RE-EXAMINATION THE EXACT CENTER OF MASS CORRECTION FOR LONGITUDINAL ELECTRO-EXCITATION FROM FACTORS OF ^{13}C NUCLEUS

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Abstract

The inelastic longitudinal electron scattering form factors are calculated for the low-lying excited states of ^{13}C with J^{π} T= 3/2$^-$ 1/2 (3.684 MeV) and 5/2$^-$ 1/2 (7.55 MeV). The two-body interaction of Cohen and Kurath is used to generate the 1p-shell wave functions. The exact value of the center of mass correction which is calculated in the translation invariant shell model (TISM) is included, giving good results. The data are well reproduced when the core polarization effects are included through effective nucleon charge. A higher 2p-shell configuration enhances the form factors for q-values and resolves many discrepancies with the experiments. The results are compared with other theoretical models.

إعادة إختبار التصحيح الدقيق لمركز الكتلة على عوامل التشكل الطولية الاستطارة الالكترونية لنواة ^{13}C

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الخلاصة

حسبت عوامل التشكل الطوليةغير المرنة للإستطارة الألكترونية للمستويات المنخفضة لحالات **النواة** ^{13}C J^{π} T= 3/2$^-$ 1/2 (3.684 MeV) و 5/2$^-$ 1/2 (7.55 MeV) . أستخدم تفاعل الجسيمين لـ-Cohen Kurathللحصول على الدالة الموجية للغلاف 1p.أدخلت القيمة الحقيقية لتصحيح مركز الكتلة مع فضاء إنموذج الأغلفة غيرالمعتمد على حركة مركز الكتلة (TISM) معطية نتائج جيدة. البيانات تم حسابها ثانية بشكل جيد عندما أدخل تأثير أستقطاب القلب من خلال الشحنة الفعالة للنوية. أدى مساهمات المدارات العالية كالغلاف-2p الى زيادة عوامل التشكل لقيم الزخوم المنتقلة وحل العديد من التباينات مع التجارب. قورنت النتائج الحالية مع البيانات العملية وكذلك مع نتائج نماذج أخرى.

Introduction

The study of electron scattering has provided important information about electromagnetic current inside the nuclei. The scattering of electrons from nuclei gives the most precise information about nuclear size and charge distribution, since it is sensitive to the spatial dependence of the charge and current densities. There are many reasons for selecting the electron to probe the nucleus. A detailed description of the advantages of using electron scattering in studying nuclear structure is given by Walecka [1]. Good deals of works have been carried out on studying the electron scattering.

Millener *et. al.*in (1989) [2] performed an investigation of the excited states of ^{13}C below 10 MeV excitation energy by means of high-resolution electron scattering at momentum transfers between $0.4 \le q \le 2.4$ fm^{-1}. They used Wood-Saxson single-particle wave functions and different effective charges (e_p=1.2e and e_n=0.43e). The value of the transition probability B(C2) found is slightly larger than the measured value.

Wolter *et. al.*(1990) [3] studied the electromagnetic structure of 1p-shell nuclei. Their calculations included the extended (0+2)$\hbar\omega$ model space, and the effective nucleon charges. They obtained the values (e_p=1.19e and e_n= 0.06e) by fitting the electric quadrapole moments calculated in $2\hbar\omega$ space to the experimental values.

In (1999) Mihaila and Heisenberg [4] proposed a many-body expansion for the computation of the charge form factor in center of mass system. They applied their formalism to the case of the harmonic oscillator shell model, where an exact solution exists.

Radhi *et. al.*(2001) [5] studied the core polarization (CP) effects on the longitudinal form factors of 1p-shell nuclei. The modified surface-delta interaction (MSDI) was adopted as a residual interaction. Their results described the data very well in both the transition strengths and momentum transfer dependence.

Milliner [6] fitted the inelastic electron scattering form factors with polynomial times Gaussian expressions in the variable y= $(bq/2)^2$ to extract electromagnetic transition strengths at the photon point for ^{10}B nucleus.

Tomaselli *et. al.*(2004) [7] calculated the electromagnetic moment and transitions in light nuclei ($^{6,7,9}Li$, ^{9}Be and ^{13}C) in the microscopic dynamic-correlation model. The results showed that the correct treatment of the Pauli principle and the diagonalization of large dimensional spaces are not compatible with the simple picture generated by the cluster model.

In the present work, the effect of the center-of-mass correction on the longitudinal form factors is investigated. The exact center-of-mass correction of Mihaila and Heisenberg [4] has been adopted to generate the longitudinal form factors in the Born approximation picture. The center-of-mass correction that was used in other previous works was also taken into account for comparison.

