Study of Comet Tail Interactions with Solar Wind Using MHD Based Model

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Abstract

In the present work, the different possible interactions between comet tail and the solar wind in the present of the inter-planetary magnetic field (IMF) had been studied. Magneto-hydrodynamic laws (MHD) were used to simulate interactions occurring between comet tail and solar wind using one –dimensional space. The model simulation was achieved using Lax-Wendroff explicit method. The interaction between cometary tail ions and solar wind can be understood by means of spatial distribution of new-generated ions from the comet nucleus with the homogenous flow of the solar wind plasma. The difference between average molecular mass of both types along with step-increase in the number of particles can reproduce the whole shape of the comet tail, thus comparison the results of the MHD simulation can be mode fruitful to understand the specific type of particles interaction. The results show that the molecules weight distributes in such away that fallows the direction of the IMF and decreases rapidly as the distance increases away from the nucleus. Pressure results indicate that the velocity of the particles around the nucleus must decrease. That was explained on the bases of mass conservation laws. Also the effects of the IMF were explained.

الخلاصة

في البحث الحالي تمت دراسة التفاعلات المختلفة بين الرياح الشمسية وذنب المذنب بوجود المجال فالمغناطيسي ما بين الكواكب (IMF). أستخدمت قوانين المغنيتوهيدروديناميك (MHD) لمحاكاة التفاعلات التي تحدث بين ذيل المذنب والرياح الشمسية باستخدام بعد واحد. محاكاة النظام تمت باستخدام طريقة لاكس– ويندروف المباشرة. يمكن فهم التفاعل ما بين ذيل المذنب والرياح الشمسية من خلال التوزيع المكاني للأيونات الجديدة المتولدة من نواة المذنب والجريان المتجانس لبلازما الرياح الشمسية. إن الفرق بين معدل كتلة الوزن الجزيئي لكلا النوعين من البلازما مع الزيادة السريعة والمفاجئة لعدد الجسيمات يمكن ان يعيد الشكل الكامل لذيل المذنب، ولذلك مقارنة نتائج المحاكاة لنظام (MHD) تكون مثمرة لمعرفة نوع التفاعل بين الجسيمات. بينت النتائج أن توزيع الوزن الجزيئي يكون بحيث انه يتبع أتجاه المجال المغناطيسي ما بين الكواكب (IMF) ويزداد بصورة مفاجئة مع ازدياد المسافة بعيدا عن نواة المذنب. أما النتائج الخاصة بالضغط فقد بينت أن سرعة الجسيمات حول النواة يجب أن تقل. وقد تم تفسير هذا على أساس قوانين حفظ الكتلة والطاقة. وكذلك تم تفسير تأثير المجال المغناطيسي ما بين الكواكب (IMF).

Introduction

There are many models used to describe the structure of the comet nucleus [1], of which, the so called "*dirty-ice*" model [2], is the most acceptable. When the comet approaches the sun, it will suffer from higher solar wind density. This will cause generation of ions from the comet nucleus with extremely high rate. As a result, the comet will have more dense tail as it approaches the sun.

The mechanism at which the comet tail generates is as follows:-

High energy ionizing radiation of the solar wind hits the nucleus with the direction outward from the sun. Such radiation will ionize atoms and molecules of the comet nucleus, which appears as if the nucleus starts to "generate" new ions. The ions generating rate, G, varies from one comet to another depending on the shape, size, speed and composition of the nucleus. Generally speaking, most comets have $G \sim 10^{29}$ ions/sec; which indicates the extremely high change in the mass average of the plasma surrounding the comet. Due to the flow of the solar wind plasma, the ions generated from the nucleus will shape in a specific way to make the comet tail.

The shape of the tail is, therefore, a tool that can be used to study many features of the comet (such as its structure, size, and composition) and the features of the solar wind as well.

