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# Influence of Inclined MHD on Unsteady Flow of Generalized Maxwell Fluid with Fractional Derivative between Two Inclined Coaxial Cylinders through a Porous Medium

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**Abstract**

This paper presents a study of inclined magnetic field on the unsteady rotating flow of a generalized Maxwell fluid with fractional derivative between two inclined infinite circular cylinders through a porous medium. The analytic solutions for velocity field and shear stress are derived by using the Laplace transform and finite Hankel transform in terms of the generalized G functions. The effect of the physical parameters of the problem on the velocity field is discussed and illustrated graphically.

**Keywords**: Maxwell fluid, unsteady rotating flow, infinite circular cylinders, porous medium, (MHD) magnetohydrodynamic field.

# التأثير المغناطيسي المائل على مائع ماكسويل ذو المشتقات الكسريه خلال وسط مسامي بين أسطوانتين دائريه مائله

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**الخلاصة**

في هذا البحث دراسنا الحقل المغناطيسي المائل على الجريان الدوراني غير المستقر لمائع ماكسويل ذو المشتقات الكسريه بين أسطوانتين دائريه مستقيمه مائله لانهائيه خلال وسط مسامي. تم اشتقاق الحل التحليلي لمجال السرعة وإجهاد القص باستعمال تحويل لابلاس و تحويل هانكل يدلالة الدالة G المعممه .رسمنا تأثير المعلمات الكسرية على مجال السرعة بيانيا.

## 1. Introduction

The modeling of the equations governing the non-Newtonian fluids gives rise to a nonlinear differential equation .Such nonlinear fluids are considered to play a more important and appropriate role in technological applications in comparison with Newtonian fluids. Recently, the most important applications in differential equations, integral equations, physics, fluid mechanics, viscoelasticity, mathematical biology, and electrochemistry can be described by using the subject of fractional

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calculus. There is no doubt that the fractional calculus is becoming an interesting spot and an exciting new way to solve the diverse problems in mathematics, science and engineering. In fluid mechanics, many models have been proposed to describe the response of the fluids, which are classified as fluids with a mathematical model of differential type with fractional derivatives. The applications of non-Newtonian fluids in engineering problems such as magnetohydrodynamic (MHD ) have become the focus of an extended number of studies; for example plastics, polymer fluids, exotic lubricant, food stuffs and polymers are handled extensively by chemical industries, whereas biological and rheological properties of many materials are described by their constitutive equation.

In recent years, the fractional derivatives are found to be quite flexible in describing the behaviors of the viscoelastic fluid and are studied by many mathematicians considering various motions of such fluids . In these studies, the constrictive equation for generalized non-Newtonian fluids are modified from the well-known fluid models by replacing the time derivative of an integer order by precisely non-integer order integrals or derivatives. The fractional derivative models of the viscoelastic fluids are obtained by researchers. Especially, the problems of the motion of a fluid in rotating or translating cylinder are of interest to both theoretical and practical domains . The first exact solution for non-Newtonian fluids which flows in cylindrical problems are those of Srivastava [9] for Maxwell fluids and Waters and King [8] for Oldroyd –B fluids in a straight circular tube . Fetecau [2] studied some helical flows of Maxwell and Oldroyd -B fluids within an infinite cylinder .The exact solutions of generalized Maxwell fluid flow due to oscillatory and constantly accelerating plate were obtained by Zheng [4]. Also, Hayat and Asgher [6] studied the exact solution for MHD flow of a generalized Oldroyd –B fluid with modified Darcy’s law. Nazar and Corina Fetecau [7] considered a note on the unsteady flow of a generalized second-grade fluid through a circular cylinder subject to a time-dependent shear stress. Liancun and Zhang [8] studied the unsteady rotating flows of a viscoelastic generalized Maxwell fluid with oscillating pressure gradient between coaxial cylinders. Sundos and Ahmed [10] studied the effects of MHD on the unsteady rotating flow of a generalized Maxwell fluid with oscillating gradient between coaxial cylinders.

In this paper, we studied the effects of inclined magnetic field on the unsteady rotating flow of a generalized Maxwell fluid with fractional derivative between two inclined infinite straight circular cylinders. The velocity field and the shear stress are obtained by means of discrete Laplace transform and finite Hankel transform. The exact solutions for the velocity field and the shear stress were obtained by integral and series form in terms of the generalized G functions. Graphs were plotted to show the effects of the fractional parameter on the fluid dynamic characteristics with MHD on the velocity field.

**2. Mathematical Formulation**

The constitutive equations of an unsteady inclined magnetic hydrodynamic incompressible flow of generalized Maxwell fluid with fractional derivative between two inclined coaxial cylinders through a porous medium are given by [2,5]

$$\mathrm{T} = -pI + \mathrm{S} + \rho g sin\theta + J \times B + R \,, S + \lambda \frac{DS}{Dt} = \mu A \tag{1}$$

where T is the Cauchy stress tensor, - $pI$ is the indeterminate spherical stress, S is the extra-stress tensor , $\rho$ is the fluid density , $g$ is the external body force, $A = \mathrm{L} + \mathrm{L}^{\mathrm{T}}$ is the first Rivlin-Ericksen tensor with L=grad V, $\boldsymbol{B}$ is the magnetic field, and $J$ is the current density (or conduction current). $\mu$ is the dynamic viscosity of the fluid, $\lambda$ is the material constant, and $\frac{\mathrm{DS}}{\mathrm{Dt}}$ is defined by

$$\frac{\mathrm{DS}}{\mathrm{Dt}} = \mathrm{D}_t^{\alpha}\mathrm{S} + \mathrm{V}.\nabla\mathrm{S} - \mathrm{LS} - \mathrm{SL}^{\mathrm{T}} \tag{2}$$

where $\nabla$ is gradient operator, $\alpha$ is the fractional calculus parameter such that $0 \leq \alpha \leq 1$, and $\mathrm{D}_t^{\alpha}$ is the fractional differential operator based on Riemann-Liouville's, defined as

$$\mathrm{D}_t^{\alpha} f(\mathrm{t}) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau, \quad 0 \leq \alpha \leq 1 \tag{3}$$

where $\Gamma(.)$ is the Gamma function and

$$\mathrm{D}_{\mathrm{t}}^{2\alpha} = \mathrm{D}_{\mathrm{t}}^{\alpha}(\mathrm{D}_{\mathrm{t}}^{\alpha}\mathrm{S}) \tag{4}$$

