DIGIT RECOGNITION USING FRACTAL AND MOMENT INVARIANTS

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Abstract

This paper presents quantitative and qualitative methods based on fractal geometry and invariants moment to recognize the printed or handwritten digits. The fractal features are computed for the area of image segment within the frame. The analysis of the results showed that the fractal dimension can be recognized different digits printed at same font size, but it can not distinguish the printed digits or handwritten at different font sizes. The lacunarity appears high ability in recognizing printed or handwritten digits at different font sizes. The investigation of fractal description efficiency necessitates comparing the fractal performance with a common moment descriptor used in this field. The comparison proved that the fractal geometry possess high digit recognition capabilities and it gave 93% score of recognizing the printed digits during 6.6 s and 71% to recognize handwritten digit during 6.8 s, which is greater than the corresponding values 81% and 64% scores during 1.5s and 1.7 s, respectively given by the moments.

تمييز الارقام باستخدام الكسوريات والعزوم الثابتة

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الخلاصة

في هذا البحث تمت دراسة الوصف الكمي والنوعي لهندسة الكسوريات والعزوم الثابتة لتمييز ارقام مطبوعة و المكتوبة باليد . وقد تم حساب الخواص الكسورية لمساحة الصورة المجزاة داخل الاطار. اظهرت نتائج التحليل ان البعد الكسوري يمكن ان يميز ارقام مطبوعة مختلفة في نفس حجم الخط لكن لايميز الارقام المطبوعة او المكتوبة باليد في حجوم خط مختلفة.لقد اظهرت استخدام الفجوة قابلية عالية في تمييز الارقام في حجوم خط مختلفة لبحث كفاءة الوصف الكسوري يستلزم مقارنة اداء الوصف الكسوري مع وصف العزم المستخدم في هذا المجال. اضهرت نتائج المقارنة ان الهندسة الكسورية تمتلك قابلية غير عالية وقد اعطت 93% لتمييز الارقام المطبوعة خلال 6.6ثانية و71% خلال 6.8ثانية وهي اكثر من 81% و64% خلال 1.5 و1.7 ثانية على التولي المعطاة باستخدام العزوم.

Introduction

In purpose of digit recognition, many approaches have been improved including decision theory; feature selection, optimization, learning, and so on [1]. These approaches include template matching, image signature, image geometric features, or shape-based image invariant [2]. Among them, the shape-based image invariants are of particular interests as they have the invariance property which can improve the recognition performance even the image have undergone various image transformations such as translation, scaling, and rotation. There are two types of shape-image invariant; boundary-based and region-based. The most common boundary-based image invariant includes Fourier descriptors or chain codes. The region-based image invariants take the whole image area as the research object. Region based-image invariant includes various moment based invariant such as Hu's seven moments or fractal descriptor invariants. Such technique computes the orientation, size and position, features vector with help of nonlinear invariant functions [3]. Several of studies have been reported in the literature of digits recognition, Jain A. K. and Douglas Z. [4] investigated the application of deformable templates to boundary-based recognizing hand-printed digits. Two digits are matched by deforming the contour of one to fit the edge strengths of the other**.** Reena B. et al. [5] provided an efficient and reliable technique for region-based recognition of handwritten numerals. Three different types of features have been used for classification of numerals. Asymmetrical segmentation pattern was proposed to obtain the feature vector [6]. Since many algorithms depend on digit identity, a Neural Network is used to prevail over this dependence. Inputs of this Network are the moment of inertia and the center of gravity which do not depend on digit identity. Automatic system was presented for word recognition using real Turkish word signals [7]. It deals with combination of the feature extraction and classification from real Turkish word signals. Ahmad A.and Hammad M. [8] proposed an efficient structural approach for recognizing on-line handwritten digits**.** The proposed method is tested on a sample of 3000 digits written by 100 different persons; each person wrote the 10 digits three times each. In this work, we study the performance of fractal and moment invariant techniques to recognize the printed and handwritten digits. Also a comparison between both techniques was introduced.

