Some Results on (σ,τ) – Lie Ideals in Prime rings

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Abstract

Let R be a prime with characteristic not equal two, $\sigma,\tau : R \rightarrow R$ be two automorphisms of R. and d be a nonzero derivation of R commuting with σ,τ .It is proved that :

1) Assume U ba a(σ,τ)-left Lie ideal of R.

(a) If $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$ and $[U,U]=(0)$,then $U \subset Z(R)$.

(b) If $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then $U \subset Z(R)$.

(c) If $\sigma(v)+\tau(v) \notin Z(R)$, for some $v \in U$,then there exists a nonzero left ideal A of R and a nonzero right ideal B of R such that $[R,A]_{\sigma,\tau} \subset U$, $[R,B]_{\sigma,\tau} \subset U$ but $[R,A]_{\sigma,\tau} \not\subset C_{\sigma,\tau}$ and $[R,B] \not\subset C_{\sigma,\tau}$.

(d) If $ad(U)=(0)$ $(or \quad d(U)a=(0))$ for $a \in R$, then $a=0$ or $\sigma(u)+\tau(u) \notin Z(R)$, for all $u \in U$.

2) If U be a(σ,τ)-Lie ideal of R for $a \in R, d(U)a \subset C_{\sigma,\tau} (or \quad ad(U) \subset C_{\sigma,\tau})$, $a \in Z(R)$, then $a=0$ or $U \subset Z(R)$.

Also, in this paper we study some results when characteristic of R equal two and we show that the condition characteristic of R not equal two can not be excluded.

الخلاصة

لتكن R حلقة أولية ممثلها لايساوي 2 $\sigma,\tau: R \rightarrow R$ دالتين متكافئة تشاكليا و وكانت $d: R \rightarrow R$ تمثل مشتقة في R تتبادل مع الدالتين σ ,τ.

1) ليكن U (σ,τ)- مثالي لي يساري في R .

أ) إذا كانت $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, $[U,U]=(0)$, فانه $U \subset Z(R)$.

ب) إذا كانت $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, فانه $U \subset Z(R)$.

ج) ليكن U (σ,τ)- مثالي لي يساري في R بحيث ان $\sigma(v)+\tau(v) \notin Z(R)$, لبعض $v \in U$, فانه يوجد على الاقل مثالي يساري A غير صفري ومثالي يميني B غير صفري بحيث انه $[R,A]_{\sigma,\tau} \subset U$ و $[R,B]_{\sigma,\tau} \subset U$ فان $[R,A]_{\sigma,\tau} \not\subset C_{\sigma,\tau}$ و $[R,B]_{\sigma,\tau} \not\subset C_{\sigma,\tau}$.

د) ليكن U (σ,τ)- مثالي لي يساري في R وكانت d تمثل مشتقة في R بحيث إن $ad(U)=(0)$ أو $d(U)a=(0)$, $a \in R$,فانه أما $a=0$ أو $\sigma(u)+\tau(u) \notin Z(R)$, لكل $u \in U$.

2) ليكن U (σ,τ)- مثالي في R وكانت d تمثل مشتقة في R, لأي $a \in R, ad(U) \subset C_{\sigma,\tau}$ أو $d(U)a \subset C_{\sigma,\tau}$ و $a \in Z(R)$, فانه اما $a=0$ او $U \subset Z(R)$.

أيضا ,خلال هذا البحث سندرس بعض تلك النتائج عندما يكون ممثل الحلقة R يساوي 2 , وسنلاحظ إن ممثل الحلقة R لا يساوي 2 شرط لايمكن الاستغناء عنه .

Introduction

Let R be a ring, U be an additive subgroup of R ,σ,τ : $R \rightarrow R$ be two mappings .Then

1) U is called a (σ,τ)- right Lie ideal of R if $[U,R]_{\sigma,\tau} \subset U$.
2) U is called a (σ,τ)- left Lie ideal of R if $[R,U]_{\sigma,\tau} \subset U$.
3) U is called a (σ,τ)- Lie ideal of R if U is both (σ,τ)- right Lie ideal and (σ,τ)- left Lie ideal of R .