We assume that the velocity and shear stress in cylindrical coordinates $(r, \theta, z)$ are given by

$$\mathrm{V} = u(r, \mathrm{t})\, e_{\theta\,,} \;\; \mathrm{S} = \mathrm{S}(r, \theta) \tag{5}$$

where $e_\theta$ is the unit vector in the $\theta$- axis and $u$ is the velocity. Since V is dependent on $r$ and t, we also assume that S depends only on $r$ and $\theta$ . If the fluid is being at rest at t=0, then

$$\mathrm{V}(r,0)=0\ ,\ \mathrm{S}(r,0)=0 \tag{6}$$

We can obtain

$$\tau(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})=\mu\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)u \tag{7}$$

According to our problem,
$S_{rr}=S_{zz}=S_{rz}=S_{\theta z}=S_{\theta\theta}=0\ ,\tau(r,\mathrm{t})=S_{r\theta}\ (r,\mathrm{t})$ is the shear stress .

We consider a generalized Maxwell fluid between two inclined infinite circular cylinders through a porous medium .The pressure gradient in the $\theta$ axial direction and the balance of the linear momentum leads to the relevant and meaningful equation

$$\rho\frac{\partial u}{\partial \mathrm{t}}=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\frac{1}{r^2}\frac{\partial(r^2\tau)}{\partial r}-\sigma\beta_0^2\,u\,sin\theta-\rho g sin\theta-\frac{\mu\varphi}{\mathrm{K}}\mathrm{u}, \tag{8}$$

where $\rho$ is the constant density of the fluid , K is the permeability and $\varphi$ is the porosity of the porous medium .

Now, by eliminating $\tau(r,t)$ between Eqs, (7) and (8), we get the governing equation of motion, as follows:

$$(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})\frac{\partial u}{\partial \mathrm{t}}=-\frac{1}{r\,\rho}(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})\frac{\partial p}{\partial\theta}+\frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\right)u$$
$$-\left(\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})u-\rho g\ (1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})\ sin\theta \tag{9}$$

Let $\upsilon=\frac{\mu}{\rho}$ be the kinematic viscosity and $\frac{\partial p}{\partial\theta}=-\rho\,p_0\cos(\omega t)$ or $\frac{\partial p}{\partial\theta}=-\rho\,p_0\sin(\omega t)$ where $p_0$ is constant, then we get the governing equation

$$(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})\frac{\partial u}{\partial \mathrm{t}}=\frac{p_0}{r}(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})\cos(\omega t)\ +\upsilon\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)$$
$$-\left(\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)(1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha})u-\rho g sin\theta\ (1+\lambda\ \mathrm{D}_{\mathrm{t}}^{\alpha}) \tag{10}$$

and the appropriate initial and boundary conditions are as follows:

$$u(r,0)=\frac{\partial u(r,0)}{\partial \mathrm{t}}=0\ ,\quad r\,\epsilon[R_1\,,R_2] \tag{11}$$

$$u(R_1,t)=fe^{at}\ ,\ u(R_2,t)=\ 0,\ \ \mathrm{t}>0 \tag{12}$$

where $f$ is constant.

**3.Calculation of the velocity field**

In this section ,the velocity field will be calculated for different cases of pressure gradient.

**3.1. Case one**

To find the solution for Eq(10) subject to boundary condition and $\frac{\partial p}{\partial\theta}=-\rho\,p_0\,\mathrm{Cos}(\omega t)$ , we frist take the Laplace transformation to Eqs. (10)-(12) and, using the Laplace transform of the sequential fractional derivatives [6], we find that

$$(q+\lambda q^{\alpha+1})\bar{u}=\frac{p_0}{r}\left[\left(1+\lambda\,\omega^{\alpha}\cos\left(\frac{\pi}{2}\,\alpha\right)\right)\frac{q}{q^2+\omega^2}-\lambda\,\omega^{\alpha}\sin\left(\frac{\pi}{2}\,\alpha\right)\frac{\omega}{q^2+\omega^2}\right]$$
$$+\upsilon\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)\bar{u}-\left[\left(\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\bar{u}-\rho g sin\theta\right](1+\lambda q^{\alpha})\,,r\,\epsilon[R_1\,,R_2] \tag{13}$$

$$\bar{u}(r,0)=0\ ,\quad r\,\epsilon[R_1\,,R_2] \tag{14}$$

$$\bar{u}(R_1,q)=\frac{f}{(q-a)}\ ,\ \ \bar{u}(R_2,q)=\ 0,\ \ \mathrm{t}>0 \tag{15}$$

