GENERALIZED JORDAN HOMOMORPHISMS AND JORDAN TRIPLE HOMOMORPHISMS ONTO PRIME RINGS

Abdul Rhman. H.Majeed , *Rajaa C.SHaheen
Department of Mathematics ,College of Science, University of Baghdad. Baghdad- Iraq
Department of Mathematics, College of Science, University of Al-Qadisiy. Al-Qadisiy- Iraq

Abstract
In this paper we initiate the study of generalized Jordan homomorphism and generalized Jordan triple homomorphism onto prime rings.

تشاكلات جوردان المعممه وتشاكلات جوردان الثلاثية المعممة على الحلقات الاولية

عبد الرحمن حميد مجيد، *رجاء شفات شاهين
قسم الرياضيات، كلية العلوم، جامعة بغداد. بغداد – العراق.
قسم الرياضيات، كلية التربية، جامعة القادسية. القادسية – العراق.

الخلاصة
في هذا البحث سنتطرق الى دراسة تشاكلات جوردان المعممة وتشاكلات جوردان الثلاثية المعممة على الحلقات الاولية.

1. Introduction

An additive mapping θ of a ring R into a ring Ŕ is called a Jordan homomorphism if $\theta(ab+ba)=\theta(a)\theta(b)+\theta(b)\ \theta(a)$ for all a ,b ε R. And an additive mapping θ of a ring R into a ring $\acute{R}$ which satisfies $\theta(aba)=\theta(a)\theta(b)\theta(a)$,for all a,b ε R ,will be called a Jordan triple homomorphism . Recall that R is prime ring if $aRb=0$ implies that either $a=0$ or $b=0$. I.N.Herstein in [1] had proved that every Jordan homomorphism onto prime ring of characteristic not 2 and 3 is either a homomorphism or an anti-homomorphism .While Smiley [2] had given a brief proof of this result ,and at the same time had removed the requirement that the characteristic of R be not 3.After this, I.N. Herstein in [3] had proved that every Jordan homomorphism from R onto a 2 - torsion free prime ring $\acute{R}$ is either a homomorphism or an anti- homomorphism.

In this paper we shall introduce new definitions which are the definition of generalized Jordan homomorphism and the definition of generalized Jordan triple homomorphism and study them onto prime rings as follows.

Definition 1.1.

Let $R, \acute{R}$ be rings and $\delta: R\rightarrow \acute{R}$ an additive map, if there is a homomorphism $\theta: R\rightarrow \acute{R}$ such that
$\delta(a\ b)= \delta(a)\ \theta(b)$, for all a ,b ε R .
Then δ is called a **generalized homomorphism** and θ is called the relating homomorphism.

Definition 1.2.

Let R , $\acute{R}$ be rings and $\delta : R\rightarrow \acute{R}$ an additive map , if there is an anti-homomorphism $\theta: R\rightarrow \acute{R}$ such that
$\delta(ab) = \delta(b)\theta(a)$, for all a ,b ε R.

Then δ is called a **generalized anti - homomorphism** and θ is called the relating anti-homomorphism.

Definition 1.3.

Let R, $\acute{R}$ be rings and $\delta: R \rightarrow \acute{R}$ an additive map if there is a Jordan homomorphism $\theta: R\rightarrow \acute{R}$ such that

$\delta(ab+ba) = \delta(a)\theta(b) + \delta(b)\theta(a)$, for all $a, b \in R$.

Then δ is called a **generalized Jordan homomorphism** and θ is called the relating Jordan homomorphism .

Remark that if Ŕ is a 2-torsion free ring, and $\delta:R\rightarrow \acute{R}$ is a generalized Jordan homomorphism it is also satisfy

$\delta(a^2)= \delta(a)\theta(a)$, for all $a \in R$.

Definition 1.4.

Let R ,$\acute{R}$ be rings and $\delta:R\rightarrow \acute{R}$ an additive map, if there is a Jordan triple homomorphism $\theta: R\rightarrow \acute{R}$ such that

$\delta(aba) =\delta(a)\theta(b)\theta(a)$, for all $a ,b \in R$.

Then δ is called a **generalized Jordan triple homomorphism** and θ is called the relating Jordan triple homomorphism.

In this present paper section two, we extend some results concerning Jordan homomorphism onto prime ring to a generalized Jordan homomorphism. And we shall prove that every generalized Jordan homomorphism onto 2 - torsion free prime ring is either a generalized homomorphism or a generalized anti-homomorphism.

