Introduction

Throughout this paper, G denotes a zero – symmetric left – near-ring with a multiplicative center \( Z(G) \). For a \( \Gamma \)-near-ring \( G \), the set \( G_0=\{s\in G: \psi s=0, \forall \psi \in \Gamma \} \) is called a zero symmetric part of \( G \). If \( G=G_0 \), then \( G \) is called a zero symmetric ring [1,2,3,4]. An additive mapping \( h:G\rightarrow G \) is called a \( \Gamma\)-derivation on a \( \Gamma \)-near-ring \( G \) if there exist two automorphisms mapping \( \lambda, \delta: G \rightarrow G \) such that \( h(spr)=h(s)p\lambda(r)+\delta(s)p\rho(r) \), for every \( s, r \in G \) and \( \rho \in \Gamma \) [4,5]. A \( \Gamma \)-near-ring \( G \) is said to have a prime \( \Gamma \)-near-ring if \( sGt=0 \) implies \( s=0 \) or \( t=0 \), for every \( s, r \in G \), and it is said to be a semiprime if \( sGt=0 \) implies \( s=0 \) for every \( s \in G \) [5,6]. Further, an element \( s \in G \) is called constant if \( h(s)=0 \) [4,7]. A non-empty subset \( K \) of \( G \) is called semi-group ideal if \( K \cap G \subseteq K \) and \( K \cap G \subseteq K \) [8]. For \( s, r \in G \) and \( \rho \in \Gamma \), the symbol \( [s, r]^\rho_\lambda, \delta \) will denote \( \delta(s)p-r \rho \lambda(s) \), as previously described[4,9]. The other commutators are \( [s, r]=spr-rps \) and \( (s, r)=s+r-s-r \) which denote the additive-group commutator [4,9].

The purpose of this paper is to study and generalize some results of previous authors [4,7,9,10] on the commutativity of the prime \( \Gamma \)-near-ring. Some recent results on rings deal with commutativity of prime and semiprime rings admitting suitably constrained derivations. For further details on prime near-ring, we refer to some previous articles [11-15].

As a generalization of near-ring, the \( \Gamma \)-near-ring was discussed by Satyanarayana [6], while Booth and Groenewald [5,13] surveyed various portions in the \( \Gamma \)-near-ring. In this paper, we investigate the condition for a \( \Gamma-(\lambda, \delta) \)-derivation on a prime \( \Gamma \)-near-ring to be commutative.
2. The Main Results

In this section, we investigate some results of a semi-group ideal of a $\Gamma$-\textit{near-ring} admitting a $\Gamma-(\lambda, \delta)$-derivation.

To prove the main theorems, we need the following lemmas.

**Lemma 2.1.** Let $h$ be a $\Gamma-(\lambda, \delta)$ – derivation on a prime $\Gamma$-near-ring and $K$ a semi-group ideal of $G,$ if and only if $h(snr) = \delta(s)\eta h(r) + h(s)\eta \lambda(r),$ for all $s, r \in K$ and $\eta \in \Gamma.$

**Proof.** \( \forall s, r \in K \) and \( \eta \in \Gamma, \) we have $s\eta(r + r) = snr + snr.$ By applying \( h \) for both sides we obtain

$$h(s\eta(r + r)) = h(s)\eta\lambda(r + r) + \delta(s)\eta h(r + r)$$

$$= h(s)\eta\lambda(r) + h(s)\eta\lambda(r) + \delta(s)\eta h(r).$$

and

$$h(snr + snr) = h(snr) + h(snr)$$

$$= h(s)\eta\lambda(r) + \delta(s)\eta h(r) + h(s)\eta\lambda(r) + \delta(s)\eta h(r).$$

By comparing the two relations, we have

$$h(s)\eta\lambda(r) + \delta(s)\eta h(r) = \delta(s)\eta\lambda(r) + h(s)\eta\lambda(r)$$

$$h(snr) = \delta(s)\eta\lambda(r) + h(s)\eta\lambda(r)$$

\( \forall s, r \in K \) and \( \eta \in \Gamma. \)

Conversely, assume for every $s, r \in K$ and $\eta \in \Gamma,$ that

$$h(snr) = \delta(s)\eta\lambda(r) + h(s)\eta\lambda(r) .$$

then,

$$h(s\eta(r + r)) = \delta(s)\eta h(r + r) + h(s)\eta\lambda(r + r)$$

$$= \delta(s)\eta h(r) + \delta(s)\eta h(r) + h(s)\eta\lambda(r) + h(s)\eta\lambda(r).$$

and

$$h(snr + snr) = h(snr) + h(snr)$$

$$= \delta(s)\eta h(r) + h(s)\eta\lambda(r) + \delta(s)\eta h(r) + h(s)\eta\lambda(r).$$

