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$sp[\gamma,\gamma^*]$-Open Sets and $sp[\gamma,\gamma^*]$-Compact Spaces

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Abstract:
In this work, we present the notion of $sp[\gamma,\gamma^*]$-open set, $sp[\gamma,\gamma^*]$-closed, and $sp[\gamma,\gamma^*]$-closure such that several properties are obtained. By using this concept, we define a new type of spaces named $sp[\gamma,\gamma^*]$-compact space.

Keywords: $sp[\gamma,\gamma^*]$-open set, $sp[\gamma,\gamma^*]$-closed, $sp[\gamma,\gamma^*]$-closure, $sp[\gamma,\gamma^*]$-regular space, $sp[\gamma,\gamma^*]$-compact space.

المجموعات المفتوحة من النمط $sp[\gamma,\gamma^*]$ و الفضاءات المتراصة من النمط $sp[\gamma,\gamma^*]$

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الخلاصة
في هذا البحث قمنا بدراسة بمفهوم المجموعة المفتوحة من صنف $sp[\gamma,\gamma^*]$ و المجموعة المغلقة من صنف $sp[\gamma,\gamma^*]$ وذلك من خلال استخدام عاملين احداهما شبه مفتوح و الاخر شبه اولي حيث أعطينا عدة خواص وبرهنا عدة نظريات حول هذه المجموعات وكذلك قمنا بتعريف الفضاءات و المجموعات المرصوصة من صنف $sp[\gamma,\gamma^*]$ حيث درسنا بعض الخواص المهمة لهذه الفضاءات كذلك درسنا تأثير الدوال المفتوحة من صنف $sp[\gamma,\gamma^*]$ على تلك الفضاءات .

1-Introduction

Levine [1] defined the semi-open set in topological space and investigated some properties of semi-continuous functions. Mashhour. [2] introduced the notion of pre-open set such that several results are obtained. The concept of operation was initiated by Kasahara [3] and discussed α-closed graphs. Van and others [4] studied the operation pre-open sets in topological space and investigated several properties of γ_p-T_i spaces ($i = 0, {}^1/_2, 1$). Hariwan [5] defined the concept of γ-semi open set and used it to define new types of functions such as γ-semi continuous and weakly γ-semi continuous functions. Later, Maki and Noiri [6] introduced the notion $[\gamma,\gamma^*]$-open set in topological space. Carpintro, Rajesh, and Rosas [7] defined $[\gamma,\gamma^*]$-semi open sets and studied $[\gamma,\gamma^*]$-semi continuous functions such that several important properties are given.

In this work, we present a new type of bi-operation open sets that we named as $sp[\gamma,\gamma^*]$-open set, by using operation γ defined on the collection of semi-open sets and operation γ^* defined on the collection of pre-open sets. We studied the relations between $sp[\gamma,\gamma^*]$-open sets with other types of bi-operation open sets. Moreover, the present work introduced $sp[\gamma,\gamma^*]$-compact spaces and sets, then investigated some important results from these spaces.

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2-Preliminaries

Definition 2.1A subset A of topological space (X,τ) is named semi-open [1] (resp., pre-open set [2] if $A \subseteq cl\ int\ (A)$(resp., $A \subseteq int\ cl(A)$). We use $SO(X)$ and $PO(X)$ to denote, respectively, the family of semi-open and pre-open sets on topological space X.

Definition 2.2 [8]. A topological space (X,τ) is called extremally disconnected if the closure of any open subset of X is open.

Proposition 2.3 [8]. In extremally disconnected space, every semi-open set is pre-open.

Definition 2.4 [9]. An operation γ on topology τ is mapping $\gamma: \tau \to P(X)$ from τ to the power set $P(X)$ of X such that $V \subseteq V^{\gamma}$ for each $V \in \tau$, where V^{γ}denotes the value of γ at V.

Definition 2.5 [10]. Let (X,τ) be a topological space and let $\gamma: PO(X) \to P(X)$ be an operation defined on $PO(X,\tau)$. A non empty subset A of (X,τ) is called γ pre-open if for each point $x \in A$, there exists a pre-open set U such that $x \in U$ and $U^{\gamma} \subseteq A$

Definition 2.6 [5]. Let(X,τ) be a topological space and let $\gamma: SO(X) \to P(X)$ be an operation defined on $SO(X,\tau)$. A non empty subset A of (X,τ) is called γ semi-open if for each point $x \in A$, there exists a semi-open set U such that $x \in U$ and $U^{\gamma} \subseteq A$

Definition 2.7 [11]. Let(X,τ) be a topological space, an operation $\gamma: SO(X) \to P(X)$is named by semi-γ-regular, if for every semi-open sets S and T containing x, there exists a semi-open V containing x such that $V^{\gamma} \subseteq S^{\gamma} \cap T^{\gamma}$.

Definition 2.8 [10]. Let(X,τ) be a topological space, an operation $\gamma: PO(X) \to P(X)$ is named by pre-γ-regular, if for every pre-open sets U and V containing x, there exists a pre-open P containing x such that $P^{\gamma} \subseteq U^{\gamma} \cap V^{\gamma}$.

