DOI: 10.24996/ijs.2018.59.1A.21

ISSN: 0067-2904

# Evaluation of Orbital Maneuvers for Transition from Low Earth Orbit to Geostationary Earth Orbit

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**Abstract**

The transition from low Earth orbit 200-1500 (km) to geostationary Earth orbit 42162 (km) was studied in this work by many methods of transfer. The delta-v requirement (Δv), the time of flight (Δt), the mass ratio of propellant consume (Δm/m) and total mass was calculated for many values altitude in the same plane also when the plane is change. The results from work show that (Δv) that required for transfer when the plane of orbit change is large than (Δv) required when the transfer in coplanar maneuvers while the bi-elliptical transfer method need time of transfer longer than a Hohmann transfer method. The most energy efficiency was determined when the transfer in coaxial between elliptical orbits, the result show the most efficiency transfer orbit occur at apogee on the original orbit where the total of velocity required is (0.7864 km/s) that least from total velocity at perigee (0.7975 km/s).

**Keywords:** Hohmann transfer, satellite orbit, elliptical, coaxial, coplanar maneuvers.

# تقييم للمناورات المدارية للانتقال من المدار الارضي المنخفض الى المدار الجغرافي الارضي الثابت

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**الخلاصة**

تم دراسة الانتقال من المدار الارضي المنخفض (200−1500 ) كم الى المدار الجغرافي الثابت (42162) كم من خلال عدة طرائق للانتقال . كذلك تم حساب التغير المطلوب في السرعة اضافة الى مدة الطيران و نسبة الكتلة الدافعة والكتلة الكلية عند قيم مختلفة للارتفاعات في حالة نفس المستوي وفي حالة تغير المستوي. تبين نتائج البحث ان اقل تغيرمطلوب في السرعة في حالة تتغير المستوى تكون اكبر من قيمتها في حالة نفس المستوي، بينما الانتقالات الثنائية الاهليجية تحتاج وقت اطول من انتقالات هوهمان، كما بينت النتائج ان الانتقال الاكثر كفاءة للطاقة يحدث عند الانتقال من اوج المدار االابتدائي حيث كانت السرعة (0.7864) كم/ثا بينما الانتقال من الحضيض كانت السرعة المطلوبة (0.7975) كم /ثا

## 1. Introduction.

A satellite accomplishes orbital maneuvers to rectification the shape, size, or the site of a satellite [1]. In some stage of the lifetime for most satellites, we need to change the orbital elements of a satellite. While a transfer happen at an original orbit to target orbit, also to adjust the orbital elements

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when treatment the path results from the perturbations. We want to adjust the altitude of orbit, plane of orbit, or both of them to rectification the orbit of a satellite, the variation in the velocity vector obtained in the direction or in the magnitude. The propulsion measurements applied at the shortened time, if it's unite with the orbital rotation; therefore during an impulsive maneuver the position of satellite remained to be constant, the alteration of orbit satellite happens in a locust in which the old orbit cross with the new orbit [2]. Low earth orbits has height typically ranging from (200 - 1500) km. Low Earth orbit that near to polar orbits is called sun-synchronous, therefore they always measure the same two- time of the day and generally precess when local time of the observations is drift [3]. Geostationary Earth orbit is the orbit on which a satellite appears stationary relative to the objects on the Earth. When a satellite is on the geostationary Earth orbit, the antennas of a ground stations keep pointed to the satellite automatically, because of the Earth is rotating with same the period of the satellite [4].Geostationary Earth orbit has a 24-hour period, also the satellite in this orbit sits over same longitude of the Earth so the orbit covered entire the diurnal cycle, the circular geostationary orbits that the equatorial of orbits with an inclination is $0^{\circ}$ [3].

**2. Orbit Altitude Changes.**

The change of orbit altitude that means correct obtain in the same plane and the maneuver would vary the size and energy of an orbit. The maneuvers that change the orbit of a satellite happens in locus of the first orbit cross with the second orbit, if the orbits of satellite does not cross together, in this situation, we need to use an intermediate orbit on which the orbits intersects together as a Hohmann transfer [2].

**2.1. Hohmann transfer.**

The Hohmann transfer method is an oval orbit that contacts both inner and outer orbits, while the transfer orbit is present radii of both orbits in perigee and apogee respectively as show in Figure-1. The Hohmann transfer method is most energy efficiency for two- impulse maneuver when in transit happens in two-circular orbits that linked in a popular focus. The velocity change required to transfer between circular orbits for a two-burn corresponds to use an ellipse on which the intermediate orbit tangent to inner and outer circles. When the original and target orbits $r_1$, $r_2$, the determination of the total velocity accomplish through use the equations below [4].

$$a_t = \frac{r_1 + r_2}{2} \quad \text{.......(1)}$$

$$\varepsilon_t = -\frac{\mu}{2a_t} = -\frac{\mu}{r_1 + r_2} \quad \text{.......(2)}$$