Invariant Recognition Features

Digit recognition depends on the variability in the feature values for digits in the same category, relative to the difference between feature values for digits in different categories [9]. Features should be distinct for different digit images, so that the computer can extract the correct model from the library without confusion. Meanwhile, also it is wanted the features to be robust enough. This found when they not be affected by any distortion source [10].In the present work, the proposed features fractal and moments are used in digit recognition system due to they are remains invariant whenever the recognizable digits are resizable drawn or orientation variant through sequence of images. In the following section a detailed explanation about both adopted invariant features is given.

1. Fractal-based invariant

In fact, fractal geometry provides a suitable mathematical framework to study the nature irregularity shapes since it allows to easily describing such complex patterns. In particular, it has been shown that most natural surfaces are fractals and that intensity images of these surfaces are also fractal. The main characteristics of fractal images are that they show a fine detailed at any arbitrarily small scale. A fundamental concept of fractal geometry is the fractal dimension, one can use the fractal dimension as a recognizable fractal feature because it is measure of complexity in fractal patterns, the higher fractal dimension the more complex the fractal pattern. Fractal dimension originated as a scale invariant metric to distinguish between different patterns, it can be explained by the following simple way [11].

If we take a set of dimension *D*, one would expect to say that it consists of *N* parts, each scaled by a factor of *r*. one have;

$$N\,r^D = C \qquad (1)$$

Where, *C* is constant, solving one get;

$$D = \frac{\log(N)}{\log(1/r)} \qquad (2)$$

where, *D* is the fractal dimension. Because *D* can takes non-integer (i.e., $2<D<3$) number, it is difficult to recognize plenty of classes well. Mandelbrot suggests new fractal measure "Lacunarity" to handle the case of sharing two different classes with same fractal dimension [12]. Lacunarity is an accurate similarity measure. It is a set of points (curve) computed by the same method of computing the fractal dimension "Box counting method". Box counting method is an accurate one for computing the fractal dimension and the lacunarity, but it has some of complexity. Box counting method based on defining the box dimension D of a set S to be contained in R^n as follows; for any L>0, let $N(S)$ be the minimum number of η-dimensional cubes of side length L needed to cover S. If there is a number D so that;

$N(S) \approx 1/L^D$ as $L \to 0$ (3)
Where, D is the box dimension of S.

The box dimension is D if there is some positive constant K, so that,

$$\lim_{L\to 0} \frac{N_L(S)}{1/S^D} = K \qquad (4)$$

Since both sides of the equation (4) are positive, it will still hold if one takes the logarithm of both sides to obtain;

$$\lim_{L\to 0}(\ln(N_L(S) + D \times \ln(S))) = \ln(K) \ (5)$$

Solving for D gives

$$D = \lim_{L\to 0} \frac{\ln(K) - \ln(N_L(S))}{\ln(S)} = -\lim_{L\to 0} \frac{\ln(N_L(S))}{\ln(S)}$$
(6)

From equation (6) it clear that $ln(k)$ drops out, because it is constant while the denominator becomes infinite as $L \to 0$. Also, since 0<L<1, $ln(L)$ is negative, so D is positive as one would expect [11].

2. Moment-based invariants

Moment invariants are important shape descriptors in computer vision. Hu[13] dedicated the moment invariants regarding the algebraic images theory, evolved the algebraic values of two-dimensional images obtained by non-liner combination equation to describe the geometric characteristics[14]. To translate, scale and rotate images, the specified values can evolve invariant to describe the geographic characterization. Therefore, it broadly has been made use to identify aircraft fighters, word fonts and other related images these years, while the computer technology commenced to bloom. The basic definition for moment invariants is as follows [15]:-
Two-dimensional moments of a digitally sampled $M \times M$ image that has gray function

$f(x,y),(x,y = 0,............M-1)$ is given as,

$$m_{pg} = \sum_{x=0}^{M-1}\sum_{y=0}^{M-1} x^p y^q f(x,y) \quad \text{p, q} = 0,1,2,3.....$$
.

