0034, kc_3 = -0.0038, kd_3 = 0.0087.$$

$$ka_4 = f_4(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$

$$kb_4 = f_4(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$

$$kc_4 = f_4(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$

$$kd_4 = f_4(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3).$$

$$ka_4 = -0.0014, kb_4 = -0.0033, kc_4 = -0.0036, kd_4 = 0.0083.$$

5. Results and Discussion

Approximate and numerical solutions for the nonlinear smoking habit model applied in Spain are discussed and analyzed in this section. Table 3 is to validate the real and predicted values (2006-2009) with approximate solutions, where $h=\{1, 0.5, 0.25\}$ and interval [1,3], of years under study from 2006 to 2009. In addition, table 4 demonstrates the predicted and approximate values for $a(t), b(t), c(t)$ and $d(t)$ by using ADM, VIM, FDM and RK4 for the period 2006-2022, with $h \in \{1, 0.5, 0.25\}$ in interval [1,16].

Table 3-Approximate solutions of the smoking habit model from 2006 to 2009

Model Variables	Real data [1]		Predicted values [1]		t	ADM	VIM (3 iter.)	Step size h (year)	FDM (3 iter.)	RK4 (3 iter.)
	2006	2009	2006	2009						
a(t)	0.499 7	0.4835	0.5041	0.5049	3	0.5583247 0	0.49939633	1	0.49929684	0.49940233
								0.5	0.49941218	0.49940232
								0.25	0.49940479	0.49940232
b(t)	0.185 6	-	0.1902	0.1906	3	0.1950015 8	0.19437918	1	0.19416778	0.19442567
								0.5	0.19446101	0.19442561
								0.25	0.19443454	0.19442561
c(t)	0.109 4	-	0.1264	0.1240	3	0.1437209 0	0.14372145	1	0.14356661	0.14369647
								0.5	0.14369579	0.14369651
								0.25	0.14369626	0.14369651
d(t)	0.205 3	0.2017	0.1773	0.1805	3	0.1625028 1	0.16250304	1	0.16296876	0.16247554
								0.5	0.16243102	0.16247556
								0.25	0.16246441	0.16247556

For the purpose of comparison, the absolute error for $a(t), b(t), c(t)$ and $d(t)$ between the real data and the ADM, VIM, FDM and RK4 methods from 2006 to 2009 are shown numerically in Table

-5. The absolute error for $a(t)$ has the smallest value with $h = 1$ compared with the other methods under study when $h = \{1, 0.5, 0.25\}$. On the other hand, the absolute error for $d(t)$ of VIM has the smallest value compared with the absolute error for the other methods (ADM, RK4 and FDM) when $h = \{1, 0.5, 0.25\}$.

Table-6 shows the measure error, indicating that the difference measure error for $a(t)$ of FDM has the smallest value when $h = 0.5$ than that when $h = 1$ and 0.25 , and compared with the other methods under study with the different step size $h = \{1, 0.5, 0.25\}$. In addition, the difference measure errors of $b(t)$, $c(t)$ and $d(t)$ in VIM have the smallest errors compared with ADM, FD and RK4 methods when $h = \{1, 0.5, 0.25\}$.

Figures 1 describes the trend of the smoking habit from 2006 to 2022. In Figure-1 (a) that is related to non-smoke people $a(t)$, the curve of ADM rises, indicating an increase in numbers of non-smokers through 16 years, while is that was stable with using the other methods, because they have the same iterative nature. The results from the application of these methods (VIM, FDM and RK4) agree with the results of a previous study [1].

Figure-1(b), that is related to normal smoke people $b(t)$, shows that the curves of the four used methods are near to the predicted values from 2006 until 2013. After that, all the curves gradually decreased on a yearly basis until 2022. However, the curve of VIM showed a higher decrease from 2013 until 2022 than the others curves.

Table 4-Expected approximate solutions of the smoking habit model from 2006 to 2022

Model Variables	t	ADM	VIM (16 iter.)	Step size h yearly	FDM (16 iter.)	RK4 (16 iter.)
$a(t)$	16	0.77521396	0.48643685	1	0.49049314	0.49025842
				0.5	0.49031746	0.49025841
				0.25	0.49027319	0.49025841
$b(t)$	16	0.16532272	0.14627349	1	0.17438741	0.17011593
				0.5	0.17128695	0.17011592
				0.25	0.17041579	0.17011591
$c(t)$	16	0.12413672	0.12457661	1	0.11159163	0.11499751
				0.5	0.11404419	0.11499750
				0.25	0.11475214	0.11499750
$d(t)$	16	0.24252661	0.24427130	1	0.22352783	0.22462814
				0.5	0.22435139	0.22462817
				0.25	0.22455887	0.22462817

Table 5- Absolute error for ADM, VIM, FDM and RK4 with the real data [1] from 2006 to 2009.

Model variables	t	ADM 2009	VIM (3 iter.) 2009	Step size, h (year)	FDM (3 iter.) 2009	RK4 (3 iter.) 2009
$a(t)$	3	0.07482470	0.01589633	1	0.01579684	0.01590233
				0.5	0.01591217	0.01590232
				0.25	0.01590479	0.01590232
$d(t)$	3	0.03919719	0.03919696	1	0.29759684	0.29770233
				0.5	0.29771218	0.29770232
				0.25	0.29770479	0.29770232

Table 6- Difference measure error for ADM, VIM, FDM and RK4 solutions with the predicted values [1] from 2006 to 2009.