A remarkable event that occurs for comet tail is the "*disconnection event*", a temporary separation in the flow of the comet tail that is followed by regeneration of the tail in some slightly different direction. The disconnection events take place when the comet crosses an inter-planetary magnetic field (IMF) sector boundary. Thus the disconnection event provides a more significant tool to study the structure of the IMF. The disconnection event was explained as follows [3, 4]

1- the comet tail becomes narrow due to change of the IMF direction.

2- the tail is totally disconnected from the comet, indicating the large force applied on the ions of the tail due to the magnetic pressure.

3-in the new region of the IMF, the ions of the comet tail reproduce the shape of the tail. This means that a "new tail" is formed. The direction of the new tail is slightly different from the direction of the old tail (few degrees).

4-the comet returns to its normal appearance.

However, other explanations of the disconnection event were proposed based on the plasma flow disturbances [5]. Such disturbances include:

1- Temporary momentum increase due to sudden changes in the flow of the solar wind.

2- Shock waves generated due to different plasma velocities.

3- Reconnection of the IMF in the tail-side of the comet.

Niedner [6] and Brandt [7] used the IMF sector boundary crossing idea to explain the disconnection event observed at Halley comet in 1986. On the other hand, Brosius et al. [8] tried to explain the disconnection event using solar wind properties as measured experimentally by different spacecraft missions, but the large distance between the measurement and the comet make the relation uncertain between the disconnection event occurring in the comet tail and solar wind characteristics.

Ogino et al [9] tried to use both models, i.e., magnetic sectors crossing and solar wind disturbances, to explain the disconnection event. In all cases the magnetohydrodynamic (*MHD*) equations are used to describe the system. Due to the nature of the comet, MHD equations are modified by adding source terms to them, so that these equations can describe plasma flow in the presence of the magnetic field and taking into account that there are new ions added to the system from the comet nucleus.

The nucleus of the comet is made of ice contaminated by other chemical impurities, according to the "*dirty-ice model*" [1]. When the nucleus approaches the sun, solar wind radiation will ionize the molecules of the surface of the comet nucleus. The size of the nucleus is of order of few kilometers; therefore, the production rate of ions from is relatively high comparing to the density of the solar wind.

The new ions will leave the surface of the comet with speed of order of few kilometers per second, since the new ions are of mush larger weight than the particles of the solar wind. These ions will further suffer from successive ionization process until become fully ionized plasma, i.e., undistinguishable from the particles of the solar wind. This process takes time of order of 10^6 seconds [10]. Thus, the ionization region around the comet nucleus is of order of $\sim 10^6$ kilometers [11]. As a result, comet appears much larger than the size of the nucleus when interacting with the solar wind.

In the ionization region, source term play the important rule. Therefore, each comet is characterized by specific properties of the source term. Ionization mainly occurs due to photo-ionization and electron impact, and with less probability due to heavy ions in elastic scattering (protons, neutrons and alpha particles of the solar wind).

In the present paper, we solve the magnetohydrodynamic (MHD) system for cometary plasma interactions with the solar wind in the presence of the interplanetary magnetic field (IMF). The simulation was prepared for boundary conditions and using appropriate scaling laws for simulating Halley comet. Method of calculation was based on Lax-Wendroff second order accurate method for one dimension in space.

The MHD Equations

Let's denote the plasma mass density by ρ, particle velocity u, number of particles n, pressure p, magnetic field B, and the internal energy E. Then the MHD equations of the system are:

1-Mass Conservation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\vec{u}\rho) = \rho_o \qquad (1)$$

Where ρ_o is the source term for density, which is given, for spherical symmetry, by [10]:

$$\rho_o = \frac{\sigma G\, m_c}{4\pi r^2 v_c} e^{\frac{-\sigma r}{v_C}} \qquad (2)$$

G is the ion production rate, m_c is the mean molecular mass, v_c is the mean molecular velocity, σ is the ionization rate, and the distance is r from the nucleus of the comet.