In the original circle, the circular velocity is

$$v_1^2 = \mu\left(\frac{2}{r_1} - \frac{1}{a_t}\right) \quad \text{.......(3)}$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}} \quad \text{.......(4)}$$

The velocity of increment in an one burn is

$$\Delta v_1 = v_1 - v_{c1} \quad \text{.......(5)}$$

The velocity required in target orbit is

$$v_2^2 = \mu\left(\frac{2}{r_2} - \frac{1}{a_t}\right) \quad \text{.......(6)}$$

$$v_{c2} = \sqrt{\frac{\mu}{r_2}} \quad (7)$$

$$\Delta v_2 = v_{c2} - v_2 \quad (8)$$

$$\Delta v_{tot} = \Delta v_1 + \Delta v_2 \quad (9)$$

The time required for flight or transfer is

$$\Delta t = \pi \sqrt{\frac{a_t^3}{\mu}} \quad (10)$$

Where
$a_t$ = semi major axis of transfer ellipse.
$v_1$ = the velocity of initial orbit.
$v_2$ = the velocity of target orbit.
$v_{c1}$ =.the velocity at A on the orbit 1.
$v_{c2}$ = the velocity at B on the orbit 2.
$\Delta t$ = the time of flight.
$\mu$ = 398600.44 $km^3/s^2$ is present the Earth's mass gravitational constant.
$\varepsilon_t$ = the specific energy of orbit [4].

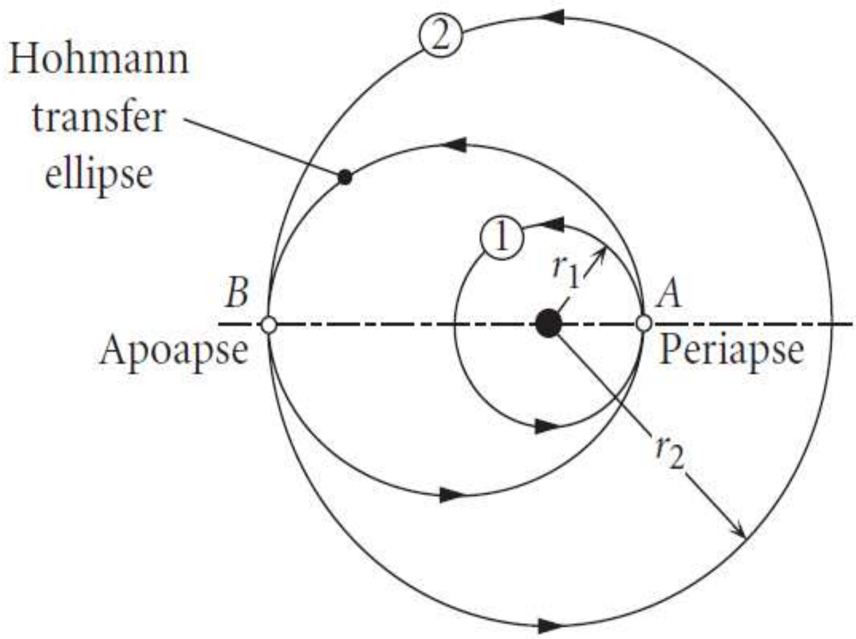


**Figure 1-** Hohmann Transfer[5].

**2.1.1 The Steps of solution.**
1. Choose the method of transfer orbit.
2. Calculate velocity circular for the orbit that has radius $r_1$ that presents the initial velocity.
3.Calculate velocity of intersects with the initial orbit that present the velocity transfer orbit at perigee.
4. Calculate the initial burn ($\Delta v_1$) to maneuver into the transfer orbit.
5. Calculate velocity of intersects with the target orbit that present the velocity transfer orbit at apogee.
6. Calculate velocity circular for the orbit that has radius $r_2$ that present the target velocity
7. Calculate the final burn ($\Delta v_2$) to maneuver into the target orbit.

8. Calculate the total of velocity (Δv total).
9. Calculate the time of flight or transfer (Δt).
10. Calculate the mass consume for propellant (Δm).

for a given satellite mass (500 kg) and specific impulsive $I_{sp}$ (435), the propellant mass consume that need to complete the transfers can be calculated from the Δv total , also can be determined total mass. The equation required to calculate propellant mass consume is shown below [5].

$$\frac{\Delta m}{m} = 1 - e^{-\frac{\Delta v}{I_{sp} g_o}} \qquad \text{.....(11)}$$

Where $m$ is the satellite mass before burn.
$\Delta m$ is the consume mass for propellant.
$\Delta v$ is the delta velocity required for transfer.
$g_o$ is the gravity standard of acceleration.
$I_{sp}$ is the impulsive specific of the propellants.