Definition 2.9 [6]. Let (X,τ) be a topological space and A be a non-empty subset of X, we named A is$[\gamma,\gamma^*]$-open if there are two open sets $U$and V containing x such that $U^{\gamma} \cap V^{\gamma^*} \subseteq A$.

Definition 2.10. Let (X,τ) be a topological space and A be a non-empty subset of X, we named A is pre $[\gamma,\gamma^*]$-open if there are two pre-open sets $U$and V containing x such that $U^{\gamma} \cap V^{\gamma^*} \subseteq A$.

Definition 2.11 [6]. A function $f: (X,\tau) \longrightarrow (Y,\psi)$ is said to be $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous if for each point $x \in X$ and each open neighborhood W and S of $f(x)$, there exists open neighborhoods U and V of x such that $f(U^{\alpha} \cap V^{\alpha^*}) \subseteq W^{\gamma} \cap S^{\gamma^*}$

Theorem 2.12 [6]. A function $f: (X,\tau) \longrightarrow (Y,\psi)$ is said to be $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous if the inverse image of every $[\gamma,\gamma^*]$-open set in Y is $[\alpha,\alpha^*]$ - open set in X

Definition 2.13. Let (X,τ) be a topological space and A be a non-empty subset of X, we named A is pre $[\gamma,\gamma^*]$-open if there are two pre-open sets $U$and V containing x such that $U^{\gamma} \cap V^{\gamma^*} \subseteq A$.

Definition 2.14. Let (X,τ) be a topological space and A be a non-empty subset of X, we named A is semi $[\gamma,\gamma^*]$-open if there are two semi-open sets $U$and V containing x such that $U^{\gamma} \cap V^{\gamma^*} \subseteq A$.

3-$sp[\gamma,\gamma^*]$-open set

Definition 3.1. Let (X,τ) be a topological space and A be a non-empty subset of X, we named A is $sp[\gamma,\gamma^*]$-open if for each $x \in A$, there are a semi-open set $U$and pre-open set V containing x such that $U^{\gamma} \cap V^{\gamma^*} \subseteq A$.

Proposition 3.2. In extremely disconnected, every $sp[\gamma,\gamma^*]$-open is semi$[\gamma,\gamma^*]$-open (resp., pre $[\gamma,\gamma^*]$-open set).

Proof: Follows from Proposition 2.3.

Proposition 3.3. Every $[\gamma,\gamma^*]$-open set is $sp[\gamma,\gamma^*]$-open.

Proof: Follows from the fact that every open set is semi-open (resp., pre-open).

But the converse is not true generally as showed in the next example

Example 3.4. Let $X = \{a,b,c\}$ and let $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ be a topology defined on. Let $\gamma: SO(X) \longrightarrow P(X)$ and $\gamma^*: PO(X) \longrightarrow P(X)$ be two operators defined as follows

$$A^{\gamma} = \begin{Bmatrix} cl(A) & if\ A = \{a\} \\ A & if\ A \neq \{a\} \end{Bmatrix}$$

$$A^{\gamma^*} = \begin{Bmatrix} A & if\ A = \{b\} \\ A \cup \{a\} & if\ A \neq \{b\} \end{Bmatrix}$$

Then $\{b\}$ is $sp[\gamma,\gamma^*]$-open set, however, it is not $[\gamma,\gamma^*]$-open set

Proposition 3.5. The union of $sp[\gamma,\gamma^*]$-open sets is also $sp[\gamma,\gamma^*]$-open set.

Proof: Let $\{V_i: i \in I\}$be the collection of $sp[\gamma,\gamma^*]$-open sets of topological space (X,τ) . Let $x \in \bigcup_{i\in I} V_i$, then there is $sp[\gamma,\gamma^*]$-open set $V_i$containing x and so, there are semi-open set S and pre-open set P containing x such that $S^\gamma \cap P^{\gamma^*} \subseteq V_i \subseteq \bigcup_{i\in I} V_i$. Hence $\bigcup_{i\in I} V_i$ is $sp[\gamma,\gamma^*]$-open set.

Proposition 3.6. Let(X,τ) be a topological space. If A is γ semi-open and γ^* pre-open subsets of X, then it is $sp[\gamma,\gamma^*]$-open set

Proof: Let $x \in X$ and since Aisγ semi-open containing x, then there exists a semi-open set U containing x such that $x \in U^\gamma \subseteq A$. And, since A is γ^* pre-open set, then there exists a pre-open set V such that $x \in V^{\gamma^*} \subseteq A$. It follows that $x \in U^\gamma \cap V^{\gamma^*} \subseteq A$. Hence A is $sp[\gamma,\gamma^*]$-open set.

Proposition 3.7. Let$A$ and B are non-empty subsets of X. If A is γ semi-open set and B isγ^* pre-open set, then $A \cap B$ is $sp[\gamma,\gamma^*]$-open set.

Proof: Similar to the proof of Proposition 3.6.

Proposition 3.8. Let $\gamma: SO(X) \to P(X)$ be semi-γ-regular and $\gamma^*: PO(X) \to P(X)$ be pre-γ^*-regular operation. If A and B are $sp[\gamma,\gamma^*]$-open sets, then $A \cap B$ is $sp[\gamma,\gamma^*]$-open set.

