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Coefficient Bounds for a New Integral Operator Using a Weibull Function

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Abstract

In this paper we introduced a new integral operator which is derived from a Weibull function by depending on the analytic and normalized function in open unit disk denoted by D . Also, we defined new two classes, and conducted a study of the coefficient bounds for Taylor's series, specifically focusing on $|a_2|$ and $|a_3|$. We satisfied a new results and properties.

Keywords: Weibull function, Analytic function, Univalent function, Integral operator.

القيود الكافية لمؤثر تكاملي جديد باستخدام دالة ويبيل

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الخلاصة

في هذا البحث قدمنا معاملاً تكاملياً جديداً المشتق من دالة ويبيل، بالاعتماد على دالة تحليلية و نورمال ضمن قرص الوحدة المفتوح و الذي رمزنا له بالرمز D . كما عرفنا صفتين جديديين على المؤثر و أجرينا دراسة لحدود معاملات سلسلة تايلور بالتركيز على المعاملين $|a_2|$ و $|a_3|$. و قد حققنا نتائج و خواص جديدة.

1. Introduction

Let $\mathcal{B}$ symbolized the class of all functions that have the form

$$f(z) = z + \sum_{\eta=2}^{\infty} a_{\eta} z^{\eta}, \quad (1)$$

where that analytic in the open unit disk $D = \{z \in \mathbb{C}: |z| < 1\}$, and satisfies the normalization condition as $f(0) = f'(0) - 1 = 0$ in D . Assume the subclass $\mathcal{S}$ of $\mathcal{B}$ which consists of all univalent functions in D with the form in (1), for details, see [1]. Let $\mathcal{F}$ be a subclass of each function $h(z)$ of the form

$$h(z) = 1 + h_1 z + h_2 z^2 + h_3 z^3 + \dots \quad (2)$$

That is analytic in the open disk D with $\Re(h(z)) > 0, z \in D$.

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The Koebe $\frac{1}{4}$ -Theorem [2] asserts that every univalent function $f \in S$ has an inverse f^{-1} define by

$$f^{-1}(f(z)) = z, (z \in D) \text{ and } f(f^{-1}(\omega)) = \omega \quad \left(|\omega| < r_0(f), r_0(f) \geq \frac{1}{4}\right).$$

For inverse of f , denoted $g = f^{-1}$, which given as

$$g(\omega) = f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 - (5a_2^3 - 5a_2a_3 + a_4)\omega^4 + \dots \quad (3)$$

Said a bi-univalent for $f \in \mathcal{B}$ if both f, f^{-1} be univalent in D . We denoted to the class of bi-univalent in D by $\hat{\Sigma}$. Some examples for the functions in $\hat{\Sigma}$ can be found in [3]. The estimates of the Taylor-Maclaurian coefficients are an important concern in geometric function theory since the information is provided on these properties.

Lewin [4] proposed the class $\hat{\Sigma}$ of bi-univalent functions, he showed that $|a_2| < 1.51$ for Taylor-Maclaurin coefficients. Later on, Barnna et al., [5] conjectured that $|a_2| \leq \sqrt{2}$.

Netanyahu [6], an additional way investigated

$$\max_{f \in \hat{\Sigma}} |a_2| = \frac{4}{3}.$$

The various studies provided examples of $\hat{\Sigma}$. Refer to the recent worked which presented by Srivastava et al., [7].

Recently, a lot of work has been showed devoted to developing the study of bi-univalent class Σ and obtaining non-sharp estimates on the coefficient a_2 with a_3 on $\hat{\Sigma}$ (see [8] and [9]).

More studies have pointed out for essential subclasses of $\hat{\Sigma}$, that is the most two important subclasses of univalent functions in classes $C^*(\alpha)$ and $\zeta(\alpha)$ of order α (see [10] and [11]). These classes can define as

$$\zeta(\alpha) = \left\{ f \in \mathcal{B} : \Re \left(\frac{zf'(z)}{f(z)} \right) > \alpha, z \in D \text{ and } \alpha \in [0,1] \right\},$$

$$C^*(\alpha) = \left\{ f \in \mathcal{B} : \Re \left(1 + \frac{zf''(z)}{f'(z)} \right) > \alpha, z \in D \text{ and } \alpha \in [0,1] \right\}.$$

For $0 \leq \alpha < 1$, let f and f^{-1} be starlike and convex function for order α , then the function $f \in \hat{\Sigma}_\rho$, which belongs to classes $C_{\hat{\Sigma}}^*(\alpha)$ or, $\zeta_{\hat{\Sigma}}(\alpha)$. These classes are investigated by [6].