Theory

The longitudinal form factor for a given multipolarity J and momentum transfer $\vec{q}$ is expressed as [8]:

$$\left|F_J^{coul}(\vec{q})\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\left\langle J_f \left\| \hat{T}_J^{coul}(\vec{q}) \right\| J_i \right\rangle\right|^2 \times \left|F_{c.m}(\vec{q})\right|^2 \times \left|F_{f.s}(\vec{q})\right|^2 \qquad (2.1)$$

where $F_{f.s} = e^{\frac{-0.43q^2}{4}}$ is the finite size of the nucleon correction, and $F_{c.m.} = e^{\frac{q^2b^2}{4A}}$ is the center of mass correction [9, 10]. The reduced matrix element of the longitudinal electron scattering operator $\hat{T}^{coul}$ is expressed as a sum of the one body density matrix (OBDM) $\chi_{J_iJ_f}^{JT}(\alpha,\beta)$ times the single-particle elements and is given by:

$$\left\langle J_F \left\| \hat{T}_{JT}^{coul} \right\| J_i \right\rangle = \sum_{\alpha\beta} \chi_{J_iJ_f}^{J}(\alpha,\beta) \left\langle \alpha \left\| \hat{T}_J^{coul} \right\| \beta \right\rangle \qquad (2.2)$$

where α and β label single-particle states (isospin is included) for the model space.

The exact value of the center of mass correction in the translationally invariant shell model TISM $F_{ex.}$ [4] longitudinal form factor can be written as:

$$\left|F_J^{coul}(\vec{q})\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\left\langle J_f \left\| \hat{T}_J^{coul}(\vec{q}) \right\| J_i \right\rangle\right|^2 \times \left|F_{ex.}(\vec{q})\right|^2 \times \left|F_{f.s}(\vec{q})\right|^2 \quad (2.3)$$

$$F(\vec{q}) = F_{c.m.}(\vec{q})\, F_{ex.}(\vec{q}) \quad (2.4)$$

When the 1p-shell model space is extended to include the 2p-shell model space, the wave functions of the initial (i) and final (f) states will be written as:

$$|i\rangle = \alpha\left|i\,(1p)\right\rangle + \sqrt{1-\alpha^2}\left|i\,(2p)\right\rangle \quad (2.5)$$

$$|f\rangle = \gamma\left|f\,(1p)\right\rangle + \sqrt{1-\gamma^2}\left|f\,(2p)\right\rangle \quad (2.6)$$

where α and γ are mixing parameters. Since the C-K interaction depends on the angular parts only, the same OBDM are used for both 1p and 2p shells. The reduced transition probability is given by [11]:

$$B(CJ) = \frac{\left[(2J+1)!!\right]^2}{4\pi}\frac{Z^2}{k^{2J}}\left|F_J^{col}(\vec{q}=\vec{k})\right|^2 \quad (2.7)$$

where $q = k = \dfrac{E_x}{\hbar c}$.

Results and Discussion

The inelastic transitions from the ground state J^π T= 1/2$^-$ 1/2 to the J^π T= 3/2$^-$ 1/2 (3.684 MeV) and 5/2$^-$ 1/2 (7.55 MeV) states are dominated by C2 multipole. The single-particle wave functions of the harmonic oscillator potential with size parameter b=1.628fm, chosen to reproduce the measured root mean square (rms) charge radii of the nuclei, are considered in the present work. The core- polarization effects (CP) are included through the effective charges. Modifications of the wave functions used are considered by the admixture of higher 2p-shell configurations with some percentage.

In the present work, the results of 1p-shell and (1p+2p)-shells with CP effects including the exact value of the center of mass correction (c.m.) will be denoted by red dashed and solid curves respectively, while that results without exact value of the c.m. correction will be denoted by blue dashed and solid curves respectively.

1. The 3.684 MeV (3/2$^-$ 1/2) State

The longitudinal C2 form factors with free nucleon charges are shown in Figure (1).

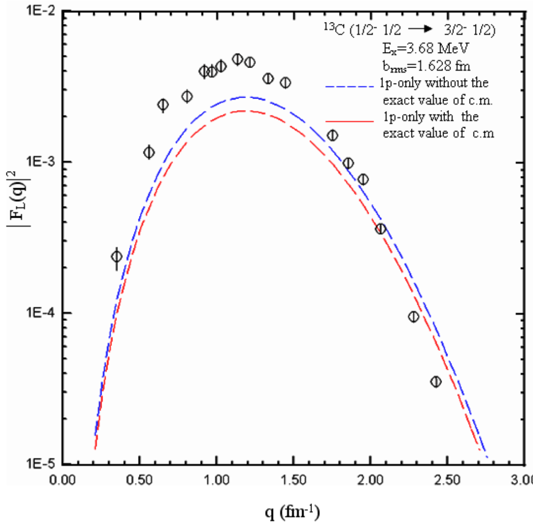


Figure 1: The longitudinal form factors for (3/2$^-$ 1/2) state in ^{13}C calculated in 1p-shell model space only. The results calculated with exact value of c.m. correction (red dashed curve) and without the exact value of c.m. correction (blue dashed curve). The experimental data are taken from Ref.[5] (circles).