Next, we denote the finite Hankel transform of $\bar{u}$ [7] , defined as follows

$$\bar{u}_H=\int_{R_1}^{R_2} r\,\bar{u}\,B_1(r\,r_n)dr\ ,n=1,2,3\ldots\ldots\ldots \tag{16}$$

where $r_n$ are the positive roots of equation $B_1(R_1\,r)=0$ and

$$B_1(r\,r_n) = J_1(r\,r_n)Y_1(R_2\,r_n) - J_1(R_2\,r_n)Y_1(r\,r_n) \tag{17}$$

where $J_n(.)$ and $Y_n(.)$ are Baseel functions of the first and second kind of order n, respectively. Now, by multiplying both sides of Eq. (13) by $r\,B_1(r\,r_n)$ and then integrating with respect to $r$ from $R_1$ to $R_2$ and taking into account the conditions (14) and (15) as well as the identity

$$\int_{R_1}^{R_2} r\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)\bar{u}\,B_1(r\,r_n)dr = -\frac{2}{\pi}\frac{f}{(q-a)}\frac{J_1(R_2\,r_n)}{J_1(R_1\,r_n)} - {r_n}^2\,\bar{u}_H \tag{18}$$

we find that

$$\bar{u}_H = p_0\left(1+\lambda\,\omega^\alpha\cos\left(\frac{\pi}{2}\,\alpha\right)\right)\frac{\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)}{r_n(q^2+\omega^2)(1+\lambda q^\alpha)}\frac{q}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}$$
$$-\lambda\,\omega^\alpha\sin\left(\frac{\pi}{2}\,\alpha\right)\frac{\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)}{r_n(q^2+\omega^2)(1+\lambda q^\alpha)}\frac{\omega}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}-\frac{2}{\pi\,r_n^2}\frac{f}{(q-a)}$$
$$\times\frac{J_1(R_2\,r_n)}{J_1(R_1\,r_n)}-\frac{\left[\frac{2}{\pi}\frac{f}{r_n^2}\frac{J_1(R_2\,r_n)}{J_1(R_1\,r_n)}\left(q+\upsilon\left(\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{k}\right)\right)-\rho g sin\theta\right]}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)} \tag{19}$$

where $\bar{B}_1(r\,r_n) = J_0(r\,r_n)Y_1(R_2\,r_n) - J_1(R_2\,r_n)Y_0(r\,r_n)$ [1].

By taking inverse Hankel transformation of Eq.(19), we get

$$\bar{u} = \frac{R_1(R_2^2-r^2)f}{(R_2^2-R_1^2)r(q-a)}+\frac{\pi^2 p_0\left(1+\lambda\,\omega^\alpha\cos\left(\frac{\pi}{2}\,\alpha\right)\right)q}{2(1+\lambda q^\alpha)(q^2+\omega^2)}\sum_{n=1}^{\infty}\frac{r_n\,J_1^2(R_1\,r_n)B_1(r\,r_n)}{\{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)\}}$$
$$\times\frac{\left(\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)\right)}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}-\frac{\pi^2 p_0\,\lambda\,\omega^\alpha\sin\left(\frac{\pi}{2}\,\alpha\right)}{2(1+\lambda q^\alpha)(q^2+\omega^2)}\sum_{n=1}^{\infty}\frac{r_n\,J_1^2(R_1\,r_n)B_1(r\,r_n)}{\{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)\}}$$
$$\times\frac{\left(\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)\right)}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}+\pi f\sum_{n=1}^{\infty}\frac{J_1(R_1\,r_n)J_1(R_2\,r_n)B_1(r\,r_n)}{\{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)\}(q-\mathrm{a})}$$
$$\times\frac{\left(q+\upsilon\left(\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}-\frac{\pi}{2}\sum_{n=1}^{\infty}\frac{r_n^2\,J_1^2(R_1\,r_n)B_1(r\,r_n)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}$$
$$\times\frac{\rho g sin\theta}{\left(q+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)} \tag{20}$$

where

$$\bar{u} = \frac{\pi}{2}\sum_{n=1}^{\infty}\frac{r_n^2\,J_1^2(R_1\,r_n)B_1(r\,r_n)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}\,\bar{u}_H \tag{21}$$

Now, by taking discrete Laplace transformation of Eq.(20) and using the following property of inverse Laplace transformation

$$L^{-1}\left\{\frac{q^b}{(q^a-d)^c}\right\}=G_{a,b,c}(d,\tau) \tag{22}$$

where [11]

$$G_{a,b,c}(d,\tau)=\sum_{j=0}^{\infty}\frac{(c)_j d^j t^{(j+c)a-b-1}}{j!\,\Gamma[(j+c)a-b]} \tag{23}$$

is the generalized $G$ function and $(c)_j$ is Pochhammer polynomial [3] .

Finally, we get the expressions for the velocity field as follows:

$$u=\frac{R_1(R_2^2-r^2)f}{(R_2^2-R_1^2)r}e^{at}-\sum_{k=0}^{\infty}\frac{(-1)^{\mathrm{k}}\pi^2 p_0\left(1+\lambda\,\omega^{\alpha}\cos\left(\frac{\pi}{2}\,\alpha\right)\right)}{2\lambda\left(\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}}\int_0^{\mathrm{t}}Cos\big(\omega(t-\tau)\big)$$

$$\times G_{\alpha,k,1}\left(-\lambda^{-1},\tau\right)\mathrm{d}\tau\sum_{n=1}^{\infty}\frac{r_n J_1^2(R_1\,r_n)B_1(r\,r_n)\left(\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)\right)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}$$

$$-\int_0^{\mathrm{t}}Sin\big(\omega(t-\tau)\big)\,G_{\alpha,k,1}\left(-\lambda^{-1},\tau\right)\mathrm{d}\tau\sum_{k=0}^{\infty}\frac{(-1)^{\mathrm{k}}\pi^2 p_0\,\lambda\,\omega^{\alpha}\sin\left(\frac{\pi}{2}\,\alpha\right)}{2\lambda\left(\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}}\times$$

$$\sum_{n=1}^{\infty}\frac{r_n J_1^2(R_1\,r_n)B_1(r\,r_n)\left(\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)\right)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}+\sum_{\mathrm{n}=1}^{\infty}\frac{\pi f J_1(R_1\,r_n)J_1(R_2\,r_n)B_1(r\,r_n)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}\times$$

$$\left\{\sum_{k=0}^{\infty}\left(-\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}}\int_0^t\left\{(k-1)!\,e^{\alpha(t-\tau)}\tau^{k-1}\right\}d\tau+\sum_{k=0}^{\infty}\sum_{m=0}^{k}\frac{(-1)^{\mathrm{k}}k!\,r_n^{2(\mathrm{k-m})}}{m!\,(m-k)!}\right.$$

$$\left.\left(\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}\int_0^t\left\{(k-1)!\,e^{\alpha(t-\tau)}\tau^{k-1}\right\}d\tau\right\}-\frac{\pi}{2}\sum_{n=1}^{\infty}\frac{r_n^2 J_1^2(R_1\,r_n)B_1(r\,r_n)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}$$

$$\times\rho g sin\theta\sum_{k=0}^{\infty}\left(-\boldsymbol{\upsilon}\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\mathrm{t}\right)^{\mathrm{k}}k! \tag{24}$$