Proof: Let $x \in X$ such that $x \in A \cap B$. Since $x \in A$, and A is $sp[\gamma,\gamma^*]$-open, then there are a semi-open S_1 and pre-open P_1 containing x such that $S_1^\gamma \cap P_1^\gamma \subseteq A$

and since, $x \in B$, and B is $sp[\gamma,\gamma^*]$-open, then there exists a semi-open S_2 and pre-open P_2 containing x such that $S_2^\gamma \cap P_2^\gamma \subseteq B$.

By hypothesis, γ is semi-γ-regular, thus there exists a semi-open set S_3 containing x such that $S_3^\gamma \subseteq S_1^\gamma \cap S_2^\gamma$.

Similarly, γ^* is pre-γ^*-regular operation, then there exists a pre-open set P_3 containing x such that $P_3^{\gamma^*} \subseteq P_1^{\gamma^*} \cap P_2^{\gamma^*}$. It follows that $S_3^\gamma \cap P_3^{\gamma^*} \subseteq \left(S_1^\gamma \cap P_1^{\gamma^*}\right) \cap \left(S_2^\gamma \cap P_2^{\gamma^*}\right) \subseteq A \cap B$. Hence $A \cap B$ is $sp[\gamma,\gamma^*]$-open set.

Proposition 3.9. If $\gamma: SO(X) \to P(X)$ be semi-γ-regular and $\gamma^*: PO(X) \to P(X)$ be pre-γ^*-regular operations, then the collection of $sp[\gamma,\gamma^*]$-open sets forms a topology.

Proof: Obviously $\phi$is $sp[\gamma,\gamma^*]$-open set. Let $x \in X$ and since $X^\gamma \cap X^{\gamma^*} \subseteq X$. The union and intersection conditions follow from Proposition 3.5 and Proposition 3.8.

Example 3.10. Let $X = \{a,b,c\}$ and let $\tau = \{\phi, X, \{a\}\}$ be a topology defined on . Let $\gamma: SO(X) \longrightarrow P(X)$ and $\gamma^*: PO(X) \longrightarrow P(X)$are two operations defined as following$A^\gamma = A$ and

$$A^{\gamma^*} = \begin{Bmatrix} A & if\ A = \{b\} \\ \phi & if\ A \neq \{b\} \end{Bmatrix}$$

Clearly, γ and γ^* aresemi-γ-regular andpre-γ^*-regular operations, respectively. Then, the family of $sp[\gamma,\gamma^*]$-open setswhich listed as $\phi, X, \{a\}, \{a,b\}, \{a,c\}$ forms a topology defined on X.

Definition 3.11. A topological space(X,τ) is named by $sp[\gamma,\gamma^*]$-regular space if for each $x \in X$ and every semi-open set A containing x, there are semi-open set S and pre-open set P containing x such that $S^\gamma \cap P^{\gamma^*} \subseteq A$.

Proposition 3.12. A topological space(X,τ) is $sp[\gamma,\gamma^*]$-regular space if and only if for each $x \in X$ and every semi-open set U of X, there a $sp[\gamma,\gamma^*]$-open set V such that $x \in V$ and $V \subseteq U$.

Proof: Let $x \in X$ and let U be a semi-open set containing x. Since X is $sp[\gamma,\gamma^*]$-regular space, then there are a semi-open S and pre-open P containing x such that $\left(S^\gamma \cap P^{\gamma^*}\right) \subseteq U$.

Conversely, suppose that A is a semi-open set containing x.By hypothesis, there is $sp[\gamma,\gamma^*]$-open set V such that $x \in V$ and $V \subseteq A$. So, there are a semi-open S and pre-open Pcontaining x such that $S^\gamma \cap P^{\gamma^*} \subseteq V \subseteq A$. Hence (X,τ) is $sp[\gamma,\gamma^*]$-regular space.

Proposition 3.13A topological space(X,τ) is $sp[\gamma,\gamma^*]$-regular space if and only if $SO(X) = sp[\gamma,\gamma^*]O(X)$.

Proof: straightforward.

Proposition 3.14. Let $id\gamma: SO(X) \to P(X)$ and $id\gamma^*: PO(X) \to P(X)$be two identity operators, then every semi-open and pre-open set is $sp[\gamma,\gamma^*]$-open set.

Proof: obvious.

Definition 3.15. Let γ and γ^* be two operations defined on $SO(X)$ and $PO(X)$, respectively, then a subset A of$X$is named$sp[\gamma,\gamma^*]$-closed if its complement is $sp[\gamma,\gamma^*]$-open set.