The 1p-shell model space results with and without the exact value of the c.m. correction (red and blue dashed curves respectively), underestimate the experimental data of Millener et al.[2](circles) at q<1.9 fm^{-1}. This discrepancy could be resolved by introducing the higher contributions as shown in Figure (2.a) (red and blue solid curves). The inclusion of core-polarization effects (with e_p=1.24e and e_n=0.24e) and 2p-shell admixture (with $\alpha = \gamma = -0.99$) without the exact value of c.m. correction (blue solid curve) reproduce the experimental data very well over all regions of momentum transfer except at 0.7 < q < 1.7 fm^{-1}, and the results obtained with the exact value of c.m. correction (red solid curve) reproduce the experimental data very well over all regions of momentum transfer. The present results are compared with the (1p+CP) results of Radhi et al. [5] (cross symbol curve), this comparison is shown in Figure (2.b).

The present results give a reasonable agreement with the experimental data for all regions of q, and are close to the results of Radhi et al. [5] only at q < 0.5 fm^{-1}. The predicted value of the reduced transition probability B(C2) in both calculations with and without exact value of c.m. correction are 15.17 $e^2.fm^4$ and 18.64 $e^2.fm^4$, respectively. These values are slightly less than the experimental value (19.65 $\pm$0.55) $e^2.fm^4$ [2].

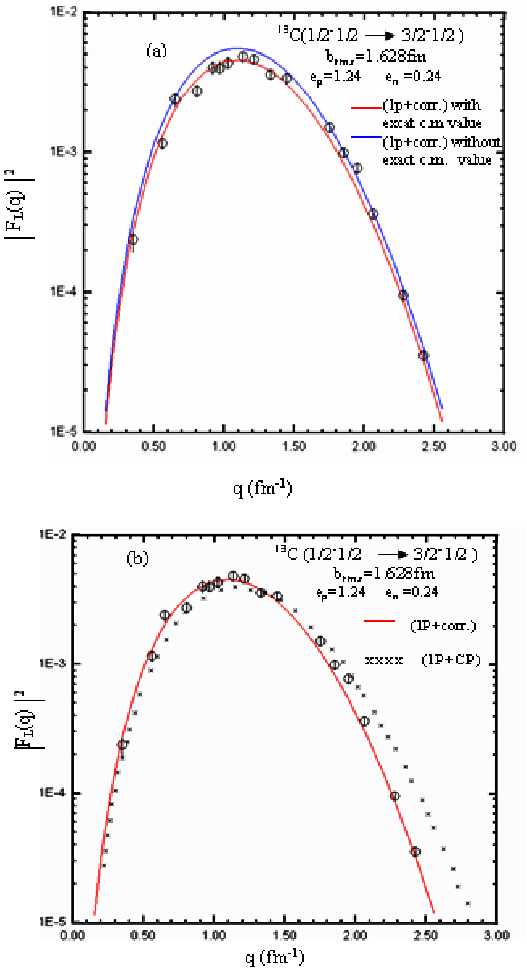


Figure 2: The longitudinal form factors for (3/2$^-$ 1/2) state in ^{13}C calculated with (1p+*corr). (a) The present results with and without exact value of c.m. correction (red and blue solid curves respectively). (b) The present results (red solid curve) are compared to those of Ref. [7] (cross symbole curve).

*corr. $\Rightarrow$brief of (1p+2p+CP) results of present work

2. The 7.55 MeV (5/2$^-$ 1/2) State

The longitudinal C2 form factors with free nucleon charges are shown in Figure (3).