2- Particles' Number Conservation

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (\vec{u}n) = n_o \qquad (3)$$

Where n_o is source term for the number of particles related to ρ_o as [10]:

$$n_o = \rho_o / m_c. \qquad (4)$$

m_c being the average ion mass.

3- Momentum Conservation

$$\frac{\partial(\rho\, \vec{u})}{\partial t} = \vec{\nabla}.(\rho\vec{u})\vec{u} + \vec{\nabla}p - \frac{1}{4\,\pi}(\vec{\nabla} \times \vec{B}) \times \vec{B} = \vec{q}_o \quad (5)$$

And q_o is the source term of the momentum. This term is negligible in comet calculations [10], thus we put

$$q_o \approx 0 \qquad (6)$$

From eq. (5), we see that since the time-dependent part, $\frac{\partial(\rho\, \vec{u})}{\partial t}$, includes both ρ and v, and eq.(1) describes ρ, then eq.(5) can be used to find the velocity v only which describes the conservation of momentum.

4- Magnetic Field Conservation

$$\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times (\vec{u} \times \vec{B}) = 0 \qquad (7)$$

Of course, there is no source term for B in nature [10].

5- Energy Conservation

$$\frac{\partial E}{\partial t} + \vec{\nabla}.\left((E + p_g)\vec{u} - \frac{(\vec{u}.\vec{B})\vec{B}}{4\pi}\right) = e_o \qquad (8)$$

Where p_g is the total plasma pressure [10],

$$p_g = p + \frac{B^2}{8\pi} \qquad (9)$$

And the term $\frac{B^2}{8\pi}$ is usually known as the "*Magnetic Pressure*". The source term, e_o in eq. (8) is given by,

$$e_o = \frac{1}{2}\rho_o v_c^2 \qquad (10)$$

Scaling Laws

In order to make the system normalized to the properties of the solar wind (namely, mass density ρ_Θ, and velocity of the solar wind u_Θ); suitable scaling is applied to the MHD equations (equations 1, 3, 5, 7, and 8). Defining R_I as the interaction length, given by

$$R_I = \frac{\sigma G\, m_c}{4\pi v_c\, \rho_\Theta\, u_\Theta} \qquad (11)$$

R_I determines the interaction region size. Then we can re-define MHD equations using

$$\left.\begin{aligned} r_1 &= \frac{r}{R_I} \\ t_1 &= \frac{t\,u_\Theta}{R_I} \\ \rho_1 &= \frac{\rho}{\rho_\Theta} \\ \vec{u} &= \frac{\vec{u}}{u_\Theta} \\ p_1 &= \frac{p}{\rho_\Theta u_\Theta^2} \\ n_1 &= \frac{m_c\; n}{\rho_\Theta} \end{aligned}\right\} \quad (12)$$

So, after suitable re-arrangement of MD equations, one can see that for scaled system, the general form of all these equations will be the same except multiplying the source (eq.2 and 10) with suitable constant.

We note that for distance $r \ll v_c/\sigma$, the mass source (eq.2) will be dependent on r^{-2} only because the *exp* $(-r\,\sigma/v_c) \approx 1$. then the scaling laws will mean that any comet differs from the other by *G* only.

Note that the magnetic field can also be scaled using [4]

$$B_1 = \frac{B}{\sqrt{\rho_\Theta u_\Theta^2}} \quad (13)$$

This relation means that the magnetic pressure is being scaled to ρ_Θ and u_Θ. Also we know that when *B* is small, any terms having (B^2) can be negligible in the MHD system. Therefore, the IMF will be build up and are described by eq.(7) in all cases.