Comparing the two relation provides that:

$$\delta(s)\eta h(r) + h(s)\eta\lambda(r) = h(s)\eta\lambda(r) + \delta(s)\eta h(r)$$

$$h(snr) = h(s)\eta\lambda(r) + \delta(s)\eta h(r)$$

**Lemma 2.2.** If \( h \) be a $\Gamma-(\lambda, \delta)$ – derivation on a $\Gamma$-near-ring $G,$ $K$ a semi-group ideal of $G,$ and $\lambda(K)=K,$ then

$$h(s)\eta\lambda(r) + \delta(s)\eta h(r)pv = h(s)\eta\lambda(r)pv + \delta(s)\eta h(r)pv ,$$

for all $s, r, v \in K$ and $\eta, \rho \in \Gamma.$

**Proof.** Assume that $\forall s, r, v \in K$ and $\eta, \rho \in \Gamma.$

$$h(snr)pv = h(snr)\rho\lambda(v) + \delta(snr)\rho h(v)$$

$$= (h(s)\eta\lambda(r) + \delta(s)\eta h(r))\rho\lambda(v) + \delta(s)\eta \delta(r)\rho h(v)$$

and

$$h(snrpv) = h(snr)\rho\lambda(rv) + \delta(snrpv)\rho h(rv)$$

$$= h(s)\eta\lambda(r)\rho\lambda(v) + \delta(s)\eta\lambda(r)\rho\lambda(v) + \delta(s)\eta\delta(r)\rho h(v).$$

Comparing the two relations above of $h(snrpv), \forall s, r, v \in K$ and $\eta, \rho \in \Gamma,$ and since $\lambda(K)=K,$ implies that

$$h(s)\eta\lambda(r) + \delta(s)\eta h(r)pv = h(s)\eta\lambda(r)pv + \delta(s)\eta h(r)pv .$$

**Lemma 2.3.** If \( h \) be a $\Gamma-(\lambda, \delta)$ – derivation on a $\Gamma$-near-ring $G$ and $K$ a semi-group ideal of $G$ such that $h([s, r]) = [s, r]_p,$ $\lambda(K)=K,$ and $\delta(K)=K,$ then

(i) $h(v) = v$, for every commutator $v$ in $K.$

(ii) $h(k)v[s, r]p = [s, r]p\gamma h(k),$ for every $s, k \in K,$ \( r \in G \) and $\rho, \gamma \in \Gamma.$
Proof. (i) Let \( v = [s, r]_\rho \), where \( s \in K \), \( r \in G \) and \( \rho \in \Gamma \).

\[
 h([s, r]_\rho) = [s, r]_\rho, \quad \text{for every} \quad s \in K, r \in G \text{ and } \rho \in \Gamma.
\]
Thus, \( h(v) = v \), for each commutator \( v \) in \( K \).

(ii) By the hypothesis that \( h([s, r]_\rho) = [s, r]_\rho, \) we have

\[
 -[s, r]_\rho \gamma k + h([s, r]_\rho) \gamma k = -k \gamma [s, r]_\rho + h(k \gamma [s, r]_\rho), \quad \forall \ s, k \in K, \ r \in G \text{ and } \rho, \gamma \in \Gamma.
\]

By using Lemma 2.1, we have

\[
 -[s, r]_\rho \gamma k + h([s, r]_\rho) \gamma k = -k \gamma [s, r]_\rho + h(k \gamma [s, r]_\rho) + \delta(k) \gamma h([s, r]_\rho)
\]

By applying \( \phi \) (i) and as \( \lambda(K) = K, \delta(K) = K \), \( \lambda \) is an automorphism, we obtain:

\[
 [s, r]_\rho \gamma h(k) = h(k) [s, r]_\rho, \quad \forall \ s, k \in K, \ r \in G \text{ and } \rho, \gamma \in \Gamma.
\]

**Lemma 2.4.** If \( h \) is a \( \Gamma \)-derivation on a \( \Gamma \)-near-ring \( G \), \( K \) is a nonzero semi-group ideal of \( G \), and \( h([s, r]_\rho) = [s, r]_\rho \), \( \lambda(K) = K, \delta(K) = K \), then

(i) If \( v \) is a commutator in \( K \) and \( w \nu \gamma = z \mu \nu \), where \( w, z \in K \) and \( \mu, \nu \in \Gamma \), then \( \nu h(w-z) = 0 \).

(ii) If \( v_1 \) and \( v_2 \) are commutators in \( K \) with \( v_1 \mu v_2 = 0 \), then \( v_1 = 0 \) or \( v_2 = 0 \).