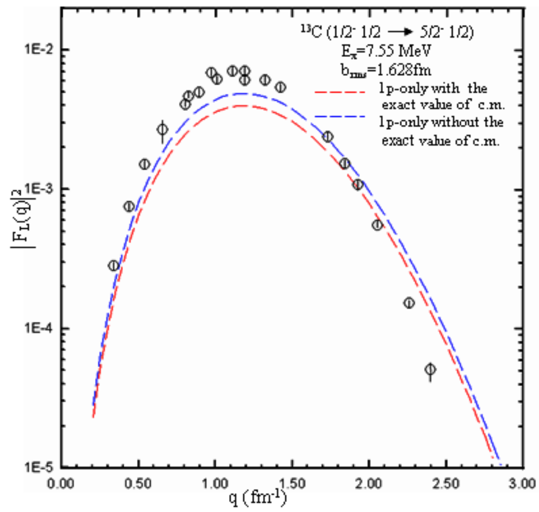
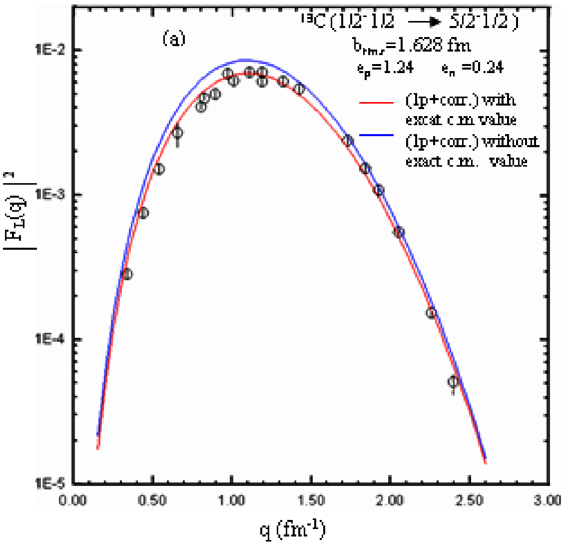


Figure 3: The longitudinal from factors for (5/2$^-$ 1/2) state in ^{13}C calculated in 1p – shell model space only.The results calculated with exact value of c.m. correction (red dashed curve),and without the exact value of c.m. correction (blue dashed curve).the experimental data are taken from Ref.[2](circles).

The 1p-shell model space results with and without exact value of the c.m. correction (red and blue dashed curves respectively) underestimate the experimental data at q < 1.7 fm^{-1} and overestimate the experimental data at higher of momentum transfer. The experimental data of Millener et al. [2] (circles), are compared to the present results.

The inclusion of core-polarization effects (with e_p=1.24e and e_n=0.24e), and 2p-shell admixture (with $\alpha = \gamma = -0.99$) is shown in Figure.(4.a). The results of calculations without the exact value of c.m. correction (blue solid curve) overestimate the experimental data at 0.7 < q < 1.7 fm^{-1}, and reproduce the experimental data for other regions of momentum transfer while the results with the exact value of c.m. correction reproduce the experimental data very well at all regions of q.

The experimental data are compared to the present results,and those of Radhi et. al. [5]. This comparison is shown in Figure (4. b). The present results and the results of Radhi et al. [5] (cross symbol curve), are close to each other at q < 0.5 fm^{-1}, and they are different for all other q-values. The results of the above two models give the same location of the maximum (at q ~ 1.1 fm^{-1}), but they are different in the magnitudes of the measured C2 form factors.



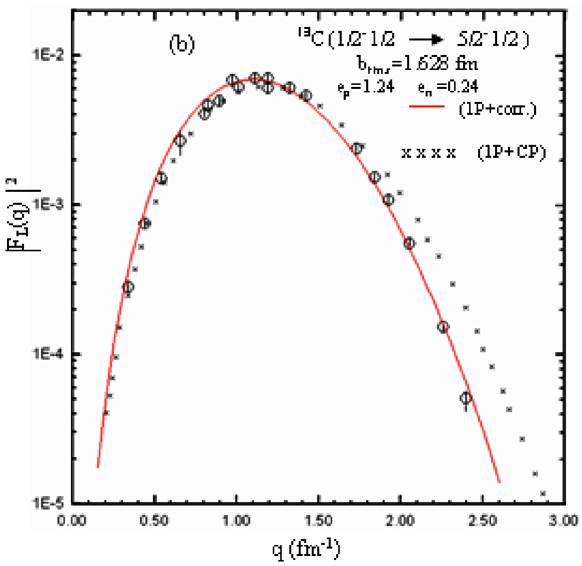


Figure 4: The longitudinal form factors for (5/2- 1/2) state in 13C calculated with (1p+*corr). (a) The present results with and without exact value of c.m. correction (red and blue solid curves respectively). (b) The present results (red solid curve) are compared to that of Ref. [5] (cross symbol curve).

The predicted value of reduced transition probability B(C2) in both calculations with and without exact value of c.m. correction are 22.89 and 28.14 $e^2.fm^4$, respectively is larger than the experimental value (19.65 ± 0.55) $e^2. fm^4$ [2].

Conclusions

The most important conclusions of the present work can be briefly summarized as follows:-

1. The longitudinal inelastic electron scattering form factors are fairly well predicted with the CP effects.
2. For both C2 transitions, the inclusion of effective charges (e_p=1.24e and e_n= 0.24e) are adequate to obtain a good agreement between the predicted and measured form factors.
3. The inclusion of the exact value of c.m. correction has a remarkable role on the B(C2) value and minor role on q dependence form factors.
4. The predicted values of B(C2) are reduced with the exact value of c.m. correction.

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