Lax-Wendroff Explicit Method

It is based on the following:

The general form of the MHD equations is $\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$ then using Taylor expansion in time,

$$u(x, t+\Delta t) =$$

$$u(x,t) + \Delta t\frac{\partial u}{\partial t} + \frac{(\Delta t)^2}{2!}\frac{\partial^2 u}{\partial t^2} + O((\Delta t)^3)$$

$$u_i^{n+1} = u_i^n + \Delta t\frac{\partial u}{\partial t} + \frac{(\Delta t)^2}{2!}\frac{\partial^2 u}{\partial t^2} + O((\Delta t)^3)$$

Then, if one takes time derivatives,

$$\frac{\partial^2 u}{\partial t^2} =$$

$$-a\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) = -a\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) = a^2\frac{\partial^2 u}{\partial x^2}$$

Therefore,

$$u_i^{n+1} = u_i^n + \Delta t(-a\frac{\partial u}{\partial t}) + \frac{(\Delta t)^2}{2!}(a^2\frac{\partial^2 u}{\partial t^2})$$

And after using centered difference scheme,

$$\therefore u_i^{n+1} = u_i^n - \frac{c}{2}\left(u_{i+1}^n - u_{i-1}^n\right) + \frac{c^2}{2}\left(u_{i+1}^n - 2u_i^n + u_{i+1}^n\right) \quad (14)$$

Where *c* is Courant Number, $c = a\,\frac{\Delta t}{\Delta x}$.

Equation (14) is the Lax-Wendroff approximation. It is of accuracy of $O((\Delta t)^2, (\Delta x)^2)$. Stability condition is $c \le 1$.

When applying this scheme to the MHD equations, the numerical form of the equations is obtained. Below is the numerical form of the density equation only:

$$\rho_i^{n+1} = -\left(\frac{\Delta t}{2\,\Delta x}\right)\left[(\rho_{i+1}^n u_{i+1}^n - \rho_{i-1}^n u_{i-1}^n)\right] + \left(\frac{(\Delta t)^2}{8(\Delta x)^2}\right)\left[(u_{i+1}^n - u_i^n)(\rho_{i+1}^n u_{i+1}^n - \rho_i^n u_i^n) - (u_i^n - u_{i-1}^n)(\rho_i^n u_i^n - \rho_{i-1}^n u_{i-1}^n)\right] \quad (15)$$

The code for programming was MatLab 7.0. The numerical grid was based on equally-divided space for one dimension, taking it as the sun-comet line. Steps on the space axis were of units of R_I and the system of equations was scaled as described above.

Results and Discussion

In the present research, continuity equations governing the behavior of a comet moving in the presence of IMF have been used to study the interaction of cometary ions of the tail with solar wind. Solving the MHD system with specific boundary conditions made possible to investigate the behavior of comet tail due to such interaction.

The effect of the source term in the MHD equations represents the effect of the comet nucleus [11]. Such effect will highly re-generate the properties of the comet tail, and its shape changes considerably due to the specifications of the comet nucleus properties, as well as the properties of the solar wind. Table (1) lists the boundary conditions used to perform the present simulation, as well as the properties of the solar wind as collected experimentally [10]. These values are for comet Halley.

The numerical grid was assumed to have 200 nodes to cover a region about 20x10^6 kilometers. R_I is found to be 10^6 kilometers. The nucleus position was assumed at distance x=10 (units of R_I) in the computational grid. The effect of the source terms is included until x=180 (units of R_I). The interaction length, R_I, is comparable to the bow shock radius, R_B, where found to have the value 1.4156x10^9. This indicates between the bow shock and the nucleus of the comet, the plasma flow is slowed down in a continuous manner because of the additional mass added from the nucleus. The ions generated from the nucleus, when added to the plasma in front of the comet, will make different interactions, of which the hydrodynamic interactions are the most important. This can be seen from the results of the present simulation, where in Fig.(1) the results for mass density and number of particles are presented.