**2.1.2 The results of Hohmann transfer.**

From the table below we note that when increase the altitude of satellite, the semi major of transfer and the time of the transfer required will be increase as shown in Figures-(4, 7), while delta-v requirement and mass consumed decrease as shown in Figures-(5, 6).

**Table 1-** The results of transfer by Hohmann transfer.

| LEO altitude (km) | $a_t$ (km) | Δv (km/s) | Δt (h) | Δm (kg) | Δm/m | total of mass (kg) |
|---|---|---|---|---|---|---|
| 200 | 24371 | 3.93 9 | 5.2588 | 799.6 | 1.5992 | 1299.6 |
| 300 | 24421 | 3.8926 | 5.2750 | 746 | 1.522 | 1246 |
| 400 | 24471 | 3.8530 | 5.2912 | 743.5 | 1.487 | 1234.5 |
| 500 | 24521 | 3.8160 | 5.3075 | 723.8 | 1.4476 | 1223.8 |
| 600 | 24571 | 3.7788 | 5.3237 | 713.2 | 1.4264 | 1213.2 |
| 700 | 24621 | 3.7422 | 5.3400 | 702.8 | 1.4056 | 1202.8 |
| 800 | 24671 | 3.7062 | 5.3562 | 692.8 | 1.3856 | 1192 |
| 900 | 24721 | 3.6708 | 5.3725 | 682.8 | 1.3656 | 1182.8 |
| 1000 | 24771 | 3.6360 | 5.3888 | 673.2 | 1.3464 | 1173 |
| 1100 | 24821 | 3.6017 | 5.4052 | 663.8 | 1.3276 | 1163.8 |
| 1200 | 24871 | 3.5680 | 5.4215 | 654.6 | 1.3092 | 1154.6 |
| 1300 | 24921 | 3.5348 | 5.4379 | 645.7 | 1.2914 | 1145.7 |
| 1400 | 24971 | 3.5022 | 5.4542 | 636.9 | 1.2738 | 1136.9 |
| 1500 | 25021 | 3.4700 | 5.4706 | 628.4 | 1.2568 | 1128.4 |

**2.2 Hohmann transfers method between a coaxial elliptical orbits.**

Hohmann transfer methods contain two-impulsive maneuvers happen between elliptical orbits is called as a coaxial elliptical orbits, the transition occur in the original orbit to target orbit in two cases either from perigee or apogee show in Figure-2, to find which of two-transfer required low energy must calculated the individual total change velocity requirement for the perigee and the apogee that determined below in tables, where the orbital parameters of the orbits $r_A$ = 6858 km; $r_{A'}$= 7818 km, $r_B$ = 10218 km $r_{B'}$= 8298 km [5] .

$$h = \sqrt{2\mu}\sqrt{\frac{r_a r_p}{r_a + r_p}}$$ …………………………………………………………………….……. (12)

Where h in terms of perigee and apogee radius represents the angular momentum of orbits, $r_a$ ,$r_p$ are represent radius at the perigee and the apogee [5].

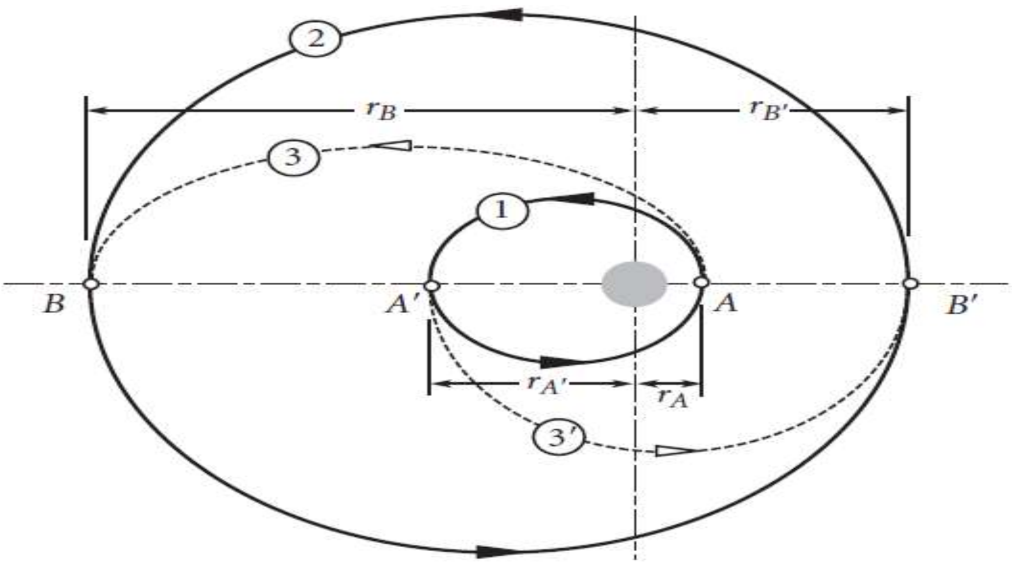


**Figure 2-** Hohmann transfers method between coaxial elliptical orbits [5].

**2.2.1. The results of Hohmann transfer between the coaxial elliptical orbits.**

The results that shown in Tables-(2,3,4, 5) are evaluate the angler momentum for orbit 1 and 2 and also the angular momentum in the term of perigee and apogee for orbit 3 and 3´and determined the velocity required for these orbits after that calculated delta-vs required and final determined the total velocity for individual two-transfer, the result show the most efficiency transfer orbit that occur in apogee on the original orbit where the total of velocity required is (0.7864 km/s) that least from total velocity in perigee (0.7975 km/s).