Let R be a prime with characteristic not equal two, d : $R \rightarrow R$ be a derivation of R.In [1] Aydin N. proved that if U is a (σ,τ)-left Lie ideal of R such that $[U,U]_{\sigma,\tau}$ =(0) and $[U,U]$=(0) , then $U \subset Z(R)$,and if for $\sigma(v)+\tau(v) \notin Z(R)$, for some $v \in U$,then there exists a nonzero left ideal A of R and a nonzero right ideal B of R such that $[R,A]_{\sigma,\tau} \subset U$, $[R,B]_{\sigma,\tau} \subset U$ but $[R,A]_{\sigma,\tau} \not\subset Z(R)$ and $[R,B] \not\subset Z(R)$.In[4] Aydin,N.,Kaya,K. , Golbasi,O.proved that if U is a noncentral (σ,τ)- left Lie ideal of R *and* if ad(U)=0, then a=0 or $\sigma(u)+\tau(u) \notin Z(R)$, for all $u \in U$ .In [5] Aydin ,N. and Soyturk,M. proved that if U is called a (σ,τ)- Lie ideal of R for $a \in R, d(U)a = (0)$ $(or\ ad(U) = (0))$, then $a = 0$ or $U \subset Z(R)$.

In this paper, we generalized and extended these results, and we study some results in [1],[2],[5],[6],[7],when characteristic of R equal two and we show that the condition characteristic of R not equal two can not be excluded.

§1 Basic Lemmas

In this section we recall some results that interesting in our study.

Lemma (1.1):-[1]

Let U be both a (σ,τ)-left Lie ideal of R and a subring of R, then either $\sigma(u) + \tau(u) \in Z(R)$, for all $u \in U$ or U contains a nonzero left ideal and a nonzero right ideal of R .

Lemma (1.2):-[1]

Let U is a (σ,τ)-left Lie ideal of R such that $\sigma(v) + \tau(v) \notin Z(R)$,for some $v \in U$ and $a,b \in R$. If $aUb = (0)$,then a=0 or b=0.

Lemma (1.3):-[5]

Let U be a (σ,τ)-Lie ideal of R. If $d(U) \subset C_{\sigma,\tau}$, then $U \subset Z(R)$.

Remark (1.4):-[2]

Let R be a ring and let U be a nonzero (σ,τ)-left Lie ideal of R. We shall define the set $T(U) = \{a \in R \ / \ [R,a]_{\sigma,\tau} \subset U\}$. Clearly $U \subset T(U)$. On the other hand, if $a,b \in T(U)$ and $x \in R$, then $[x,ab]_{\sigma,\tau} = [x\sigma(a),b]_{\sigma,\tau} + [\tau(b)x,a]_{\sigma,\tau} \in U$. That is, $[x,ab]_{\sigma,\tau} \in U$. Therefore, $T(U)$ is both a subring and a (σ,τ)-left Lie ideal of R.

Lemma (1.5):-[7, theorem2]

Let U be a (σ,τ)- right Lie ideal of R .If $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then either $U \subset Z(R)$ or $U \subset C_{\sigma,\tau}$.

Lemma (1.6):-[7, theorem3]

If U is both a nonzero (σ,τ)-right Lie ideal of R and a subring of R, then one of the following holds:-

(i)$U \subset Z(R)$.(ii) $U \subset C_{\sigma,\tau}$.(iii) U contains a nonzero ideal of R .

Lemma (1.7)[2,lemma2]

If U is a (σ,τ)-right Lie ideal of R and $a \in R$ such that $[U,a]_{\sigma,\tau} \subset C_{\sigma,\tau}$,then $a \in Z(R)$ or $U \subset C_{\sigma,\tau}$.