Table (1): Comet Properties

1. Initial, Boundaries and Source Terms used in the Simulation			
Property	**Initial value**	**Boundary value**	**Source**
ρ	ρ_Θ gm/cm^3	10xρ_Θ gm/cm^3	ρ_c
n	ρ_Θ / m_p cm^{-3}	10 x ρ_Θ / m_p cm^{-3}	ρ_c / m_p
v	10 km /sec	5 km /sec	None
B	4.0 nT	5.0 nT	None
p	10^{-12}dyn/cm^2	10^{-11} dyn/cm^2	None
E	1x10^{-9} erg	4 x 10^{-9} erg	E_c
2. Comet Nucleus Properties			
$\sigma = 10^{-6}$ sec^{-1}	γ = 1.667	G=10^{30} sec^{-1}	R_I=10^9 m
3.Solar Wind Properties			
$\rho_\Theta = 5\ m_p$	u_Θ = 400 km /sec	m_c = 20 x m_p	v_c =10^3 km/sec

Both curves increase rapidly near the nucleus position, as expected from the effect of the source term for the mass density (eq.2). When the ions generate from the nucleus of the comet and added to the solar wind plasma, this increase in the plasma density occurs, hence the number of particles also increases. This indicates that the interaction in this region is made by adding the new ions to the solar wind. Such addition is, however, not homogeneously distributed when concerning the velocity, as seen from Fig. (2). Note the similar behavior of both mass density and number of particles' curves in the region near the nucleus, which is a fair indication of the linear relation between these two variables.

From Fig.(2) we see that the particles' velocity distribution near the comet nucleus is not as smooth as the distribution of the mass density (or number of particles), where a sudden decrease in the velocity curve occurs just near the nucleus and continue almost along the tail. This can be justified because of the change of the molecular weight of the plasma. From momentum conservation equation (eq. 5) it was assumed that the source term for velocity is neglected. This is conventional assumption because the only addition to momentum in the MHD system is due to inelastic scattering, which adds considerably small effect. In the region near the comet nucleus where ionization rate reaches the maximum, source term for velocity may add some effect, however, that effect was found to be less than ~ 2% [10]. Since the interaction region is of order of ~ 10^6 km and the nucleus size is of order of few kilometers only, the effect of the source term for velocity can be entirely ignored. Therefore, the behavior of velocity shown in Fig.(2) indicates that the total linear momentum of the particles in the plasma is decreased. Being a vector quantity, the velocity reaching the minimum also means that the average linear velocity is at minimum. This also shades light on the random behavior of the new ions near the comet, a reasonable situation if we knew that the new ions generate in spherical symmetric manner, i.e., emission probability is the same in all directions. So when adding this to the plasma flow, the result will be as in Fig.(2). In order for the momentum to be conserved, the velocity must decrease as the mass density increases, satisfying: Linear momentum is proportional to $\rho\, u$. As the distance increases away from the nucleus, the velocity increases returning to the normal state of the solar wind, i.e., homogenously distributed with the velocity of the solar wind particles. This is due to successive collisions with solar wind particles making the velocity uniformed to them.

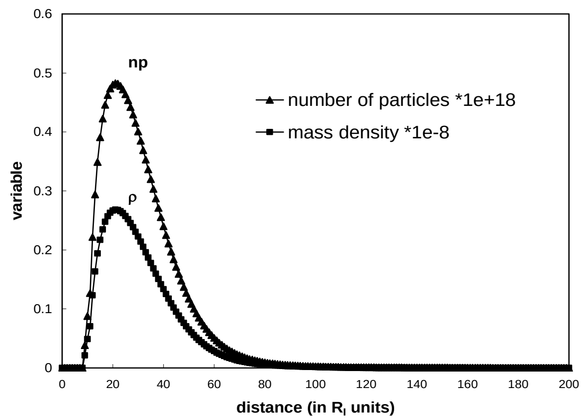

Fig. (1): Spatial distribution of MHD plasma density and number of particles.