An easy computation showes that every Jordan homomorphism is also a Jordan triple homomorphism. I.N.Herstien [4] proved that a Jordan triple homomorphism θ of a ring R onto a ring $\acute{R}$ of a characterstic different from 2 and 3 is of the form $\theta =\pm \Phi$ where Φ is a homomorphism or an anti- homomorphism *of* R onto $\acute{R}$. While M.Bresar [5,Theorem3.3] generalized the above result by removing the requirement that the characteristic be different from 3and proved that every Jordan triple homomorphism θ of a ring R onto a 2- torsion free prime ring $\acute{R}$ is of the form $\theta =\pm\Phi$,where Φ is a homomorphism or an anti-homomorphism of R onto $\acute{R}$.

In this present paper section three , we extend some result concerning Jordan triple homomorphism onto prime ring to generalized Jordan triple homomorphism. And we shall prove that every generalized Jordan triple homomorphism δ of a ring R onto a 2 - torsion free prime ring is of the form $\delta = \pm \Psi$, where Ψ is a generalized homomorphism or a generalized anti-homomorphism of R onto $\acute{R}$.

2. Generalized Jordan Homomorphism Onto Prime Ring

Throughout this section, R will be a ring and $\acute{R}$ will be a 2 – torsion free ring. Let $\delta: R\rightarrow\acute{R}$ be a generalized Jordan homomorphism and $\theta: R\rightarrow\acute{R}$ be the relating Jordan homomorphism.

Lemma 2.1.

For all $a, b, c \in R$ .The following statements hold:

1- $\delta(aba) = \delta(a)\theta(b)\theta(a)$

2- $\delta(abc+cba) = \delta(a)\theta(b)\theta(c) + \delta(c)\theta(b)\theta(a)$.

Proof (1).

Let $W = \delta(a(ab+ba) + (ab+ba)a)$.Then

$W= \delta(a)\theta(ab+ba) + \delta(ab+ba)\theta(a)$

$= \delta(a)\theta(a)\theta(b)+ \delta(a)\theta(b)\theta(a)+\delta(a)\theta(b)\theta(a)+\delta(b)\theta(a)\theta(a)= \delta(a)\theta(a)\theta(b) +2\delta(a)\theta(b)\theta(a)+ \delta(b)\theta(a)\theta(a)$.

On the other hand

$W= \delta(a^2b+ba^2+2aba)$

$= \delta(a^2b+ba^2) +2\delta(aba)$

$= \delta(a^2)\theta(b) +\delta(b)\theta(a^2)+2\delta(aba)$

$= \delta(a)\theta(a)\theta(b) +\delta(b)\theta(a)\theta(a)+ 2\delta(aba)$.

By comparing the two expression of W and since $\acute{R}$ is a 2-torsion free ring, we have

$\delta(aba) = \delta(a)\theta(b)\theta(a)$, for all $a, b \in R$.

Proof (2)

By linearizing (1) on a we obtain

$\delta(abc+cba) = \delta(a)\theta(b)\theta(c) + \delta(c)\theta(b)\theta(a)$ for all $a, b,c \in R$.

Remark 2.2 .

For the purpose of this section we shall write

1- $a^b = \delta(ab) -\delta(a)\theta(b)$

2- ${}^a b = \delta(ab) -\delta(b)\theta(a)$.

Remark that if $a^b=0$ then δ is a generalized homomorphism and if ${}^a b=0$ then δ is a generalized anti-homomorphism.

Remark 2.3.

For all $a, b \in R$

(1) $a^b = - b^a$

(2) ${}^a b = - b_a$.

Proof .
$a^b + b^a = \delta(ab) - \delta(a)\theta(b) + \delta(ba) - \delta(b)\theta(a)$
$= \delta(ab+ba) - (\delta(a)\theta(b) + \delta(b)\theta(a)) = 0.$
Then $a^b + b^a = 0$.
Therefore $a^b = -b^a$ for all $a, b \in R$.

Theorem 2.4.
If $\delta : R \to \acute{R}$ be a generalized Jordan homomorphism onto a 2- torsion free prime ring . Then δ is either a generalized homomorphism or a generalized anti-homomorphism.