**Table 2-** The angular momentum for the orbits

| $h_1$ (km² / s) | $h_2$ (km² / s) | $h_3$ (km² / s) | $h_{3'}$ (km² / s) |
|---|---|---|---|
| 53967 | 60422 | 57197 | 56649 |

**Table 3-** The velocities for these orbits.

| $(v_A)_1$ (km / s) | $(v_A)_3$ (km / s) | $(v_B)_2$ (km / s) | $(v_B)_3$ (km / s) | $(v_{A'})_1$ (km / s) | $(v_{A'})_{3'}$ (km / s) | $(v_{B'})_2$ (km / s) | $(v_{B'})_{3'}$ (km / s) |
|---|---|---|---|---|---|---|---|
| 7.8692 | 8.3402 | 5.9131 | 5.5977 | 6.9029 | 7.2459 | 7.2813 | 6.8268 |

**Table 4-** The delta- v (Δv) for orbits.

| $\Delta(v_A)$ (km / s) | $\Delta(v_B)$ (km / s) | $\Delta(v_{A'})$ (km / s) | $\Delta(v_{B'})$ (km / s) |
|---|---|---|---|
| 0.4710 | 0.3430 | 0.3154 | 0.4545 |

**Table 5-** The total Δv requirement for transfer from apogee and perigee.

| Δv total 3 | Δv total 3´ |
|---|---|
| 0.7864 | 0.7975 |

**2.3 Bi-Elliptic Transfer**

Bi-elliptical transfer is the dotted ellipse located inside the outer orbit but outside the inner orbit, in which touching the both orbits. Bi-elliptic transfer required two-transfer orbit and three-impulse burn. Bi-elliptic transfer applied two-coaxial semi ellipses 2, 3 in which extend beyond outer orbit as show in Figure-3 [5],

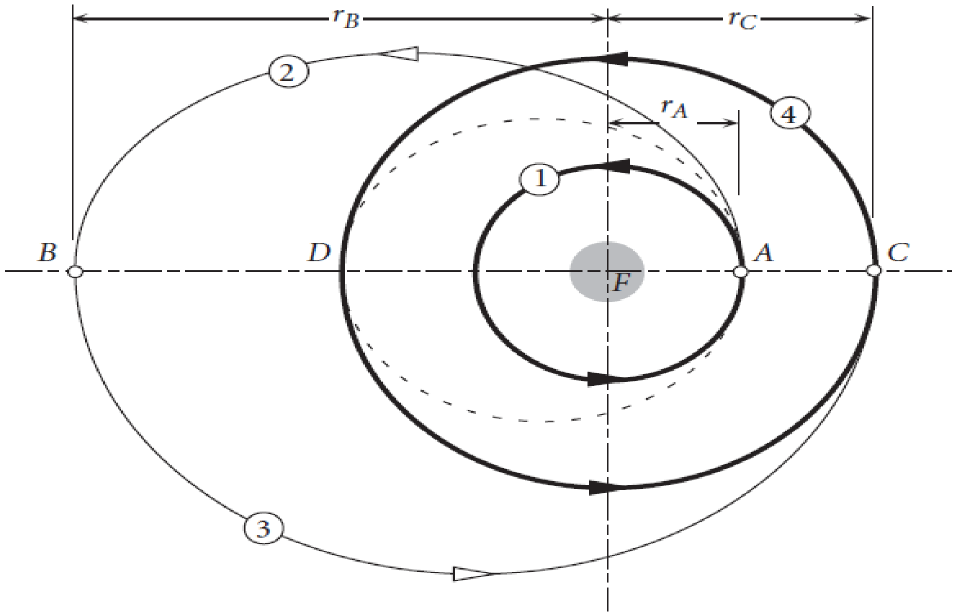


**Figure 3-** Bi-elliptic transfer [5].

**2.3.1. The results of bi-elliptical transfer.**

In a bi-elliptic transfer the time of transfer and velocity become more great values when the semi major axis increase , the results of transfer is show in Figures-(8, 9) below.