Added to that, the coefficients $|a_2|$ and $|a_3|$ for Tylor's series in classes $C_{\hat{\Sigma}}^*(\alpha)$ and $\zeta_{\hat{\Sigma}}(\alpha)$ be found. It is verified the conditions

$$\left| \arg \left(\frac{zf'(z)}{f(z)} \right) \right| < \frac{\alpha\pi}{2} \quad \text{and} \quad \left| \arg \left(\frac{wg'(w)}{g(w)} \right) \right| < \frac{\alpha\pi}{2}.$$

Recent research focuses on studying the class $\hat{\Sigma}$ of bi-univalent functions and obtaining non-sharp bounds on the factors $|a_2|$ and $|a_3|$ for Taylor-Maclaurin. Notably, Srivastava et al., [12] advanced the study of various subclasses within the class $\hat{\Sigma}$ and discovered bounds on $|a_2|$ and $|a_3|$. Subsequently, by depending on the ground-breaking research by [12], and focusing on coefficient problems for different subclasses of the analytic and the class Σ , (see [13-22], for other details [23-33]).

In this paper, we introduced two classes on a new integral operator by using a Weibull function which be defined as $\psi f(z) = \int_0^\infty \frac{k}{\lambda} \left(\frac{t}{\lambda} \right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k} dt$. (see [34] and [35]). We found the estimates for the factors $|a_2|$ and $|a_3|$ in new subclasses.

Lemma 1.1: [36] If $h \in \mathcal{F}$, where $h(z)$ is given by (2), then $|h_\kappa| \leq 2$ for all $\kappa \in \mathbb{N}$.

2. Coefficient estimates of $\mathcal{L}_{\Sigma(\zeta, \mu, \alpha)}^\alpha$

An integral operator can be get as:

Lemma 2.1: Let $f \in \mathcal{B}$, $\lambda > 0$, $k \in \mathbb{N}$, $t \geq 0$ and $\alpha \in \mathbb{N}$. The integral operator ψ^α defined as

$$\psi^\alpha f(z): \mathcal{B} \rightarrow \mathcal{B},$$

$$\psi^\alpha f(z) = \int_0^\infty \frac{k}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k} f(\lambda z) dt = z + \sum_{\eta=2}^\infty a_\eta \lambda^{\eta-1} z^\eta.$$

In general

$$\psi^\alpha f(z) = z + \sum_{\eta=2}^\infty a_\eta (\lambda^{\eta-1})^\alpha z^\eta.$$

Proof:

$$\begin{aligned} \psi f(z) &= \int_0^\infty \frac{k}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k} f(\lambda z) dt = \int_0^\infty \frac{k}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k} (\lambda z + \sum_{\eta=2}^\infty a_\eta \lambda^\eta z^\eta) dt \\ &= \int_0^\infty \frac{k}{\lambda} \frac{t}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k} \lambda z dt + \sum_{\eta=2}^\infty a_\eta z^\eta \lambda^\eta \int_0^\infty \frac{k}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k} dt. \\ &= \int_0^\infty k \frac{\lambda}{t} \left(\frac{t}{\lambda}\right)^k e^{-\left(\frac{t}{\lambda}\right)^k} z dt + \sum_{n=2}^\infty a_n z^n \lambda^{n-1} \int_0^\infty k \frac{\lambda}{t} \left(\frac{t}{\lambda}\right)^k e^{-\left(\frac{t}{\lambda}\right)^k} dt. \end{aligned}$$

Suppose $\theta = \left(\frac{t}{\lambda}\right)^k$.