Proof.
Let
$W = \delta(abxba + baxab)$,for all $a, b, x \in R$.
$= \delta(ab)\theta(x)\theta(ba) + \delta(ba)\theta(x)\theta(ab)$.
On the other hand
$W = \delta(a)\theta(bxb)\theta(a) + \delta(b)\theta(axa)\theta(b)$
$= \delta(a)\theta(b)\theta(x)\theta(b)\theta(a) + \delta(b)\theta(a)\theta(x)\theta(a)\theta(b)$.
Then by comparing these two expression of W. We get
$\delta(ab)\theta(x)\theta(ba) + \delta(ba)\theta(x)\theta(ab) - \delta(a)\theta(b)\theta(x)\theta(b)\theta(a) - \delta(b)\theta(a)\theta(x)\theta(a)\theta(b) = 0$…..........*
Since θ is Jordan homomorphism onto a 2 – torsion free prime ring then θ is either a homomorphism or an anti - homomorphism [3,Theorem 3.1]. Then we have two cases.

Case1.
If θ is a homomorphism then* becomes
$\delta(ab)\theta(x)\theta(ba) + \delta(ba)\theta(x)\theta(ab) - \delta(a)\theta(b)\theta(x)\theta(ba) - \delta(b)\theta(a)\theta(x)\theta(ab) = 0$. Then
$(\delta(ab) - \delta(a)\theta(b))\theta(x)\theta(ba) + (\delta(ba) - \delta(b)\theta(a))\theta(x)\theta(ab) = 0$.
Then $a^b\theta(x)\theta(ba) + b^a\theta(x)\theta(ab) = 0$
By Properties 2.3,we get
$a^b\theta(x)\theta(ba) - a^b\theta(x)\theta(ab) = 0$
$a^b\theta(x)\theta(ba-ab) = 0$,for all $a, b, x \in R$
By [5,Lemma1.2]
$a^b\acute{R}\theta(cd-dc) = 0$ for all $a, b \in R$.
$a^b\acute{R} = 0$ for all $a, b \in R$.
Then $a^b = 0$ for all $a, b \in R$. So by Remark 2.2
$\delta(ab) = \delta(a)\theta(b)$ for all $a, b \in R$
So δ is a generalized homomorphism.

Case 2.
If θ is an anti- homomorphism, then (*) becomes
$\delta(ab)\theta(x)\theta(ba) + \delta(ba)\theta(x)\theta(ab) - \delta(a)\theta(b)\theta(x)\theta(ab) - \delta(b)\theta(a)\theta(x)\theta(ba) = 0$
$(\delta(ab) - \delta(b)\theta(a))\theta(x)\theta(ba) + (\delta(ba) - \delta(a)\theta(b))\theta(x)\theta(ab) = 0$
${}^a b\theta(x)\theta(ba) - {}^a b\theta(x)\theta(ab) = 0$
$-{}^a b\theta(x)\theta(ab-ba) = 0$,for all $a, b, x \in R$
Then we obtain.
$a_b\acute{R}\theta(ab-ba) = 0$,for all $a, b \in R$.
By [5,Lemma1.2]
$a_b\acute{R}\theta(cd-dc) = 0$, for all $a, b \in R$.
$a_b\acute{R} = 0$ for all $a, b \in R$.
Then ${}^a b = 0$ for all $a, b \in R$, so by Remark2.2 ,we get
$\delta(ab) = \delta(b)\theta(a)$,for all $a, b \in R$.
Thus δ is a generalized anti- homomorphism .This completes the proof of the above theorem■

3.Generalized Jordan triple homomorphism onto prime ring

Throughout this section, R, $\acute{R}$ will be rings . Let $\delta: R \to \acute{R}$ be a generalized Jordan triple homomorphism and $\theta: R \to \acute{R}$ the relating Jordan triple homomorphism.

Lemma 3.1 .
If $\delta: R \to \acute{R}$ be a generalized Jordan triple homomorphism and $\theta: R \to \acute{R}$ the relating Jordan triple homomorphism .Then δ satisfy $\delta(abc+cba) = \delta(a)\theta(b)\theta(c) + \delta(c)\theta(b)\theta(a)$, for all $a, b, c \in R$.