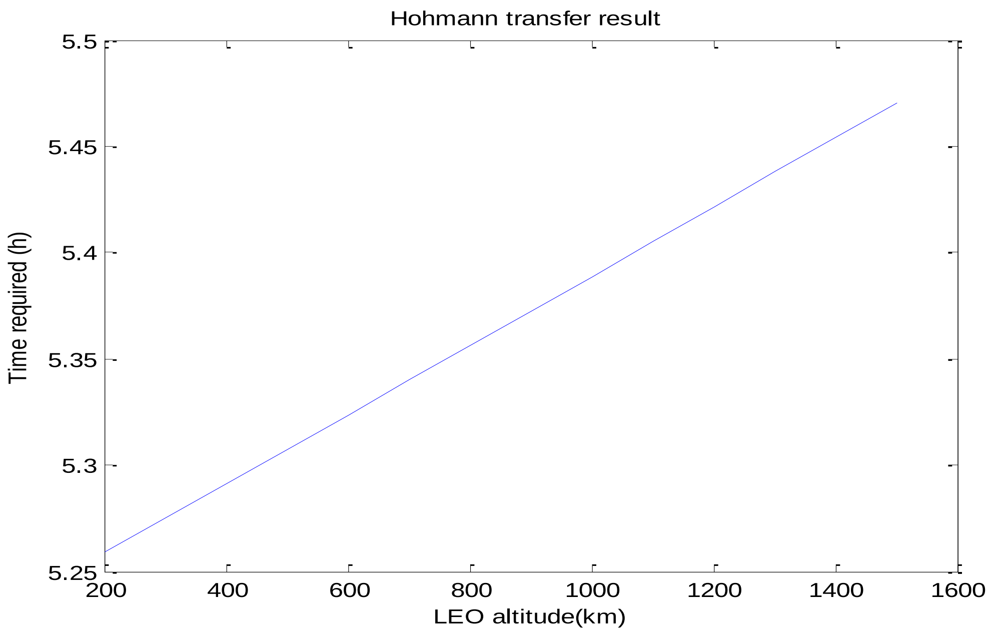


**Figure 4-** The time of transfer required for Hohmann transfer

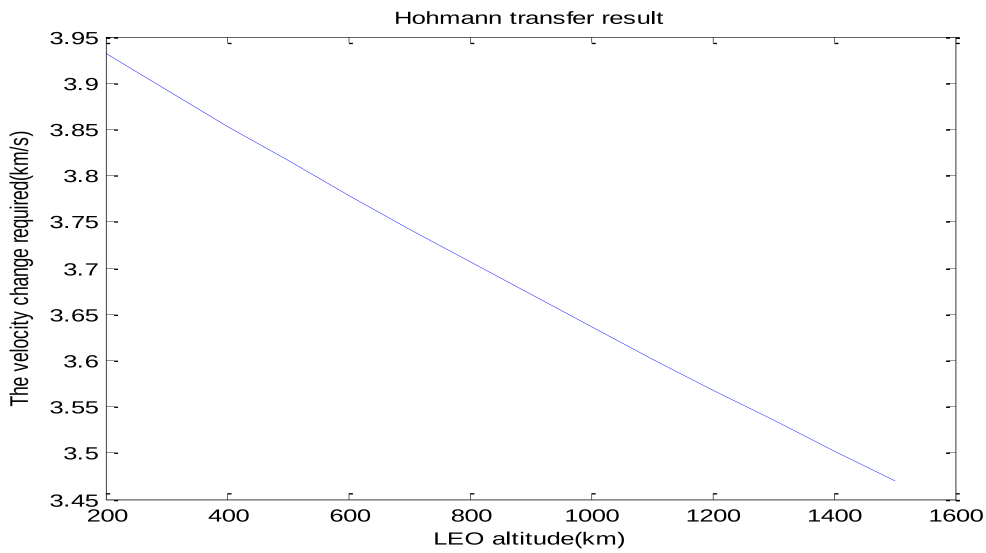


**Figure 5-** The velocity change for Hohmann transfer method.

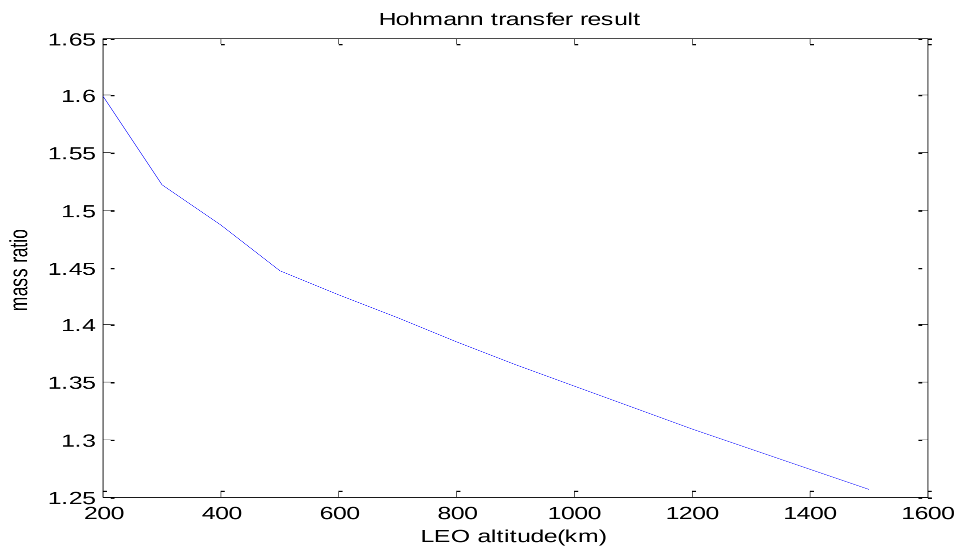


**Figure 6-** the mass ratio required for Hohmann transfer

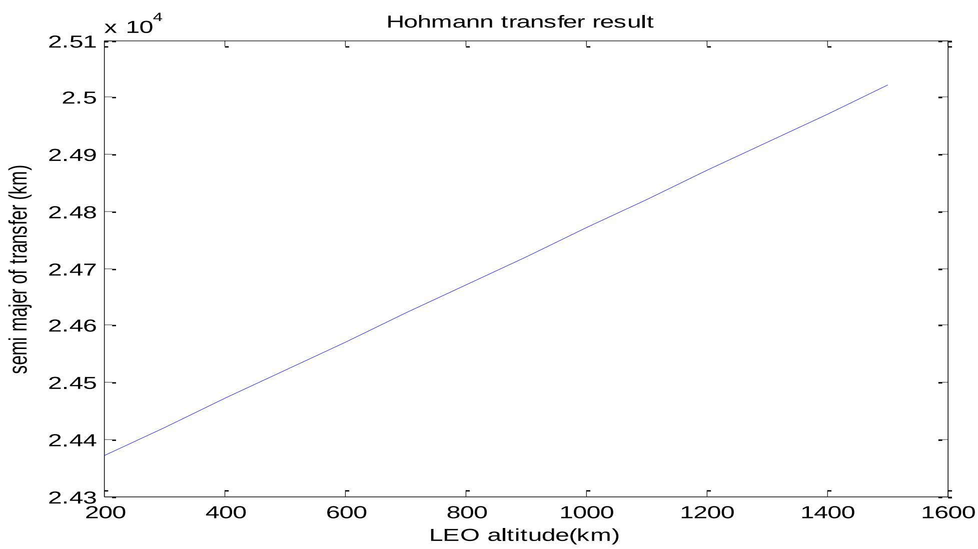


**Figure 7-** Semi major of transfer for Hohmann transfer

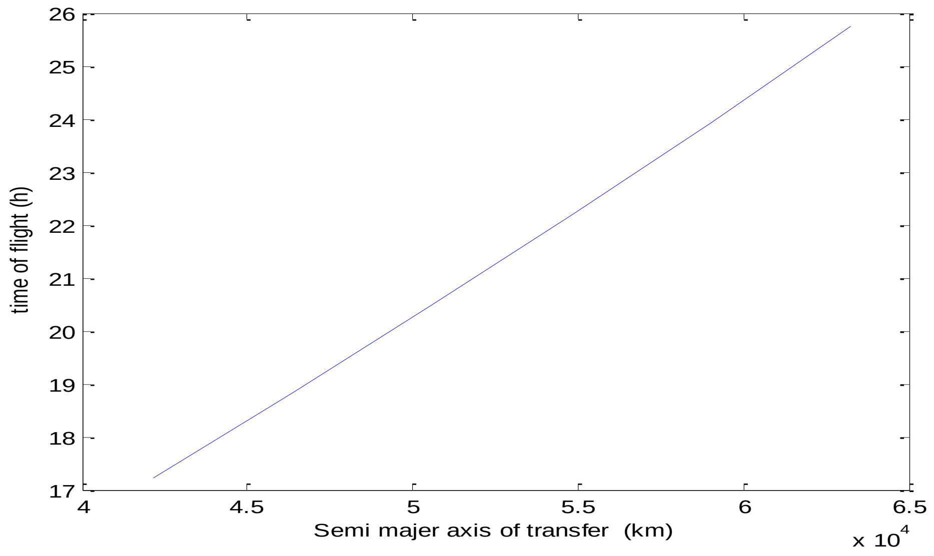


**Figure 8-** The time of transfer required for bi-elliptic transfer.

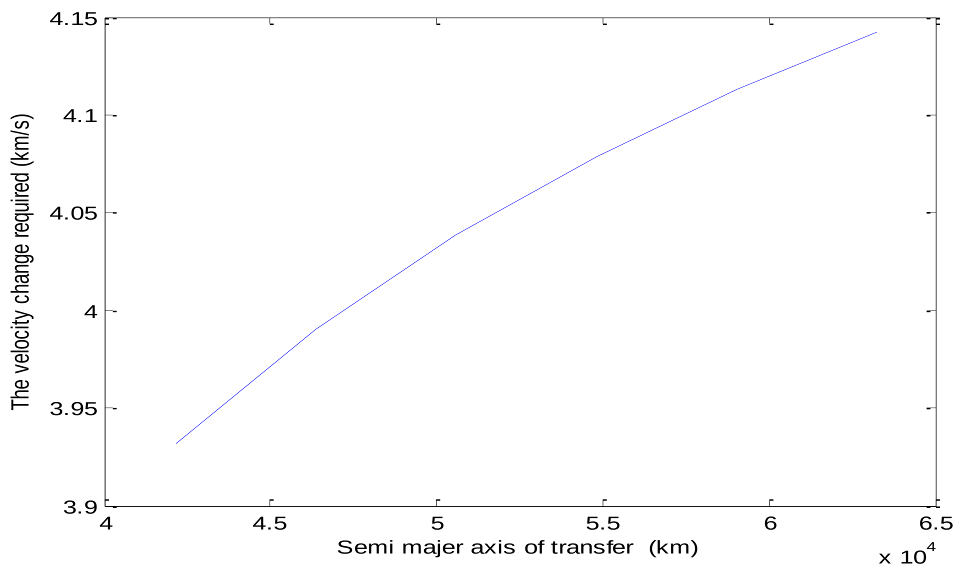


**Figure 9-** The velocity required for bi-elliptic transfer.

**3. Orbital plane change.**

The simple inclination to vary the maneuver that obtain when two orbits is intersect. The change of inclination for a satellite orbits plane needs to made change in the velocity vector, when size of orbits remain immutable, when calculated velocity change through use the equation that show below, Δv for Separation of Inclination required[1]