The moments $f(x, y)$ translated by an amount (a, b), are defined as,

$$\mu_{pq} = \sum_x\sum_y (x+a)^p.(y+b)^q f(x,y) \qquad (7)$$

Thus the central moments m'_{pq} or μ'_{pq} can be computed from (7) on substituting $a = -\bar{x}$ and $b = -\bar{y}$ as

$$\bar{x} = \frac{m_{10}}{m_{00}} \text{ and } \bar{y} = \frac{m_{01}}{m_{00}},$$

$$\mu_{pq} = \sum_x\sum_y (x-\bar{x})^p.(y-\bar{y})^q f(x,y) \qquad (8)$$

When a scaling normalization is applied the central moments change as,

$$\eta_{pq} = \mu_{pq} / \mu^{\gamma}{}_{00} \qquad \gamma = [(p+q)/2]+1 \qquad (9)$$

In particular, Hu[13],defines seven values, computed by normalizing central moments through order three, that are invariant to object scale, position, and orientation. In terms of the central moments, the seven moments are given as:

$$\begin{aligned}
M_1 &= (\eta_{20} + \eta_{02}), \\
M_2 &= (\eta_{20} - \eta_{02})^2, +4\eta_{11}^2, \\
M_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2, \\
M_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2, \\
M_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 \\
&\quad -3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} _ \eta_{03})(\eta_{21} + \eta_{03}) \\
&\quad [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2], \qquad (10) \\
M_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
&\quad + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}), \\
M_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 \\
&\quad -3(\eta_{21} + \eta_{03})^2] - (\eta_{30} + 3\eta_{12})(\eta_{21} + \eta_{03}) \\
&\quad [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2],
\end{aligned}$$

The Proposed System

The proposed system necessitate using images contain printed or handwritten black digits on white background. The digits in test images may be seen disarranged, rotated, or in different sizes. It is wanted to suggest a recognition scheme utilized at classifying any digit in the acquired image. This carried out through sequential processes shown in Figure. 1. The work branched into two streams; training and recognition. The training stream goes to enrol the database of the considered ten digits in different situations of size, pose, and orientation. The parameter that determines the direction of work is considered to be Boolean "phase". If there is enough training data collected in the system, the phase is true and the implementation direct to the recognition stream. The recognition streams concerned with recognize the digit essence by finding the best match between the digit in the acquired image and the database. A detailed explanation about the proposed technique is presented in the following sections:

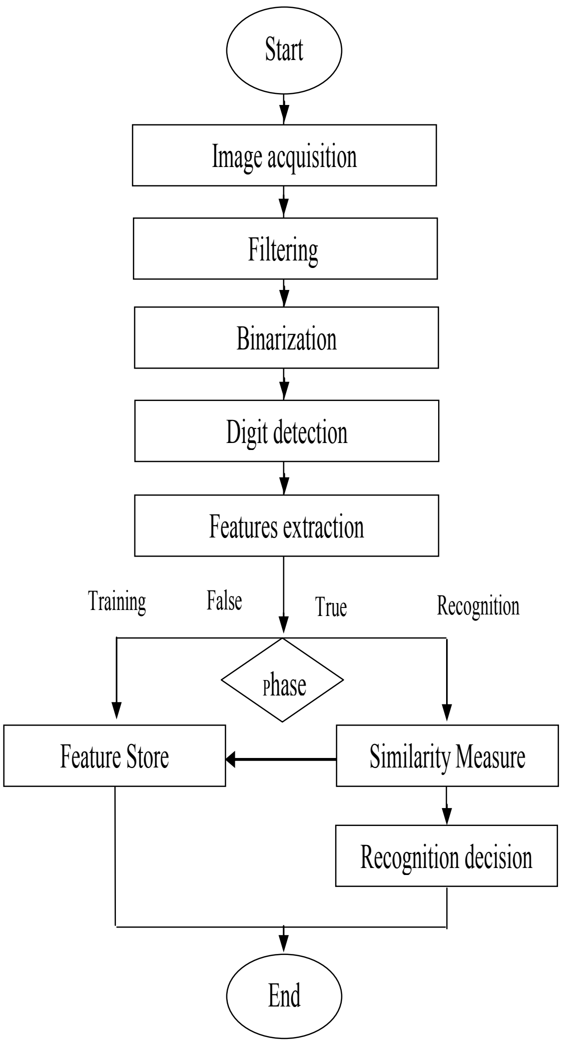


Figure 1: The proposed digit recognition scheme.

A-Image Acquisition

Image that contains digits to be recognized is acquired using scanner and a bitmap image with gray levels will be saved for further processing. The image is made to support the training and recognition processes of the system, in handwritten digits the training image contain all 10 digits (0, 1, 2,…, 9) on a white paper. Images are scanned with 150, 200, and 300 dpi resolution with different orientation and saved as 8 bit per pixel bitmap.

B-Filtering

The noise exists in the image is almost come from the digitization process in the scanner; such noise can be skewed digit recognition accuracy potentially. Because of this noise is distributed naturally (white noise), the Gaussian filter is proposed to discard the noise in the image using the following relationship:

$$if\ \ f(x,y) \geq (\bar{f} + \sigma)\ then\ \ f(x,y) = \bar{f} + \sigma$$

$$elseif\ \ f(x,y) < (\bar{f} - \sigma)\ then\ \ f(x,y) = \bar{f} - \sigma$$

Where f(x,y) is gray level value of the pixel in x, and y position, $\bar{f}$ is the mean of all pixels in the image, and σ is the standard deviation of the image pixels.

C-Binarization

Binarization is the process of converting the image into binary mode. The goal of the binarization is to separate the digit from the background in the gray image, also to reduce the image levels into black and white. A gray level image may be converted to a binary mage by threshold process. If a pixel's value is higher than the global threshold, it is assigned to 255 otherwise it is assigned to 0 value.

D-Digits detection

The improved idea of the present work upon previous studies is that the acquired image may include several digits within. Thus, it should be first determine the area of each digit in the image. The raster scan search method [16] can be utilized to detect each digit from others in same image, and then determine the positional coordinates of the frame that enclose to the digit. This process gives the chance for the classifier to use just the image segment within the frame in the next process.

E-Features extraction

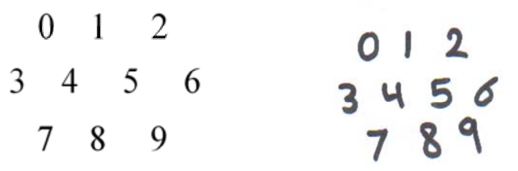
This process transforms the image data into a reduce representation set of features. It concerns with selecting the characteristic features of each digit. In the present work, the employed features are fractal dimension, lacunarity, and Hu's moments. Both fractal and moment techniques have the theoretical basis of some properties as invariance by rotation, scale, and translation of the digit. It is expected that the reduced representation of the fractal or moment set will describe the relevant information from the input data in order to perform the desired recognition task instead of the full size input.

F-Training and Recognition

In order to recognize any digit in the test image, we need to make a training test. In the training test, the fractal features and Hu's moments are computed separately. The average of each of them is stored in feature store for same digit in different occurrence. There are a pointer corresponding to each digit saved in the feature store, it refers to the target digit in the recognition process. The recognition concerns with the decision criteria for the digit recognition among one of its known digits in the knowledge database using the previous features. This is accomplished by searching a match between the features extracted from the given digit's image and the database. The matching carried out by computing the normalized MSE, which give a percentage ratio describes the amount of similarity between the two sides. According to this ratio, the recognition decision issued to classify the digit under consideration to be identified one of the ten known digits in the database.