Definition 3.16. Let A be a subset of topological space (X,τ), the intersection of all $sp[\gamma,\gamma^*]$-closed sets containing A is named $sp[\gamma,\gamma^*]$-closure of A and is denoted by $sp[\gamma,\gamma^*]-cl(A)$.
Proposition 3.17. The intersection of any $sp[\gamma,\gamma^*]$-closed sets is also $sp[\gamma,\gamma^*]$-closed set.
Proof: Follows from Proposition 3.5.
Proposition 3.18. Let A and B are two sets in topological space $(X,\tau)$and let γ and γ^* be two operations defined on $SO(X)$ and $PO(X)$, respectively, then we have the following
1)$A \subseteq sp[\gamma,\gamma^*]-cl(A)$.
2)A is $sp[\gamma,\gamma^*]$-closed if and only if $A = sp[\gamma,\gamma^*]-cl(A)$
3) If $\subseteq B$, then $sp[\gamma,\gamma^*]-cl(A) \subseteq sp[\gamma,\gamma^*]-cl(B)$
4)$sp[\gamma,\gamma^*]-cl(A\cap B) \subseteq sp[\gamma,\gamma^*]-cl(A)\cap sp[\gamma,\gamma^*]-cl(B)$
5)$sp[\gamma,\gamma^*]-cl(A) \subseteq [\gamma,\gamma^*]-cl(A)$
6)$sp[\gamma,\gamma^*]-cl\big(sp[\gamma,\gamma^*]-cl(A)\big) = sp[\gamma,\gamma^*]-cl(A)$
Proposition 3.19. For each a$x \in X$, $x \in sp[\gamma,\gamma^*]-cl(A)$ if and only if $V\cap A \neq \phi$ for each $sp[\gamma,\gamma^*]$-open V containing x.
Proof: Obvious.
Proposition 3.20. If γ and γ^* are semi-γ-regular and pre-γ^*-regular operations defined on $SO(X)$ and $PO(X)$respectively, then $sp[\gamma,\gamma^*]-cl(A\cup B) = sp[\gamma,\gamma^*]-cl(A)\cup sp[\gamma,\gamma^*]-cl(B)$.
Proof: Clearly $sp[\gamma,\gamma^*]-cl(A)\cup sp[\gamma,\gamma^*]-cl(B) \subseteq sp[\gamma,\gamma^*]-cl(A\cup B)$. Assume that $x \notin [sp[\gamma,\gamma^*]-cl(A)\cup sp[\gamma,\gamma^*]-cl(B)]$. Since $x \notin sp[\gamma,\gamma^*]-cl(A)$, then by proposition, there is $sp[\gamma,\gamma^*]$-open set U containing x such that $U\cap A = \phi$. Similarly, $x \notin sp[\gamma,\gamma^*]-cl(B)$, then by Proposition 3.19, there is $sp[\gamma,\gamma^*]$-open set V containing x such that $V\cap A = \phi$. By Proposition 3.8, $U\cap V$ is$sp[\gamma,\gamma^*]$-open set containing x such that $(U\cap V)\cap(A\cup B) = \phi$. It follows that $sp[\gamma,\gamma^*]-cl(A\cup B)$.
Definition 3.21. Let A be a subset of topological space (X,τ), then $x \in spcl-[\gamma,\gamma^*](A)$ if $\left(S^\gamma\cap P^{\gamma^*}\right)\cap A \neq \phi$ for every semi-open S and pre-open P containing x .
Proposition 3.22. Let A be a subset of topological space (X,τ), then
1) $spcl-[\gamma,\gamma^*](A) \subseteq sp[\gamma,\gamma^*]-cl(A)$
2) $sp[\gamma,X]-cl(A) \subseteq S\gamma cl(A)$
3)$spcl-[\gamma,\gamma^*](A\cup B) \subseteq scl_\gamma(A)\cup pcl_{\gamma^*}(B)$
4) If γ and γ^* are semi-open and pre-open operations defined on$spcl-[\gamma,\gamma^*](spcl-[\gamma,\gamma^*]) = spcl-[\gamma,\gamma^*](A)$
Proof: 1) let $x \notin sp[\gamma,\gamma^*]-cl(A)$, then there exists a $sp[\gamma,\gamma^*]$-open set U containing x such that $U\cap A = \phi$. , then there are semi-open S and pre-open Pcontaining x such that $S^\gamma\cap P^{\gamma^*} \subseteq U$ and so,$\left(S^\gamma\cap P^{\gamma^*}\right)\cap A = \phi$. Hence $x \notin spcl-[\gamma,\gamma^*](A)$.
2) Let $x \notin S\gamma cl(A)$, then there is γ-semi open U containing x such that $U\cap A = \phi$, and since$(U\cap X)\cap A = \phi$ by proposition 3.6, $U\cap X$ is $sp[\gamma,\gamma^*]$-open set containing x and so, $x \notin w[\gamma,X]-cl(A)$.
3) Follows from Definition 3.21.
4) Follows from Proposition 3.23 and Proposition 3.18.
Proposition 3.23. Let γ and γ^* are semi-open and pre-open operations defined on $SO(X)$ and $PO(X)$, respectively, then $spcl-[\gamma,\gamma^*](A) = sp[\gamma,\gamma^*]-cl(A)$
Proof: By Proposition 3.22 (1), $spcl-[\gamma,\gamma^*](A) \subseteq sp[\gamma,\gamma^*]-cl(A)$. It is remaining to prove that $sp[\gamma,\gamma^*]-cl(A) \subseteq spcl-[\gamma,\gamma^*](A)$. Let$x \notin spcl-[\gamma,\gamma^*](A)$, then there are semi-open set S and pre-open set Pcontaining x such that$\left(S^\gamma\cap P^{\gamma^*}\right)\cap A = \phi$. Since γ is a semi-open operation, then there is γ-semi open set U containing x such that $U \subseteq S^\gamma$and since γ^* is a pre-open operation, then there isγ^*-pre open set V containing x such that $V \subseteq P^{\gamma^*}$. It follows that $U\cap V \subseteq S^\gamma\cap P^{\gamma^*}$and by Proposition 3.6, $U\cap V$ is $sp[\gamma,\gamma^*]$-open set containing x such that$(U\cap V)\cap A = \phi$. Hence $x \notin sp[\gamma,\gamma^*]-cl(A)$.