$$\ln \theta = k \ln \left(\frac{t}{\lambda}\right)$$

$$\frac{1}{\theta} d\theta = k \frac{\lambda}{t} dt.$$

By subtracting, we can get

$$\begin{aligned} &= \int_0^\infty \frac{1}{\theta} \theta e^{-\theta} z d\theta + \sum_{\eta=2}^\infty a_\eta z^\eta \lambda^{\eta-1} \int_0^\infty \frac{1}{\theta} \theta e^{-\theta} d\theta = z \int_0^\infty e^{-\theta} d\theta + \sum_{\eta=2}^\infty a_\eta z^\eta \lambda^{\eta-1} \int_0^\infty e^{-\theta} d\theta \\ &= z \left[-(e^{-\theta}) \right]_0^\infty + \sum_{\eta=2}^\infty a_\eta z^\eta \lambda^{\eta-1} \left[-(e^{-\theta}) \right]_0^\infty = -z(0-1) - \sum_{\eta=2}^\infty a_\eta z^\eta \lambda^{\eta-1} (0-1) \\ &= z + \sum_{\eta=2}^\infty a_\eta \lambda^{\eta-1} z^\eta. \end{aligned}$$

In general, we have

$$\psi^\alpha f(z) = z^\eta + \sum_{\eta=2}^\infty a_\eta (\lambda^{\eta-1})^\alpha z^\eta. \quad (4)$$

Definition 2.2: A function f with form as (1) is in class $\mathcal{L}_{\Sigma(\zeta, \mu, \alpha)}^\alpha$, if holding the condition

$$\left| \arg \left((1 + \mu) z \psi^\alpha f''(z) + z \zeta \left(\frac{\psi^\alpha f(v)}{z} \right)' \right) \right| < \frac{\alpha \pi}{2}, (z \in D), \quad (5)$$

and

$$\left| \arg \left((1 + \mu) \omega \psi^\alpha f''(\omega) + \omega \zeta \left(\frac{\psi^\alpha f(\omega)}{\omega} \right)' \right) \right| < \frac{\alpha \pi}{2}, (\omega \in D), \quad (6)$$

where $(\zeta \geq 0, 0 \leq \mu \leq 1, 0 < \alpha \leq 1)$. The function g has the form (3).

Theorem 2.3: Let f given by (1), if f is in $\mathcal{L}_{\Sigma(\eta, \mu, \alpha)}^\alpha$. Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{\lambda^{2\alpha} [\alpha(3+3\mu+\zeta) + (1-\alpha)(1+\mu+\zeta)^2]}}. \quad (7)$$

And

$$|a_3| \leq \frac{2\alpha}{\lambda^{2\alpha}(3+3\mu+\zeta)} + \frac{2\alpha^2}{\lambda^{2\alpha}(1+\mu+\zeta)^2}. \quad (8)$$

Proof: An inequality given in (7) and (8) are equivalent to

$$(1 + \mu)z\psi^\alpha f''(z) + z\zeta \left(\frac{\psi^\alpha f(z)}{z} \right)' = [\rho(z)]^\alpha, \quad (9)$$

$$(1 + \mu)\omega\psi^\alpha f''(\omega) + \omega\zeta \left(\frac{\psi^\alpha f(\omega)}{\omega} \right)' = [q(\omega)]^\alpha, \quad (10)$$

from (2), then $\rho(z)$ and $q(\omega)$ given by

$$\rho(z) = 1 + \rho_1 z + \rho_2 z^2 + \rho_3 z^3 + \dots, \quad (11)$$

$$q(\omega) = 1 + q_1 \omega + q_2 \omega^2 + q_3 \omega^3 + \dots. \quad (12)$$

Now, by equating the coefficient in (7) and (8), we get

$$2\lambda^\alpha(1 + \mu + \zeta)a_2 = \alpha\rho_1, \quad (13)$$

$$2\lambda^{2\alpha}(3 + \mu + \zeta)a_3 = \alpha\rho_2 + \frac{\alpha(\alpha-1)}{2}\rho_1^2, \quad (14)$$

$$2\lambda^\alpha(1 + \mu + \zeta)a_2 = -\alpha q_1, \quad (15)$$

$$2\lambda^{2\alpha}(3(1 + \mu) + \zeta)(2a_2^2 - a_3) = \alpha q_2 + \frac{\alpha(\alpha-1)}{2}q_1^2. \quad (16)$$