$$\Delta V = 2V_i \sin(\alpha/2) \quad \text{…… (13)}$$

The orbit when plane change is composed with the circularization burn at GEO; the Δv is calculated as following:

$$\Delta V = 2V_i \sin(\alpha/2) - \left[\left(V_f - V_i + 2V_f \sin(\alpha/2)\right) - \sqrt{V_i^2 + V_f^2 - 2V_iV_f \cos(\alpha)}\right] \quad \text{…… (14)}$$

Δv is presents for combined Hohmann and Plane inclination change, when inclination of change maneuver is applied along with the second burn of the Hohmann transfer then the Δv required will be less than Δv for Hohmann transfer followed by inclination change maneuver,

where:
$\Delta V$ = the delta-V requirement when the plane change.
$V_i$ = the velocity for the original orbit.
$V_f$ = the velocity for the target orbit.
Assumed α =28$^o$ represent the inclination.

**3.1 The results of the transfer when the plane changes.**

The Δv that required for a transfer when plane change is large than Δv required when transfer in coplanar maneuvers and the mass of consume that need to transfer is great then that required for transfer in the same plane The results from a transfer shown in Table-6.

**Table 6-** The results of transfer when the plane is change.

| LEO altitude (km) | Δv (km/s) | Δm (kg) | Δm/m | total of mass (kg) |
|---|---|---|---|---|
| 200 | 5.4195 | 1282.7 | 2.5654 | 1782.7 |
| 300 | 5.3802 | 1266.3 | 2.5326 | 1766.3 |
| 400 | 5.3416 | 1250.4 | 2.5008 | 1750.4 |
| 500 | 5.3037 | 1234. | 2.4698 | 1734.9 |
| 600 | 5.2664 | 1219.8 | 2.4396 | 1719.8 |
| 700 | 5.2298 | 1205.1 | 2.4102 | 1705.1 |
| 800 | 5.1938 | 1190.7 | 2.3814 | 1690.7 |
| 900 | 5.1584 | 1176.8 | 2.3536 | 1676.8 |
| 1000 | 5.1236 | 1163.1 | 2.3262 | 1663.1 |
| 1100 | 5.0894 | 1149.8 | 2.2996 | 1649.8 |
| 1200 | 5.0557 | 1136.8 | 2.2736 | 1636.8 |
| 1300 | 5.0225 | 1124.1 | 2.2482 | 1624.1 |
| 1400 | 4.9898 | 1111.7 | 2.2234 | 1611.7 |
| 1500 | 4.9577 | 1099.6 | 2.1992 | 1599.6 |

the results of transfer when the plane changes as shown in Figures-(10,11) below.