$sp[\gamma,\gamma^*]$-compact space and set4-

Definition 4.1. A subset A of topological space $(X,\tau)$is $sp[\gamma,\gamma^*]$-compact set, if every cover $\{V_i: i \in I\}$ of X by $[\gamma,\gamma^*]$-open sets, there exists a finite subset I_O of I such that $A \subseteq \bigcup_{i\in I_O} sp[\gamma,\gamma^*]-cl(V_i)$. And topological space (X,τ) is named $sp[\gamma,\gamma^*]$-compact if $X = \bigcup_{i\in I_O} sp[\gamma,\gamma^*]-cl(V_i)$.

Definition 4.2. A subset A of topological space $(X,\tau)$is$[\gamma,\gamma^*]$-compact set if every cover $\{V_i: i\in I\}$ of X by $[\gamma,\gamma^*]$-open sets, there exists a finite subset I_O of I such that $A\subseteq\bigcup_{i\in I_O}V_i$. And topological space (X,τ) is named $[\gamma,\gamma^*]$-compact space if $X=\bigcup_{i\in I_O}V_i$

It is clear that every $[\gamma,\gamma^*]$-compact is $sp[\gamma,\gamma^*]$-compact space

Proposition 4.3. Let γ and γ^* be two operations defined on $SO(X)$ and $PO(X)$ and let A be any proper subset of X. If A and $X\,/\,A$ are $sp[\gamma,\gamma^*]$-compact sets, then X is $sp[\gamma,\gamma^*]$-compact.

Proof: Let $\varphi=\{U_i: i\in I\}$ be $[\gamma,\gamma^*]$-open cover of X, then $\varphi=\{U_i: i\in I\}$ is$[\gamma,\gamma^*]$-open cover of A and $X\,/\,A$. Since A and $X\,/\,A$ are $sp[\gamma,\gamma^*]$-compact sets, then there are finite sub-collection I_0 and $I_1$of I such that $A\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(U_i)$ and $X\,/\,A\subseteq\bigcup_{i\in I_1}sp[\gamma,\gamma^*]-cl(U_i)$, therefore $X=A\bigcup X\,/\,A\subseteq\bigcup_{i\in I_0\cup I_1}sp[\gamma,\gamma^*]-cl(U_i)$. Hence X is $sp[\gamma,\gamma^*]$-compact.

Proposition 4.4. The finite union of any $sp[\gamma,\gamma^*]$-compact sets is $sp[\gamma,\gamma^*]$-compact set.

Proof: Similar to the proof of Proposition 4.3.

Proposition 4.5. Let γ and γ^* be two operations defined on $SO(X)$ and (X) , then a topological space (X,τ) is $sp[\gamma,\gamma^*]$-compact if and only if every proper $[\gamma,\gamma^*]$-closed subset of X is $sp[\gamma,\gamma^*]$-compact.

Proof: Let F be a proper $[\gamma,\gamma^*]$-closed set in X and let $\varphi=\{U_i: i\in I\}$ be $[\gamma,\gamma^*]$-open cover of F, then$\{U_i: i\in I\}\bigcup X\,/\,F$ is $[\gamma,\gamma^*]$-open cover of X. Since X is $sp[\gamma,\gamma^*]$-compact, then there is finite sub-collection$I_0$of I such that $X=\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(U_i\bigcup X\,/\,F)=\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(U_i)\bigcup X\,/\,F$ and so, $F\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(U_i)$. Hence F is $sp[\gamma,\gamma^*]$-compact.

Conversely, suppose that every proper $[\gamma,\gamma^*]$-closed subset of X is $sp[\gamma,\gamma^*]$-compact and let $\psi=\{V_i: i\in I\}$ be $[\gamma,\gamma^*]$-open cover of X such that $V_{j_0}$is a proper $[\gamma,\gamma^*]$-open subset of X for $j_0\in I$, then $X\,/\,V_{i_0}$isa proper $[\gamma,\gamma^*]$-closed set and by hypothesis $X\,/\,V_{j_0}$ is $sp[\gamma,\gamma^*]$-compact, then there is finite sub-collection $I_0$of I such that $X\,/\,V_{j_0}\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(V_i)$, it follows that $X=V_{j_0}\bigcup X\,/\,V_{j_0}\subseteq V_{j_0}\bigcup\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(V_i)\subseteq sp[\gamma,\gamma^*]-cl(V_{j_0})\bigcup\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(V_i)\subseteq\bigcup_{i\in I_0\cup j_0}sp[\gamma,\gamma^*]-cl(V_i)$. Hence X is $sp[\gamma,\gamma^*]$-compact.