From (13) and (15) we have

$$\rho_1 = -q_1. \quad (17)$$

Now, by adding (14) with (16), then

$$(4\lambda^{2\alpha}(3 + 3\mu + \zeta))a_2^2 = \alpha(\rho_2 + q_2) + \frac{\alpha(\alpha-1)}{2}(\rho_1^2 + q_1^2), \quad (18)$$

$$(4\lambda^{2\alpha}(3 + 3\mu + \zeta))a_2^2 = \alpha(\rho_2 + q_2) + \frac{(\alpha-1)}{\alpha}4\lambda^{2\alpha}(1 + \mu + \zeta)^2 a_2^2, \quad (19)$$

$$a_2^2 = \frac{\alpha^2(\rho_2 + q_2)}{\alpha 4\lambda^{2\alpha}(3 + 3\mu + \zeta) - (\alpha-1)4\lambda^{2\alpha}(1 + \mu + \zeta)^2}, \quad (20)$$

$$a_2^2 \leq \frac{2\alpha}{\sqrt{\lambda^{2\alpha}[\alpha(3 + 3\mu + \zeta) + (1-\alpha)(1 + \mu + \zeta)^2]}}, \quad (21)$$

$$4\lambda^{2\alpha}(3 + 3\mu + \zeta)(a_3 - a_2^2) = \alpha(\rho_2 - q_2), \quad (22)$$

$$a_3 = \frac{\alpha(\rho_2 - q_2)}{4\lambda^{2\alpha}(3 + 3\mu + \zeta)} + \frac{\alpha^2(\rho_1^2 + q_1^2)}{4\lambda^{2\alpha}(1 + \mu + \zeta)^2}, \quad (23)$$

$$a_3 \leq \frac{2\alpha}{\lambda^{2\alpha}(3 + 3\mu + \zeta)} + \frac{2\alpha^2}{\lambda^{2\alpha}(1 + \mu + \zeta)^2}. \quad (24)$$

Hence, this proof is finished.

Now, let $\mu = 1, \lambda = 1$, in Theorem 2.3, then we get as:

Corollary 2.4: If $f \in \mathcal{B}$ which be a form in (1), suppose f in $\mathcal{L}_{\Sigma(\eta,1)}^\alpha$. Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{[\alpha(6 + \zeta) + (1-\alpha)(2 + \zeta)^2]}} \quad (25)$$

and

$$|a_3| \leq \frac{2\alpha}{(6 + \zeta)} + \frac{2\alpha^2}{(2 + \zeta)^2}. \quad (26)$$

Assuming $\alpha = 1$ and in Corollary 2.4, then we have the following corollary:

Corollary 2.5: Let $f \in \mathcal{B}$ which be a form in (1), suppose f be in $\mathcal{L}_{\Sigma(0,\zeta,1)}^\alpha$. Then

$$|a_2| \leq \frac{2}{\sqrt{(6 + \zeta)}}, \quad (27)$$

and

$$|a_3| \leq \frac{2}{(6 + \zeta)} + \frac{2}{(2 + \zeta)^2}. \quad (28)$$

Assuming $\zeta = 0$ in Theorem 2.3, then we have the following corollary:

Corollary 2.6: Let $f \in \mathcal{B}$ which be a form in (1), suppose f be in $\mathcal{L}_{\Sigma(\mu,0,\alpha)}^\alpha$. Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{\lambda^{2\alpha}[\alpha(3 + 3\mu) + (1-\alpha)(1 + \mu)^2]}}, \quad (29)$$

and

$$|a_3| \leq \frac{2\alpha}{\lambda^{2\alpha}(3+3\mu)} + \frac{2\alpha^2}{\lambda^{2\alpha}(1+\mu)^2}. \quad (30)$$

If $\alpha = 1, \lambda = 1$ in above corollary, then we have the result:

Corollary 2.7: Let $f \in \mathcal{B}$ which be a form in (1), suppose f be in $\mathcal{L}_{\Sigma(\mu)}^\alpha$. Then

$$|a_2| \leq \frac{2}{\sqrt{(3+3\mu)}}, \quad (31)$$

and

$$|a_3| \leq \frac{2}{3(1+\mu)} + \frac{2}{(1+\mu)^2}. \quad (32)$$

3. Coefficient estimates of $\mathcal{L}_{\Sigma(\zeta, \mu, \gamma)}^\alpha$

Definition 3.1: Suppose f which given as (1), then f be in the class $\mathcal{L}_{\Sigma(\zeta, \mu, \gamma)}^\alpha$, if satisfy the conditioning

$$\left| \arg \left((1+\mu)z\psi^\alpha f''(z) + z\zeta \left(\frac{\psi^\alpha f(z)}{z} \right)' \right) \right| > \gamma, (z \in D), \quad (33)$$

and

$$\left| \arg \left((1+\mu)\omega\psi^\alpha f''(\omega) + \omega\zeta \left(\frac{\psi^\alpha f(\omega)}{\omega} \right)' \right) \right| > \gamma, (\omega \in D). \quad (34)$$

Where ($\zeta \geq 0, 0 \leq \mu \leq 1, 0 \leq \gamma < 1$). The function g has the form (3).