**Proof.** (i) Let \( v = [s, r]_\rho \), \( \forall s \in K, r \in G \) and \( \rho \in \Gamma \).

Then, the hypothesis provides that \( w \mu [s, r]_\rho = z \mu [s, r]_\rho \), \( \forall s, w, z \in K, r \in G \) and \( \rho, \mu \in \Gamma \).

Applying \( h \) for both sides, implies that

\[
 h(w \mu [s, r]_\rho) = h(z \mu [s, r]_\rho), \quad \forall s, w, z \in K, \ r \in G \text{ and } \rho, \mu \in \Gamma.
\]

Thus,

\[
 h(w) \mu \lambda([s, r]_\rho) + \delta(w) \mu h([s, r]_\rho) = h(z) \mu \lambda([s, r]_\rho) + \delta(z) \mu h([s, r]_\rho).
\]

Using Lemma 2.3 (i,ii) provides that:

\[
 h(w) \mu \lambda([s, r]_\rho) = h(z) \mu \lambda([s, r]_\rho), \quad \forall s, w, z \in K, \ r \in G \text{ and } \rho, \mu \in \Gamma.
\]

So, \( [s, r]_\rho \mu h(w-z) = 0 \). Thus, \( v_1 h(w-z) = 0 \), for every commutator \( v \) in \( K \), \( w, z \in K \) and \( \mu \in \Gamma \).

(ii) If \( v_1 \mu v_2 = 0 = 0 \mu v_2 \), since \( v_2 \) is a commutator in \( K \), (i) yields

\[
 v_2 h(v_1) = 0.
\]

By using Lemma 2.3 (i), since \( v_1 \) is a commutator in \( K \), we obtain

\[
 v_2 \mu h(v_1) = 0.
\]

B substituting \( r \gamma v_1 \) for \( v_1 \), where \( r \in K, \gamma \in \Gamma \) in equation (1), we obtain:

\[
 v_2 h(r \gamma v_1) = 0 = v_2 h(r \gamma \lambda(v_1)) + v_2 \mu \delta(r) \gamma h(v_1).
\]

Using Lemma 2.3 (ii) and equation (2) in equation (3) provides that:

\[
 v_2 \mu \delta(r) \gamma h(v_1) = 0, \quad \text{for every commutator} \quad v_1, v_2 \text{ in } K, \ r \in K, \text{ and } \mu, \gamma \in \Gamma.
\]

Hence, \( v_2 \Gamma K \mu h(v_1) = 0 \).

By using Lemma 2.3 (i), since \( v_1 \) is commutator, we obtain \( v_2 \Gamma K \gamma v_1 = 0 \).

Since \( K \) is a nonzero semi-group ideal of \( G \) and \( G \) is a prime \( \Gamma \)-near-ring, we obtain \( v_1 = 0 \) or \( v_2 = 0 \).

**Lemma 2.5.** If \( G \) be a prime \( \Gamma \)-near-ring and \( K \) is a nonzero semi-group ideal of \( G \), then \( Z(K) \subseteq Z(G) \).

**Proof.** Suppose that \( t \in Z(K) \), this means that, \( [t, s]_\rho = 0, \quad \forall s \in K \text{ and } \rho \in \Gamma \).

Replacing \( s \) by \( s \mu r \), so \( r \in G \) in the above equation, we obtain
Thus, \( K_{[t,r]} = 0 \). Since \( K \) is a nonzero semi-group ideal of \( G \) and \( G \) is a prime \( \Gamma \)-near-ring, we get \( [t,r] = 0, \forall t \in K, r \in G \) and \( r \in \Gamma \). Hence, \( t \in Z(G) \).

**Lemma 2.6.** If \( h \) is a \( \Gamma \)-\( (\lambda, \delta) \) - derivation on a prime \( \Gamma \)-near-ring \( G \) and \( K \) be a semi-group ideal of \( G \).

(i) If \( u \) is a nonzero element in \( Z(G) \), then \( u \) is not a zero divisor.

(ii) If there exists a nonzero element \( u \) of \( Z(G) \) such that \( u + u \in Z(G) \), then \( (K,+ ) \) is an abelian.

**Proof.**

(i) If \( u \in Z(G) \backslash \{0\} \) and \( ups = 0, \forall s \in K \) and \( \eta \in \Gamma \).

Then, left multiplication of this equation by \( ty \), where \( t \in G \) and \( \gamma \in \Gamma \), provides that \( t\eta u s = 0 \). Since \( G \) is a multiplicative with the center \( Z(G) \), it implies that

\[ u t \eta s = 0, \forall t \in G \text{ and } s \in K, \text{ thus } u t G I s = 0. \]

Since \( G \) is a prime \( \Gamma \)-near-ring and \( u \) is a nonzero element, it shows that \( s = 0 \).