Proposition 4.6. Let K be a subset of topological space (X,τ), and let γ and γ^* be two operations defined on $SO(X)$ and $PO(X)$, such that $w[\gamma,\gamma^*]_K-cl(G\bigcap K)=sp[\gamma,\gamma^*]-cl(G)\bigcap K$ for every G is $[\gamma,\gamma^*]$-open set in X, then K is $sp[\gamma,\gamma^*]$-compact if and only if K is $sp[\gamma,\gamma^*]_K$-compact.

Proof: Suppose that K is $sp[\gamma,\gamma^*]$-compact and let $\varphi=\{G_i\bigcap K: i\in I\}$ be$[\gamma,\gamma^*]_K$ –open cover of K, then$K\subseteq\bigcup_i(G_i\bigcap K)\subseteq\bigcup_i G_i$. But K is $sp[\gamma,\gamma^*]$-compact, thus there is a finite subset $I_0$of I such that$K\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(G_i)$. It follows that $K\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(G_i)\bigcap K=\bigcup_{i\in I_0}sp[\gamma,\gamma^*]_K-cl(G_i\bigcap K)$and so K is $sp[\gamma,\gamma^*]_K$-compact.

Conversely, suppose that $\psi=\{U_i: i\in I\}$be is$[\gamma,\gamma^*]$-open cover of , then $\varphi^*=\{U_i\bigcap K: i\in I\}$ be $sp[\gamma,\gamma^*]_k$-open cover of K. Since K is $[\gamma,\gamma^*]_k$-compact set, thenthere is a finite subset $I_0$of I such that$K\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(U_i\bigcap K)\subseteq\bigcup_{i\in I_O}sp[\gamma,\gamma^*]-cl(U_i)\bigcap K\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(U_i)$.

Hence K is $sp[\gamma,\gamma^*]$-compact.

Definition 4.7. A topological space (X,τ) is named $sp[\gamma,\gamma^*]$-Urysohn space if for every two distinct points x and y, there are two$[\gamma,\gamma^*]$-open sets U and V containing x and y such that $sp[\gamma,\gamma^*]-cl(U)\bigcap sp[\gamma,\gamma^*]-cl(V)=\phi$

Proposition 4.8. Let γ be semi- γ-regular and γ^*be pre-γ^*-regular operators defined on $SO(X)$ and $PO(X)$. If $X$is $sp[\gamma,\gamma^*]$-Urysohn space and K be$sp[\gamma,\gamma^*]$-compact subset of topological space(X,τ), then K is $sp[\gamma,\gamma^*]$-closed.

Proof: We want to prove that $X\,/\,K$ is $sp[\gamma,\gamma^*]$-open set. Let $x\in X\,/\,K$, then for each $y\in K$, there are two $[\gamma,\gamma^*]$-open sets U and$V$ containing x and y such that $sp[\gamma,\gamma^*]-cl(U_x)\bigcap sp[\gamma,\gamma^*]-cl(V_y)=\phi$

Take $\varphi=\{V_y: y\in K\}$ be $[\gamma,\gamma^*]$-open cover of $K$and since K is $sp[\gamma,\gamma^*]$-compact, then there is a finite sub-collection of $I_0$of Isuch that$K\subseteq\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(V_{yi})$, let $\bigcup_{i\in I_0}sp[\gamma,\gamma^*]-cl(V_{yi})=sp[\gamma,\gamma^*]-cl(V)$and let $U=\bigcap_{i=1}^{n}U_{xi}$, such that $U\bigcap sp[\gamma,\gamma^*]-cl(V)=\phi$ then by Proposition 2.8, U is $[\gamma,\gamma^*]$-open set and so, $x\in U\subseteq X\,/\,K$, that is $X\,/\,K$ is $[\gamma,\gamma^*]$-open set. Hence K is $sp[\gamma,\gamma^*]$-closed.

Proposition 4.9. Let γ be semi- γ-regular and γ^* be pre-γ^*-regular operators defined on $SO(X)$ and $PO(X)$. If A is $sp[\gamma,\gamma^*]$-compact and U is $[\gamma,\gamma^*]$-open and $sp[\gamma,\gamma^*]$-closed sets in topological space(X,τ)such that $U \subseteq A$, then A / U is $sp[\gamma,\gamma^*]$-compact.

Proof: Let $\varphi = \{V_i: i \in I\}$ be $[\gamma,\gamma^*]$-open cover of A / U.Since U is $[\gamma,\gamma^*]$-open set, then $\varphi \cup U$ is $[\gamma,\gamma^*]$-open cover of A, and since A is $w[\gamma,\gamma^*]$-compact, then there is a finite sub-collection of I_0 of Isuch that $A \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(V_i \cup U) = \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(V_i) \cup sp[\gamma,\gamma^*] - cl(U) = \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(V_i) \cup U$ and so, $A / U \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(V_i)$. Hence A / U is $sp[\gamma,\gamma^*]$-compact.