Lemma (1.8))[2,theorem1]

If U is a (σ,τ)-right Lie ideal of R and $a \in R$ such that $[U,a]$ = (0), then $a \in Z(R)$ or $U \subset C_{\sigma,\tau}$.

Lemma (1.9)[2,corollary2]

Let U be a nonzero (σ,τ)-Lie ideal of R such that $U \not\subset Z(R)$ and $U \not\subset C_{\sigma,\tau}$ and $a,b \in R$. If $a\ U\ b$ = (0), then a =0 or b=0.

Lemma (1.10)[1,theorem5]

Let U be a nonzero (σ,τ)-Lie ideal of R such that $U \subset C_{\sigma,\tau}$. Then σ = τ or R is commutative.

Lemma (1.11)[1,theorem2]

Let U be a (σ,τ)-left Lie ideal of R, such that $[U,U]_{\sigma,\tau}$ = (0) and $[U,U]$=(0). Then $U \subset Z(R)$.

Lemma (1.12)[5,theorem1]

Let U be a (σ,τ)-right Lie ideal of R *and* d be a derivation of R. If $d(U) \subset C_{\sigma,\tau}$,then R is commutative or $U \subset C_{\sigma,\tau}$.

Lemma (1.13)[5,lemma5]

Let U be a (σ,τ)-Lie ideal of R d be a derivation of R and $a \in R$. If $d(U)a = (0)$ or $ad(U) = (0)$, then a =0 or $U \subset Z(R)$.

§2Extensions and generalizations

In this section we shall extend and generalized some results and give their details.

Theorem (2.1):-

Let U be a (σ,τ)-left Lie ideal of R such that $(0) \neq [U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$ and $[U,U]$=(0) . Then $U\subset Z(R)$.

Proof:-

By assumption $0 \neq [U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then for all $u,v\in U$, $x\in R$, we have $C_{\sigma,\tau} \ni [[x\sigma(u),u]_{\sigma,\tau},v]_{\sigma,\tau} = [[x,u]_{\sigma,\tau}\sigma(u),v]_{\sigma,\tau}$

$$= [x,u]_{\sigma,\tau}[\sigma(u),\sigma(v)] + [[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(u)$$

. Since $[U,U]$=(0) , we get $[[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(u) \in C_{\sigma,\tau}$. That is, $[[[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(u),r]_{\sigma,\tau} = 0$, for all $r\in R$. This implies

$$[[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(u)\sigma(r) - \tau(r)[[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(u) = 0$$

Then

$$[[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(u)\sigma(r) - [[x,u]_{\sigma,\tau},v]_{\sigma,\tau}\sigma(r)\sigma(u) = 0$$

and we have $[[x,u]_{\sigma,\tau},v]_{\sigma,\tau}[\sigma(u),\sigma(r)] = 0$, for all $u,v\in U$, $x,r\in$R. By a primeness of R and $[[x,u]_{\sigma,\tau},v]_{\sigma,\tau} \neq 0$, we get $\sigma([u,r]) = 0$. This implies $u\in Z(R)$, for all $u\in U$. Hence, $U\subset Z(R)$.

Remark (2.2):-

In Theorem (2.1), we can exclude the condition $[U,U]$=(0) as below.

Theorem (2.3):-

Let U be a (σ,τ)-left Lie ideal of R such that $(0) \neq [U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$.Then $U\subset Z(R)$.

Proof:-

Since $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then for any $u \in U$, we have

$$[[\tau(u)\sigma(u),u]_{\sigma,\tau},u]_{\sigma,\tau} =$$
$$[[\tau(u),u]_{\sigma,\tau}\sigma(u),u]_{\sigma,\tau} =$$
$$[[\tau(u),u]_{\sigma,\tau},u]_{\sigma,\tau}\sigma(u) \in C_{\sigma,\tau}$$