(ii) Let \( u \in Z(G) \backslash \{0\} \) be an element, such that \( u + u \in Z(G) \).

Let \( s, r \in K \) and \( \rho \in \Gamma \) so,

\[ (s + r)\rho(u + u) = (u + u)\rho(s + r) \]

\[ spu + spu + rpu + rpu = u\rho sp + u\rho rp + u\rho sp + u\rho rp \]

Since \( u \in Z(G) \), we get \( ups + u\rho sp + u\rho rp = u\rho sp + u\rho rp \)

Thus, \( up(s + r - s - r) = 0, \forall s, r \in K \) and \( \rho \in \Gamma \).

Left multiplication this equation by \( ay \), where \( a \in G, \gamma \in \Gamma \), provides that:

\[ a\eta u p(s,r) = 0, \forall s, r \in K, a \in G \text{ and } \gamma, \rho \in \Gamma. \]

Because \( G \) is a multiplicative with the center \( Z(G) \), this provides that:

\[ u\eta p(s,r) = 0. \] Hence, \( u t G I s = 0 \)

Because \( G \) is a prime \( \Gamma \)-near-ring and \( u \) is a nonzero element, it implies that \( s = 0, \forall s, r \in K \). Thus, \((K,+)\) is an abelian. \( \square \)

**Lemma 2.7.** If \( h \) be a nonzero \( \Gamma \)-\( (\lambda, \delta) \)-derivation on a prime \( \Gamma \)-near-ring \( G \) and \( K \) be a nonzero semi-group ideal of \( G \). Then \( s t G h(K) = 0 \), which implies that \( s = 0 \) and \( h(K) = 0 \), which means that \( s = 0 \), where \( s \in G \).

**Proof.** Assume that \( s t G h(K) = 0, \forall r \in G, \gamma, b \in \Gamma \).

Then, \( s t G h(t r) = 0 \), showing that:

\[ s t G h(t \beta r) + s t G h(t) \beta r = 0 \]

Therefore, \( \forall s, r \in G, t \in K \) and \( \eta b \in \Gamma \), we have \( s t G h(t) \beta r = 0 \).

Since \( \delta(K) = K \), then \( s t G h(r) = 0 \).

Since \( K \) is a nonzero semi-group ideal and \( G \) is a prime \( \Gamma \)-near-ring, \( h \neq 0 \), it implies that \( s = 0 \).

Similarly, we can show that if \( h(K) = 0, \forall s \in G \), it implies that \( s = 0 \). \( \square \)

**Lemma 2.8.** If \( G \) is a 2 -torsion free prime \( \Gamma \)-near-ring, \( h \) be a nonzero \( \Gamma \)-\( (\lambda, \delta) \)-derivation of \( G \), and \( K \) be a nonzero semi-group ideal of \( G \). If \( h^2(K) = 0 \) and \( \lambda, \delta \) commute with \( h \), then \( h(K) = 0 \).

**Proof.** \( \forall s, r \in K \) and \( \rho \in \Gamma \).

\[ 0 = h^2(s)\rho = h(s(\rho s)) = h(h(s))\rho \lambda(r) + h(\delta(s))\rho \lambda(r) \]

\[ = h(s)\rho \lambda(r) + h(\delta(s))\rho \lambda(r) \]

\[ = h^2(s)\rho \lambda^2(r) + \delta(s)\rho \lambda(\rho \lambda(r)) + h(\delta(s))\rho \lambda(\rho \lambda(r)) \]

By the hypothesis, we obtain that \( 2h(\delta(s))\rho \lambda(\rho \lambda(r)) = 0, \forall s, r \in K \) and \( \rho \in \Gamma \).

Because \( G \) is a 2 - torsion free and \( \lambda(K) = K \), this provides \( h(\delta(s))\rho \lambda(K) = 0 \)

By using Lemma 2.7, we obtain that \( h = 0 \). \( \square \)

**Lemma 2.9.** Let \( G \) be a prime \( \Gamma \)-near-ring and \( K \) be a nonzero semi-group ideal of \( G \). If \( K \) is a commutative then \( G \) is a commutative ring.

**Proof.** \( \forall s, r \in K, [s, r] = 0 \).

By taking \( s t a \) instead of \( s \) and \( r t b \) instead of \( r \), where \( a, b \in G \) and \( \gamma \in \Gamma \), we obtain that \( [s t a, r t b] = 0 \). Since \( K \) is a commutative and semi-group ideal of \( G \), this provides
Because \( K \) is a a nonzero semi-group ideal of \( G \), \( G \) is a prime \( \Gamma \)-near–ring , thus 

\[ [a,b]_\rho = 0, \forall a,b \in G. \] 

Thus, \( G \) is a commutative ring.