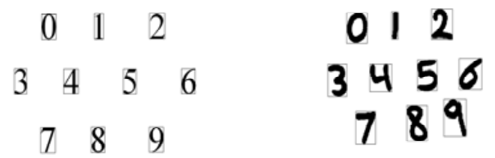
Results and Evaluation

In this work, there are several images were trained, each of which contains ten digits (0-9) in different sizes. Figure. 2 shows one sample of acquired images. Applying digit detection process was succeeded, and each detected digit appears surrounded by a frame contains just the digit as shown in Figure. 3. Image segment within frame was used lonely to enroll the database features of each digit. The database was established to contain 10 training models belong to the ten digits



(a) Printed digits. (b) Handwritten digits.

Figure 2: Sample of an acquired image.



(a) Printed digits. (b) Handwritten digits.

Figure 3: Digital detection process.

In the recognition; another images of printed or handwritten digits were acquired. The recognizer detects the digits and then computes the fractal dimension for each digit using box counting method. The results show the fractal dimension has relatively invariant description to inform the class of the digit. This description was found invariant only when all test digits are drawn in same font size. Whereas the fractal dimension was slightly differs with increasing the font size and it appears invariant behavior with varying the position or orientation of the target digit. The results analysis show that the increasing in the font size does not affect the shape of distributing the fractal dimension values belongs to certain digit, but it shift its distribution by an amount proportional to the increase in the font size as shown in figure. 4. Therefore, the use of several font sizes for same digit make the overall distribution of fractal dimension of certain digit (class) takes same shape with greater width of separation. This almost leads to little interference between fractal distributions of adjacent digit classes. Further font sizes leads to make a significant confusion in the recognition, since greater associated interference lead to appear greater error percentage in the recognition decision. Figure. 5

shows the behavior of overall fractal dimension distributions of same class for different sets of font sizes. The first set uses (12 and 14) font sizes only, while the second set uses (12, 14, 16, 18, 20, and 22) font sizes.
Figure. 6 shows the behaviour of the fractal dimension as function of individual font size for the digits of interest. It is clear that the fractal dimension values increases relatively by increasing the font size until reaching a region of less increment at higher font sizes. In addition, the behaviour of the fractal dimension belongs to certain class close to other behaviours of different classes at region of higher font sizes; this is because of expanding the separation width of each class. Therefore, the recognition percentage degraded by increasing the number of contributed fonts. This result proved that the fractal dimension can not be given invariant situation against scale since the confusion became greater at using further font sizes.

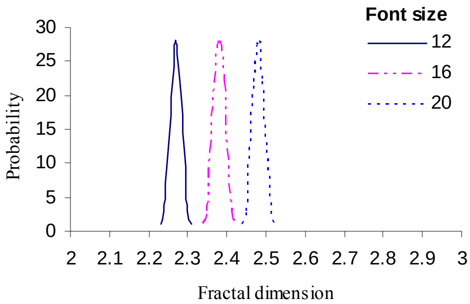


Figure 4: Fractal dimension distribution of the same digit for three font sizes.

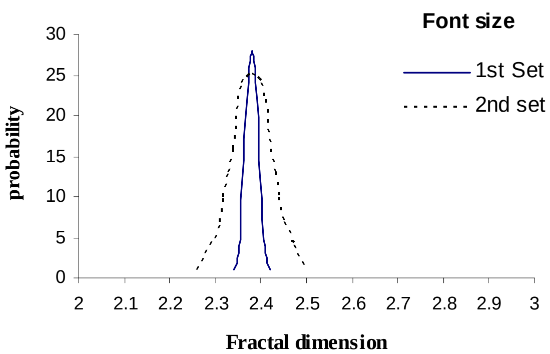


Figure 5: Fractal dimension distribution of the same digit for two sets of font sizes.