That is, for all $r \in R$, we have $[[[\tau(u),u]_{\sigma,\tau},u]_{\sigma,\tau}\sigma(u),r]_{\sigma,\tau} = 0$. This implies

$$[[\tau(u),u]_{\sigma,\tau},u]_{\sigma,\tau}[\sigma(u),\sigma(r)] +$$
$$[[[\tau(u),u]_{\sigma,\tau},u]_{\sigma,\tau},r]_{\sigma,\tau}\sigma(u) = 0$$

Also, we have

$[[\tau(u),u]_{\sigma,\tau},u]_{\sigma,\tau}[\sigma(u),\sigma(r)] = 0$. Since R is a prime ring and $(0) \neq [U,U]_{\sigma,\tau}$,then $\sigma([u,r]) = 0$ for all $u \in U, r \in R$. Thus, $[u,r] = 0$ for all $u \in U, r \in R$. Hence, $U\subset Z(R)$.

Theorem (2.4):-

Let U be a (σ,τ)-left Lie ideal of R such that $\sigma(v) + \tau(v) \notin Z(R)$, for some $v \in U$. Then there exists a nonzero left ideal A of R and a nonzero right ideal B of R such that $[R,A]_{\sigma,\tau} \subset U$ and $[R,B]_{\sigma,\tau} \subset U$ but $[R,A]_{\sigma,\tau} \not\subset C_{\sigma,\tau}$ and $[R,B]_{\sigma,\tau} \not\subset C_{\sigma,\tau}$.

<u>Proof</u>:-

Let $T(U) = \{x \in R : [R,x]_{\sigma,\tau} \subset U\}$. By Remark (1.4) , $T(U)$ is both a (σ,τ)-left Lie ideal and a subring of R such that $U \subset T(U)$. Since $\sigma(v) + \tau(v) \notin Z(R)$ for some $v\in U$, then by Lemma (1.1), $T(U)$ must contains a nonzero left ideal A of R and a nonzero right ideal B of R . From the definition of $T(U)$, we obtain $[R,A]_{\sigma,\tau} \subset U$ and $[R,B]_{\sigma,\tau} \subset U$. If $[R,A]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then for any $a \in A$, $x \in R$, we have $[\tau(a)x,a]_{\sigma,\tau} = \tau(a)[x,a]_{\sigma,\tau} \in C_{\sigma,\tau}$, that is, $[\tau(a)[x,a]_{\sigma,\tau},r]_{\sigma,\tau} = 0$, for all $r \in R$. This implies that

$0=\tau(a)[[x,a]_{\sigma,\tau},r]_{\sigma,\tau}+[\tau(a),\tau(r)][x,a]_{\sigma,\tau}$
$=[\tau(a),\tau(r)][x,a]_{\sigma,\tau}$, for all $r\in R$.

Taking ry, $y\in R$ instead of r in the last equation, then $[\tau(a),\tau(r)]R[x,a]_{\sigma,\tau}=(0)$. By a primeness of R we get either $[\tau(a),\tau(r)]=0$ or $[x,a]_{\sigma,\tau}=0$.

If $[\tau(a),\tau(r)]=0$, then $a\in Z(R)$. Therefore $A\subset Z(R)$. If $[x,a]_{\sigma,\tau}=0$, for all $x\in R$.

Replacing x by xy , $y\in R$, such that $[xy,a]_{\sigma,\tau}=0$. That is, $0=[xy,a]_{\sigma,\tau}=$
$x[y,a]_{\sigma,\tau}+[x,\tau(a)]y=[x,\tau(a)]y$

for all $x,y\in R$. Hence, $[R,\tau(a)]R=(0)$. By the primeness of R we get $a\in Z(R)$. Therefore $A\subset Z(R)$. So, for all $x,y\in R, a\in A$, we have

$0=[x,ya]=y[x,a]+[x,y]a=[x,y]a$ .Hence, $[R,R]A=(0)$, by primeness of R we have R is commutative, then $U\subset Z(R)$ so, $\sigma(u)+\tau(u)\in Z(R)$, for all $u\in U$ and this is a contradiction. That is , $[R,A]_{\sigma,\tau}\not\subset C_{\sigma,\tau}$.