**Lemma 2.10.** If \( G \) is a prime \( \Gamma \)-near-ring and \( K \) is a nonzero semi-group ideal of \( G \). If \( (K,+)_\rho \) is an abelian, then \((G,+)_\rho \) is an abelian.

**Proof.** Since \((K,+,\rho)\) is an abelian, we obtain that \( z + c = c + z, \forall z, c \in K \).

By substituting \( s\eta z \) for \( z \) and \( r\eta z \) for \( c \), for \( s,r \in G \) and \( \eta \in \Gamma \), we have

\[ s\eta z + r\eta z = r\eta z + s\eta z, \forall z \in K, s,r \in G \text{ and } \eta \in \Gamma. \]

Which gives \( (s+r-s-r)\eta z = 0 \).

Thus, \((s,r)_\Gamma K = 0 \). Since \( K \neq 0 \) is a semi-group ideal and \( G \) is a prime, then

\( (s,r) = 0, \forall s,r \in G \). Then \((G,+)_\rho \) is abelian.

**Lemma 2.11.** If \( h \) be a \( \Gamma \)-\((\lambda,\delta)\) – derivation on a prime \( \Gamma \)-near-ring \( G \) and \( K \) is a semi-group ideal of \( G \). Suppose that \( t \in K \) is not a left zero divisor. If \[ [t,h(t)]^{\rho}_{(\lambda,\delta)} = 0, \] then \((s,t)_\Gamma \) is a constant for every \( s \in K \) and \( \beta \in \Gamma \).

**Proof.** From \[ tf(s+t) = tf(s) + tf(t), \forall t \in K \text{ and } \beta \in \Gamma. \]

By applying \( h \) for both sides, we have

\[ h(tf(s+t)) = h(tf(s)) + h(tf(t)) \]

and

\[ h(tf(s) + tf(t)) = h(tf(s)) + h(tf(t)). \]

Which gives \( h(tf(s)) + h(tf(t)) = h(tf(s)) + h(tf(t)) \).

By using the hypothesis, we have \( \delta(t)\beta h((s,t)) = 0 \).

By substituting \( \delta(t) \) by \( \delta(t)\gamma m \), where \( m \in K \) and \( \gamma \in \Gamma \), we get \( \delta(t)\gamma m h((s,t)) = 0 \).

Hence, \( \delta(t)\Gamma K h((s,t)) = 0, \forall t,s \in K \).

Because \( t \) is not a left zero divisor and \( \delta(K) = K \), \( K \) is a semi-group ideal and \( G \) is a prime \( \Gamma \)-nearring, we obtain that

\( h((s,t)) = 0 \). Thus, \((s,t)_\Gamma \) is a constant for every \( s \in K \).

**Now we can prove the main theorems.**

**Theorem 2.12.** Let \( h \) be a \( \Gamma \)-\((\lambda,\delta)\) – derivation of a prime \( \Gamma \)-near-ring \( G \) and \( K \) is a semi-group ideal of \( G \) which has no nonzero divisors of zero, where \( h \) is commuting on \( K \), \( \lambda(K) = K \), then \((G,+)\) is an abelian.

**Proof.** Let \( v \) be any additive commutator in \( K \).

So, the application of Lemma 2.11 yields that \( v \) is a constant.

For any \( s \in K \), \( svy \) is also an additive commutator in \( K \). Then, \( syv \) is also a constant.

Therefore, \( 0 = h(svy) = h(s)v\lambda(y) + \delta(s)v\lambda(v) = h(s)v\lambda(v), \forall s \in K \) and \( \gamma \in \Gamma \).

Because \( h(s) \neq 0 \), for some \( s \in K \), and \( K \) has no nonzero divisors of zero, Which gives \( \lambda(v) = 0 \), thus \( v = 0 \), for every additive commutator \( v \) in \( K \).

Hence, \((K, +)\) is an abelian. By using Lemma 2.10, we obtain that \((G, +)\) is an abelian. \( \Box \)
We need the following lemma to prove the main theorem.

**Lemma 2.13.** Let \( h \) be a nonzero \( \Gamma \)-derivation on a prime \( \Gamma \)-near-ring \( G \) and \( K \) is a semigroup ideal of \( G \), so \( \lambda yh = h\lambda y, \delta yh = h\delta \delta \) for every \( \gamma \in \Gamma \), \( \lambda (K) = K \), where \( h(K) \subseteq Z(G) \), then \( (K,+) \) is an abelian. If \( G \) is a 2-\( \Gamma \)-torsion free and \( h(K) \subseteq K \), then \( K \) is a central ideal.