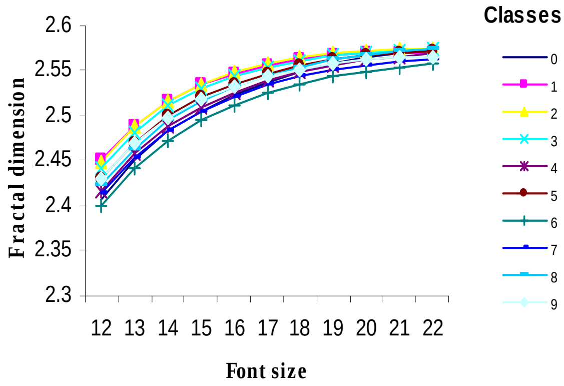


Figure 6: Fractal dimension behaviours versus individual font size.

It can be concluded that the results achieved by fractal dimension are inefficient because of the fractal dimension is merely fractional number utilized to describe huge different shapes by just fraction differences. Therefore, it was better to leave the fractal dimension and direct to its twin parameter "lacunarity". Practically, the lacunarity gave powerful recognition results. Each digit in the image had a relatively different lacunarity shape from others. Figure. 7 presents the training results of lacunarity curves for each class of digits computed by box counting method. It is clear that the value of Lacunarity is changing by changing the box size therefore; it gives different behavior by variation of classes.

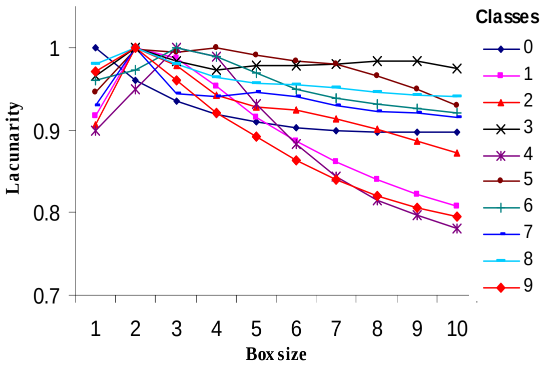


Figure 7: Lacunarity curves of the ten digits used in the lookup table.

Figure.8 presents a sample of resulted lacunarity curves belong to same class in the recognition process. It is shown that the variation of the font size show insignificant effect on the shape of lacunarities. The increasing in the font size lead to shift the lacunarity curve upward, the amount of the shift is proportional to the increase in the font size. Thus, it was necessity to normalize the resulted lacunarities. One can notes that the resulted lacunarity shapes of same digit at different font size does not exactly identifying lacunarities of the lookup table, but there are small differences in between. This may give an impress about the stability of lacunarity shape for each digit.

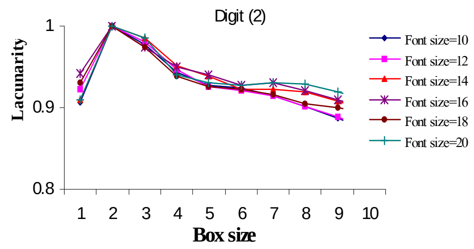


Figure 8: Lacunarity shapes of the same digit at different font size.

The identification between the lacunarity of the test digit with the set of lacunarities stored in lookup table is carried out by utilizing the following formula:

$$S_k = \left(1 - \left|\sum_{k=1}^{N} (Lac - LacL_k)\right|\right) \times 100\%$$

where, N=10, which is the number of digits of interest, k is pointer refers to the current class, Lac is the lacunarity of current digit, and LacLk is the lacunarity of the kth class in the lookup table. Then the test digit can be classified according to its similarity measure. The least similarity measure is the most similar class belongs to the test digit. It is noticeable that the normalized lacunarity curves lead to make that the resulted similarity measure is normalized too. So, the similarity measure takes a normalized value in between 0 and 1. Where 1 refers to the two compared curves are absolutely similar to each other, while the 0 means that they are absolutely different and the fraction between them intend the similarity between them. Practically the use of the lacunarity “only” in the recognition make the recognition score reaches high levels; about 93% for printed digit and about 71% for handwritten digits. It is found that the similarity measure that less than 63% leads almost to different classes, otherwise leads to similar classes. Tables 1 and 2 present the results of the printed and handwritten digit recognition by fractal lacunarity, respectively. These tables reflect the behavior of the recognition process. The high score of the recognition results may tell the most about the efficiency of fractal geometry to describe shapes of digits.