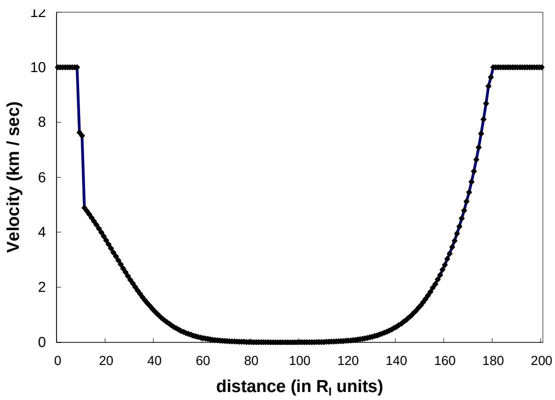

Fig. (2): Spatial distribution of MHD plasma velocity.

Fig. (3) illustrates the results of the MHD magnetic field of the system. It can be noted that the magnetic field suffers from sudden and sharp increase at the region before the nucleus position. This is due to the pile up of magnetic field just in front of the comet nucleus due to the effect of the massive gathering of charged particles. This disturbance plays an important rule in forming the bow shock in front of the comet. The magnetic pressure, $p=B^2/8\pi$ in this region will force the new particles to push away with non-uniform velocity, i.e., disturb the plasma flow in that region. This is also seen from Fig. (2) for regions very close to the comet nucleus. It was suggested [Ip reference] that such disturbances cause the disconnection events in the comet tail, where sudden condensation of these disturbances may progress to reach the body of the tail itself leading to disconnect it from the comet. As the distance increases, the magnetic field decreases for minimal values indicating the high chaos of the charged particles. This increase in the magnetic pressure must add a good contribution to the total pressure. This is clearly seen from Fig. (4).

Another feature is seen in that figure, that is the large peak just behind the comet. The maximum peak is 10^4 times larger than the initial value (Table 1). Such increment indicates the high internal energy at that accompanies the increment of the mass-average molecular weight of the new ions generated from the comet, a clear behavior shown in the energy results plotted in the same figure. Although this is joined with velocity decrease, but the rapid increase in the number of particle -hence the total mass- contributes in the internal energy and makes the energy increases. Increasing the energy in a fixed volume will certainly increase the pressure for an ideal system, however in the case of comet, one expects that for such behavior of the pressure and energy, the electron temperature must also increase with proportional manner.

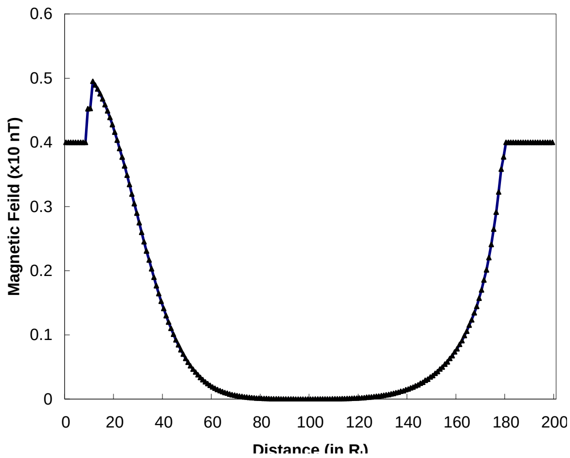

Fig. (3): Spatial distribution of MHD plasma magnetic field.

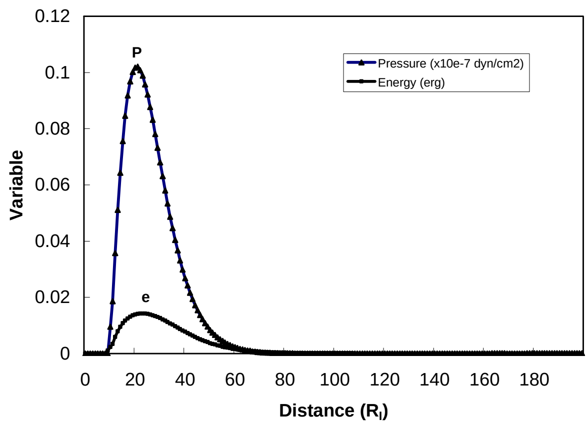

Fig. (4): Spatial distribution of MHD plasma presure.

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