### 3.2. Case two

By a similar method and using $\frac{\partial \mathrm{p}}{\partial \theta}=-\rho\,\mathrm{p}_0\,\mathrm{Sin}(\omega\mathrm{t})$, we can obtain the solution in the following form

$$u=\frac{R_1(R_2^2-r^2)f}{(R_2^2-R_1^2)r}e^{at}-\sum_{k=0}^{\infty}\frac{(-1)^{\mathrm{k}}\pi^2 p_0\left(1+\lambda\,\omega^{\alpha}\cos\left(\frac{\pi}{2}\,\alpha\right)\right)}{2\lambda\left(\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}}\int_0^{\mathrm{t}}Sin\big(\omega(t-\tau)\big)$$

$$G_{\alpha,k,1}\left(-\lambda^{-1},\tau\right)\mathrm{d}\tau\sum_{n=1}^{\infty}\frac{r_n J_1^2(R_1\,r_n)B_1(r\,r_n)\left(\bar{B}_1(R_1\,r_n)-\bar{B}_1(R_2\,r_n)\right)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}-\int_0^{\mathrm{t}}Cos\big(\omega(t-\tau)\big)$$

$$G_{\alpha,k,1}\left(-\lambda^{-1},\tau\right)\mathrm{d}\tau\sum_{k=0}^{\infty}\frac{(-1)^{\mathrm{k}}\pi^2 p_0\,\lambda\,\omega^{\alpha}\sin\left(\frac{\pi}{2}\,\alpha\right)}{2\lambda\left(\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}}+\sum_{\mathrm{n}=1}^{\infty}\frac{\pi f J_1(R_1\,r_n)J_1(R_2\,r_n)B_1(r\,r_n)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}$$

$$\left\{\sum_{k=0}^{\infty}\left(-\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}}\int_0^t\left\{(k-1)!\,e^{\alpha(t-\tau)}\tau^{k-1}\right\}d\tau+\sum_{k=0}^{\infty}\sum_{m=0}^{k}\frac{(-1)^{\mathrm{k}}k!\,r_n^{2(\mathrm{k-m})}}{m!\,(m-k)!}\right.$$

$$\left.\left(\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}\int_0^t\left\{(k-1)!\,e^{\alpha(t-\tau)}\tau^{k-1}\right\}d\tau\right\}-\frac{\pi}{2}\sum_{n=1}^{\infty}\frac{r_n^2 J_1^2(R_1\,r_n)B_1(r\,r_n)}{J_1^2(R_1\,r_n)-J_1^2(R_2\,r_n)}$$

$$\rho g sin\theta \sum_{k=0}^{\infty}\left(-\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\mathrm{t}\right)^{\mathrm{k}} k! \tag{25}$$

**4. Calculation of the shear stress**

In this section, the shear stress is calculated for different types of pressure gradient.

**4.1 Case one**

The shear stress can be calculated from Eq(7) and $\frac{\partial p}{\partial \theta} = -\rho\, p_0 \cos(\omega t)$, by taking Laplace transform of Eq(7), we get

$$(1+\lambda q^{\alpha})\bar{\tau} = \mu\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\bar{u} \tag{26}$$

$$\bar{\tau} = \frac{1}{(1+\lambda q^{\alpha})}\mu\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\bar{u} \tag{27}$$

$$\bar{\tau} = \frac{-2\mu R_1 R_2^2 f}{(R_2^2-R_1^2)r(q-a)(1+\lambda q^{\alpha})} + \frac{\mu\,\pi^2 p_0\left(1+\lambda\,\omega^{\alpha} cos\left(\frac{\pi}{2}\,\alpha\right)\right)}{2}$$
$$\times\sum_{n=1}^{\infty}\frac{\{r_n J_1^2(R_1 r_n)(\bar{B}_1(R_1 r_n)-\bar{B}_1(R_2 r_n))\}q\,(r_n\bar{B}_1(R_1 r_n)-\left(\frac{1}{r}\right)B_1(r\,r_n))}{\{J_1^2(R_1 r_n)-J_1^2(R_2 r_n)\}(q^2+\omega^2)\left(1+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}$$
$$-\frac{\mu\pi^2 p_0\,\lambda\,\omega^{\alpha} sin\left(\frac{\pi}{2}\,\alpha\right)}{2}\times\sum_{n=1}^{\infty}\frac{r_n J_1^2(R_1 r_n)(\bar{B}_1(R_1 r_n)-\bar{B}_1(R_2 r_n))}{\{J_1^2(R_1 r_n)-J_1^2(R_2 r_n)\}}\frac{\omega}{(q^2+\omega^2)}\times$$
$$\frac{(r_n\bar{B}_1(R_1 r_n)-\left(\frac{1}{r}\right)B_1(r\,r_n))}{\left(1+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}+\sum_{n=1}^{\infty}\frac{\pi f\,J_1(R_1 r_n)J_1(R_2 r_n)(r_n\bar{B}_1(R_1 r_n)-\left(\frac{1}{r}\right)B_1(r\,r_n))}{\{J_1^2(R_1 r_n)-J_1^2(R_2 r_n)\}(q-a)}\times$$
$$\frac{(1+\lambda q^{\alpha})\left(1+\upsilon\left(\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}{\left(1+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)}-\frac{\rho g sin\theta}{\left(1+\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta+\frac{\mu\varphi}{\mathrm{K}}\right)\right)} \tag{28}$$