**Proof.** Since \( h(K) \subseteq Z(G) \) and \( h \) is a nonzero \( \Gamma \)-\((\lambda,\delta)\)-derivation.

There exists a nonzero element \( t \) in \( K \), such that \( u = h(t) \in Z(G) \setminus \{0\} \).

And, \( u + u = h(t) + h(t) = h(t + t) \in Z(G) \).

Therefore, \( (K,+) \) is an abelian by Lemma 2.6 (ii).

Using the hypothesis, \( \forall s, r \in K, c \in G \) and \( \beta, \gamma \in \Gamma \) gives \( \lambda (c) yh(s\beta r) = h(s\beta r)\gamma \lambda (c) \).

Using Lemma 2.2, it provides

\[
\lambda (c) yh(s) \beta \gamma (r) + \lambda (c) y\delta (s) \beta h(r) = h(s) \beta \lambda (r) \gamma \lambda (c) + \delta (s) \beta h(r) \gamma \lambda (c).
\]

Now, by using \( h(K) \subseteq Z(G) \) and since \( (K,+) \) is an abelian, \( \lambda yh = h\lambda y, \delta yh = h\delta \delta \), it shows that

\[
h(s) \beta \lambda (c) \gamma \lambda (r) - h(s) \beta \lambda (r) \gamma \lambda (c) = h(r) \beta \delta (s) \gamma \lambda (c) - h(r) \beta \lambda (c) \gamma \delta (s).
\]

Then, \( h(s) \beta \lambda (c, r) = h(r) \beta [(\delta (r), \lambda (c))] \), \( \forall s, r \in K, c \in G \) and \( \gamma, \beta \in \Gamma \).

Suppose that \( K \) is not a central ideal.

By choosing \( r \in K \) and \( c \in G \), such that \( [c, r] \neq 0 \).

And since \( h(K) \subseteq K \) let \( s = h(x) \in Z(G) \), where \( x \in K \), which gives

\[
h^{2}(x) \beta \lambda ([c, r], c) = h(r) \beta [(\delta (r), \lambda (c))] \), \( \forall x, r \in K, c \in G \) and \( \gamma, \beta \in \Gamma \).

Then, \( h^{2}(x) \beta \lambda ([c, r], c) = 0 \).

By Lemma 2.6 (i), the central element \( h^{2}(x) \) cannot be a nonzero divisor of zero, then we conclude that

\[
h^{2}(x) = 0, \forall x \in K.
\]

By using Lemma 2.8, we obtain that \( h(x) = 0 \).

This contradicts that \( h \) is a nonzero \( \Gamma \)-\((\lambda,\delta)\)-derivation on \( G \).

So, we obtain that \( \lambda ([c, r], c) = 0, \forall x \in K, c \in G \).

Because \( \lambda (K) = K \), this gives a contradiction with the assumption. Then \( K \) is a central ideal. \( \square \)

**Theorem 2.14.** Let \( h \) be a nonzero \( \Gamma \)-\((\lambda,\delta)\)-derivation on a prime \( \Gamma \)-near-ring \( G \) and \( K \) a semigroup ideal of \( G \), so \( \lambda yh = h\lambda y, \delta yh = h\delta \delta \) for every \( \gamma \in \Gamma \), \( \lambda (K) = K \), where \( h(K) \subseteq Z(G) \), then \( (G,+) \) is an abelian. If \( G \) is a 2-\( \Gamma \)-torsion free and \( h(K) \subseteq K \), then \( G \) is a commutative ring.

**Proof.** By using Lemma 2.13, it gives that \( (K,+) \) is an abelian.

By using Lemma 2.10, it gives that \( (G,+) \) is an abelian.

Now, assume that \( G \) is a 2-\( \Gamma \)-torsion free. The application of Lemma 2.13 shows that \( K \) is a central ideal.

Thus, \( K \) is commutative. By Lemma 2.9, it implies that \( G \) is a commutative ring. \( \square \)

**Theorem 2.15.** Let \( h \) be a nonzero \( \Gamma \)-\((\lambda,\delta)\)-derivation on a prime \( \Gamma \)-near-ring \( G \), and \( K \) is a nonzero semigroup ideal of \( G \), so \( h([s, r]) = [s, r] \), and \( \gamma \lambda = \lambda \gamma \), for every \( \gamma, k \in K \), \( \gamma, \beta \in \Gamma \), then \( (G,+) \) is an abelian. If \( G \) is a 2-\( \Gamma \)-torsion free and \( h(K) \subseteq K \), then \( G \) is a commutative ring.