Theorem 3.2: Let $f \in \mathcal{B}$ which be a form in (1), suppose f be in $\mathcal{L}_{\Sigma(\zeta, \mu, \gamma)}^\alpha$. Then

$$|a_2| \leq \frac{2(1-\gamma)}{\sqrt{\lambda^{2\alpha}[\alpha(3+3\mu+\zeta)+(1-\alpha)(1+\mu+\zeta)^2]}} \quad (35)$$

and

$$|a_3| \leq \frac{2(1-\gamma)}{\lambda^{2\alpha}(3+3\mu+\zeta)} + \frac{2(1-\gamma)}{\lambda^{2\alpha}(1+\mu+\zeta)^2}. \quad (36)$$

Proof: Assume $f \in \mathcal{L}_{\Sigma(\zeta, \mu, \gamma)}^\alpha$, then

$$(1+\mu)z\psi^\alpha f''(z) + z\zeta \left(\frac{\psi^\alpha f(z)}{z} \right)' = \gamma + (1-\gamma)\rho(z). \quad (37)$$

$$(1+\mu)\omega\psi^\alpha f''(\omega) + \omega\zeta \left(\frac{\psi^\alpha f(\omega)}{\omega} \right)' = \gamma + (1-\gamma)q(\omega). \quad (38)$$

Now, by equating the coefficient in (37) and (38), we getting

$$2\lambda^{2\alpha}(1+\mu+\zeta)a_2 = (1-\gamma)\rho_1. \quad (39)$$

$$2\lambda^{2\alpha}(3+3\mu+\zeta)a_3 = (1-\gamma)\rho_2. \quad (40)$$

$$2\lambda^{2\alpha}(1+\mu+\zeta)a_2 = -(1-\gamma)q_1. \quad (41)$$

$$2\lambda^{2\alpha}(3(1+\mu)+\zeta)(2a_2^2 - a_3) = (1-\gamma)q_2. \quad (42)$$

From (39) and (41) we have

$$\rho_1 = -q_1.$$

Now, by adding (40) with (42), then

$$(4\lambda^{2\alpha}(3+3\mu+\zeta))a_2^2 = (1-\gamma)(\rho_2 + q_2). \quad (43)$$

$$a_2^2 = \frac{(1-\gamma)(\rho_2 + q_2)}{4\lambda^{2\alpha}(3+3\mu+\zeta)}. \quad (44)$$

$$|a_2| \leq \sqrt{\frac{2(1-\gamma)}{4\lambda^{2\alpha}(3+3\mu+\zeta)}}. \quad (45)$$

$$4\lambda^{2\alpha}(3+3\mu+\zeta)(a_3 - 2a_2^2) = (1-\gamma)(\rho_2 - q_2). \quad (46)$$

$$|a_3| = \frac{2(1-\gamma)}{4\lambda^{2\alpha}(3+3\mu+\zeta)} + \frac{2(1-\gamma)}{4\lambda^{2\alpha}(1+\mu+\zeta)^2}. \quad (47)$$

$$|a_3| \leq \frac{2(1-\gamma)}{\lambda^{2\alpha}(3+3\mu+\zeta)} + \frac{2(1-\gamma)}{\lambda^{2\alpha}(1+\mu+\zeta)^2}. \quad (48)$$

Hence, the proof of this theorem is completed.

Now, let $\zeta = 0, \gamma = 0$, in Theorem 2.3, then we have:

Corollary 3.3: Let f given by (1), if f in $\mathcal{L}_{\Sigma}^{\alpha}(\mu)$. Then

$$|a_2| \leq \sqrt{\frac{2}{4\lambda^{2\alpha}(3+3\mu)}}, \quad (49)$$

$$a_3 \leq \frac{2}{\lambda^{2\alpha}(3+3\mu)} + \frac{2}{\lambda^{2\alpha}(1+\mu)^2}. \quad (50)$$

4. Conclusions

This paper, shown and defined a new integral operator by using Weibull function, that introduced two new classes on this operator. We studied the coefficient bounds between $|a_2|$ and $|a_3|$ for Taylor's series. Then we get some interest results and properties on this operator.

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