By applying discrete inverse Laplace transform for Eq(28), we obtain the shear stress in the following form

$$\tau = \frac{-2\mu R_1 R_2^2}{(R_2^2-R_1^2)r}\left[e^{at}-\int_0^t\left\{e^{\alpha(t-\tau)}G_{\alpha,\alpha,1}\left(-\lambda^{-\mathbf{1}},\tau\right)\right\}d\tau\right]+\frac{\mu\pi^2 p_0\left(1+\lambda\,\omega^{\alpha} cos\left(\frac{\pi}{2}\,\alpha\right)\right)}{2}$$
$$\times\sum_{n=1}^{\infty}\frac{r_n J_1^2(R_1 r_n)(r_n\bar{B}_1(R_1 r_n)-\left(\frac{1}{r}\right)B_1(r\,r_n))\left(\bar{B}_1(R_1 r_n)-\bar{B}_1(R_2 r_n)\right)}{\{J_1^2(R_1 r_n)-J_1^2(R_2 r_n)\}}\sum_{\mathrm{k}=0}^{\infty}\left(-\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta\right.\right.$$
$$\left.\left.+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}}\times\int_0^t\left\{Cos\left(\omega(t-\tau)\right)\tau^{\mathrm{k}}(k-1)!\right\}d\tau-\frac{\mu\pi^2 p_0\,\lambda\,\omega^{\alpha} Sin\left(\frac{\pi}{2}\,\alpha\right)}{2}$$
$$\times\sum_{n=1}^{\infty}\frac{r_n J_1^2(R_1 r_n)(r_n\bar{B}_1(R_1 r_n)-\left(\frac{1}{r}\right)B_1(r\,r_n))\left(\bar{B}_1(R_1 r_n)-\bar{B}_1(R_2 r_n)\right)}{\{J_1^2(R_1 r_n)-J_1^2(R_2 r_n)\}}\sum_{\mathrm{k}=0}^{\infty}\left(-\upsilon\left(r_n^2+\frac{\sigma\beta_0^2}{\rho}sin\theta\right.\right.$$
$$\left.\left.+\frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}}\times\int_0^t\left\{Sin\left(\omega(t-\tau)\right)\tau^{\mathrm{k}}(k-1)!\right\}d\tau$$

$$\times \sum_{n=1}^{\infty} \frac{r_n J_1^2(R_1\, r_n)(r_n \bar{B}_1(R_1\, r_n) - \left(\frac{1}{r}\right) B_1(r\, r_n))\left(\bar{B}_1(R_1\, r_n) - \bar{B}_1(R_2\, r_n)\right)}{\{J_1^2(R_1\, r_n) - J_1^2(R_2\, r_n)\}} \sum_{\mathrm{k}=0}^{\infty} \left(-\upsilon\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}} \times \int_0^t \left\{Sin\big(\omega(t-\tau)\big)\tau^{\mathrm{k}}(k-1)!\right\} d + \mu\pi f\tau$$

$$\sum_{n=1}^{\infty} \frac{J_1(R_1\, r_n) J_1(R_2\, r_n) \left(r_n \bar{B}_1(R_1\, r_n) - \left(\frac{1}{r}\right) B_1(r\, r_n)\right)}{J_1^2(R_1\, r_n) - J_1^2(R_2\, r_n)} \left\{\sum_{k=1}^{\infty} \frac{(-1)^{\mathrm{k}}\lambda}{\left(\upsilon\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}}\right.$$

$$\left. + \sum_{k=1}^{\infty} \frac{(-1)^{\mathrm{k}}\boldsymbol{\lambda}\left(\frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)}{\left(\boldsymbol{\upsilon}\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}} \right\} \times \int_0^t \left\{e^{\alpha(t-\tau)} G_{\alpha,k,-1}\left(\boldsymbol{\lambda}^{-1}, \tau\right)\right\} d\tau - \rho g sin\theta$$

$$\times \sum_{\mathrm{k}=0}^{\infty} \left(-\boldsymbol{\upsilon}\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}} \ \mathrm{t}^{\mathrm{k}}(k-1)! \tag{29}$$

### 4.2 Case two

By proceeding in the same approach as before and using $\frac{\partial p}{\partial \theta} = -\rho\, p_0\, \mathrm{Sin}(\omega t)$, we can find the solution in the following form

$$\tau = \frac{-2\mu R_1 R_2^2}{(R_2^2 - R_1^2) r} \left[e^{at} - \int_0^t \left\{e^{\alpha(t-\tau)} G_{\alpha,\alpha,1}\left(-\lambda^{-\mathbf{1}}, \tau\right)\right\} d\tau\right] + \frac{\mu\pi^2 p_0 \left(1 + \lambda\, \omega^{\alpha} \cos\left(\frac{\pi}{2}\, \alpha\right)\right)}{2}$$

$$\times \sum_{n=1}^{\infty} \frac{r_n J_1^2(R_1\, r_n)(r_n \bar{B}_1(R_1\, r_n) - \left(\frac{1}{r}\right) B_1(r\, r_n))\left(\bar{B}_1(R_1\, r_n) - \bar{B}_1(R_2\, r_n)\right)}{\{J_1^2(R_1\, r_n) - J_1^2(R_2\, r_n)\}} \sum_{\mathrm{k}=0}^{\infty} \left(-\upsilon\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}} \times \int_0^t \left\{Sin\big(\omega(t-\tau)\big)\tau^{\mathrm{k}}(k-1)!\right\} d\tau - \frac{\mu\pi^2 p_0\, \lambda\, \omega^{\alpha}\, Sin\left(\frac{\pi}{2}\, \alpha\right)}{2}$$