Model Variables	t	ADM 2009	VIM (3 iter.) 2009	Step size, h (year)	FDM (3 iter.) 2009	RK4 (3 iter.) 2009
$a(t)$	3	0.0534247	0.00550367	1	0.00560316	0.00549767
				0.5	0.00548782	0.00549768
				0.25	0.00549521	0.00549767
$b(t)$	3	0.00440158	0.00377918	1	0.30869684	0.30880233
				0.5	0.30881218	0.30880232
				0.25	0.30880479	0.30880232
$c(t)$	3	0.0197209	0.01972145	1	0.37529684	0.37540233
				0.5	0.37541217	0.37540232
				0.25	0.375404789	0.37540232
$d(t)$	3	0.01799719	0.01799696	1	0.31879684	0.31890233
				0.5	0.31891218	0.31890232
				0.25	0.31890479	0.31890232

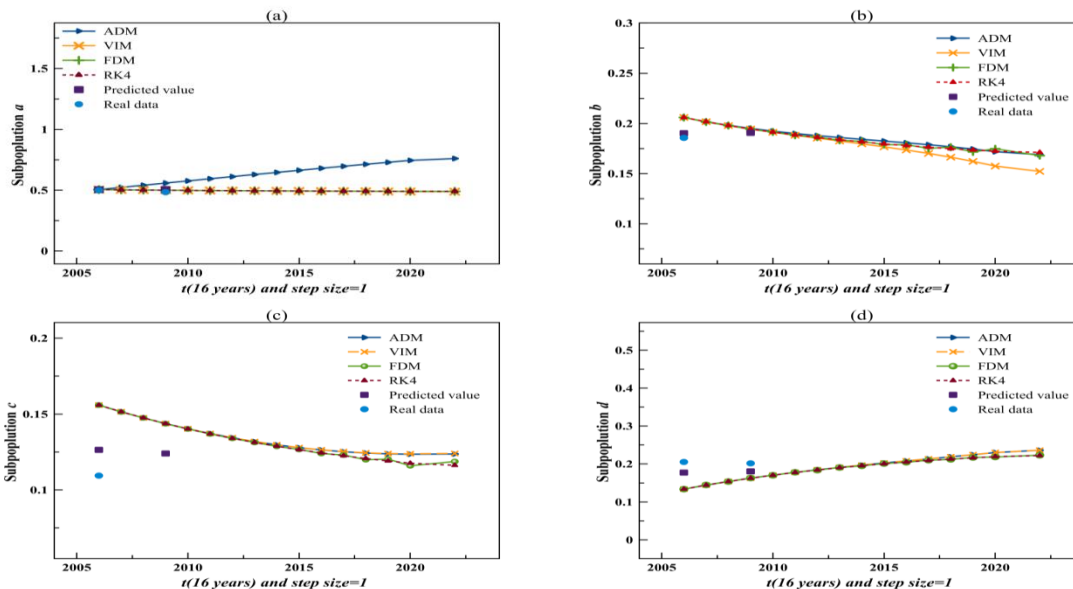


Figure 1-Variation of approximate and numerical solutions by using ADM, VIM, FDM and RK4 around predicted values [1] of (a) $a(t)$, (b) $b(t)$, (c) $c(t)$ and (d) $d(t)$ from 2006 to 2022 when $h=1$.

Figure-1(c). that is related to excessive smokers $c(t)$, demonstrates a decreasing curve in all methods applied, with the numerical methods (FDM and RK4) showing a higher decrease than the analytical methods (ADM and VIM). These results agree with those of a previous study [1].

Figure-1(d), that is related to ex-smokers $d(t)$, demonstrates an increase in all curves for the period from 2006 to 2022. However, the curves obtained by the analytical methods (ADM and VIM) showed a higher increase than those of the numerical methods (FDM and RK4) for the period from 2013 until 2022. The nature trend in the proportion of ex-smokers in the current study agrees with that reported in a previous study [1].

6. Conclusion

In the current study, the trend of the harmful and social habit of smoking was analyzed using the nonlinear epidemic model through sixteen years (from 2006 to 2022). In our work, some reliable approximate methods were used for solving a non-linear system of epidemic models for ordinary differential equations of the first order. There was a convergence in the results of the analytic methods, which were Adomian decomposition and variation iteration, along with numerical methods represented by the Finite difference and Runge-Kutta that examined the nonlinear case. The use of the analytic, ADM and VIM, with the numerical, FDM and RK4, methods assisted in analyzing the effects of the harmful social habit by the smoking habit model. The results showed that category $a(t)$ of the

non-smokers stayed stable along fifteen years, except with the ADM curve. While category $b(t)$ of the normal-smokers and category $c(t)$ of the excessive smokers were gradually declining until 2022. However the category $d(t)$ of ex-smokers was increasing until 2022, that refers to raise smoking habit in this region. The most predicted values around the ADM, VIM, FDM and RK4 curves indicated the reliability of the obtained results.

Other methods might be able to solve systems such as the one under study, including the homotopy perturbation method, homotopy analysis method semi analytical iterative method of Temimi and Ansari, as well as the iteration methods.

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