Proposition 4.10. Let γ be semi- γ-regular and γ^* be pre-γ^*-regular operators defined on $SO(X)$ and $PO(X)$ and let U be $sp[\gamma,\gamma^*]$-compact subset of $sp[\gamma,\gamma^*]$-Urysohn space$X$, for every $x \in U$ and any $[\gamma,\gamma^*]$-open, $sp[\gamma,\gamma^*]$-closed set$V$ such that $x \in V \subseteq U$, then there is $[\gamma,\gamma^*]$-open set$G$ such that $x \in G \subseteq sp[\gamma,\gamma^*] - cl(G) \subseteq V$.

Proof: Let $x \in U$ and let V any $[\gamma,\gamma^*]$-open, and $sp[\gamma,\gamma^*]$-closed set in X such that $x \in V \subseteq U$. For every $y \in U / V$ in $sp[\gamma,\gamma^*]$-Urysohn space X, then there are $[\gamma,\gamma^*]$-open sets $G_x$and $H_y$containing x and , thus $\{H_y: y \in U / V\}$ is $[\gamma,\gamma^*]$-open cover of U / V and since V is $[\gamma,\gamma^*]$-open, and $w[\gamma,\gamma^*]$-closed set, then by Proposition 3.9, U / V is $sp[\gamma,\gamma^*]$-compact, and so $U / V \subseteq \bigcup_{i=1}^{n} sp[\gamma,\gamma^*] - cl(H_{yi})$=$sp[\gamma,\gamma^*] - cl(\bigcup_{i=1}^{n} H_{yi})$= $sp[\gamma,\gamma^*] - cl(H)$. Assume that$G_{xi} \subseteq A$, set $G = \bigcap_{i=1}^{n} G_{xi} \subseteq A$ with $sp[\gamma,\gamma^*] - cl(G) \cap sp[\gamma,\gamma^*] - cl(H) = \phi$. It follows $sp[\gamma,\gamma^*] - cl(G) \cap H = \phi$. Since U is $sp[\gamma,\gamma^*]$-compact subset of $sp[\gamma,\gamma^*]$-Urysohn space$X$, then U is $sp[\gamma,\gamma^*]$-closed and since $G \subseteq V \subseteq U$, then $sp[\gamma,\gamma^*] - cl(G) \subseteq U$, therefore $U/V \subseteq U \cap sp[\gamma,\gamma^*] - cl(H) \subseteq B \cap (X/sp[\gamma,\gamma^*] - cl(G)) = B / X/sp[\gamma,\gamma^*] - cl(G)$. Hence $x \in G \subseteq sp[\gamma,\gamma^*] - cl(G) \subseteq V$

Proposition 4.11. Let γ be semi- γ-regular and γ^* be pre-γ^*-regular operators defined on $SO(X)$ and (X) , and let A and B are two subsets of topological space X. If A is $sp[\gamma,\gamma^*]$-compact and B is $[\gamma,\gamma^*]$-closed, then $A \cap B$ is $sp[\gamma,\gamma^*]$-compact

Proof: Let $\{U_i: i \in I\}$ be $[\gamma,\gamma^*]$-open cover of $A \cap B$. Since B is $[\gamma,\gamma^*]$-closed, then X / B is $[\gamma,\gamma^*]$-open set and so,$\{U_i: i \in I\} \cup X / B$ is $[\gamma,\gamma^*]$-open cover of A.But $A$is $sp[\gamma,\gamma^*]$-compact, thus there is a finite sub-collectionI_0 of Isuch that $A \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(U_i \cup X / B) = \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(U_i) \cup sp[\gamma,\gamma^*] - cl(X/B) = \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(U_i) \cup X/B$

That is $A \cap B \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(U_i)$. Hence $A \cap B$ is $sp[\gamma,\gamma^*]$-compact.

Definition 4.12. A function $f: (X,\tau) \longrightarrow (Y,\psi)$ is said to be $sp([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous if the inverse image of each$sp[\gamma,\gamma^*]$-open set in Y is $sp[\alpha,\alpha^*]$ - open set in X. Equivalently, the inverse image of each$sp[\gamma,\gamma^*]$-closed set in Y is $sp[\alpha,\alpha^*]$ - closed set in$X$.

Lemma 4.13. A function $f: (X,\tau) \longrightarrow (Y,\psi)$ is $sp([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous if and only if $f(sp[\alpha,\alpha^*] - cl(U)) \subseteq sp[\gamma,\gamma^*] - cl(f(U))$for each subset U of X.