$$\times \sum_{n=1}^{\infty} \frac{r_n J_1^2(R_1\, r_n)(r_n \bar{B}_1(R_1\, r_n) - \left(\frac{1}{r}\right) B_1(r\, r_n))\left(\bar{B}_1(R_1\, r_n) - \bar{B}_1(R_2\, r_n)\right)}{\{J_1^2(R_1\, r_n) - J_1^2(R_2\, r_n)\}} \sum_{\mathrm{k}=0}^{\infty} \left(-\upsilon\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}} \times \int_0^t \left\{Cos\big(\omega(t-\tau)\big)\tau^{\mathrm{k}}(k-1)!\right\} d\tau$$

$$\times \sum_{n=1}^{\infty} \frac{r_n J_1^2(R_1\, r_n)(r_n \bar{B}_1(R_1\, r_n) - \left(\frac{1}{r}\right) B_1(r\, r_n))\left(\bar{B}_1(R_1\, r_n) - \bar{B}_1(R_2\, r_n)\right)}{\{J_1^2(R_1\, r_n) - J_1^2(R_2\, r_n)\}} \sum_{\mathrm{k}=0}^{\infty} \left(-\upsilon\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}} \times \int_0^t \left\{Cos\big(\omega(t-\tau)\big)\tau^{\mathrm{k}}(k-1)!\right\} d + \mu\pi f\tau$$

$$\sum_{n=1}^{\infty} \frac{J_1(R_1\, r_n) J_1(R_2\, r_n) \left(r_n \bar{B}_1(R_1\, r_n) - \left(\frac{1}{r}\right) B_1(r\, r_n)\right)}{J_1^2(R_1\, r_n) - J_1^2(R_2\, r_n)} \left\{\sum_{k=1}^{\infty} \frac{(-1)^{\mathrm{k}}\lambda}{\left(\upsilon\left(r_n^2 + \frac{\sigma\beta_0^2}{\rho} sin\theta + \frac{\mu\varphi}{\mathrm{K}}\right)\right)^{\mathrm{k}+1}}\right.$$

$$
+\sum_{\square=1}^{\infty}\frac{(-1)^{k}\lambda\left(\frac{\square\square_{0}^{2}}{\square}\square\square\square\square+\frac{\square\square}{K}\right)}{\left(\upsilon\left(\square_{\square}^{2}+\frac{\square\square_{0}^{2}}{\square}\square\square\square\square+\frac{\square\square}{K}\right)\right)^{k+1}}\Bigg\}\times\int_{0}^{\square}\{\square^{\square(\square-\square)}\square_{\square,\square,-1}(\lambda^{-\square},\square)\}\,\square\square-\square\square\square\square\square\square
$$

$$
\times\sum_{k=0}^{\infty}\left(-\upsilon\left(\square_{\square}^{2}+\frac{\square\square_{0}^{2}}{\square}\square\square\square\square+\frac{\square\square}{K}\right)\right)^{k}t^{k}(\square-1)! \qquad (30)
$$

**5**. Numerical Results and Conclusions

In this paper, we have established the influence of inclined magnetic field on the unsteady rotating flow of a generalized Maxwell through two inclined infinite straight circular cylinders. The analytical solutions for velocity field and shear stress are obtained using Hankel and Laplace transforms of sequential fractional. The solution was determinedand written under integral and series from in terms of generalized G-function. Diagrams are plotted to show the behavior of different parameters involved in the expressions of the velocity field □.

The velocity and shear stresses are plotted about the case $\frac{\partial p}{\partial \theta}=-\rho\, p_0\, cos(\omega t)$ using the Mathematica package. Figure-1 is sketched to show the velocity field of Maxwell model with fractional derivative at different values of time. From this figure, it is obvious that velocity is decreasing as the time increases. Figure-2 displays the impact of the parameter $p_0$ on the velocity field. It can be seen that the velocity becomes similar with increasing the value of $p_0$. Fig (3) is depicted to show the changes in the velocity with the parameter $f$ and magnetic field. The velocity decreases with increasing the parameter $f$. Figure-4 displays the behavior of the parameter $\sigma$ with the magnetic field. It is observed that the velocity is increasing in the interval $(0 \leq \sigma < 0.6)$ but it maintains the same value when $\sigma = 0.6$. It can be also seen that the velocity is decreasing in the interval $(0.6 \leq \sigma < 1)$. Figure-5 provides the graphical illustrations for the effects of the non- integer fractional parameter $\alpha$ on the velocity fields. The velocity is increasing with the increase of the parameter $\alpha$. Figure-6 displays the behavior of parameter $\lambda$ with the magnetic field. It is observed that the velocity keeps similar values with the increase of the parameter $\lambda$. Figure-7 is prepared to show the effect of the kinematic viscosity on the velocity field with the magnetic field. The velocity decreases with the increase of □. Figure-8) is prepared to show the effect of the parameter $\theta$ on the velocity field with the magnetic field. The velocity decreases with the increase of the parameter□$\theta$. Figure-9 is established to show the behavior of different values of time with $\lambda$=8, $\alpha \rightarrow 1$ and magnetic field. The velocity field decreases with the increase of time. Figure-10 provides the graphical illustration for the effects of different values of $r$ with the magnetic field. The velocity field is increasing with the increase of $r$. Figures-(11, 12) provide the graphical illustration for the effects of different values of □ and □□□respectively with the magnetic field. The velocity field decreases with the increase in the values of □ and □□□respectively. Figure-13) provides the graphical illustration for the relationship of velocity field with parameter K. The velocity field is increasing with the increase of K. Figure-14 is established to show the effects of different value of $\boldsymbol{g}$ with the magnetic field on the velocity field. The velocity field is decreasing with the increase of $\boldsymbol{g}$.