Similarly, using the identity $[x,\sigma(b),b]_{\sigma,\tau}=[x,b]_{\sigma,\tau}\sigma(b)$ to prove that $[R,B]_{\sigma,\tau}\not\subset C_{\sigma,\tau}$.

Theorem (2.5):-
Let U be a (σ,τ)-Lie ideal of R. If $d(U)a\subset C_{\sigma,\tau}$ (or $ad(U)\subset C_{\sigma,\tau}$) and $a\in Z(R)$, then a =0 or $U\subset Z(R)$.

Proof:-
For all $u\in U, x\in R$, we have $[ad(u),x]_{\sigma,\tau}=0$. Hence,
$0=[ad(u),x]_{\sigma,\tau}=$
$a[d(u),x]_{\sigma,\tau}+[a,\tau(x)]d(u)=a[d(u),x]_{\sigma,\tau}$
That is, $0=a[d(U),x]_{\sigma,\tau}$, for all $x\in R$. By a primenss of R , we get either a =0 or $d(U)\subset C_{\sigma,\tau}$. If $d(U)\subset C_{\sigma,\tau}$, then by Lemma (1.3), we have $U\subset Z(R)$.

By the same way when $d(U)a\subset C_{\sigma,\tau}$, we can show that either a=0 or $U\subset Z(R)$.

Theorem (2.6)
Let U be a (σ,τ)-left Lie ideal of R and $a\in R$. If $ad(U)$=(0) (or $d(U)a$=(0)), then $\sigma(u)+\tau(u)\in Z(R)$, for all $u\in U$ or a=0 .

Proof:-
Assume $v\in U$ such that $\sigma(v)+\tau(v)\notin Z(R)$ .Then, by assumption $ad(U)=(0)$, we have
$0=ad([r\sigma(u),u]_{\sigma,\tau})=ad([r,u]_{\sigma,\tau}\sigma(u))$
$=ad([r,u]_{\sigma,\tau})\sigma(u)+a[r,u]_{\sigma,\tau}d(\sigma(u))=a[r,u]_{\sigma,\tau}d(\sigma(u))$
Replace $r\sigma(u)$ instead of r in the last equation , we get
$0=a[r\sigma(u),u]_{\sigma,\tau}d(\sigma(u))=a[r,u]_{\sigma,\tau}\sigma(u)d(\sigma(u))$ .Hence, $a[r,u]_{\sigma,\tau}\sigma(u)d(\sigma(u))=0$.
Also, we get $0=\sigma^{-1}(a[r,u]_{\sigma,\tau})ud(u)$ for all $u\in U$. Therefore, $\sigma^{-1}(a[r,u]_{\sigma,\tau})Ud(u)=(0)$ .By Lemma (1.2), we obtain either $d(u)$=0 or $a[r,u]_{\sigma,\tau}=0$
If $d(u)$=0, then $d(U)$=(0) . That is, $d(U)\subset Z(R)$, so $\sigma(u)+\tau(u)\in Z(R)$,for all $u\in U$ and that is, a contradiction . If $a[r,u]_{\sigma,\tau}=0$. Replace rx instead of r , we get $0=a[rx,u]_{\sigma,\tau}=ar[x,\sigma(u)]+a[r,u]_{\sigma,\tau}x=ar[x,\sigma(u)]$. That is, $ar[x,\sigma(u)]=0$.
Since R is a prime ring , then we have a=0 .

§3Examples when Chr=2 :-

In this section we study the results in [1],[2],[5],[6],[7], when characteristic of R equal two and we show that the condition characteristic of R not equal two can not be excluded.

Let $R=\left\{\begin{pmatrix} x & y \\ z & t\end{pmatrix} x,y,z,t\in F\right\}$, where F is a field of ChF=2 be a ring of 2x2 matrices with respect to the usual operation of addition and multiplication, then R is a prime ring ,see[6].