**Proof.** Since \( [s, s\gamma r] = s\gamma [s, r] \), \( \forall s \in K, r \in G \) and \( \gamma \in \Gamma \).

By using Lemma 2.3 (ii), we have

\[
h(\beta) [s, r] = s\gamma [s, r] \beta h(v) = s\gamma h(v) \beta [s, r] \lambda, \forall s, v \in K, r \in G \) and \( \gamma, \beta \in \Gamma \).
\]

By using Lemma 2.4 (i), we have

\[
[s, r] \beta h([s, r], r) = 0, \forall s, v \in K, r \in G \) and \( \gamma, \beta \in \Gamma \).
\]

Hence, \( [s, r] \beta h([s, r], r) = 0 \).

By Lemma 2.3 (ii), we obtain that \([s, r] \beta h([s, r], s) = 0 \).

The application of Lemma 2.4 (ii) gives

either \([s, r] = 0 \) or \([h(v), s] = 0 \), \( \forall s, v \in K \), \( r \in G \) and \( \gamma, \beta \in \Gamma \).

If \([h(v), s] = 0 \), \( \forall s, v \in K \) and \( \gamma \in \Gamma \).

Hence, \( h(K) \subseteq Z(K) \). By Lemma 2.5, we obtain that \( h(K) \subseteq Z(G) \). Then, by Theorem 2.14, we complete this theorem.

So, if \([s, r] = 0 \), \( \forall s \in K, r \in G \) and \( \gamma \in \Gamma \).
We substitute \( s\gamma h(v) \) for \( s \), where \( v \in K \), which gives
\[
[s\gamma h(v), r]_P = s[h(v), r]_P, \forall s, v \in K, r \in G \text{ and } \rho, \gamma \in \Gamma
\]
Which gives, \( K\gamma h(v), r]_P = 0, \forall v \in K, r \in G \) and \( \rho, \gamma \in \Gamma \)
Because \( K \) is a nonzero semi-group ideal of \( G \) and \( G \) is a prime \( \Gamma \)-near-ring, this implies that
\[
[h(v), r]_P = 0, \forall v \in K, r \in G \text{ and } \rho \in \Gamma.
\]
So, \( h(K) \subseteq Z(G) \). By Theorem 2.14, the proof will be complete. \( \square \)

**Theorem 2.16.** Let \( h \) be a nonzero \( \Gamma \) - \((\lambda, \delta)\) - derivation on a prime \( \Gamma \)-near-ring \( G \), and \( K \) is a nonzero semi-group ideal of \( G \), so \( [s, r]_P = [s, r]_P \) and \( \rho \lambda k \beta u = t \beta k \gamma u \) for every \( t, k, u \in K \) and \( \gamma, \beta \in \Gamma \). Let \( \lambda(K) = K \), then \( G \) is a commutative ring.

**Proof.** Since \( h([s, r]_P) = -s\rho r + r\rho s = (-s\rho r + r\rho s)\gamma \), \( \forall \gamma \in \Gamma \), \( s \in K, r \in G \) and \( \rho, \gamma \in \Gamma \).

And,
\[
h([s, r]_P) = h([s, r]_P)\lambda = h([s, r]_P)\lambda = (s\rho r + r\rho s)\gamma
\]

It follows from the two expressions for \( h([s, r]_P) \) that
\[
\delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r
\]
Replacing \( r \) by \( r\gamma s \) in equation (4), we get
\[
h([s, r\gamma s]_P) = -s\rho r + r\rho s = (-s\rho r + r\rho s)\gamma
\]

Left multiplicative equation (4) by \( \delta(v) \beta \), gives
\[
\delta(v)\delta(v)\beta = \delta(v)\delta(v)\beta = \delta(v)\delta(v)\beta = \delta(v)\delta(v)\beta = \delta(v)\delta(v)\beta = \delta(v)\delta(v)\beta
\]

The combining of equation (5) and equation (6) gives
\[
\delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r = \delta(s)\rho r
\]
Since \( \delta(K) = K \), and \( \delta \) is automorphism on \( G \).

Hence, \( [s, v]_P, \Gamma\Gamma\beta h(s) = 0 \).

Because \( G \) is a prime \( \Gamma \)-near-ring, this implies that \( h(s) = 0 \) or \( [s, v]_P = 0 \).

Since \( h \neq 0 \), \( [s, v]_P = 0, \forall s, v \in K \) and \( \beta \in \Gamma \).

Thus, \( K \) is a commutative ring. By Lemma 2.9, we obtain that \( G \) is a commutative ring. \( \square \)

**Lemma 2.17:** Let \( h \) be a nonzero \( \Gamma \) - \((\lambda, \delta)\) - derivation on a prime \( \Gamma \)-near-ring \( G \), and \( K \) is a nonzero semi-group ideal of \( G \), if \( [s, r]_P = [h(s), h(r)]_P \), then the constant in \( K \) is in \( Z(G) \).