Proof: Suppose that f is $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous. Since $f(U) \subseteq sp[\gamma,\gamma^*] - cl(f(U))$, then $U \subseteq f^{-1}\left(sp[\gamma,\gamma^*] - cl(f(U))\right)$. Since $sp[\gamma,\gamma^*] - cl(f(U))$ is $sp[\gamma,\gamma^*]$-closed in Y and since f is $sp([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, then$f^{-1}\left(sp[\gamma,\gamma^*] - cl(f(U))\right)$ is $w[\gamma,\gamma^*]$-closed in X and so, $sp[\alpha,\alpha^*] - cl(U) \subseteq f^{-1}\left(sp[\gamma,\gamma^*] - cl(f(U))\right)$. Hence $f(sp[\alpha,\alpha^*] - cl(U)) \subseteq sp[\gamma,\gamma^*] - cl(f(U))$

Conversely, suppose that $f(sp[\alpha,\alpha^*] - cl(U)) \subseteq sp[\gamma,\gamma^*] - cl(f(U))$for each subset U of X. Let F be $sp[\gamma,\gamma^*]$-closed in Y, and so $f^{-1}(F)$ be a subset of X. By hypothesis, $f\left(sp[\alpha,\alpha^*] - cl(f^{-1}(F))\right) \subseteq sp[\gamma,\gamma^*] - cl\left(f(f^{-1}(F))\right)$. It follows $f\left(w[\alpha,\alpha^*] - cl(f^{-1}(F))\right) \subseteq sp[\gamma,\gamma^*] - cl(F)$ and so, $sp[\alpha,\alpha^*] - cl(f^{-1}(F)) \subseteq f^{-1}(F)$. Then $f^{-1}(F)$ is $sp[\gamma,\gamma^*]$-closed in X, Hence f is $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous.

Proposition 4.14 Let $f: (X,\tau) \longrightarrow (Y,\psi)$ is $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, $sp([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, and one to one function. If K is $sp[\alpha,\alpha^*]$-compact set in X , then $f(K)$ is $sp[\gamma,\gamma^*]$-compact set in Y.

Proof: Let $\varphi = \{V_i: i \in I\}$ be $[\gamma,\gamma^*]$-open cover of $f(K)$, then $V_i = U_i \cap f(K)$where U_i is $[\gamma,\gamma^*]$-open set in Y. Since f is $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, then $f^{-1}(V_i) = f^{-1}(U_i) \cap K$, $f^{-1}(U_i)$ is$[\gamma,\gamma^*]$-

open set in X, it follows $\{f^{-1}(V_i): i \in I\}$is$[\alpha,\alpha^*]$-open cover of K. Since K is $sp[\alpha,\alpha^*]$-compact, then there is finite sub-collectionI_0 of I such that $K \subseteq \bigcup_{i\in I_0} sp[\alpha,\alpha^*] - cl\big(f^{-1}(V_i)\big)$ and so, $f(K) \subseteq f\big(\bigcup_{i\in I_0} sp[\alpha,\alpha^*] - cl\big(f^{-1}(V_i)\big)\big) = \bigcup_{i\in I_0} f\left(sp[\alpha,\alpha^*] - cl\big(f^{-1}(V_i)\big)\right) \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl\left(f\big(f^{-1}(V_i)\big)\right) \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(V_i)$. hence $f(K)$ is $sp[\gamma,\gamma^*]$-compact set.

Corollary 4.15 Let γ be semi- γ-regular and γ^* be pre-γ^*-regular operators defined on $SO(X)$ and $PO(X)$, and let $f:(X,\tau) \longrightarrow (Y,\psi)$ is $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, $sp([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, and one to one function. If A is $sp[\gamma,\gamma^*]$-compact and B is $[\gamma,\gamma^*]$-closed sets in topological space X, $f(A \cap B)$ is $sp[\gamma,\gamma^*]$-compact set in Y.

Proof: Follows from Proposition 4.11, and Proposition 4.14.

Definition 4.16 A function $f:(X,\tau) \longrightarrow (Y,\psi)$ is said to be $([\alpha,\alpha^*],[\gamma,\gamma^*])$-continuous, if the image of each$[\alpha,\alpha^*]$ - open set in X is $[\gamma,\gamma^*]$-open set in Y

Proposition 4.17 Let $f:(X,\tau) \longrightarrow (Y,\psi)$ be $([\alpha,\alpha^*],[\gamma,\gamma^*])$- continuous and bijective function, If K is $[\gamma,\gamma^*]$-compact set in Y, then $f^{-1}(K)$ is $sp[\alpha,\alpha^*]$-compact set in X.

Proof: Suppose that$\varphi = \{V_i: i \in I\}$ be $[\alpha,\alpha^*]$-open cover of $f^{-1}(K)$, then $\varphi^* = \{f(V_i): i \in I\}$ is $[\gamma,\gamma^*]$-open cover of K and since K is $[\gamma,\gamma^*]$-compact set, then there is a finite sub-collection I_0 of I such that $K \subseteq \bigcup_{i\in I_0} f(V_i)$ then $f^{-1}(K) \subseteq f^{-1}\big(\bigcup_{i\in I_0} f(V_i)\big) = \bigcup_{i\in I_0} f^{-1}\big(f(V_i)\big) \subseteq \bigcup_{i\in I_0} V_i \subseteq \bigcup_{i\in I_0} sp[\gamma,\gamma^*] - cl(V_i)$. Hence $f^{-1}(K)$ is $sp[\alpha,\alpha^*]$-compact set in X.

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