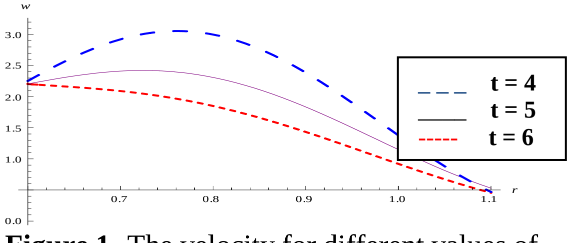


**Figure 1-** The velocity for different values of time $t$ : {$\lambda$=10, $\upsilon$=0.2, a=−0.2, $f$ =4, $\beta_0$=0.1, $\sigma$=0.1, $\omega$=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, $\rho$=0.02, $\theta = \pi/2$ , $p_0$=2, $\sigma$ = 0.1, $\mu$=1, $\varphi$ =1, K =1, $g$ =1}

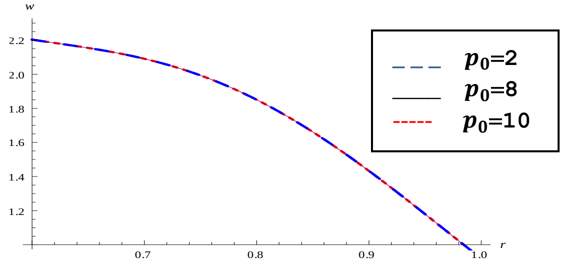


**Figure 2-** The velocity for different values of time $p_0$ : {$\lambda$=10, $\upsilon$=0.2, a=−0.2, $f$ =4, $\beta_0$=0.1, $\sigma$=0.1, $\omega$=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, $\rho$=0.02, $\theta = \pi/2$ , $\sigma$ = 0.1, $\mu$=1, $\varphi$ =1, K =1, $g$ =1}

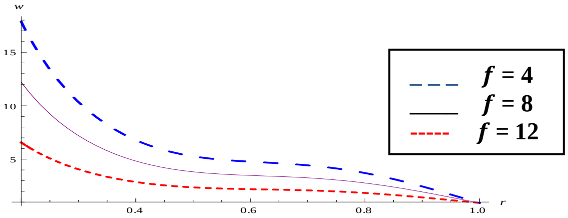


**Figure 3-** The velocity for different values of $f$ : {λ=10, υ=0.2, a=−0.2, $\beta_0$=0.1, σ=0.1, ω=0.1, $\alpha = 0.5$ $r_n$=7.8999, , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, θ =π/2, $p_0$=2, σ = 0.1, μ=1, $\varphi$ =1, K =1, $g$ =1}

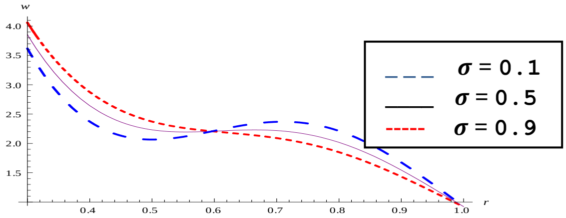


**Figure 4-** The velocity for different values of σ : {λ=10, υ=0.2, a=−0.2, $f$=4, $\beta_0$=0.1, ω=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, θ = π/2, $\alpha = 0.5$ , $p_0$=2, μ=1, $\varphi$ =1, K =1, $g$=1}

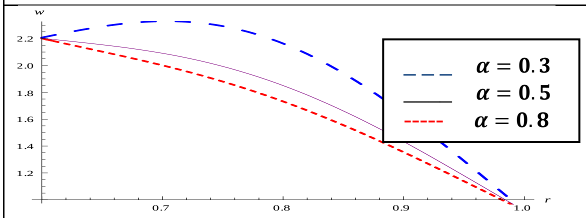


**Figure 5-** The velocity for different values of $\alpha$ : {λ=10, υ=0.2, a=−0.2, $f$ =4, $\beta_0$=0.1, ω=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, θ = π/2, $p_0$=2, μ=1, $\varphi$ =1, K =1, $g$ =1}

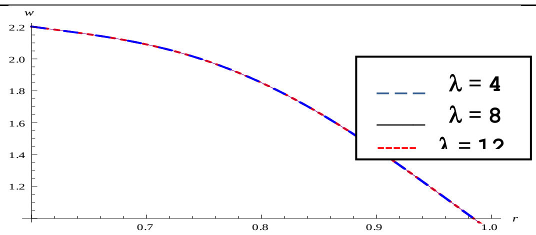


**Figure 6-** The velocity for different values of λ : {υ=0.2, a=−0.2, $f$ =4, $\beta_0$=0.1, σ=0.1, ω=0.1, $\alpha = 0.5$ $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, θ = π/2 , $p_0$=2, μ=1, $\varphi$ =1, K =1, $g$ =1}

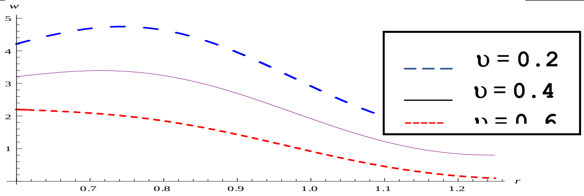


**Figure 7- The velocity for different values of υ : {λ=10, a=−0.2, $f$ =4, $\beta_0$=0.1, ω=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, θ = π/2, $\alpha = 0.5$ , $p_0$=2, σ = 0.1, μ=1, $\varphi$ =1, K =1, $g$=1}**