Example (3.1):-

Let $U=\left\{\begin{pmatrix} a & b \\ 0 & a\end{pmatrix} a,b\in F\right\}$ be an additive subgroup of R . Assume σ,τ: : $R\to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}$. Then,

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix} - \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} =$$

$$\begin{pmatrix} ax+bz & ay+bt \\ az & at \end{pmatrix} - \begin{pmatrix} xa & xb-ya \\ -za & -zb+ta \end{pmatrix}$$

$$= \begin{pmatrix} bz & bt-xb \\ 0 & zb \end{pmatrix} \in U.$$

The hypothesis of Lemma (1.5) is satisfied but $U \not\subset Z(R)$ and $U \not\subset C_{\sigma,\tau}$.

Example (3.2):-

Let $U = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \; a,b \in F \right\}$ be an additive subgroup of R . Assume $\sigma,\tau: : R \to R$ be two automophisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}.$$

The hypothesis of Lemma (1.6) is satisfied , but $U \not\subset Z(R)$, $U \not\subset C_{\sigma,\tau}$ and U doesn't contain a nonzero ideal of R.

Example (3.3):-

Let $U = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \; a,b \in F \right\}$ be an additive subgroup of R . Assume $\sigma,\tau: : R \to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}.$$

Let $a \in R$ such that $a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Assumption of Lemma (1.7) is satisfied , but $a \notin Z(R)$ and $U \not\subset C_{\sigma,\tau}$.

Example (3.4):-

Let $U = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \; a,b \in F \right\}$ be an additive subgroup of R . Assume $\sigma,\tau: : R \to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}.$$

Let $a \in R$ such that $a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. The hypothesis of Lemma (1.8) is satisfied, but $a \notin Z(R)$ and $U \not\subset C_{\sigma,\tau}$.

Example (3.5):-

Let $U = \left\{ \begin{pmatrix} u & v \\ 0 & u \end{pmatrix} \; u,v \in F \right\}$ be an additive subgroup of R . Assume $\sigma,\tau: : R \to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}$.Let

$$a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

The hypothesis oh Lemma (1.9) is satisfied but $a \neq 0$ and $b \neq 0$.

Example (3.6):-

Let $U = \left\{ \begin{pmatrix} \boldsymbol{a} & 0 \\ 0 & \boldsymbol{a} \end{pmatrix} , a \in F \right\}$ be an additive subgroup of R .

Assume $\sigma,\tau: : R \to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}.$$

The hypothesis of Lemma (1.10) is satisfied, but $\sigma \neq \tau$ and R is not commutative .

Example (3.7):-

Let $U=\left\{\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \; a,b \in F\right\}$ be an additive subgroup of R. Assume σ,τ: $: R \to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

$$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}.$$

The hypothesis of Lemma (1.11) is satisfied, but $U \not\subset Z(R)$.

Example (3.8):-

$U = \left\{\begin{pmatrix} b & a \\ a & 0 \end{pmatrix} \; , a,b \in F\right\}$ be an additive subgroup of R .Let σ, τ : $R \to R$ be two automorphisms , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix}, \; \tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} t & -z \\ -y & x \end{pmatrix}$$

Also, $d : R \to R$, defined by

$$d\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} 0 & -y \\ z & 0 \end{pmatrix},$$ be a derivation of *R*.

The assumption of Lemma (1.12) is satisfied, but $U \not\subset C_{\sigma,\tau}$ and R is not commutative.

Example (3.9):-

$U = \left\{\begin{pmatrix} u & v \\ 0 & u \end{pmatrix}, u,v \in F\right\}$ be an additive subgroup of R . Assume $\sigma,\tau : R \to R$ be two automorphisms such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix} \text{and}$$

$$\tau\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & -y \\ -z & t \end{pmatrix}.$$

Also $d : R \to R$, defined by

$$d\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} 0 & -y \\ z & 0 \end{pmatrix},$$ is a derivation of *R*.

Let $a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Assumption of Lemma (1.13) is satisfied, but $U \not\subset Z(R)$ and $a \neq 0$.

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