Table 1: The printed digit recognition by fractal lacunarity

Estimated class → / True class ↓	0	1	2	3	4	5	6	7	8	9
0	92%	0%	0%	0%	0%	0%	1%	0%	1%	1%
1	0%	96%	0%	0%	0%	0%	0%	2%	0%	0%
2	0%	1%	95%	1%	1%	3%	0%	1%	0%	1%
3	1%	0%	0%	92%	1%	1%	1%	0%	2%	1%
4	0%	1%	0%	0%	95%	0%	0%	2%	0%	0%
5	0%	0%	1%	2%	1%	93%	2%	0%	1%	1%
6	2%	0%	1%	1%	0%	1%	90%	0%	2%	4%
7	0%	2%	2%	0%	2%	0%	0%	95%	0%	0%
8	3%	0%	0%	3%	0%	1%	1%	0%	93%	1%
9	2%	0%	1%	1%	0%	1%	5%	0%	1%	91%

Table 2: The handwritten digit recognition by fractal lacunarity.

Estimated class → True class ↓	0	1	2	3	4	5	6	7	8	9
0	72%	0%	2%	1%	1%	2%	1%	0%	9%	2%
1	0%	75%	0%	2%	6%	1%	0%	7%	1%	0%
2	2%	3%	69%	5%	2%	3%	4%	3%	2%	1%
3	3%	2%	4%	73%	1%	3%	2%	2%	4%	2%
4	1%	5%	3%	2%	72%	2%	1%	5%	1%	4%
5	2%	1%	6%	4%	3%	71%	5%	2%	3%	6%
6	3%	2%	3%	2%	4%	8%	68%	3%	4%	12%
7	2%	8%	7%	4%	7%	2%	1%	73%	2%	1%
8	11%	1%	3%	5%	1%	3%	3%	1%	71%	2%
9	4%	3%	3%	2%	3%	5%	15%	4%	3%	70%

For purpose of further investigation, the central moments are computed using the centroid of the image, which is equivalent to the regular moments of an image whose centre has been shifted to coincide with its centroid. The moment method showed efficiency in digits recognition, this give the chance to evaluate the capabilities of the moment in comparison with fractal method. The test results of the moments in both training and recognition streams show invariant description between classes. The difference between classes can be utilized to distinguish between digits. Figure 9 presents the behaviour of Hu's moments for each digit. Multiple tests show the moments of high order gave higher recognition ability than that of lower order. The use of just M5, M6, and M7 in the recognition gave a score of 81% to recognize printed digits and 64% to recognize handwritten digits. Tables 3 and 4 present the results of the printed and handwritten digit, respectively using only high order Hu's moments

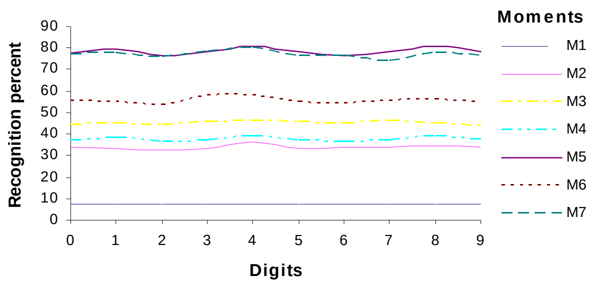


Figure 9: Behaviours of Hu's seven moments for one class of digits

Table 3: The printed digit recognition by Hu's moments.