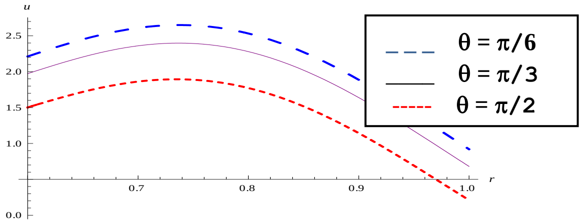


**Figure 8-**. The velocity for different values of θ : {λ=10, υ=0.2, a=−0.2, $f$ =4, $\beta_0$=0.1, σ=0.1, ω=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, $\alpha = 0.5$ , $p_0$=2, μ=1, $\varphi$ =1, K =1, $g$ =1}

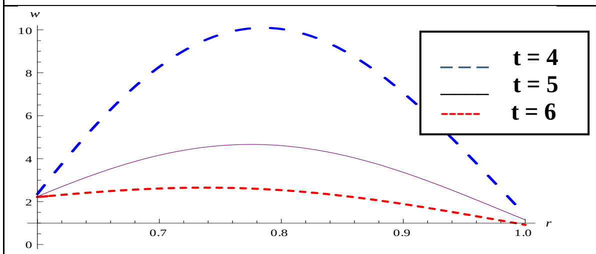


**Figure 9-** The velocity for different values of time t with $\lambda = 8, \sigma = 1$: {υ=0.2, a=−0.2, $f$=4, $\beta_0$=0.1, ω=0.1, $\alpha = 0.5$ $r_n$=7.8999, $R_1$=0.5, $p_0$=2, $\varphi$ =1, K =1, $R_2$=1, μ =1 ,ρ=0.02, θ = π/2, $g$ =1}

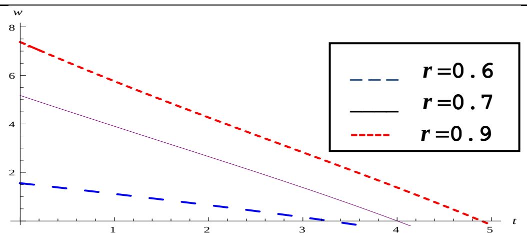


**Figure 10-** The velocity for different values of r {λ=10, υ=0.2, a=−0.2, $f$ =4, $\beta_0$=0.1, ω=0.1, $r_n$=7.8999 , $R_1$=0.5 , $R_2$=1, t=6, ρ=0.02, θ = π/2, $\alpha = 0.5$ , $p_0$=2, σ = 0.1, μ=1, $\varphi$ =1, K =1, $g$ =1}

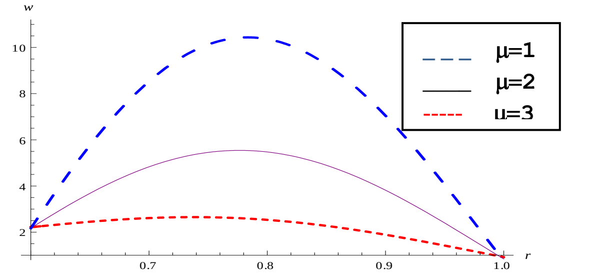


**Figure 11-** The velocity for different values of $\mu$ $\{\lambda=10, \upsilon=0.2, a=-0.2, f=4, \beta_0=0.1, \omega=0.1, r_n=7.8999, R_1=0.5, R_2=1, t=6, \rho=0.02, \theta=\pi/2, \alpha=0.5, p_0=2, \sigma=0.1, \varphi=1, K=1, g=1\}$

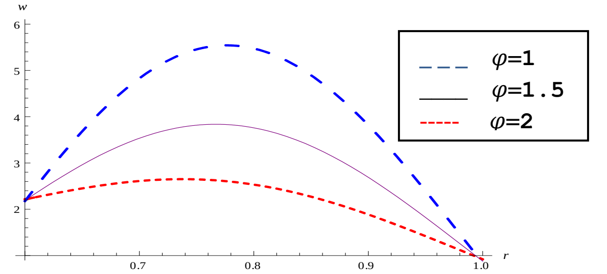


**Figure 12-** The velocity for different values of $\varphi$ $\{\lambda=10, \upsilon=0.2, a=-0.2, f=4, \beta_0=0.1, \sigma=0.1, \omega=0.1, r_n=7.8999, R_1=0.5, R_2=1, t=6, \rho=0.02, \theta=\pi/2, p_0=2, \sigma=0.1, \mu=1, K=1, g=1, \alpha=0.5\}$

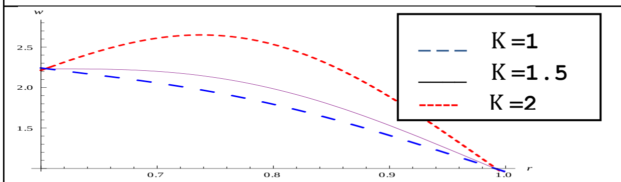


**Figure 13-** The velocity for different values of K $\{\lambda=10, \upsilon=0.2, a=-0.2, f=4, \beta_0=0.1, \omega=0.1, r_n=7.8999, R_1=0.5, R_2=1, t=6, \rho=0.02, \theta=\pi/2, \alpha=0.5, p_0=2, \sigma=0.1, \mu=1, \varphi=1, g=1\}$

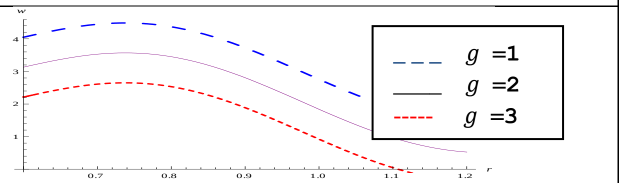


**Figure 14-** The velocity for different values of q $\{\lambda=10, \upsilon=0.2, a=-0.2, f=4, \beta_0=0.1, \omega=0.1, r_n=7.8999, R_1=0.5, R_2=1, t=6, \rho=0.02, \theta=\pi/2, \alpha=0.5, p_0=2, \sigma=0.1, \mu=1, \varphi=1, K=1\}$

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