Proof .
Since $\delta(aba) = \delta(a)\theta(b)\theta(a)$, By Definition 1.4.Replace a by $a+c$
$W = \delta((a+c)b(a+c)) = \delta(a+c)\theta(b)\theta(a+c) =$
$(\delta(a) + \delta(c))\theta(b)(\theta(a) + \theta(c)) = \delta(a)\theta(b)\theta(a) + \delta(a)\theta(b)\theta(c) + \delta(c)\theta(b)\theta(a) + \delta(c)\theta(b)\theta(c)$.
On the other hand
$W = \delta((a+c)b(a+c)) = \delta(aba + cbc + abc + cba) = \delta(aba) + \delta(cbc) + \delta(abc+cba)$.
By comparing the two expression of W, we get
$\delta(abc+cba) = \delta(a)\theta(b)\theta(c) + \delta(c)\theta(b)\theta(a)$ for all $a, b, c \in R$.

Remark 3.2 .
We shall write in this section
$S(a,b,c) = \theta(abc) - \theta(a)\theta(b)\theta(c)$
$T(a,b,c) = \theta(abc) - \theta(c)\theta(b)\theta(a)$
$A(a,b,c) = \delta(abc) - \delta(a)\theta(b)\theta(c)$
$B(a,b,c) = \delta(abc) - \delta(c)\theta(b)\theta(a)$

For the purpose of this section .We list a few elementary properties of A and B.
1- $S(a,b,c) + S(c,b,a) = 0$
2-$T(a,b,c) + T(c,b,a) = 0$
3- $A(a,b,c) + A(c,b,a) = 0$
4- $B(a,b,c) + B(c,b,a) = 0$.

Lemma 3.3.

If $\delta:R\rightarrow \acute{R}$ be a generalized Jordan triple homomorphism, then $A(a,b,c)\theta(x)T(a,b,c)+B(a,b,c)\theta(x)S(a,b,c)= 0$, for all $a,b,c,x \in R$.

Proof .

Let $W= \delta(abcxcba+cbaxabc)$ for all $a,b,c,x \in R$, then

$$W= \delta(abc)\theta(x)\theta(cba)+\delta(cba)\theta(x)\theta(abc) = \delta(abc)\theta(x)(\theta(a)\theta(b)\theta(c) + \theta(c)\theta(b)\theta(a) - \theta(abc))+\delta(cba)\theta(x)\theta(abc).$$

and so by Remark 3.2 we have

$$W = \delta(abc)\theta(x)\theta(a)\theta(b)\theta(c)+\delta(abc)\theta(x)\theta(c)\theta(b)\theta(a)-\delta(abc)\theta(x)\theta(abc))+\delta(cba)\theta(x)\theta(abc)$$
$$= \delta(abc)\theta(x)\theta(a)\theta(b)\theta(c)+\delta(abc)\theta(x)\theta(c)\theta(b)\theta(a)-\delta(abc)\theta(x)\theta(abc))+\delta(a)\theta(b)\theta(c) + \delta(c)\theta(b)\theta(a)-\delta(abc))\theta(x)\theta(abc)=\delta(abc)\theta(x)\theta(a)\theta(b)\theta(c)+\delta(abc)\theta(x)\theta(c)\theta(b)\theta(a) -\delta(abc)\theta(x)\theta(abc)+\delta(a)\theta(b)\theta(c)\theta(x)\theta(abc)+\delta(c)\theta(b)\theta(a)\theta(x)\theta(abc)-\delta(abc)\theta(x)\theta(abc).$$

On the other hand

$$W= \delta(abcxcba+cbaxabc) = \delta(a)\theta(bcxcb)\theta(a)+\delta(c)\theta(baxab)\theta(c) = \delta(a)\theta(b)\theta(c)\theta(x)\theta(c)\theta(b)\theta(a)+\delta(c)\theta(b)\theta(a)\theta(x)\theta(a)\theta(b)\theta(c).$$

By comparing these two expressions of W, we get

$$(\delta(abc)-\delta(c)\theta(b)\theta(a))\theta(x)\theta(a)\theta(b)\theta(c)+(\delta(abc)-\delta(a)\theta(b)\theta(c))\theta(x)\theta(c)\theta(b)\theta(a)-(\delta(abc)-\delta(a)\theta(b)\theta(c))\theta(x)\theta(abc)-(\delta(abc)-\delta(c)\theta(b)\theta(a))\theta(x)\theta(abc)=0$$

Then

$$B(a,b,c)\theta(x)\theta(a)\theta(b)\theta(c)+A(a,b,c)\theta(x)\theta(c)\theta(b)\theta(a)-A(a,b,c)\theta(x)\theta(abc)-B(a,b,c)\theta(x)\theta(abc) = 0.$$

Then

$$-B(a,b,c)\theta(x)(\theta(abc)-\theta(a)\theta(b)\theta(c))-A(a,b,c)\theta(x)(\theta(abc)-\theta(c)\theta(b)\theta(a))=0.$$

Then

$B(a,b,c)\theta(x)S(a,b,c)+A(a,b,c)\theta(x)T(a,b,c)=0$ for all $a,b,c,x \in R$.