Estimated class → / True class ↓	0	1	2	3	4	5	6	7	8	9
0	82%	0%	0%	0%	0%	1%	1%	0%	8%	1%
1	0%	83%	2%	1%	3%	0%	0%	8%	0%	0%
2	1%	2%	80%	6%	2%	2%	1%	1%	1%	2%
3	1%	1%	5%	81%	1%	1%	0%	5%	7%	1%
4	2%	6%	0%	0%	82%	3%	1%	2%	0%	2%
5	3%	1%	10%	1%	4%	79%	1%	1%	2%	1%
6	1%	1%	0%	1%	2%	5%	82%	0%	1%	13%
7	0%	5%	3%	2%	1%	1%	0%	83%	0%	0%
8	9%	0%	0%	7%	0%	2%	3%	0%	81%	1%
9	1%	1%	0%	1%	5%	6%	11%	0%	2%	79%

Table 4: The handwritten digit recognition by Hu's moments.

Estimated class → / True class ↓	0	1	2	3	4	5	6	7	8	9
0	62%	1%	1%	2%	1%	5%	6%	2%	12%	9%
1	0%	68%	2%	0%	9%	2%	2%	11%	7%	2%
2	1%	2%	62%	8%	4%	11%	4%	3%	1%	5%
3	3%	1%	7%	65%	2%	2%	4%	1%	8%	3%
4	2%	12%	2%	2%	66%	3%	3%	11%	1%	1%
5	4%	1%	13%	4%	1%	62%	3%	1%	2%	4%
6	7%	2%	5%	3%	2%	6%	69%	1%	3%	11%
7	1%	9%	2%	2%	11%	3%	2%	68%	1%	2%
8	12%	1%	1%	9%	1%	1%	4%	1%	62%	7%
9	8%	2%	5%	4%	3%	5%	17%	1%	3%	59%

The influence of total recognition score and spent time for both techniques is shown in Figure 10. It is observed that the printed digits collect higher recognition scores than that of handwritten. Also, the fluctuations appear in the behaviours of printed digits recognition was less than that appear in the behaviours belong to handwritten digit recognition. This indicates that the more stable decision occur in recognizing the printed digit than handwritten digits

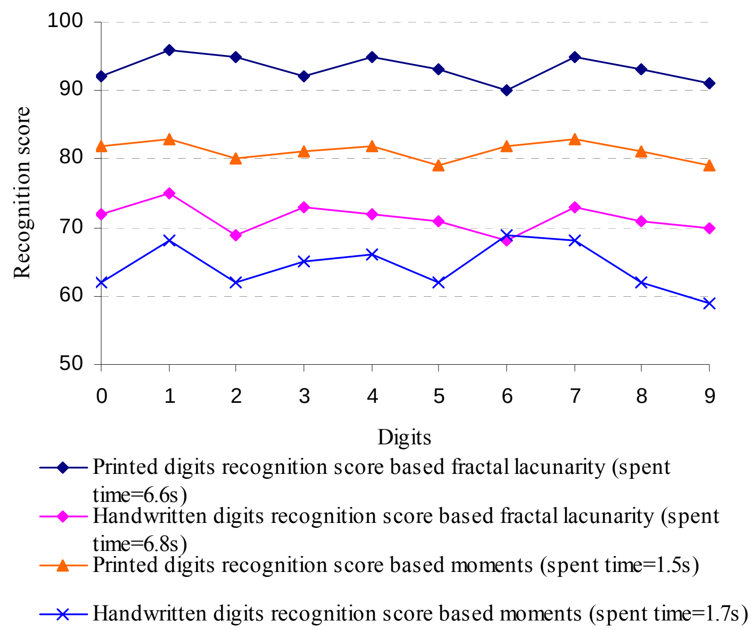


Figure 10: The Recognition scores versus digits

Conclusions

The performance of fractal and moment invariant techniques to recognize the printed and the handwritten digits was evaluated .The moment technique gave lower and faster recognition in comparison with the fractal technique. The latter consumes a time of about 5 times longer than the former. The time estimated to recognize a digit in acquired image of 512x512 resolution using moment technique was about 1.6 *s* which is less than 6.7 *s* using fractal technique. The test of both fractal and moment techniques show quite extensive investigation since the fractal technique giving robust results at long time match in comparison with the moment technique.

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