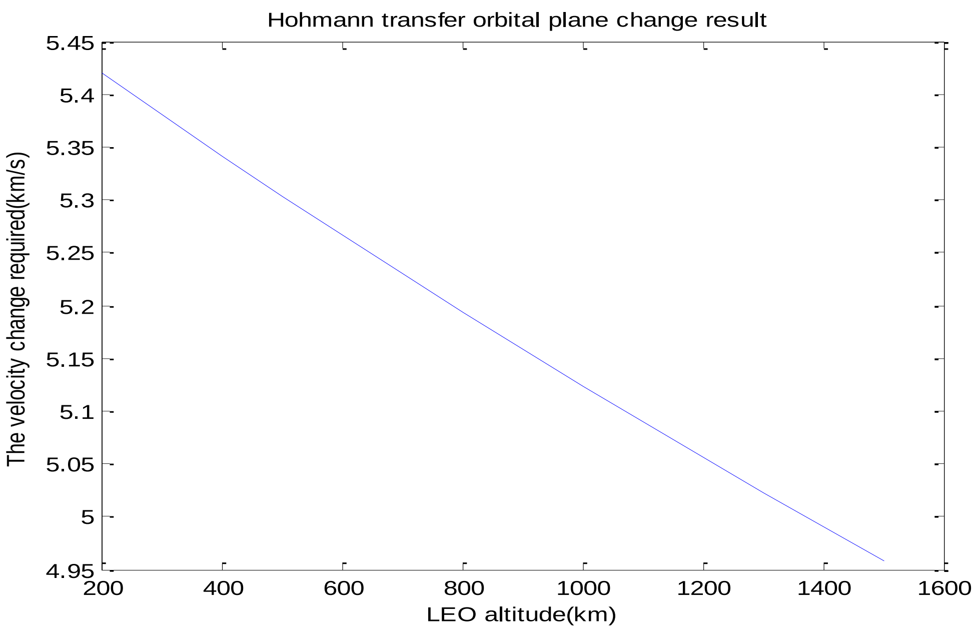


**Figure 10-** The velocity required for Hohmann transfer

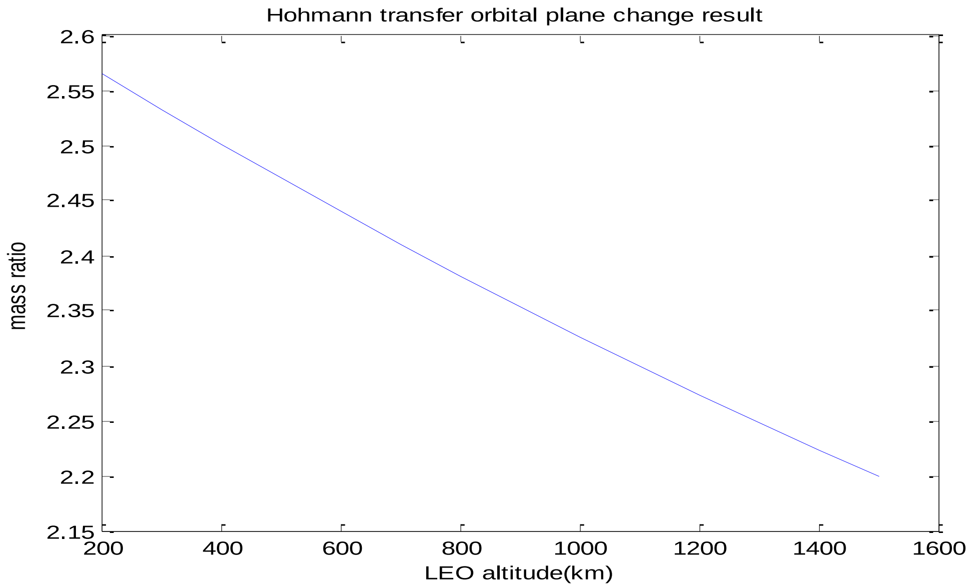


**Figure 11-** The mass ratio required for Hohmann transfer

**4. Conclusions**

1. The Hohmann transfer method requires Δv less than other transfers.
2. The bi-elliptic transfer method requires Δt longer than Hohmann maneuvers.
3. The Δv that required for the transfer when a plane change is large than Δv required when transfer in coplanar maneuvers.
4. The most efficient transfer orbit that occur in apogee on the original orbit when a transfer in coaxial between elliptical orbits.
5. The total mass required for the transfer when plane change is large than when a transfer occur in the same plane.

**References**


1. Gerald, R.H. **2015.** *Orbital Mechanics and astrodynamics*, Department of astronomical Engineering, University of Southern California, Los Angeles, CA, USA.
2. www.braeunig.us
3. Sutton, G. P. and Biblarz, O. **2001**. *The Rocket Propulsion Elements*, Seventh Edition Wiley.
4. Chobotov, V. A. **2002.** *Orbital Mechanics*, Third edition Reston, Virginia, American Institute of Aerobatics.
5. Howard, D.C. **2007.** *Orbital Mechanics for Engineering Students*. Aeronautical University Daytona Beach, Florida.