Theorem 3.4.

If $\delta: R\rightarrow \acute{R}$ is a generalized Jordan triple homomorphism onto a 2- torsion free prime ring then $\delta=\pm\Psi$, where Ψ is either a generalized homomorphism or ageneralized anti-homomorphism.

Proof.

Since $\delta: R\rightarrow \acute{R}$ is a generalized Jordan triple homomorphismthen there exist $\theta : R\rightarrow \acute{R}$ is Jordan triple homomorphism by Definition 1.4 . Then by [1Theorem3.3] . We have $\theta = \pm \Phi$, where Φ either a homomorphism or an anti-homomorphism .Now we have four cases:-

Case 1:-If $\theta= +\Phi$, where Φ is homomorphism.Then for all $a,b,c \in R$.

$$\theta(abc) = \theta((ab)c) = \theta(ab)\theta(c) = \theta(a)\theta(b)\theta(c)$$

since $S(a,b,c)= \theta(abc)- \theta(a)\theta(b)\theta(c)$, for all $a, b ,c \in R$.

Then $S(a, b, c) = 0$,for all $a, b, c \in R$. And so by Lemma 3.3,we get

$A(a, b, c)\theta(x)T(a, b, c) =0$,for all $a, b, c, x \in R$. Since $\acute{R}$ is prime ring, then either

$T(a, b, c) =0$ or $A(a, b, c) =0$.

If $T(a, b, c) =0$, then

$\theta(abc) = \theta(c)\theta(b)\theta(a)$

Which implies that $\theta=\pm \Phi$ is anti-homomorphism which is contradicts the assumption of θ as homomorphism .

Then $A(a,b,c) = 0$,for all $a,b,c \in R$

i.e. $\delta(abc)= \delta(a)\theta(b)\theta(c)$. for all $a,b,c \in R$ And therefore

$$W= \delta(abxab) = \delta(ab)\theta(x)\theta(ab) = \delta(ab)\theta(x)\theta(a)\theta(b).$$

On the other hand

$$W= \delta(abxab)= \delta(a)\theta(bxa)\theta(b) = \delta(a)\theta(b)\theta(x)\theta(a)\theta(b).$$

By comparing the two expression of W, we have

$(\delta(ab)- \delta(a)\theta(b))\theta(x)\theta(a)\theta(b) =0$

.for all $a, b, x \in R$

$(\delta(ab)- \delta(a)\theta(b))\acute{R} =0$ .Thus

$\delta(ab)- \delta(a)\theta(b) =0$ .Therefore

$\delta(ab) =\delta(a)\theta(b)$,for all $a, b \in R$.

Therefore $\delta= + \Psi$, where Ψ is a generalized homomorphism.

Case 2

If $\theta = -\Phi$, where Φ is a homomorphism

$\theta(abc) =- \theta(ab)\theta(c)= \theta(a)\theta(b)\theta(c)$

for all $a ,b, c \in R$.

Then $S(a, b, c)= \theta(a b c) – \theta(a)\theta(b)\theta(c)=0$

And so byLemma3.3, we get

$A(a, b, c)\theta(x)T(a, b, c) = 0$
for all $a, b, c, x \in R$.
Since $\acute{R}$ is prime ring ,then either
$T(a, b, c) = 0$ or $A(a, b, c) = 0$
If $T(a, b, c) = 0$ then $\theta = \pm \Phi$ is anti-homomorphism which is contradicts the assumption of $\theta = -\Phi$, where Φ is a homomorphism.
Then $A(a,b,c) = 0$, for all $a, b, c \in R$.
Thus $\delta(abc) = \delta(a)\theta(b)\theta(c)$
for all $a, b, c \in R$.
Let $W = \delta(abxab)$
$= \delta(ab)\theta(x)\theta(a b)$
$= -\delta(ab)\theta(x)\theta(a)\theta(b)$.
On the other hand
$W = \delta(abxab)$
$= \delta(a)\theta(bxa)\theta(b)$
$= \delta(a)\theta(b)\theta(x)\theta(a)\theta(b)$.
By comparing the two expression of W, we have
$(\delta(ab)+\delta(a)\theta(b))\theta(x)\theta(a)\theta(b) = 0$ -
for all $a, b, x \in R$
$\delta(ab) = -\delta(a)\theta(b)$, for all $a, b \in R$.
Therefore $\delta = -\Psi$, where Ψ is a generalized homomorphism.