**Proof:** Let \( s \) be a constant in \( K \), i.e., \( h(s) = 0 \), then
\[
[s, r]_P = [h(s), h(r)]_P = [0, h(r)]_P = 0, \forall r \in K \text{ and } \rho \in \Gamma.
\]

Then, \( s \in Z(K) \). By Lemma 2.5, we obtain that \( s \in Z(G) \).

**Theorem 2.18.** Let \( h \) be a nonzero \( \Gamma \) - \((\lambda, \delta)\) - derivation on a prime \( \Gamma \)-near-ring \( G \), and \( K \) is a nonzero semi-group ideal of \( G \) that has no nonzero divisors of zero. If \( K \) has a right cancellation and \( h(s), h(r) \gamma = [s, r]_P \), \( \rho \lambda k \beta u = t \beta k \gamma u \) for every \( t, k, u \in K \) and \( \gamma, \beta \in \Gamma \), then \( h \) is commuting and \( (G, +) \) is an abelian.

**Proof.**
\[
\forall s \in K, [s, s\gamma h(s)]_P = [h(s), s\gamma h(s)]_P
\]

By using Lemma 2.1 and 2.2, the right – hand side of equation (7) equals
\[
[h(s), h(s\gamma h(s))_P = h(s)\delta(h(s))\beta(h(s)) + h(s)\delta(h(s))\beta(h(s)) - h(s)\delta(h(s))\beta(h(s)) - h(s)\delta(h(s))\beta(h(s))
\]

The left – hand side of equation (7) equals
\[
(s\gamma h(s))_P = s\gamma h(s)h^2(s) - s\gamma h^2(s)ph(s), \forall s \in K \text{ and } \rho, \gamma \in \Gamma
\]

It follows from equation (7), since \( \delta(K) = K \), it implies that
\[
s\gamma h(s)h^2(s) = h(s)ps\gamma h^2(s), \forall s \in K \text{ and } \rho, \gamma \in \Gamma.
\]

Hence, by using the hypotheses, we obtain that \( [s, h(s)]_P, h^2(s) = 0, \forall s \in K \) and \( \rho, \gamma \in \Gamma \).

If \( h^2(s) = 0, \forall s \in K \).

Then, \( h(s) \) is constant in \( K \), by using Lemma 2.17, we obtain that \( h \) is central. Thus, \( h \) is commuting in \( K \).
By Theorem 2.12, we obtain that \((G, +)\) is an abelian.

Otherwise, \(h^2(x)\) can be cancelled on the right in equation (8).

In either event, \([s, h(s)]_\rho = 0\), \(\forall s \in K\) and \(\gamma \in \Gamma\).

Then, by using Theorem 2.12, we obtain that \((G, +)\) is an abelian. \(\Box\)

**Theorem 2.19.** Let \(h\) be a nonzero \(\Gamma\)-\((\lambda, \delta)\)-derivation on a prime \(\Gamma\)-near-ring \(G\), and \(K\) be a nonzero semi-group ideal of \(G\) that has no nonzero divisors of zero. If \(h\) is commuting on \(K\) and \([s, r],_\rho = [h(s), h(r)]_\rho\), then \(G\) is a commutative ring.

**Proof.** For every \(s, r \in K\), we have

\([s, s]\rho = [h(s), h(s)]_\rho = [h(s), h(s)\beta(r) + \delta(s)\beta(h(r))]_\rho\)

By using Lemma 2.2, we have

\([h(s), h(s)\beta(r)]_\rho = h(s)\beta[h(s), \lambda(r)]_\rho + \delta(s)\beta[h(s), h(r)]_\rho\)

Since \(h\) is commuting, by using Theorem 2.12, \((G, +)\) is an abelian, and \(\delta(K) = K\), we obtain that

\([h(s), h(s)\beta(r)]_\rho = 0\), \(\forall s, r \in K\) and \(\rho \in \Gamma\).

In particular, \(\forall s, r, t \in K\), we have

\([h(s), t\beta(h(r))]_\rho = 0 = t\beta[h(s), h(r)]_\rho\).

Hence,

\(K\beta[h(s), h(r)]_\rho = 0\), \(\forall s, r \in K\) and \(\rho \in \Gamma\).

Since \(K\) is an abelian, we conclude that \([h(s), h(r)]_\rho = 0\), \(\forall s, r \in K\) and \(\rho \in \Gamma\).

Then, we conclude that \([s, r],_\rho = 0\), \(\forall s, r \in K\) and \(\rho \in \Gamma\).

Hence, \(K\) is commutative. By Lemma 2.9, we have \(G\) is a commutative ring.

**References**