Case 3

If $\theta = +\Phi$, where Φ is anti- homomorphism.
$\theta(abc) = \theta(c)\theta(ab) = \theta(c)\theta(b)\theta(a)$,
for all $a,b, c \in R$.
Then,
$T(a,b,c) = \theta(abc) - \theta(c)\theta(b)\theta(a) = 0$.
And so by Lemma 3.3
$B(a, b, c)\theta(x)S(a, b, c) = 0$
Since $\acute{R}$ is prime ring , then either
$S(a, b, c) = 0$ or $B(a,b,c) = 0$.
If $S(a, b, c) = 0$ then
$\theta(abc) = \theta(a)\theta(b)\theta(c)$, for all $a,b,c \in R$
Which implies that $\theta = \pm \Phi$ is homomorphism which is contradicts the assumption of $\theta = +\Phi$, where Φ is anti- homomorphism.
Therefore $B(a,b,c) = 0$ and so
$\delta(abc) = \delta(c)\theta(b)\theta(a)$, for all $a, b, c \in R$.
Let $W = \delta(abxab)$
$= \delta(ab)\theta(x)\theta(ab)$
$= \delta(ab)\theta(x)\theta(b)\theta(a)$.
On the other hand
$W = \delta(abxab) = \delta(b)\theta(bxa)\theta(a)$
$= \delta(b)\theta(a)\theta(x)\theta(b)\theta(a)$.
By comparing the two expression of W ,we have
$(\delta(ab) - \delta(b)\theta(a))\theta(x)\theta(b)\theta(a) = 0$
for all $a, b, x \in R$
$(\delta(ab) - \delta(b)\theta(a))\acute{R} = 0$, for all $a, b \in R$
Then $\delta(ab) = \delta(b)\theta(a)$,for all $a, b \in R$.
Therefore $\delta = +\Psi$, where Ψ is a generalized anti- homomorphism.
Case 4:- If $\theta = -\Phi$, where Φ is anti- homomorphism ,then
$\theta(abc) = -\theta(c)\theta(ab) = \theta(c)\theta(b)\theta(a)$,for all a,b,c
Then $T(a, b, c) = \theta(abc) - \theta(c)\theta(b)\theta(a) = 0$, and so
by Lemma 3.3,$B(a, b, c)\theta(x)S(a,b,c) = 0$
since $\acute{R}$ is prime ring ,then either
$S(a,b,c) = 0$ or $B(a,b,c) = 0$
If $S(a,b,c) = 0$, then we have
$\theta(abc) = \theta(a)\theta(b)\theta(c)$, for all $a,b,c \in R$.
Which implies that $\theta = \pm \Phi$ is homomorphism which is contradicts the assumption $\theta = -\Phi$, where Φ is anti- homomorphism.
Therefore , $B(a,b,c) = 0$, and so
$\delta(abc) = \delta(c)\theta(b)\theta(a)$, for all $a,b,c \in R$.
Now we suppose that
$W = \delta(abxab)$
$= \delta(ab)\theta(x)\theta(ab)$
$= -\delta(ab)\theta(x)\theta(b)\theta(a)$.
On the other hand
$W = \delta(abxab)$
$= \delta(b)\theta(bxa)\theta(a)$
$= \delta(b)\theta(a)\theta(x)\theta(b)\theta(a)$.
By comparing the two expression of W,
we have
$-(\delta(a b) + \delta(b)\theta(a))\theta(x)\theta(b)\theta(a) = 0$, for all $a ,b,x \in R$
$(\delta(a b)+\delta(b)\theta(a))\acute{R} = 0$
$\delta(ab) = -\delta(b)\theta(a)$, for all $a ,b \in R$.
Then $\delta = -\Psi$, where Ψ is a generalized anti-homomorphism.
Therefore $\delta = \pm \Psi$, where Ψ is a generalized homomorphism or a generalized anti – homomorphism .

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